The Ricardian Model

Part 2: Eaton and Kortum (2002)

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Roadmap

- Today, I will present **Eaton and Kortum (2002)**.

- The EK model extends DFS1977 by allowing for
  - many countries
  - arbitrary trade costs

- The EK model also delivers a gravity equation

$$\text{Trade Value} \propto \frac{\text{Exporter's GDP} \times \text{Importer's GDP}}{\text{Distance}^{\beta}}$$

Environment

- $j, i = 1, \ldots, N$ countries.
- A continuum of homogeneous goods $\omega \in [0, 1]$.
- Labor is the only factor of production:
  - Country i is populated by L_i workers.
  - Each worker is paid a wage w_i.
- Perfect competition + constant returns to scale.

Demand

CES Utility Function

- The representative consumer in country i has a CES utility function:

$$U_j(\mathbf{q}) = \left[\int_{\omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

- σ is the elasticity of substitution across goods.

- The Cobb-Douglas utility function in DFS1977 is a special case of the CES utility function, where $\sigma \to 1$.

Demand

CES Utility Function

- Demand for good ω

$$p_i(\omega)q_i(\omega) = \frac{p_i(\omega)^{1-\sigma}}{\int_{\omega'} p_i(\omega')^{1-\sigma} d\omega} Y_i$$

- $p_i(\omega)$: price of good ω in country i.

- $Y_i = w_i L_i$: total income in country i.

Supply

- The price of good ω if supplied by country j

$$p_{ji}(\omega) = \tau_{ji} a_j(\omega) w_j$$

  - τ_{ji}: iceberg trade cost
  - $a_j(\omega)$: unit labor cost of producing ω in country j

- Country i buys good ω from the cheapest supplier:

$$p_i(\omega) = \min\{p_{1i}(\omega), \ldots, p_{Ni}(\omega)\}$$

Technology

- Let $z_j(\omega) \equiv 1/a_j(\omega)$ denote productivity.
- Let $F_j(.)$ denote the distribution of country j's productivity:

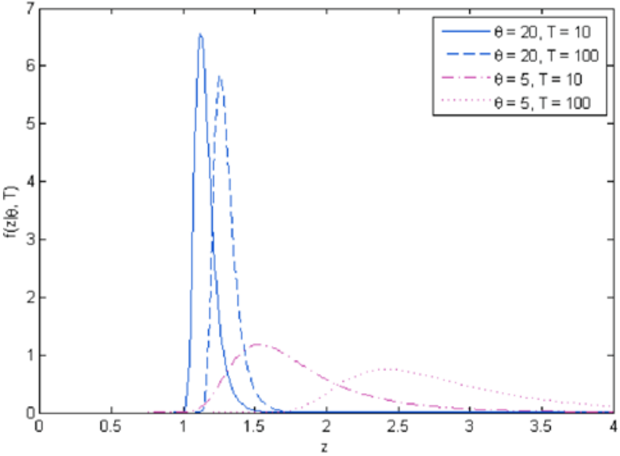
$$F_j(z) \equiv \Pr\left\{z_j(\omega) \leqslant z\right\}$$

- EK assume $F_j(z)$ is FRECHET:

$$F_j(z) = \exp\left(-T_j z^{-\theta}\right)$$

- Why FRECHET?
  - If ideas arrive with a POISSON distribution, and the technology of producing goods is determined by the best "idea," then the limiting distribution is FRECHET, where T_j reflects the country j's stock of ideas.

The Frechet Distribution



Source: Fieler (2011, Econometrica)

Equilibrium Trade Shares

- The probability that $p_{ji}(\omega) \leqslant p$ is given by

$$G_{ji}(p) \equiv \Pr\left(p_{ji}(\omega) \leqslant p\right) = \left\{\frac{w_j}{z_j(\omega)}\tau_{ji} \leqslant p\right\}$$
$$= 1 - \exp\left\{-\Phi_{ji}p^{\theta}\right\}$$

where $\Phi_{ji} \equiv T_j \left(w_j \tau_{ji}\right)^{-\theta}$.

- *Note*: The above probability is the same for all goods ω

Equilibrium Trade Shares

- The probability that good ω is supplied at a price less than p in country i

$$\begin{aligned} G_i(p) &= \Pr\{p_i(\omega) \leqslant p\} = 1 - \prod_{j \in C} \left(1 - G_{ji}(p)\right) \\ &= 1 - \exp\left\{-\Phi_i p^{\theta}\right\} \end{aligned}$$

  where $\Phi_i = \sum_{j=1}^{N} \Phi_{ji}$.

- Remember that $\Phi_{ji} \equiv T_j \left(w_j \tau_{ji}\right)^{-\theta}$.

Equilibrium Trade Shares

- The probability that country j is the lowest cost supplier of good ω to country i is

$$\pi_{ji} \equiv \Pr\left\{p_{ji}(\omega) \leqslant \min_{\ell \neq j} p_{\ell i}(\omega)\right\} = \int_0^\infty \Pr\left\{\min_{\ell \neq j} p_{\ell i}(\omega) \geqslant p\right\} dG_{ji}(p)$$
$$= \int_0^\infty \prod_{\ell \neq j} \left(1 - G_{\ell i}(p)\right) dG_{ji}(p)$$

- Substituting $G_{ji}(p) = 1 - \exp\left\{-\Phi_{ji} p^{\theta}\right\}$ in the last line, yields:

$$\lambda_{ji} = \frac{\Phi_{ji}}{\Phi_i} = \frac{T_j \left(\tau_{ji} w_j\right)^{-\theta}}{\sum_{\ell=1}^{N} T_\ell \left(\tau_{\ell i} w_\ell\right)^{-\theta}}$$

- Because (i) all goods receive *i.i.d.* draws and (ii) there are a continuum of varieties, by the law of large numbers, this probability will be equal to the fraction of goods j sells to i.

Equilibrium Trade Shares

- *Claim*: The distribution of *realized* ($\mathcal{R}$) prices for goods purchased from j is independent of country j's characteristics!

- *Proof*: Define $G_{ji}^{\mathcal{R}}(p) \equiv \left\{p_{ji}(\omega) \leqslant p \mid p_{ji}(\omega) \leqslant \min_{\ell \neq j} p_{\ell j}(\omega)\right\}$. We can easily shows that

$$G_{ji}^{\mathcal{R}}(p) = \frac{\int_0^p \prod_{\ell \neq i} \left(1 - G_{\ell i}(\tilde{p})\right) dG_{ji}(\tilde{p})}{\pi_{ji}} = G_i(p)$$

- *Implication*: the fraction of goods supplied by j is equal to the fraction of income spent on goods from j:

$$X_{ji} = \lambda_{ji} Y_i$$

Equilibrium Price Index

- The CES utility implies that the price index in country j is

$$P_i = \left(\int_\omega p_i(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} = \left(\int_0^\infty p^{1-\sigma} dG_i(p) \right)^{\frac{1}{1-\sigma}}$$

- Noting that (a) $G_i(p) = 1 - \exp\left\{-\Phi_i p^\theta\right\}$, and (b) $\Phi_i = \sum_{\ell=1}^N T_\ell \left(w_\ell \tau_{\ell i}\right)^{-\theta}$, the above expressions yields:

$$P_i = \mathcal{C} \left(\sum_{\ell=1}^N T_\ell \left(w_\ell \tau_{\ell i}\right)^{-\theta} \right)^{\frac{-1}{\theta}},$$

where $\mathcal{C} \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$ (reminder: $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$).

Equilibrium: *Definition*

Equilibrium is a vector of country-level wages, $\{w_i\}$, such that trade is balanced in all countries:

$$\underbrace{\sum_{j=1}^{N} X_{ij}(w_1, \ldots, w_N)}_{\text{total sales}} = \underbrace{Y_i(w_i)}_{\text{total spending}} \quad , \forall i$$

where

$$\begin{cases} X_{ij}(w_1, \ldots, w_N) = \frac{T_i(\tau_{ij} w_i)^{-\theta}}{\sum_{\ell=1}^{N} T_\ell(\tau_{\ell j} w_\ell)^{-\theta}} Y_j(w_j) & \forall i, j \\ Y_i(w_i) = w_i L_i & \forall i \end{cases}$$

EK versus Armington: *Gravity*

- **Eaton and Kortum (2002)**

$$X_{ji} = \frac{T_j \left(\tau_{ji} w_j\right)^{-\theta}}{\sum_{\ell=1}^{N} T_\ell \left(\tau_{\ell i} w_\ell\right)^{-\theta}} Y_i$$

  - θ: Degree of comparative advantage.
  - T_j: Stock of ideas in country j.

- **The Armington model**

$$X_{ji} = \frac{\left(\tau_{ji} a_j w_j\right)^{1-\sigma}}{\sum_{\ell=1}^{N} \left(\tau_{\ell i} a_\ell w_\ell\right)^{1-\sigma}} X_i$$

  - σ: Degree of national product differentiation
  - a_j: unit labor cost in country j.

EK versus Armington: *Welfare*

- **Eaton and Kortum (2002)**

$$\frac{w_i}{P_i} = \frac{w_i}{\mathcal{C}\left(\sum_{\ell=1}^{N} T_\ell \left(w_\ell \tau_{\ell i}\right)^{-\theta}\right)^{-1/\theta}}$$

  - θ: Degree of comparative advantage.
  - T_j: Stock of ideas in country j.

- **The Armington model**

$$\frac{w_i}{P_i} = \frac{w_i}{\left(\sum_{k=1}^{N} \left(w_\ell a_\ell \tau_{\ell i}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}$$

  - σ: Degree of national product differentiation
  - a_j: unit labor cost in country j.

Conclusions

- The EK and Armington models are observationally equivalent (i.e., isomorphic) inso far as *macro-level* implications are concerned.

- The two models differ in terms of *micro-level* predictions (can you list them?)

- The gravity equation produced by the EK model can be estimated along the same exact steps highlighted in Lecture 1.