The Multi-Industry Trade Model

International Trade (PhD), Fall 2019

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Overview

- In this lecture, we extend the standard quantitative trade model by introducing multiple industries.
- Main implications
  - the multi-industry model *typically* predicts larger gains from trade.
  - in some countries, however, trade can exacerbate pre-existing market distortions $\Longrightarrow$ smaller gains from trade
- References:
  - **perfectly competitive:** *Donaldson, Costinot, & Komunjer (2012)*
  - **imperfectly competitive:** *Kucheryavyy, Lyn, & Rodriguez-Clare (2018)*; *Lashkaripour & Lugovskyy (2019)*

Environment

- $j,\ i = 1, \ldots, N$ countries
- $k = 1, \ldots, K$ industries
- Labor is the only factor of production
- Country i is endowed with L_i units of labor
- Each country hosts many *monopolistically competitive* firms
  - firms are indexed by ω

Demand

- The representative consumer in country i buys a vector of firm-level varieties, $\mathbf{q}_{ji} \equiv \{q_{ji,k}(\omega)\}$, from each country j.
- The preferences of the representative consumer are described by a *three-tier* Cobb-Douglas-CES utility function.

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Tier 1: Cobb-Douglas aggregator across industries

$$U_i = \prod_{k=1}^{K} Q_{i,k}^{e_{i,k}}$$

- Q_k is the sub-utility from consuming industry k goods.
- $e_{i,k}$ is the *constant* share of country i's spending on industry k.

Demand

Tier 2: within-industry CES aggregator across countries

$$Q_{i,k} = \left(\sum_{j=1}^{N} \mathcal{Q}_{ji,k}{}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}$$

where index ji corresponds to *Exporter* $j \times$ *Importer* i

Tier 3: within-country CES aggregator across firms

$$\mathcal{Q}_{ji,k} = \left[\int_{\omega \in \Omega_{ji,k}} q_{ji,k}(\omega)^{\frac{\eta_k - 1}{\eta_k}} d\omega \right]^{\frac{\eta_k}{\eta_k - 1}}$$

where $\eta_k \geqslant \sigma_k$ by assumption.

Demand

- Consumer's problem (p is price, Y is income):

$$\max_{\mathbf{q}_i} U_i(\mathbf{q}_{1i}, \ldots, \mathbf{q}_{Ni})$$
$$\text{s.t.} \sum_{k=1}^{K} \sum_{j=1}^{N} \left(\int_{\omega \in \Omega_{ji,k}} p_{ji,k}(\omega) q_{ji,k}(\omega) d\omega \right) \leqslant Y_i \qquad (\mathbf{CP})$$

- Demand function implied by CP:

$$p_{ji,k}(\omega) q_{ji,k}(\omega) = \left(\frac{p_{ji,k}(\omega)}{P_{ji,k}} \right)^{1-\eta_k} \left(\frac{P_{ji,k}}{P_{i,k}} \right)^{1-\sigma_k} e_{i,k} Y_i$$

where $P_{i,k} = \left[\sum_{j=1}^{N} P_{ji,k}^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}}$ and $P_{ji,k} = \left[\int_{\omega} p_{ji,k}(\omega)^{1-\eta_k} d\omega \right]^{\frac{1}{1-\eta_k}}$ are *industry-level* and *country*×*industry-level* price indexes.

Supply: Cost Structure

- The market structure is monopolistic competition.

- Firm ω serving industry k from country j, faces 2 types of cost
  - ***entry cost***: $w_j f_k^e$
  - ***variable cost***: $\tau_{ji,k} a(\omega) w_j q_{ji,k}(\omega)$ per market i.

- Ex-ante identical firms draw $a(\omega)$ *i.i.d* from distribution $G_{j,k}(a)$

- There are no fixed exporting costs $\Longrightarrow$ no firm-selection effects
  - ***Justification:*** to tractably account for selection effects, we need to assume a Pareto firm productivity distribution. However, under the Pareto assumption, the models w/ and w/o selection effects are isomorphic.

Supply: Country-Level Export Price Indexes

- To economize the notation, define $\epsilon_k \equiv \sigma_k - 1$ and $\mu_k \equiv 1/(\eta_k - 1)$.
- firm-level profit maximization $\Longrightarrow$ $p_{ji,k}(\omega) = (1 + \mu_k)\tau_{ji,k} a_{j,k}(\omega) w_j$.
- The agg. price index of country j's exports to market i in industry k:

$$P_{ji,k} = \left(\int_{\omega} p_{ji,k}(\omega)^{-\epsilon_k} d\omega \right)^{-\frac{1}{\epsilon_k}} = (1 + \mu_k)\, \tau_{ji,k} \bar{a}_{j,k} M_{j,k}^{-\mu_k} w_j$$

  - $\bar{a}_{j,k} \equiv \left(\int_a a^{-\epsilon_k} dG_{j,k}(a) \right)^{-1/\epsilon_k}$ denotes the avg. productivity.
  - no selection effects $\Longrightarrow$ $\bar{a}_{j,k}$ is independent of trade barriers.

Supply: The Free Entry Condition

- The free-entry condition:

$$\underbrace{M_{i,k} w_i f_k^e}_{\text{total entry cost}} = \underbrace{\mu_k \sum_{j=1}^{N} P_{ij,k} Q_{ij,k}}_{\text{total profit}}$$

- Given that $P_{ji,k} Q_{ji,k} = w_i L_{i,k}$, the free-entry condition implies

$$M_{i,k} = \mu_k \frac{L_{i,k}}{f_k^e}$$

where $L_{i,k}$ is the number of industry k workers ($\sum_k L_{i,k} = \bar{L}_i$).

Industry-Level Gravity Equation

- Noting that $M_{i,k} = \mu_k \frac{L_{i,k}}{f_k^e} \implies$ the share of country i's spending on country j varieties in industry k can be expressed as

$$\lambda_{ji,k} \equiv \frac{P_{ji,k}^{-\epsilon_k}}{\sum_\ell P_{\ell i,k}^{-\epsilon_k}} = \frac{L_{j,k}^{\mu_k \epsilon_k} \left(\tau_{ji,k} \bar{a}_{j,k} w_j\right)^{-\epsilon_k}}{\sum_{\ell=1}^{N} L_{\ell,k}^{\mu_k \epsilon_k} \left(\tau_{\ell i,k} \bar{a}_{\ell,k} w_\ell\right)^{-\epsilon_k}}$$

- The above equation represents and industry-level gravity equation.

- Total industry-level exports from country j to market i are given by:

$$X_{ji,k} = \lambda_{ji,k} e_{i,k} Y_i$$

The Trade Equilibrium

Equilibrium is a vector of country-level wages, $\{w_i\}$, and industry-level labor allocations, $\mathbf{L}_k \equiv \{L_{i,k}\}$ such that:

$$w_i L_{i,k} = \sum_{j=1}^{N} \lambda_{ij,k}(\boldsymbol{w}, \mathbf{L}_k) w_j \bar{L}_j, \quad \forall i, k$$

$$\sum_{k=1}^{N} \sum_{j=1}^{N} \lambda_{ij,k}(\boldsymbol{w}, \mathbf{L}_k) w_j \bar{L}_j \;=\; w_i \bar{L}_i, \quad \forall i$$

where

$$\lambda_{ij,k}(\boldsymbol{w}, \mathbf{L}_k) = \frac{L_{i,k}^{\mu_k \epsilon_k} \left(\tau_{ij,k} \bar{a}_{i,k} w_i\right)^{-\epsilon_k}}{\sum_{\ell=1}^{N} L_{\ell,k}^{\mu_k \epsilon_k} \left(\tau_{\ell i,k} a_{\ell,k} w_\ell\right)^{-\epsilon_k}} \qquad \forall i, j$$

Performing Counterfactuals using Exact Hat-Algebra

Suppose the economy is exposed to a change in trade costs, $\{\hat{\tau}_{ji,k}\} \Longrightarrow$ counterfactual outcomes can be solved using the following system of $N + NK$ equations and $N + NK$ unknowns (namely, $\{\hat{w}_i\}$, and $\{\hat{L}_{i,k}\}$):

$$\hat{w}_i \hat{L}_{i,k} w_i L_{i,k} = \sum_{j=1}^{N} \frac{\lambda_{ij,k} \hat{L}_{i,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{ij,k} \hat{w}_i\right)^{-\epsilon_k}}{\sum_{\ell=1}^{N} \lambda_{\ell j,k} \hat{L}_{\ell,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{\ell i,k} \hat{w}_\ell\right)^{-\epsilon_k}} e_{j,k} \hat{w}_j w_j \bar{L}_j \quad , \forall i$$

$$\sum_{k=1}^{K} \sum_{j=1}^{N} \frac{\lambda_{ij,k} \hat{L}_{i,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{ij,k} \hat{w}_i\right)^{-\epsilon_k}}{\sum_{\ell=1}^{N} \lambda_{\ell j,k} \hat{L}_{\ell,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{\ell i,k} \hat{w}_\ell\right)^{-\epsilon_k}} e_{j,k} \hat{w}_j w_j \bar{L}_j = \hat{w}_i w_i \bar{L}_i \quad , \forall i$$

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– All highlighted variables are either observables or estimable parameters.

Welfare Analysis

Welfare in country $W_i = w_i/P_i$ can be expressed as[1]

$$W_i = T_i \times \underbrace{\prod_{k=1}^{K} (L_{i,k})^{\mu_k e_{i,k}}}_{\text{scale effects}} \times \underbrace{\prod_{k=1}^{K} \lambda_{ii,k}^{-e_{i,k}/\epsilon_k}}_{\text{trade openness}}$$

[1]You can use the expression for $\lambda_{ii,k} = (P_{ii,k}/P_{i,k})^{-\epsilon_k}$ and our expression for the price index to derive the above formula.

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- $r_{i,k} \equiv \frac{L_{i,k}}{\bar{L}_i}$ is the share of output produced in industry k
- $\mu_i \equiv \sum_k \mu_k e_{i,k}$
- $T_i \bar{L}_i^{\bar{\mu}_i}$ is invariant to changes in trade barriers.

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Note

- The single industry economies, considered previously, were efficient.
- The multi-industry economy is inefficient if $\mathrm{Var}(\mu_k) \neq 0$.
- Specifically, there is sub-optimal firm-entry and output in high-returns-to-scale (high-μ) industries.
- The term labeled "allocate efficiency" roughly represents economy i's distance to its efficiency frontier.

The Gains from Trade

– Let A denote *autarky* variables $\Longrightarrow r^A_{i,k} = e_{i,k}$ and $\lambda^A_{ii,k} = 1$.

– Plugging the above values in the welfare formula yields the following expression for *the gains from trade*:

$$GT_i \equiv \frac{W_i}{W_i^A} = \underbrace{\prod_{k=1}^{K} \left(\frac{r_{i,k}}{e_{i,k}}\right)^{\mu_k e_{i,k}}}_{\text{effect on production effeciency}} \times \underbrace{\prod_{k=1}^{K} \lambda_{ii,k}^{-e_{i,k}/\epsilon_k}}_{\text{pure gains from variety}}$$

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$$GT_i = \underbrace{\prod_{k=1}^{K} \left(\frac{r_{i,k}}{e_{i,k}} \right)^{(\mu_k - \bar{\mu}) e_{i,k}}}_{\text{effect on pre-existing distortions}} \times \underbrace{\prod_{k=1}^{K} \left(\frac{r_{i,k}}{e_{i,k}} \right)^{\bar{\mu} e_{i,k}}}_{\text{scale-driven gains}} \times \underbrace{\prod_{k=1}^{K} \lambda_{ii,k}^{-e_{i,k}/\epsilon_k}}_{\text{variety gains}}$$

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- **Special cases:**
  - perfectly competitive model: $\mu_k = \bar{\mu} = 0$
  - no pre-existing market distortions: $\mu_k = \bar{\mu}\ \forall k$

The Gains from Trade: *The Perfectly Comp. Case*

	% GT	
	single-industry	multi-industry
Ireland	8%	23.5%
Belgium	7.8%	32.7%
Germany	4.5%	12.7%
China	2.6%	4%
U.S.	1.8%	4.4%

Source: *Costinot & Rodriguez-Clare (2014)* based on data from the 2008 WIOD, which includes 16 traded.

Why are the Multi-Industry Gains Larger?

- **Multi-industry** PC model:

$$GT_i^{MI} = \prod_{k=1}^{K} \lambda_{ii,k}^{-e_{i,k}/\epsilon_k} = \lambda_{ii}^{-\frac{1}{\tilde{\epsilon}_i}}$$

  where $\frac{1}{\tilde{\epsilon}_i} \equiv \sum_k \frac{e_{i,k}}{\epsilon_k} \frac{\ln \lambda_{ii,k}}{\ln \lambda_{ii}}$.

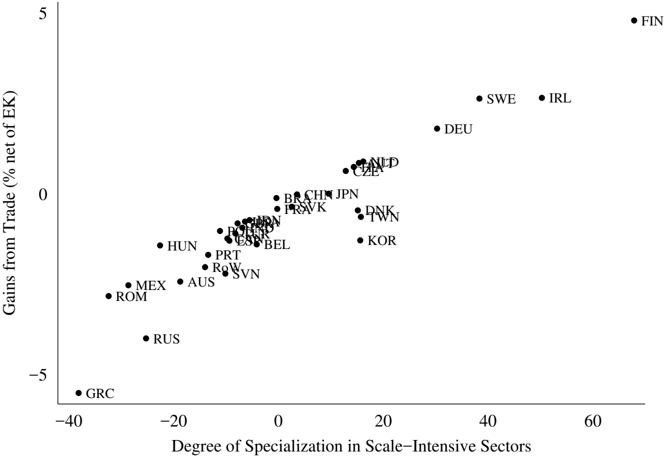
- **Single industry** model:

$$GT_i^{SI} = \lambda_{ii}^{-\frac{1}{\epsilon}}$$

  where $\epsilon \equiv \mathbb{E}(\epsilon_k)$ is a weighted arithmetic average that is implicitly estimated when using aggregate data to discipline the trade elasticity.

- Jensen's Inequality $\Longrightarrow \tilde{\epsilon}_i < \epsilon \Longrightarrow GT_i^{MI} > GT_i^{SI}$

The Effect of Trade on Pre-Existing Market Distortions



Source: the graph was created using estimated μ_k's by *Lashkaripour and Lugovskyy (2019)* and trade/production data from the 2008 WIOD.

Intuition: *The Effect of Trade on Pre-Existing Distortions*

- Recall that output is high-μ industries is already sub-optimal pre-trade liberalization.
- If a country has a comparative *disadvantage* in high-μ industries $\Longrightarrow$ it lowers its output in high-μ industries post-trade liberalization $\Longrightarrow$ pre-existing market distortion are exacerbated.
- According to data, MEXICO, GREECE, RUSSIA, and ROMANIA are in this position.

Should Countries Restrict Trade to Retain Efficiency?

- Many arguments against trade openness are based on the observation that trade can worsen pre-existing market distortions.

- So, should countries restrict trade?

Should Countries Restrict Trade to Retain Efficiency?

- Many arguments against trade openness are based on the observation that trade can worsen pre-existing market distortions.

- So, should countries restrict trade? No!

The Targeting Principle (*Bhagwati and Ramaswami, 1963*)

- Trade is not the source of inefficiency, here.
- Restricting trade is only a *second-best* solution.
- *First-best* solution: subsidize high-μ industries + liberalize trade.
- I will talk more about this when we cover trade/industrial policy.