The Muti-Industry$\times$Multi-Factor Trade Model

International Trade (PhD), Fall 2019

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Overview

- In this lecture,I present a multi-industry trade model that features multiples types of workers.
- The model is often labeled the Ricardo-Roy model as it features:
  - specialization across industries à la Ricardo
  - allocation of workers across industries à la Roy
- Main implications
  - The overall gains from trade are larger...
  - ... but trade creates winners and losers.
- Main References:
  - **parametric R-R model:** *Galle, Rodrıguez-Clare, Yi (2018)*
  - **non-parametric R-R model:** *Costinot and Vogel (2015, ARE)*

Environment

- $j, \ i = 1, \ldots, N$ countries
- $k = 1, \ldots, K$ industries
- Labor is the only factor of production
- $g = 1, \ldots, G$ groups of workers
- $\bar{L}_{i,g}$ denotes the total number of group g workers in Country i.

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Overview

- Demand/Supply of final goods is governed by a multi-industry gravity model (to fix minds, I adopt the *Eaton-Kortum* interpretation).
- Departures from the standard multi-industry model:
  - different workers have different abilities.
  - different industries in a given country pay different wages.
  - workers sort into industries in order to maximize their *productivity*×*wage* à la Roy.

Demand for Final Goods

- Cobb-Douglas utility aggregator across industries:

$$U_i(\mathbf{Q}) = \sum_k Q_k^{\beta_{i,k}}$$

$\Longrightarrow$ a constant fraction $\beta_{i,k}$ of income is spent on industry k goods.

- There are a continuum of goods $\omega \in \Omega_k$ within industry k, and the utility aggregator across these goods is CES:

$$Q_k(\mathbf{q}) = \left(\int_{\omega \in \Omega_k} q(\omega)^{\frac{\sigma_k - 1}{\sigma_k}} d\omega \right)^{\frac{\sigma_k}{\sigma_k - 1}}$$

Supply of Final Goods (*Eaton & Kortum, 2002*)

- The price at which country j can supply good $\omega \in \Omega_k$ to market i

$$p_{ji,k}(\omega) = \tau_{ji,k} w_{j,k} / z_{j,k}(\omega)$$

where $z(\omega)$'s are *Fréchet-distributed*: $\Pr\left\{z_{j,k}(\omega) \leqslant z\right\} = \exp\left(-T_{j,k} z^{-\theta}\right)$.

- Country i buys good ω from the cheapest supplier $\Longrightarrow$ the share of country i's spending on country j goods is

$$\lambda_{ji,k}\left(\boldsymbol{w}_k\right) = \frac{T_{j,k}\left(\tau_{ji,k} w_{j,k}\right)^{-\theta_k}}{\sum_\ell T_{\ell,k}\left(\tau_{\ell i,k} w_{\ell,k}\right)^{-\theta_k}}$$

where $\boldsymbol{w}_k \equiv \{w_{j,k}\}$ is a vector describing the wage per efficiency units across different countries in industry k.

Demand for Labor

- Demand for labor efficiency units in industry k of country i

$$E_{i,k}^{\mathcal{D}}(\boldsymbol{w}_k) \equiv \frac{1}{w_{i,k}} \sum_{j=1}^{N} \lambda_{ij,k}\left(\boldsymbol{w}_k\right) \beta_{j,k} Y_j$$

Demand for Labor

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- The above equation follows from the fact that

$$\text{Total Sales} = \text{Total Wage Payments} = w \times \text{eff. units}$$

Supply of Labor: *Roy Model*

- There is a constant measure $\bar{L}_{i,g}$ of group g workers in country i.
- Each individual ι from group g independently draws an efficiency vector $\boldsymbol{z}(\iota) = \{z_1(\iota), ..., z_K(\iota)\}$ from the following GEV distribution:

$$F_{i,g}(\boldsymbol{z}) = \exp\left(-\sum_{k=1}^{K} a_{i,kg} z_k^{-\kappa_g}\right)$$

- **Discrete choice problem facing individual ι:** choose an industry where wage income is maximized so that

$$\text{indidviual } \iota\text{'s income} = \max\{w_{i,1} z_1(\iota), ..., w_{i,K} z_K(\iota)\}$$

Supply of Labor: *Roy Model*

- Theorem of Extreme Value $\Longrightarrow$ share of group g workers in country i that choose to work in industry k is

$$\pi_{i,kg}(\boldsymbol{w}_i) = \frac{a_{i,kg}\, w_{i,k}^{\kappa_g}}{\sum_s a_{i,sg}\, w_{i,s}^{\kappa_g}}$$

- ***Intuition***: workers are more likely to choose
  - industries where they are inherently more capable in (high-$a_{i,kg}$)
  - industries that pay higher wages (high-$w_{i,k}$).

Supply of Labor

– The total supply of efficiency units by group g workers to industry k:

$$E^{\mathcal{S}}_{i,kg} = \pi_{i,kg}(\boldsymbol{w}_i)e_{i,kg}(\boldsymbol{w}_i)\bar{L}_{i,g}$$

Supply of Labor

- The total supply of efficiency units by group g workers to industry k:

$$E^{\mathcal{S}}_{i,kg} = \pi_{i,kg}(\boldsymbol{w}_i) \quad \underbrace{e_{i,kg}(\boldsymbol{w}_i)}_{\text{avg. productivity}} \quad \bar{L}_{i,g}$$

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where

$$e_{i,kg}(w_i) = a_{i,kg}\pi_{i,kg}(w_i)^{-\frac{1}{\kappa_g}}$$

- **Intuition:** a higher $\pi_{i,kg}$ means that more group g workers are choosing industry k, which implies that less productive individuals are choosing industry k presumably dues to higher wages $\implies$ lower avg. productivity.

Supply of Labor

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$$E^{S}_{i,kg} = \pi_{i,kg}(\boldsymbol{w}_i) \underbrace{e_{i,kg}(\boldsymbol{w}_i)}_{\text{avg. productivity}} \bar{L}_{i,g}$$

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$$e_{i,kg}(\boldsymbol{w}_i) = a_{i,kg}\pi_{i,kg}(\boldsymbol{w}_i)^{-\frac{1}{\kappa_g}}$$

– **A limitation of the model:** the avg. income of group g workers is equalized across industries:

$$y_{i,g} = w_{i,k}e_{i,kg} = \left(\sum_{k} a_{i,kg} w_{i,k}^{\kappa_g}\right)$$

Labor Market Clearing Condition

- Equilibrium is a $N \times K$ vector of wage, $\boldsymbol{w} \equiv \{w_{i,k}\}$ that satisfy $N \times K$ labor market clearing conditions:

$$\sum_g e_{i,kg}(\boldsymbol{w}_i)\pi_{i,kg}(\boldsymbol{w}_i)\bar{L}_{i,g} = \frac{1}{w_{i,k}} \sum_j \lambda_{ij,k}(\boldsymbol{w}_k)\beta_{j,k}Y_j$$

- Total income income in country i is given by

$$Y_i(\boldsymbol{w}_i) = \sum_k \sum_g w_{i,k} e_{i,kg}(\boldsymbol{w}_i)\pi_{i,kg}(\boldsymbol{w}_i)\bar{L}_{i,g}$$

Labor Market Clearing Condition

- Equilibrium is a $N \times K$ vector of wage, $\boldsymbol{w} \equiv \{w_{i,k}\}$ that satisfy $N \times K$ labor market clearing conditions:

$$\underbrace{\sum_g e_{i,kg}(\boldsymbol{w}_i)\pi_{i,kg}(\boldsymbol{w}_i)\bar{L}_{i,g}}_{\text{Supply}} = \underbrace{\frac{1}{w_{i,k}}\sum_j \lambda_{ij,k}(\boldsymbol{w}_k)\beta_{j,k}Y_j(\boldsymbol{w}_j)}_{\text{Demand}}$$

- Total income income in country i is given by

$$Y_i(\boldsymbol{w}_i) = \sum_g y_{i,g}(\boldsymbol{w}_i)\bar{L}_{i,g}$$

where $y_{i,g} = \left(\sum_k a_{i,kg} w_{i,k}^{\kappa_g}\right)$.

Welfare Analysis

– The avg. real income per worker in group g is

$$W_{i,g} = \frac{y_{i,g}}{\prod_{k=1}^{K} P_{i,k}^{\beta_{i,k}}}$$

Welfare Analysis

avg. income

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agg. price index

Welfare Analysis

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$$W_{i,g} = \frac{y_{i,g}}{\prod_{k=1}^{K} P_{i,k}^{\beta_{i,k}}}$$

- Change in welfare due to some international shocks:

$$\hat{W}_{i,g} = \prod_k \left(\frac{\hat{w}_{i,k}}{\hat{P}_{i,k}} \frac{\hat{y}_{i,g}}{\hat{w}_{i,k}} \right)^{\beta_{i,k}} = \prod_k \hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \hat{\pi}_{i,kg}^{-\frac{\beta_{i,k}}{\kappa g}}$$

Total Income

- Change in aggregate welfare

$$\hat{W}_i \equiv \frac{\hat{Y}_i}{\hat{P}_i} = \sum_{g \in G_i} \frac{Y_{i,g}}{Y_i} \hat{W}_{i,g}$$

- Plugging the expression for $\hat{W}_{i,g}$ into the above equation yields

$$\hat{W}_i = \prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \right) \cdot \left(\sum_g \frac{Y_{i,g}}{Y_i} \prod_k \hat{\pi}_{i,kg}^{-\frac{\beta_{i,k}}{\kappa_g}} \right)$$

Special Case: $\kappa_g \approx 1$

- If $\kappa_g = \kappa \approx 1 \implies$ the present model becomes isomorphic to a specific factor model (i.e., there is a fixed supply of efficiency units by group g workers to industry$k$ $E^{\mathcal{S}}_{i,kg} = a_{i,kg}\bar{L}_{i,g}$)

- The effect of an international shocks on group g workers relative to the rest of the economy:

$$\frac{\hat{W}_{i,g}}{\hat{W}_i} \approx \left(\sum_k \pi_{i,kg} \hat{r}_{i,k} \right)^{\frac{1}{\kappa}}$$

- ***Intuition***: group g workers incur a loss if they are mainly employed in shrinking (low-$\hat{r}$) industries!

Proposition (*Galle, Rodriguez-Clare, Yi, 2018*)

Assume that $\kappa_g = \kappa$ for all g, then the aggregate gains from trade are strictly higher than those that arise in the single factor model (which corresponds the special case where $\kappa \to \infty$)

Proposition (*Galle, Rodrıguez-Clare, Yi, 2018*)

Assume that $\kappa_g = \kappa$ for all g, then the aggregate gains from trade are strictly higher than those that arise in the single factor model (which corresponds the special case where $\kappa \to \infty$)

$$\hat{W}_i = \prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \right) \cdot \left(\sum_g \frac{Y_{i,g}}{Y_i} \prod_k \hat{\pi}_{i,kg}^{\frac{\beta_{i,k}}{\kappa}} \right) \quad > \quad \underbrace{\prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{e_{i,k}}{\theta_k}} \right)}_{\text{GT in single factor model}}$$

Performing Counterfactuals using Exact Hat-Algebra

– If the economy is exposed to a change in trade costs, $\{\hat{\tau}_{ji,k}\}$ and/or technology levels $\{\hat{T}_{j,k}\}$ $\Longrightarrow$ counterfactual outcomes can be solved using the following system of NK equations and unknowns:

$$\sum_{g} \hat{\pi}_{i,kg}\left(\hat{\boldsymbol{w}}\right)^{1-\frac{1}{\kappa_g}} Y_{i,gk} = \frac{1}{\hat{w}_{i,k}} \sum_{j} \hat{\lambda}_{ij,k} \lambda_{ij,k}\left(\hat{\boldsymbol{w}}\right) \beta_{j,k} \hat{Y}_j\left(\hat{\boldsymbol{w}}\right) Y_j$$

where

$$\begin{cases} \hat{Y}_i(\hat{\boldsymbol{w}}) Y_i = \sum_g \left(\sum_k \pi_{i,kg} \hat{w}_{i,k}{}^{\kappa_g}\right) Y_{i,g} \\ \hat{\pi}_{i,kg}\left(\hat{\boldsymbol{w}}\right) = \frac{\hat{w}_{i,k}{}^{\kappa_g}}{\sum_s \pi_{i,sg} \hat{w}_{i,s}{}^{\kappa_g}} \\ \hat{\lambda}_{ij,k}\left(\hat{\boldsymbol{w}}\right) = \frac{\hat{T}_{j,k}\left(\hat{\tau}_{ji,k} \hat{w}_{j,k}\right)^{-\theta_k}}{\sum_\ell \lambda_{\ell i,k} \hat{T}_{\ell,k}\left(\hat{\tau}_{\ell i,k} \hat{w}_{j,k}\right)^{-\theta_k}} \end{cases}$$

Application I: The Effect of $\hat{T}_{\text{China}}$[1]

Table 2: The Welfare Effects of the China Shock on the US

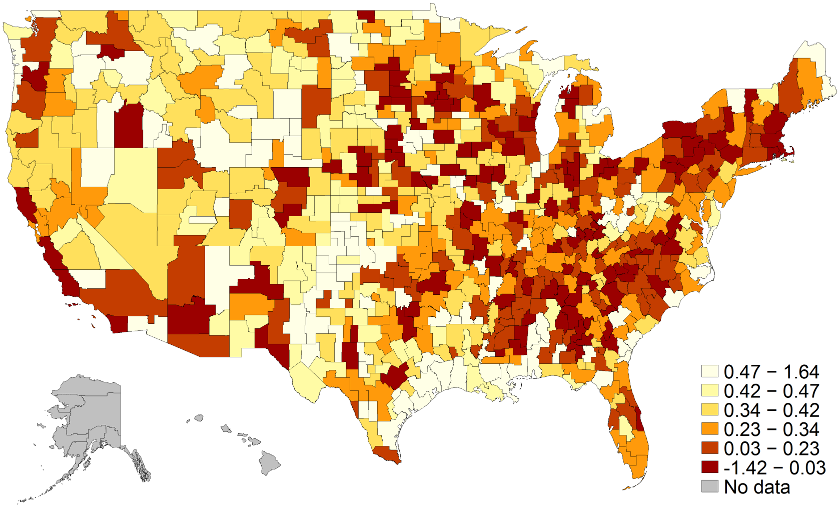
κ	Aggregate	Mean	CV	Min.	Max.	ACR
$\rightarrow 1$	0.24	0.30	1.40	-1.73	2.32	0.14
1.5	0.22	0.27	1.16	-1.42	1.64	0.15
3	0.20	0.24	0.80	-0.90	0.97	0.16
$\rightarrow \infty$	0.20	0.20	0	0.20	0.20	0.20

- $\hat{T}_{\text{China},k}$ is inferred from China's export growth to global markets.
- A worker group g is defined as a group of workers residing in one of the 722 commuting zones in the US.

[1]Source: *Galle, Rodriguez-Clare, Yi, 2018.*

Application I: The Effect of $\hat{T}_{China}$

Figure 1: Geographical distribution of the welfare gains from the rise of China



Application II: The Gains from Trade

Table 3: Aggregate and Group-level Gains from Trade

κ	Aggregate	Mean	CV	Min.	Max.	ACR
$\to 1$	1.61	1.65	0.82	-6.98	3.72	1.45
1.5	1.56	1.59	0.58	-4.19	2.97	1.45
3	1.51	1.52	0.31	-1.38	2.22	1.45
$\to \infty$	1.45	1.45	0	1.45	1.45	1.45

- The gains are calculated by setting $\hat{\tau}_{iUSA,k} \to \infty$.
- A worker group g is defined as a group of workers residing in one of the 722 commuting zones in the US.

Application II: The Gains from Trade

Figure 5: Geographical Distribution of the Gains from Trade