The Melitz-Pareto Model

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Overview

- Today, we introduce firm-level heterogeneity into the Krugman model.

- This extension preserves the gravity equation:

$$\text{Trade Value} \propto \frac{\text{Exporter's GDP} \times \text{Importer's GDP}}{\text{Distance}^{\beta}}$$

- **Main references**:
  1. Melitz (2003, Econometrica), *"The impact of trade on intra-industry reallocations and aggregate industry productivity."*
  2. Chaney (2008, AER), *"Distorted Gravity: The Intensive and Extensive Margins of International Trade."*

Why was the Melitz Model Developed?

- From the perspective of the Krugman model:
  - all firms have similar productivity levels.
  - all firms participate in exporting.

- Firm-level data suggests that:
  - there is great cross-firm heterogeneity in productivity.
  - most firms do not export: only 4% of U.S. firms exported in 2000.
  - exporters are more productive that non-exporters.

- The Melitz model was developed to account for these data regularities.

Environment

- $j,\ i = 1, \ldots, N$ countries
- The entire economy is modeled as one industry
- Labor is the only factor of production
  - Country i is endowed with L_i units of labor
- Each country hosts many *monopolistically competitive* firms
  - firms are indexed by ω
  - firms are *heterogeneous* in their productivity

Demand

The representative consumer in country i has a CES utility function:

$$U_i(\mathbf{q}_{1i}, \ldots, \mathbf{q}_{Ni}) = \left[\sum_{j=1}^{N} \left(\int_{\omega \in \Omega_{ji}} q_{ji}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}}$$

where

- Goods are differentiated at the firm-level.
- Index ji corresponds to *Exporter* j $\times$ *Importer* i
- $q_{ji}(\omega)$: quantity of firm-level variety ω originating from country j.
- $\sigma \geqslant 1$ is the cross-firm elasticity of substitution.

Demand

- Consumer's problem (p is price, Y is income):

$$\max_{\mathbf{q}} \quad U_i(\mathbf{q}_{1i}, \ldots, \mathbf{q}_{Ni})$$
$$\text{s.t.} \sum_{j=1}^{N} \left(\int_{\omega \in \Omega_{ji}} p_{ji}(\omega) q_{ji}(\omega) d\omega \right) \leqslant Y_i \qquad (\mathbf{CP})$$

- Demand function implied by CP:

$$p_{ji}(\omega) q_{ji}(\omega) = \left(\frac{p_{ji}(\omega)}{P_i} \right)^{1-\sigma} Y_i$$

- P_i is a CES price index: $P_i = \left[\sum_{j=1}^{N} \left(\int_{\omega \in \Omega_{ji}} p_{ji}(\omega)^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}}$

Supply: *Cost Function*

- The market structure is monopolistic competition.

- Firm ω located in country j, faces three types of cost
  - ***entry cost***: $w_j f^e$
  - ***fixed overhead cost***: $w_i f_{ji}$ per market i
  - ***variable cost***: $\tau_{ji} a_j(\omega) w_j q_{ji}(\omega)$ per market i

- The total cost faced by firm ω from country j:

$$TC_j(\omega) = w_j f^e + \sum_{i=1}^{N} \mathbb{1}_{ji}(\omega) \left(\tau_{ji} a_j(\omega) w_j q_{ji}(\omega) + w_i f_{ji}\right).$$

where $\mathbb{1}_{ji}(\omega)$ is an indicator of whether ω serves market i.

Entry Scheme

- There is a pool of *ex-ante* identical firms in Country j.

- Each of these firms, can pay the entry cost $w_j f^e$ to independently draw a productivity $\varphi \equiv 1/a(\omega)$ from distribution $G(\varphi)$.

- Firms enter to the point that

$$\textit{Total Expected Profit}_j = w_j f^e.$$

- After entry, a firm sells to market i from j if

$$\textit{Market-Specific Profit}_{ji} \geqslant w_i f_{ji}.$$

Supply: *Optimal Pricing*

- Productivity, φ, uniquely determines the firm-level outcomes $\Longrightarrow$ we can express all firm-level variables as a function of φ.
- A firm with productivity φ sets price to maximize variable profits

$$\pi_{ji}(\varphi) = \max_{p_{ji}(\varphi)} \left[p_{ji}(\varphi) - \tau_{ji} w_j/\varphi\right] q_{ji}(\varphi)$$

s.t. $q_{ji}(.)$ being given by the CES demand function.

- Optimal price equals *constant markup*$\times$*marginal cost*:

$$p_{ji}(\varphi) = \frac{\sigma}{\sigma - 1} \tau_{ji} w_j/\varphi$$

Entry and Selection into Markets

Zero Profit Cut-off Condition

- Firms with productivity $\varphi \geqslant \varphi^*_{ji}$ export to market i from j, where

$$\pi_{ji}(\varphi^*_{ji}) = w_i f_{ji} \quad \text{(ZPC)}$$

Free Entry Condition

- Let N_j denote the number of firms operating in country j.

- N_j is determined by the *free entry* condition (i.e., firms enter until expected profits are drawn to zero)

$$\sum_{i=1}^{N} \left[\mathbb{E}\left(\pi_{ji}(\varphi) - w_i f_{ji}\right)\right] = w_j f^e \quad \textbf{(FE)}$$

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$$\sum_{i=1}^{N} \left[\int_{\varphi^*_{ji}}^{\infty} (\pi_{ji}(\varphi) - w_i f_{ji}) \, dG(\varphi) \right] = w_j f^e \quad \text{(FE)}$$

Key Assumption

- Following Chaney (2008, AER), assume that $G(.)$ is ***Pareto***:

$$G(\varphi) = 1 - \varphi^{-\gamma}$$

- γ represents the degree of firm-level heterogeneity.
- The Pareto assumption is key to obtaining a gravity equation.

Deriving the Gravity Equation

- Export sales from country j to i are the sum of all firm-level sales:

$$X_{ji} = N_j \int_{\varphi_{ji}^*}^{\infty} p_{ji}(\varphi) q_{ji}(\varphi) dG(\varphi)$$

- The CES demand function implies that

$$\begin{aligned} X_{ji} =& N_j \int_{\varphi_{ji}^*}^{\infty} \left(\frac{p_{ji}(\varphi)}{P_i} \right)^{1-\sigma} Y_i dG(\varphi) \\ =& \gamma N_j \left(\frac{p_{ji}(\varphi_{ji}^*)}{P_i} \right)^{1-\sigma} Y_i \int_{\varphi_{ji}^*}^{\infty} \left(\frac{p_{ji}(\varphi)}{p_{ji}(\varphi_{ji}^*)} \right)^{1-\sigma} \varphi^{-\gamma-1} d\varphi \\ =& \gamma \sigma N_j w_i f_{ji} \int_{\varphi_{ji}^*}^{\infty} \left(\frac{p_{ji}(\varphi)}{p_{ji}(\varphi_{ji}^*)} \right)^{1-\sigma} \varphi^{-\gamma-1} d\varphi, \end{aligned}$$

- The last line follows from the ZPC condition: $\left(\frac{p_{ji}(\varphi_{ji}^*)}{P_i} \right)^{1-\sigma} Y_i = \sigma w_i f_{ji}$.

Deriving the Gravity Equation

- The last expression on the previous slide, can be simplified in 3 steps.
- First, we can simplify the integral by a change in variables, $\nu = \varphi/\varphi^*_{ji}$:

$$X_{ji} = N_j w_i f_{ji} \left(\varphi^*_{ji}\right)^{-\gamma} \int_1^\infty \nu^{-\sigma-\gamma} d\nu$$

- Second, we appeal to the ZPC condition to characterize φ^*_{ji}:

$$\left(\frac{p_{ji}(\varphi^*_{ji})}{P_i}\right)^{1-\sigma} Y_i = \sigma w_i f_{ji} \implies \varphi^*_{ji} = \frac{\sigma}{\sigma-1} \tau_{ji} w_j \left(\frac{Y_i P_i^{\sigma-1}}{\sigma w_i f_{ji}}\right)^{\frac{1}{1-\sigma}}$$

- Third, we can gather all *non-exporter-specific* terms into one term, A_i:

$$X_{ji} = A_i \, N_j f_{ji}^{1-\frac{\gamma}{\sigma-1}} \left(\tau_{ji} w_j\right)^{-\gamma}$$

The Gravity Equation with Firm-Selection Effects

– Combining (a) $X_{ji} = A_i N_j f_{ji}^{1-\frac{\gamma}{\sigma-1}} \left(\tau_{ji} w_j\right)^{-\gamma}$, and (b) $\sum_{i=1}^{N} X_{ji} = Y_i$, we can produce the following gravity equation:

$$X_{ji}(\mathbf{N}, \boldsymbol{w}) = \frac{N_j \left(\tau_{ji} w_j\right)^{-\gamma} f_{ji}^{1-\frac{\gamma}{\sigma-1}}}{\sum_{\ell=1}^{N} N_\ell \left(\tau_{\ell i} w_\ell\right)^{-\gamma} f_{\ell i}^{1-\frac{\gamma}{\sigma-1}}} Y_i$$

where, due to FE, $Y_i = w_i L_i$ for all i.

– $\{w_i\}$ and $\{N_i\}$ are endogenous variables; but we can use the FE condition to write N_j as a function of *structural* parameters.

An Illustration of Firm-Selection Effects

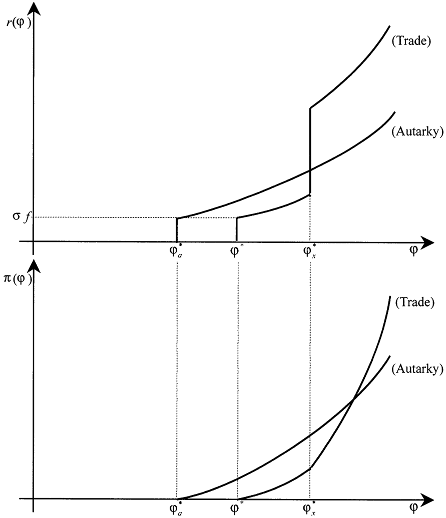

FIGURE 2.—The reallocation of market shares and profits.

Using the FE Condition to Pin Down N_i

– The free entry condition yields a closed-form solution for the number of firms:

$$\sum_{j=1}^{N}\left(\frac{X_{ij}(\boldsymbol{w},\mathbf{N})}{\sigma}-N_{ij}w_j f_{ij}\right) = N_i w_i f^e \quad ,\forall i \quad (\mathbf{FE})$$

where $N_{ij} \equiv \left[1-G(\varphi^*_{ij})\right] N_i$ for all j.

– After some tedious algebra, the above equation implies that

$$N_i = \frac{\sigma-1}{\sigma\gamma}\frac{L_i}{f^e}$$

The Trade Equilibrium

Equilibrium is a vector of wages, $\boldsymbol{w} = \{w_i\}$ that satisfy the *balanced trade* (BT) condition:

$$\sum_{j=1}^{N} X_{ij}(\boldsymbol{w}) = Y_i(w_i) \quad , \forall i \quad \text{(BT)}$$

where

$$\begin{cases} X_{ij}(\boldsymbol{w}) = \dfrac{L_i(\tau_{ji}w_i)^{-\gamma} f_{ji}^{1-\frac{\gamma}{\sigma-1}}}{\sum_{\ell=1}^{N} L_\ell(\tau_{\ell i}w_\ell)^{-\gamma} f_{\ell i}^{1-\frac{\gamma}{\sigma-1}}} Y_j(w_j) & \forall i, j \\ Y_i(w_i) = w_i L_i & \forall i \end{cases}$$

Melitz-Pareto versus Armington: *Gravity*

- **Melitz-Pareto**

$$X_{ji} = \frac{\left(\tilde{a}_{ji}\tau_{ji}w_j\right)^{-\gamma}}{\sum_{\ell=1}^{N}\left(\tilde{a}_{\ell i}\tau_{ji}w_j\right)^{-\gamma}} Y_i$$

  - γ: degree of firm-level heterogeneity.
  - $\tilde{a}_{ji} \equiv f_{ji}^{\frac{1}{\gamma}-\frac{1}{\sigma-1}} L_j^{-\frac{1}{\gamma}}$: *selection & scale-adjusted* unit labor cost.

- **The Armington model**

$$X_{ji} = \frac{\left(\tau_{ji}a_j w_j\right)^{1-\sigma}}{\sum_{\ell=1}^{N}\left(\tau_{\ell i}a_\ell w_\ell\right)^{1-\sigma}} Y_i$$

  - σ: degree of national product differentiation
  - a_j: unit labor cost.

Melitz-Pareto versus Armington: *Welfare*

- **Melitz-Pareto**

$$\frac{w_i}{P_i} = \frac{w_i}{\mathcal{C}_i \left(\sum_{\ell=1}^{N} \left(\tilde{a}_{\ell i} w_\ell \tau_{\ell i} \right)^{-\gamma} \right)^{-1/\gamma}}$$

  - γ: degree of firm-level heterogeneity.
  - $\tilde{a}_{ji} \equiv f_{ji}^{\frac{1}{\gamma} - \frac{1}{\sigma - 1}} L_j^{-\frac{1}{\gamma}}$: *selection & scale-adjusted* unit labor cost.

- **The Armington model**

$$\frac{w_i}{P_i} = \frac{w_i}{\left(\sum_{k=1}^{N} \left(w_\ell a_\ell \tau_{\ell i} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}}$$

  - σ: degree of national product differentiation
  - a_j: unit labor cost in country j.

Class Question

Why does σ not show up in the gravity equation implied by the Melitz-Pareto model?

- *Hint*: applying the Leibniz rule show that

$$\frac{\partial \ln X_{ji}}{\partial \ln \tau_{ji}} = \underbrace{\frac{\int_{\varphi_{ji}^*}^{\infty} x_{ji}(\varphi)\frac{\partial \ln x_{ji}(\varphi)}{\partial \ln \tau_{ji}} \tau_{ji} dG(\phi)}{X_{ji}}}_{\text{Extensive Margin}} + \underbrace{\frac{x_{ji}(\varphi_{ji}^*)\varphi_{ji}^* \frac{\partial \ln \varphi_{ji}^*}{\partial \ln \tau_{ji}} dG(\varphi_{ji}^*)}{X_{ji}}}_{\text{Intensive Margin}} = \gamma,$$

where $x_{ji}(\varphi) \equiv p_{ji}(\varphi) q_{ji}(\varphi)$.

Final Remarks

- *Arkolakis et. al. (2018, ReStud)* show that the CES assumption is not necessary for obtaining a gravity equation. As long as the firm-level productivity distribution is Pareto, any demand function satisfying the following functional-form will deliver gravity:

$$q_\omega(\mathbf{p}, y) = \mathcal{D}(p/P(\mathbf{p}, y))\mathcal{Q}(\mathbf{p}, y)$$

- The Melitz-Pareto and Armington models are observationally equivalent (i.e., isomorphic) inso far as *macro-level* trade values are concerned.So, the gravity equation produced by the Krugman model can be estimated along the same exact steps highlighted in Lecture 1.

- In the Melitz-Pareto model, however, the *bilateral resistance term* is driven by both the iceberg cost, τ_{ji}, and the fixed exporting cost, f_{ji}.