Profits, Scale Economies, and the Gains from Trade and Industrial Policy*

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Abstract

Trade restrictions are often used as (a) a first-best policy to manipulate the terms-of-trade or (b) a second-best policy to correct misallocation in domestic industries. We analyze the (in)effectiveness of trade restrictions at achieving these goals. To this end, we derive sufficient statistics formulas for *first-best* and *second-best* trade taxes in an important class of multi-industry, multi-country trade models where misallocation occurs due to scale economies or profit-generating markups. We discipline our formulas by estimating the key parameters that govern the gains from trade and industrial policy in open economies. Our estimates reveal that (i) trade policy is remarkably ineffective at correcting misallocation in domestic industries, reflecting a deep tension between allocative efficiency and terms-of-trade. (ii) Unilateral adoption of corrective industrial policies is also ineffective as it causes *immiserizing growth*. But (iii) industrial policies that are coordinated via a *deep agreement* are more transformative than any unilateral policy intervention.

1 Introduction

The United States will likely adopt an explicit industrial policy in the coming decade. Similar developments are well underway in other countries (Aiginger and Rodrik (2020)). And with industrial policy back on the scene, we are witnessing a revival of old-but-questionable trade policy practices. Governments are often turning to protectionist trade policy measures to pursue their industrial policy objectives—as manifested by the United States *National Trade Council*'s mission or the Chinese, *Made in China 2025* initiative.[1]

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[1]See Bhagwati (1988) and Irwin (2017) for a historical account of trade restrictions being used by governments to promote their preferred industries. A prominent example dates back to 1791, when Alexander Hamilton approached Congress with *"the Report on the Subject of Manufactures,"* encouraging the implementation of protective tariffs and industrial subsidies. These policies were intended to help the US economy catch up with Britain.

These developments have sparked new interest in old-but-open questions regarding trade and industrial policy. For instance: (i) Is trade policy an effective tool for correcting misallocation in the domestic economy? (ii) If not, should governments undertake *unilateral* domestic policy interventions to correct misallocation? or (iii) should they coordinate their industrial policies via a deep trade agreement?

To answer these questions, we characterize optimal trade and industrial policies in an important class of multi-industry, multi-country quantitative trade models where misallocation occurs due to scale economies or profit-generating markups. Guided by theory, we estimate the key parameters that govern the welfare consequences of trade and industrial policy in open economies. We then combine our estimated parameters with optimal policy formulas to quantify the ex-ante gains from trade and industrial policy among 43 major countries.

Our estimation reveals that trade policy is remarkably ineffective at correcting misallocation, reflecting a deep tension between allocative efficiency and terms of trade. Unilateral adoption of corrective industrial policies can also backfire as it often triggers *immiserizing growth*. These considerations, we argue, may have spurred a global *race to the bottom*, wherein governments either avoid corrective industrial policies or pair them with hidden trade barriers. A deep agreement can remedy this problem and deliver welfare gains that are more transformative than any unilateral policy intervention.

Section 2 presents our theoretical framework. Our baseline model is a generalized multi-industry Krugman (1980) model that features a non-parametric utility aggregator across industries and a nested CES utility aggregator within industries. This specification has an appealing property wherein the degrees of firm-level and country-level market love-for-variety can diverge. We analyze both the restricted and free entry cases of the model to distinguish between the *short-run* and *long-run* consequences of policy. With a reinterpretation of parameters, our baseline framework also nests (a) the multi-industry Melitz (2003)-Pareto model, and (b) the multi-industry Eaton and Kortum (2002) model with industry-level Marshallian externalities. We later extend our baseline model to accommodate non-parametric input-output linkages.

Section 3 derives sufficient statistics formulas for *first-best* and *second-best* trade and industrial policies. Unilaterally optimal policies in our framework pursue two objectives: First, they seek to improve the home country's *terms-of-trade (ToT)* vis-à-vis the rest of the world. Second, they seek to restore *allocative efficiency* in the domestic economy by reallocating workers towards high-returns to scale or high-profit industries.

The *first-best* optimal policy consists of misallocation-blind import tariffs and export subsidies that solely maximize ToT gains. Allocative efficiency under the first-best policy is restored via domestic Pigouvian subsidies.[2] While these insights resonate with the targeting principle, our optimal policy formulas have other implications worth highlighting. First, even though 1st-best trade policies are blind to misallocation (the dispersion in scale elasticities), they depend on the overall strength of scale economies (the level of scale elasticities). Second, optimal import tariffs are also input-output-blind when export subsidies are assigned optimally.

Our *second-best* trade policy formulas attain relevance when governments are reluctant to use industrial subsidies to correct misallocation—the root of which could be political pressures

[2]The optimal subsidy rate in each industry equals the inverse of the industry-level scale elasticity or markup.

or institutional barriers.[3] Second-best trade tax-cum-subsidies are composed of two components: a neoclassical ToT-improving component and a misallocation-correcting component. The former seeks to restrict relative exports in nationally-differentiated industries. The latter seeks to restrict imports and promote exports in high-returns-to-scale industries, mimicking the first-best Pigouvian subsidies.

Section 4 argues that, in empirically-relevant circumstances, second-best trade policies have difficulty striking a balance between ToT-improving and misallocation-correcting objectives. Put differently, trade policy interventions that seek to improve the ToT exacerbate misallocation and vice versa. This tension diminishes the possible gains from second-best trade policies to a considerable degree. In some canonical cases, optimal second-best trade policies are even industry-blind—unable to beneficially correct inter-industry misallocation or manipulate the ToT on an industry-by-industry basis.

Another consequence of this tension is that unilateral adoption of scale- or markup-correcting industrial policies can cause *immiserizing growth* and harm welfare. The logic is that unilateral and unreciprocated implementation of corrective policies severely worsens a country's ToT.[4] *Shallow* trade agreements, as a result, may prove insufficient for reaching global efficiency. Once governments agree to abandon inefficient trade restrictions under a shallow agreement, they become tangled in a coordination game involving corrective industrial policies. The outcome of this game is a *race to the bottom* where no government is willing to implement corrective industrial policies without violating their commitment to zero trade restrictions. A deep agreement can remedy this problem.

To put these matters in perspective, Section 6 estimates the structural parameters necessary for ex-ante policy evaluation. Our optimal policy formulas reveal that policy outcomes depend on a set of sufficient statistics consisting of observables and two set of parameters. Namely, (i) industry-level *trade elasticities* that govern the scope for ToT manipulation, and (ii) industry-level *scale elasticities* that govern the extent of misallocation. We develop a new methodology that *simultaneously* estimates these parameters using transaction-level trade data.

The scale elasticity parameters in our generalized Krugman (1980) model reflect the extent of *love-for-variety*—the social benefits of which are not internalized by firms' entry decisions. The trade elasticities reflect the degree of national product differentiation. Both parameters can be recovered from *firm-level* demand parameters, which we estimate by fitting a structural firm-level demand function to the universe of Colombian import transactions covering over 225,000 firms from 251 countries. A crucial advantage of our approach is its ability to *separately* identify the firm-level degree of product differentiation (that dictates the scale elasticity) from the national-level degree (that determines the trade elasticity).

The firm-level nature of our demand estimation subjects us to a less familiar identification challenge. Traditional estimations of import demand are often conducted with country-level data and use the variation in tariffs to identify demand parameters. This identification strategy

[3] Trade policy has been regularly used—in place of domestic industrial policy—to promote critical industries (Bhagwati (1988); Harrison and Rodríguez-Clare (2010); Irwin (2017)). Relatedly, see Lane (2020) for a historical account of various industrial policy practices around the world.

[4] To be clear, this is true when the scale and trade elasticity are negatively correlated across industries, which is consistent with our estimation of these elasticities.

is unsuitable for firm-level demand estimation as tariffs do not vary across firms from the same country. To navigate this issue, we construct a shift-share instrument that combines monthly exchange rate movements with lagged monthly export sales to measure exposure to exchange rate shocks at the *firm-product-year* level.

Section 7 combines our micro-level estimates, our optimal policy formulas, and macro-level data from the 2014 World Input-Output Database to quantify the ex-ante gains from policy among 43 major economies. Our analysis delivers three main findings.

First, we find that trade policy is remarkably ineffective at correcting misallocation in the domestic economy—even without factoring in the cost of retaliation by trading partners. Under free entry, second-best export subsidies and import taxes can raise the average country's real GDP by only 1.19%, which amounts to less than 4/10 of the gains attainable under the unilaterally first-best policy. Third-best import taxes are even less effective as a standalone policy, raising real GDP by a mere 0.63%. These findings corroborate the argument that trade policy has difficulty striking a balance between ToT and misallocation-correcting objectives.

Second, unilateral adoption of corrective industrial policies triggers severe immiserizing growth in most countries. The average country's real GDP declines by 2.7% if they implement scale-correcting subsidies without reciprocity by trading partners. Aversion to these consequences, we argue, we may have spurred a global race to the bottom in industrial policy implementation. To escape immiserizing growth, governments either avoid corrective policies or pair them with hidden trade barriers that breach shallow trade cooperation.[5]

Third, deep agreements can remedy the race to the bottom and deliver welfare gains that are more transformative than any unilateral intervention. To offer some perspective, corrective industrial policies coordinated via a deep agreement can elevate the average country's real GDP by 3.2%. These welfare gains rival the already-realized gains from shallow agreements for most countries. They, moreover, exceed any welfare gains achievable through unilateral trade or industrial policy interventions—even not considering that unilateralism often backfires in the form of retaliation by trading partners.

Related Literature—Our theory relates to an emerging literature on optimal policy in *distorted* open economies. In a concurrent paper, Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2019) characterize the first-best optimal policy for a small open economy in a multi-sector Ricardian model with Marshallian externalities. Relatedly, Haaland and Venables (2016) characterize optimal policy for a small open economy in two-by-two Krugman and Melitz models.[6] Beyond optimal policy, Campolmi, Fadinger, and Forlati (2018) employ a two-sector Melitz-Pareto model to elucidate the trade-offs facing countries that join shallow and deep trade agreements.[7] Our analysis of *second-best* trade policies speaks to an older literature emphasizing the firm-delocation rationale for trade restrictions (e.g., Venables (1987); Ossa (2011)),

[5]The Chinese government, for instance, pairs its domestic subsidies with hidden export taxes. These hidden barriers are applied via partial value-added tax rebates and are designed to restore China's ToT (Garred (2018)).

[6]Demidova and Rodriguez-Clare (2009) and Felbermayr, Jung, and Larch (2013) characterize optimal tariffs in a *single industry* Melitz-Pareto model. The single industry assumption ensures that markets are efficient (Dhingra and Morrow (2019)) and import and export taxes are equivalent (the Lerner symmetry). So, the unilaterally first-best can be reached with import tariffs alone. Costinot, Rodríguez-Clare, and Werning (2016) examine optimal policy in the *single industry* Melitz-Pareto model from a different lens, characterizing optimal *firm-level* taxes.

[7]Other papers have also used new or quantitative trade models to analyze piecemeal policy reforms in distorted

and supplements Bagwell and Staiger's (2001; 2004) result about the role of trade agreements in distorted economies. We also build on Kucheryavyy, Lyn, and Rodríguez-Clare (2016) to establish isomorphism between our baseline model and other workhorse models in the literature. Our *quantitative* examination of trade and industrial policy connects to two strands of literature. First, a mature line of research measuring the ex-post consequences of tariff cuts (Costinot and Rodríguez-Clare (2014); Caliendo and Parro (2015); Ossa (2014, 2016); Spearot (2016)). Second, a growing literature examining the ex-ante consequences of optimal policy. Ossa (2014), most notably, quantifies the consequences of cooperative and non-cooperative import tariffs in a multi-industry Krugman model with restricted entry. Lastly, our work relates to a vibrant literature examining the impacts of exogenous trade shocks in distorted economies (e.g., De Blas and Russ (2015); Edmond, Midrigan, and Xu (2015); Baqaee and Farhi (2019)).

We contribute to the calculus of optimal policy in open economies by developing a new dual technique for optimal policy derivation in general equilibrium quantitative trade models with many countries, increasing returns-to-scale production technologies, and input-output linkages. Our approach has applications beyond those considered in this paper. Lashkaripour (2020b), for instance, adopts a special case of this technique to characterize Nash tariffs in a monopolistic competition model with restricted entry.[8] Farrokhi and Lashkaripour (2021) extend this technique to analyze optimal carbon pricing under international climate externalities. We also contribute to the broader quantitative trade literature by developing an estimation technique that separately identifies the scale elasticity from the trade elasticity. Our demand-side approach complements the supply-side approach concurrently proposed by Bartelme et al. (2019). The main limitation of our demand-side approach is its inability to detect scale externalities unrelated to love-for-variety. Our approach, nevertheless, has the advantage of separately identifying the trade elasticity from the scale elasticity and is robust to the presence of quasi-fixed production inputs.

2 Theoretical Framework

Our baseline model is a generalized multi-industry, multi-country Krugman model with semi-parametric preferences. In Section 5 we show that our theory readily applies to alternative models featuring firm-selection à la Melitz–Chaney and external economies of scale à la Kucheryavyy et al. (2016). We also extend our theory later to accommodate arbitrary input-output networks and political economy pressures.

We consider a world economy consisting of multiple countries and industries. Countries are indexed by of $i, j, n \in \mathbb{C}$. Industries are indexed by $g, k \in \mathbb{K}$. Industries can differ in fundamentals such as the degree of scale economies or trade elasticity. Each country $i \in \mathbb{C}$ is populated by L_i individuals who supply one unit of labor inelasticity. Labor is the sole primary factor of production in each economy. Workers cannot relocate between countries but

economies or optimal policy in *non-distorted* economies—e.g., Costinot and Rodríguez-Clare (2014); Campolmi, Fadinger, and Forlati (2014); Costinot, Donaldson, Vogel, and Werning (2015); Bagwell and Lee (2018); Caliendo, Feenstra, Romalis, and Taylor (2015); Demidova (2017); Beshkar and Lashkaripour (2019, 2020).

[8] Lashkaripour (2020b) examines the cost of non-cooperative import restrictions when governments simultaneously apply their 3rd-best optimal import tariffs. To expedite the computational process, Lashkaripour (2020b) uses analytic formulas for Nash tariffs, which correspond to a special of our Theorem 3.

are perfectly mobile across industries within a country, and are paid a country-wide wage, w_i.

2.1 Preferences

Each good in our model is indexed by a triplet, which signifies its location of production (origin), it location of final consumption (destination), and the industry under which the good is classified. To give an example: Good "ji,k" denotes a good corresponding to *origin country j–destination country i–industry k*.

Cross-Industry Demand. The representative consumer in country $i \in \mathbb{C}$ faces a vector of industry-level consumer price indexes $\tilde{\mathbf{P}}_i = \{\tilde{P}_{i,k}\}$, where index $\tilde{P}_{i,k} \equiv \tilde{P}_{i,k}(\tilde{P}_{1i,k}, ..., \tilde{P}_{Ni,k})$ aggregates over industry k goods sourced from various origins. The consumer choses their demand for industry-level bundles $\mathbf{Q}_i \equiv \{Q_{i,k}\}$ to maximize a non-parametric utility function subject to a budget constraint. This choice yields an indirect utility, which is a function of the consumer's income, Y_i, and the vector of industry-level *"consumer"* price indexes in market i, $\tilde{\mathbf{P}}_i$:

$$V_i\left(Y_i, \tilde{\mathbf{P}}_i\right) = \max_{\mathbf{Q}_i} \; U_i(\mathbf{Q}_i) \qquad s.t. \sum_{k \in \mathbb{K}} \tilde{P}_{i,k} Q_{i,k} = Y_i. \tag{1}$$

Throughout this paper, the *tilde* notation on price is used to distinguish between "consumer" and "producer" prices. The former includes taxes, whereas the latter does not. Problem 1 yields an *industry-level* Marshallian demand function, which we denote by $Q_{i,k} = \mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$. This function tracks how (given prices and total income) consumers allocate their expenditure across industries. A special case of our general cross-industry demand function is the Cobb-Douglas case, wherein $U_i(\mathbf{Q}_i) = \prod_{k \in \mathbb{K}} Q_{i,k}^{e_{i,k}}$ implying that $Q_{i,k} = e_{i,k} Y_i / \tilde{P}_{i,k}$.

Within-Industry Demand. Each industry-level bundle aggregates over various origin-specific composite varieties: $Q_{i,k} \equiv Q_{i,k}(Q_{1i,k}, ..., Q_{Ni,k})$. Each origin-specific composite variety, itself, aggregates over multiple firm-level varieties: $Q_{ji,k} \equiv Q_{ji,k}(\mathbf{q}_{ji,k})$, where $\mathbf{q}_{ji,k} = \left\{q_{ji,k}(\omega)\right\}_{\omega \in \Omega_{j,k}}$ is a vector with each element $q_{ji,k}(\omega)$ denoting the quantity consumed of firm ω's output.[9] We assume that the within-industry utility aggregator, has a nested-CES structure, which enables us to abstract from variable markups and direct our attention to the scale-driven and profit-shifting effects of policy.

Assumption (A1). The within-industry utility aggregator is nested-CES. In particular,

$$Q_{i,k} = \left(\sum_{j \in \mathbb{C}} Q_{ji,k}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \qquad where \qquad Q_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} \varphi_{ji,k}(\omega)^{\frac{1}{\gamma_k}} q_{ji,k}(\omega)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega \right)^{\frac{\gamma_k}{\gamma_k - 1}},$$

with $\gamma_k \geq \sigma_k > 1$ and $\varphi_{ji,k}(\omega) > 0$ corresponding to a constant variety-specific taste shifter.

Based on (A1), the demand for the composite *national-level* variety ji,k (origin country j–destination country i–industry k) is given by

$$Q_{ji,k} = \left(\tilde{P}_{ji,k} / \tilde{P}_{i,k}\right)^{-\sigma_k} Q_{i,k}, \tag{2}$$

where $\tilde{P}_{ji,k}$ and $\tilde{P}_{i,k}$ respectively denote the origin-specific and industry-level CES price in-

[9] $\Omega_{j,k}$ denotes the set of all firms operating in origin j–industry k. In our baseline model, firms do *not* incur fixed exporting cost, so each firms in $\Omega_{j,k}$ serves market i. We relax this assumption in Section 5.

dexes.[10] Recall that $Q_{i,k}$ denotes industry-level demand, which is given by $Q_{i,k} = \mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$. The demand facing individual firms from country j is, accordingly, given by

$$q_{ji,k}(\omega) = \varphi_{ji,k}(\omega)\left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}}\right)^{-\gamma_k}\left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\sigma_k}\mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right). \quad (3)$$

Importantly, the above parameterization of demand allows for the *firm-level* and *national-level* degrees of market power to diverge. γ_k governs the degree of firm-level market power and love-for-variety, while σ_k governs the degree of national-level market power in industry k.

Elasticity of Demand Facing National-Level Varieties. Following Equation 2, the demand for aggregate variety ji,k is a function of total income in market i, Y_i, and the entire vector of *origin*$\times$*industry*-specific consumer price indexes in that market: Namely, $Q_{ji,k} = \mathcal{D}_{ji,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$. To keep track of changes in demand, we define the elasticity of demand for national-level variety ji,k w.r.t. to the price of variety ni,g as follows:

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,g}} \sim \text{price elasticity of demand}$$

Under Cobb-Douglas preferences (i.e., zero cross-substitutability between industries), the national-level demand elasticities are fully determined by the upper-tier CES parameter σ_k and national-level expenditure shares. Specifically, $\varepsilon_{ji,k}^{ji,g} = 0$ if $g \neq k$, while

$$\varepsilon_{ji,k} \sim \varepsilon_{ji,k}^{(ji,k)} = -1 - (\sigma_k - 1)(1 - \lambda_{ji,k}); \qquad \varepsilon_{ji,k}^{(\jmath i,k)} = (\sigma_k - 1)\lambda_{\jmath i,k} \quad (\jmath \neq j),$$

where $\lambda_{ji,k} \equiv \tilde{P}_{ji,k}Q_{ji,k} / \sum_{\jmath} \tilde{P}_{\jmath i,k}Q_{\jmath i,k}$ denotes the (within-industry) share of expenditure on ji,k. In the presence of cross-substitutability between industries, the demand elasticity will feature an additional term that accounts for cross-industry demand effects.

In our setup, optimal policy internalizes the entire matrix of own- and cross-demand elasticities. To present our optimal policy formulas concisely, we use the following matrix notation to track the elasticity of demand w.r.t. goods sourced from various origins and industries.

Definition (D1). Let $K = |\mathbb{K}|$ denote the number of industries. The $K \times K$ matrix $\mathbf{E}_{ji}^{(ni)}$ describes the elasticity of demand for origin $j \in \mathbb{C}$ goods w.r.t. the price of origin $n \in \mathbb{C}$ goods in market i:

$$\mathbf{E}_{ji}^{(ni)} \equiv \begin{bmatrix} \varepsilon_{ji,1}^{(ni,1)} & \cdots & \varepsilon_{ji,1}^{(ni,K)} \\ \vdots & \ddots & \vdots \\ \varepsilon_{ji,K}^{(ni,1)} & \cdots & \varepsilon_{ji,K}^{(ni,K)} \end{bmatrix}.$$

To simplify the notation, we use $\mathbf{E}_{ji} \sim \mathbf{E}_{ji}^{(ji)}$ to denote the elasticity of origin j goods w.r.t. origin j prices, and use the $K \times (N-1)K$ matrix, $\mathbf{E}_{ji}^{(-ii)} = \left[\mathbf{E}_{ji}^{(ni)}\right]_{n \neq i}$, to summarize the elasticity of demand for origin j goods w.r.t. price of all import varieties in market i (i.e., all varieties source from any origin $n \neq i$). Important for our analysis, $\mathbf{E}_{ji}$ is an invertible matrix—the proof of which is provided in Appendix E using the primitive properties of Marshallian demand.

[10] Namely, $\tilde{P}_{ji,k} = \left(\sum_{\omega \in \Omega_{ji,k}} \varphi_{ji,k}(\omega)\tilde{p}_{ji,k}(\omega)^{1-\gamma_k}\right)^{\frac{1}{1-\gamma_k}}$ and $\tilde{P}_{i,k} = \left(\sum_{j \in \mathbb{C}} \tilde{P}_{ji,k}^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}$.

2.2 Production and Firms

Each economy $i \in \mathbb{C}$ is populated with a mass $M_{i,k} = |\Omega_{i,k}|$ of single-product firms in industry $k \in \mathbb{K}$ that compete under monopolistic competition. Labor is the only factor of production. Firm entry into industry k is either free or restricted. Under restricted entry, $M_{i,k} = \overline{M}_{i,k}$ is invariant to policy. Under free entry, a pool of ex-ante identical firms can pay an entry cost $w_i f_k^e$ to serve industry k from origin i. After paying the entry cost, each firm $\omega \in \Omega_{i,k}$ draws a productivity $z(\omega) \geq 1$ from distribution $G_{i,k}(z)$, and faces a marginal cost $\tau_{ij,k} w_i / z(\omega)$ for producing and delivering goods to destination $j \in \mathbb{C}$, where $\tau_{ij,k}$ denotes a flat iceberg transport cost. Collecting these assumptions, the *"producer"* price index of composite good ij, k (which aggregates over firm-level varieties associated with *origin i–destination j–industry k*) is

$$P_{ij,k} = \frac{\gamma_k}{\gamma_k - 1} \tau_{ij,k} \bar{a}_{i,k} w_i M_{i,k}^{-\frac{1}{\gamma_k - 1}}, \tag{4}$$

where $\bar{a}_{i,k} \equiv \left[\int_1^\infty z^{\gamma_k - 1} dG_{i,k}(z)\right]^{\frac{1}{1-\gamma_k}}$ denotes the average unit labor cost in origin i.[11] Following Kucheryavyy et al. (2016), we refer to $\frac{1}{\gamma_k - 1} = -\frac{\partial \ln P_{ij,k}}{\partial \ln M_{i,k}}$ as the industry-level ***scale elasticity***:

$$\mu_k \equiv \frac{1}{\gamma_k - 1} \sim \text{scale elasticity} \sim \text{markup}$$

Considering Equation 4, μ_k represents both (a) the constant firm-level markup in industry k (*i.e.*, $1 + \mu_k = \frac{\gamma_k}{\gamma_k - 1}$), and (b) the elasticity by which (variety-adjusted) TFP increases with industry-level employment $L_{i,k}$ (noting that $L_{i,k} \propto M_{i,k}$).[12] The equivalence between markup and scale elasticity is not a universal property, but a specific feature of our baseline Krugman model. We take advantage of this equivalence to simplify notation, but it is not essential for the theoretical results that follow. As shown in Section 5, our analytical formulas for optimal policy extend to alternative models where the scale elasticity and markup levels diverge.

Expressing Producer Prices in terms of Profit-Adjusted Wages

Our optimal policy analysis reveals a tight connection between the restricted and free entry scenarios—even though misallocation stems from markup distortions in the former scenario and scale distortions in the latter. To illustrate this connection and integrate optimal policy results under free and restricted entry, we specify producer prices as a function of profit-adjusted wage rates.[13] The idea is that net profits (if any) are rebated back to workers. The profit-adjusted wage rate in country i can be, accordingly, defined as

$$\tilde{w}_i \equiv (1 + \overline{\mu}_i) w_i \sim \text{profit-adjusted wage},$$

where $\overline{\mu}_i$ denotes economy i's average profit margin across all industries. Namely,

$$\overline{\mu}_i = \begin{cases} 0 & \text{if entry is free} \\ \frac{\sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} \frac{\mu_k}{1+\mu_k} P_{ij,k} Q_{ij,k}}{\sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} \frac{1}{1+\mu_k} P_{ij,k} Q_{ij,k}} & \text{if entry is restricted} \end{cases}. \tag{5}$$

[11] Notice that $\bar{a}_{i,k}$ is constant in our baseline model. This is no longer true in the Melitz (2003) extension of oue model explored in Section 5, in which firms incur a fixed cost to serve individual markets.

[12] With free entry and constant markups, it follows immediately that $L_{i,k} = \bar{c}_{i,k} M_{i,k}$ where $\bar{c}_{i,k}$ is a constant.

[13] We should emphasize that the profit-adjusted wage is *not* a conceptual artifact without empirical foundation. It denotes a worker's overall income, which consists of her wage supplemented by her share of national profits.

Under free entry, profits are drawn to zero, resulting in $\overline{\mu}_i = 0$. Under restricted entry, the average profit margin is positive and depends on the industrial composition of country i's output—with a higher $\overline{\mu}_i$ reflecting more sales in high-markup (high-μ) industries. Appealing to our definitions for $\tilde{w}_i$ and μ_k, we can reformulate Equation 4 to express producer prices as a function of *profit-adjusted* wages:

$$P_{ij,k} = \begin{cases} \varrho_{ij,k} \left[\sum_{j\in\mathbb{C}} \tau_{ij,k} Q_{ij,k}\right]^{-\frac{\mu_k}{1+\mu_k}} \tilde{w}_i & \text{if entry is free} \\ \varrho'_{ij,k} \frac{1+\mu_k}{1+\overline{\mu}_i} \tilde{w}_i & \text{if entry is restricted} \end{cases}. \tag{6}$$

In the above formulation, $\varrho_{ij,k} \equiv (1+\mu_k)\tau_{ij,k}\bar{a}_{i,k}^{\frac{1}{1+\mu_k}} \left(\frac{\mu_k}{f_k^e}\right)^{\frac{-\mu_k}{1+\mu_k}}$ and $\varrho'_{ij,k} \equiv \tau_{ij,k}\bar{a}_{i,k}\bar{M}_{i,k}^{-\mu_k}$ are constant price shifters; and $\sum_{j\in\mathbb{C}} \left[\tau_{ij,k}Q_{ij,k}\right]$ denotes *origin i–industry* k's gross output.[14] As we explain shortly, the above formulation of producer prices is useful for tracking the gains from policy in an open economy. The gains from *firm-delocation* channel through changes in $\sum_{j\in\mathbb{C}} \left[\tau_{ij,k}Q_{ij,k}\right]$, while the gains from *profit-shifting* channel through changes in $\overline{\mu}_i$.

2.3 The Instruments of Policy

The government in country i has is afforded a complete set of revenue-raising trade and domestic policy instruments; namely,

i. ***import tax***, $t_{ji,k}$, applied to all goods imported from origin $j \neq i$ in industry k;

ii. ***export subsidy***, $x_{ij,k}$, applied to all goods sold to market $j \neq i$ in industry k;

iii. ***industrial subsidy***, $s_{i,k}$, applied to industry k's output irrespective of where it is sold.

Our specification of policy is quite flexible as it accommodates import subsidies or export taxes ($-1 \leq t < 0$ or $-1 \leq x < 0$) as well as production taxes ($-1 \leq s < 0$). We disregard *consumption taxes* as they are redundant given the availability of the other tax instruments (see Appendix A). There is a simple intuition behind this redundancy: Country $i \in \mathbb{C}$ has access to $2(N-1)+2$ different tax instruments in each industry (where $N \equiv |\mathbb{C}|$ denotes the number of countries). These $2(N-1)+2$ tax instruments can directly manipulate $2(N-1)+1$ consumer price indexes: $N-1$ export prices, $N-1$ import prices, and one price associated with the domestically-produced and consumed variety (namely, $\tilde{P}_{ii,k}$). So, by construction, one of the $2(N-1)+2$ tax instruments in each industry is redundant. Here, we treat the industry-level consumption tax as a redundant instrument.[15]

The above tax instruments create a wedge between consumer price indexes, $\{\tilde{P}_{ji,k}\}$ and producer price indexes, $\{P_{ji,k}\}$, as follows:

$$\tilde{P}_{ji,k} = \frac{1+t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji.k}, \quad \forall j,i \in \mathbb{C},\ k \in \mathbb{K}. \tag{7}$$

[14] Under free entry, the total cost of entry must equal gross profits across all markets. I particular,

$$w_i f_k^e M_{i,k} = \sum_{j\in\mathbb{C}} \left[\frac{\mu_k}{1+\mu_k} P_{ij,k}Q_{ij,k}\right] \qquad \text{(Free Entry Condition)}.$$

Replacing $P_{ij,k}$ in the above equation with 4 yields $M_{i,k} = \left(\frac{\mu_k}{f_k^e}\sum_j \left[\bar{a}_{ij,k}Q_{ij,k}\right]\right)^{\frac{1}{1+\mu_k}}$. Equation 6, then, follows from plugging the expression for $M_{i,k}$ back into Equation 4.

[15] With more than two countries ($N > 2$), Country i has access to $2(N-1)+2$ instruments per industry. These instruments can manipulate $2(N-1)+1$ price variables, which implies the same redundancy.

These tax instruments also generate/exhaust revenue for the tax-imposing country. The combination of all taxes imposed by country $i \in \mathbb{C}$ produce a tax revenue equal to

$$\mathcal{R}_i = \overbrace{\sum_{k\in\mathbb{K}} \left(\left(\frac{1}{1+s_{i,k}} - 1 \right) P_{ii,k} Q_{ii,k} \right)}^{\text{industrial subsidies}} \\ + \underbrace{\sum_{k\in\mathbb{K}} \sum_{j\neq i} \left(\frac{t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji,k} Q_{ji,k} + \left[\frac{1}{(1+x_{ij,k})(1+s_{i,k})} - 1 \right] P_{ij,k} Q_{ij,k} \right)}_{\text{import taxes + export subsidies}}. \tag{8}$$

Tax revenues are rebated to the consumers in a lump-sum fashion. After we account for tax revenues, total income in country i equals the sum of profit-adjusted wage payments, $\bar{w}_i L_i = (1+\overline{\mu}_i) w_i L_i$, and tax revenues. Namely, $Y_i = \bar{w}_i L_i + \mathcal{R}_i$, where $\mathcal{R}_i$ can be positive or negative depending on whether country i's policy consists of net taxes or subsidies.

2.4 General Equilibrium

For convenience, we refer to *profit-adjusted* wages as just wages going forward, using $\mathbf{w} \equiv \{\bar{w}_i\}$ to denote the global vector of wages. We also assume throughout the paper that the underlying parameters of the model are such that the necessary and sufficient conditions for the uniqueness of equilibrium are satisfied.[16] To present our theory, we express all equilibrium outcomes—except for wages—as a function of global taxes ($\mathbf{x}$, $\mathbf{t}$, and $\mathbf{s}$), treating wages $\mathbf{w}$ as given. As detailed in Appendix E, this formulation derives from solving a system that imposes all equilibrium conditions aside from the labor market clearing conditions. For future reference, we outline this formulation of equilibrium variables below.

Notation. For a given vector of taxes and wages $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$, equilibrium outcomes $Y_i(\mathbf{T})$, $P_{ji,k}(\mathbf{T})$, $\tilde{P}_{ji,k}(\mathbf{T})$, $Q_{ji,k}(\mathbf{T})$ are determined such that (i) producer prices are characterized by 6; (ii) consumer prices are given by 7; (iii) industry-level consumption choices are a solution to 1 with demand for national-level varieties, $Q_{ji,k}$, given by 2; and (iv) total income (which dictates total expenditure by country i) equals profit-adjusted wage payments plus tax revenues:

$$Y_i(\mathbf{T}) = \bar{w}_i L_i + \mathcal{R}_i(\mathbf{T}),$$

where tax revenues $\mathcal{R}_i(\boldsymbol{T})$ are described by Equation 8.

Considering the above formulation of equilibrium variables, welfare, too, can be expressed as a function of taxes and wages as follows:

$$W_i\left(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}\right) \equiv V_i\left(Y_i(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}), \tilde{\mathbf{P}}_i(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})\right).$$

Note that $\mathbf{w}$ is itself an equilibrium outcome. So, a vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ is feasible insofar as $\mathbf{w}$ is the equilibrium wage consistent with $\mathbf{t}$, $\mathbf{x}$, and $\mathbf{s}$. Related to this point, our goal in this paper is to study problems where the government in i choses $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ to maximize $W_i(\mathbf{T})$ subject to feasibility. So, to fix ideas, we define the set of feasible *policy–wage* vectors below.

Definition (D2). The set of feasible policy–wage vectors, $\mathbb{F}$, consists of any vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ where

[16] Following Kucheryavyy et al. (2016), this assumption holds in the two country case if $\gamma_k \geq \sigma_k$ and holds otherwise if trade costs are sufficiently small.

$\mathbf{w}$ *satisfies the labor market clearing condition in every country, given* $\mathbf{t}$*,* $\mathbf{x}$*, and* $\mathbf{s}$*:*

$$\mathbb{F} = \left\{ \mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}) \mid \bar{w}_i L_i = \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} \left[P_{ij,k}(\mathbf{T}) Q_{ij,k}(\mathbf{T}) \right]; \quad \forall i \in \mathbb{C} \right\}.$$

There is a basic reason for why we formulate equilibrium outcomes as a function of $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ instead of just $(\mathbf{t}, \mathbf{x}, \mathbf{s})$. This choice of formulation allows us to articulate an important intermediate result regarding tax neutrality. This result, which is stated below, simplifies our theoretical derivation of optimal policy to a great degree.

Lemma 1. [Tax Neutrality] *For any* a *and* $\tilde{a} \in \mathbb{R}_+$ *(i) if* $\mathbf{T} = (\mathbf{1} + \mathbf{t}_i, \mathbf{t}_{-i}, \mathbf{1} + \mathbf{x}_i, \mathbf{x}_{-i}, \mathbf{1} + \mathbf{s}_i, \mathbf{s}_{-i}; \bar{w}_i, \mathbf{w}_{-i}) \in \mathbb{F}$*, then* $\mathbf{T}' = (a(\mathbf{1} + \mathbf{t}_i), \mathbf{t}_{-i}, a(\mathbf{1} + \mathbf{x}_i), \mathbf{x}_{-i}, \frac{1}{\tilde{a}}(\mathbf{1} + \mathbf{s}_i), \mathbf{s}_{-i}; \frac{a}{\tilde{a}} \bar{w}_i, \mathbf{w}_{-i}) \in \mathbb{F}$*. Moreover, (ii) welfare is preserved under* $\mathbf{T}$ *and* $\mathbf{T}'$*:* $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ *for all* $n \in \mathbb{C}$*.*

The above lemma is proven in Appendix B, and connects two fundamental tax neutrality principles: The Lerner symmetry (Lerner (1936); Costinot and Werning (2019)) and the welfare-neutrality of uniform subsidies or markups (Lerner (1934); Samuelson (1948)). Importantly, Lemma 1 implies that there are multiple optimal tax combinations for each country i, which simplifies our forthcoming task of characterizing optimal policy. To give some detail: The contribution of general equilibrium wage and income effects to the optimal tax schedule is often summarized by aggregate terms that are industry-blind. The neutrality established by Lemma 1, simplifies the task of handling of these terms to a great degree.

Table 1: *Summary of Key Variables*

Variable	Description
$\tilde{P}_{ji,k}$	Consumer price index (origin j–destination i–industry k)
$P_{ji,k}$	Producer price index (origin j–destination i–industry k)
Y_i	Total income in country i
$\mathcal{R}_i$	Total tax revenue in country i (Equation 8)
w_i and $\bar{w}_i$	pure and profit-adjusted wage rates in country i: $\bar{w}_i = (1 + \overline{\mu}_i) w_i$
$x_{ji,k}$	Export subsidy applied to good ji, k (if $j \neq i$)
$t_{ji,k}$	Import tax applied on good ji, k (if $j \neq i$)
$s_{i,k}$	Industrial subsidy applied to all goods from *origin i–industry k*
$\lambda_{ji,k}$	Within-industry expenditure share (good ji, k): $\tilde{P}_{ji,k} Q_{ji,k} / \sum_{\jmath} \tilde{P}_{\jmath i,k} Q_{\jmath i,k}$
$r_{ji,k}$	Within-industry sales share (good ji, k): $P_{ji,k} Q_{ji,k} / \sum_{\iota} P_{j\iota,k} Q_{j\iota,k}$
$e_{i,k}$	Industry-level expenditure share (destination i–industry k)
$\rho_{i,k}$	Industry-level sales share (origin i–industry k)
μ_k	industry-level markup ~ industry-level scale elasticity
$\overline{\mu}_i$	Average profit margin in origin i (Equation 5)
σ_k	Cross-national CES parameter ~ (1 + trade elasticity)
$\varepsilon_{ji,k}^{(ni,g)}$	Elasticity of demand for good ji, k w.r.t. the price of ni, g
$\omega_{ji,k}$	Inverse of good ji, k's supply elasticity (Equation 27)

3 Sufficient Statistics Formulas for Optimal Policy

This section derives sufficient statistics formulas for optimal trade and industrial policies. These formulas are later employed to quantify the ex-ante gains from policy among many countries. Before proceeding to the derivation, let us highlight the two rationales for policy intervention in our setup. A non-cooperative, welfare-maximizing government seeks to (i) restrict trade and reap unexploited *terms-of-trade (ToT)* gains vis-à-vis the rest of the world, and (ii) correct misallocation in the domestic economy. Misallocation, notice, stems from the cross-industry heterogeneity in markups or scale elasticities, leading to inefficiently low output in high-profit or high-returns-to-scale (high-μ) industries. A crucial difference between these policy objectives is that ToT manipulation is inefficient from a global standpoint—as it disrupts allocative efficiency to transfer surplus from the rest of the world to the tax-imposing country.

3.1 Efficient Policy from a Global Standpoint

As a useful benchmark, we first characterize the efficient policy from a global standpoint. Efficient policies, by definition, are the solution to a central planner's problem that maximizes global welfare via taxes and lump-sum international transfers. Let δ_i denote the Pareto weight assigned to country i in the planner's objective function. The globally efficient policy solves the following planning problem *subject to* the availability of lump-sum transfers:

$$\max_{(\mathbf{t},\mathbf{x},\mathbf{s};\mathbf{w})\in\mathbb{F}} \quad \sum_{i\in\mathbb{C}} \delta_i \log W_i\left(\mathbf{t},\mathbf{x},\mathbf{s};\mathbf{w}\right).$$

Keep in mind that the above problem affords the planner enough instruments to obtain their first-best. Good-specific taxes allow the planner to restore allocative efficiency, while lump-sum transfers allow her to redistribute inter-nationally based on the Preto weights, δ_i. This point is expanded on in Appendix F, were it is shown that the efficient tax policy involves *zero* trade taxes and Pigouvian subsidies that restore marginal-cost-pricing globally:[17]

$$t^{\star}_{ji,k} = x^{\star}_{ji,k} = 0 \quad \forall ji,k; \qquad 1+s^{\star}_{i,k} = 1+\mu_k \quad \forall i,k \tag{9}$$

The above characterization applies to both the free and restricted entry cases—with the understanding that μ_k assumes different interpretations in each case. Appealing to this result, Appendix E establishes a basic point about international cooperation: Welfare-maximizing governments will settle on the efficient policy only if they are unable to influence consumer/producer prices in the rest of the world. Otherwise, they will defect to take advantage of terms-of-trade (ToT) gains. This result indicates that the pursuit of ToT gains is the sole reason welfare-maximizing governments deviate from efficient policy choices—at least when they are afforded sufficient policy instruments.[18] This result echos the argument in Bagwell and Staiger (2001, 2004), generalizing it to settings with many countries and differentiated industries.

[17] To be specific, the implementation of the efficient allocation involves the above taxes plus lump-sum international transfers based on Pareto weights. The logic is that the planner maximizes global output by restoring marginal-cost pricing and redistributes the corresponding income gains between countries via efficient transfers. Absent transfers, implementing $\{\mathbf{t}^{\star},\mathbf{x}^{\star},\mathbf{s}^{\star}\}$ would deliver a Kaldor-Hicks improvement (Kaldor (1939); Hicks (1939)), but not necessarily a Pareto improvement relative to Laissez-faire. Though, the resulting policy equilibrium would constitute a point on the Pareto-efficient frontier—see Appendix F for more details.

[18] This need not be true if government are prohibited from using domestic taxes and afforded only import tax

In the next section, we characterize the unilaterally optimal policy of non-cooperative governments. This exercise elucidates two issues. First, it determines how governments deviate from the cooperative policy choice when ToT considerations are taken into account. Second, it clarifies how governments approach industrial policy when they view 1st-best Pigouvian subsidies as politically-infeasible. Once we settle these two issues, we argue that the implementation of globally efficient policies requires both a *"shallow agreement"* to discipline trade policy choices and a *"deep agreement"* to coordinate industrial policy implementation.

3.2 Unilaterally Optimal Policy Choices

First-Best: Unilaterally Optimal Trade and Domestic Policies

We now characterize a non-cooperative country's *unilaterally* optimal policy. We consider cases where a non-cooperative country $i \in \mathbb{C}$ selects taxes, $\mathbf{t}_i \equiv \{t_{ji,k}\}$, $\mathbf{x}_i \equiv \{x_{ij,k}\}$, and $\mathbf{s}_i \equiv \{s_{i,k}\}$, taking policy choices elsewhere as given. Countries in the rest of the world are passive in their use of taxes but actively maintains internal cooperation—i.e., countries other than i preserve the balance of market access concessions among themselves. To elaborate: country i's policy could, in principle, disrupt the balance of concessions in the rest of the world, leading to a deterioration of one cooperative country's ToT relative to another. Cooperation *within* the rest of the world requires that these extraterritorial ToT effects be neutralized via buffers that preserve w_n/w_j for all $n, j \neq i$—see Appendix G for further details.

We begin with the unilaterally *first-best* case where the government in i is afforded all possible tax instruments. The first-best unilaterally optimal policy solves the following problem:[19]

$$\max_{(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w})} \quad W_i\left(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}\right) \qquad s.t.\ \left(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}\right) \in \mathbb{F} \qquad \text{(P1)}.$$

We analytically solve Problem (P1) under both the restricted and free entry cases. We perceive the restricted entry case to be a more appropriate benchmark if governments are concerned with ***short-run*** gains from policy. The free entry case, on the other hand, is more relevant if governments are concerned with ***long-run*** gains. These two cases exhibit an important difference: Producer prices respond differently to contractions in export supply under restricted and free entry—as we elaborate next.

Conditional Export Supply Elasticity. The terms-of-trade gains from policy, in our framework, channel through changes in the price of imported and exported goods. The government in $i \in \mathbb{C}$ cannot directly dictate the *producer* price of say good, ji, k, that is imported from origin $j \neq i$. Instead, it can deflate its producer price ($P_{ji,k}$) *indirectly* by contracting or expanding its export supply ($Q_{ji,k}$). The contraction in $Q_{ji,k}$ also affects the producer price of goods supplied by other locations through general equilibrium linkages. Our theory indicates that, for optimal policy analysis, the ***conditional inverse export supply elasticity*** is sufficient to track these effects. To present this elasticity, let $\tilde{\mathbb{P}}_i$ contain the consumer price of all goods either produced by or consumed in country i. These are prices that country i's government can fully control via

instruments. Following Venables (1987) and Ossa (2011), welfare-maximizing governments will erect tariffs in that case, even if they perceive world prices as invariant to their policy choice. By doing so, they improve allocative efficiency in the domestic economy but impose a negative firm-delocation externality on the rest of the world.

[19] Given that the rest of the world is passive in their use of taxes (i.e.,$\mathbf{t}_{-i} = \mathbf{x}_{-i} = \mathbf{s}_{-i} = \mathbf{0}$), we condense the notation by specifying equilibrium variables as a function of only $(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i, \mathbf{w})$.

taxes. We define the *conditional* inverse export supply elasticity of good ji,k as

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_{g\in\mathbb{K}} \left[\frac{\tilde{w}_i L_i}{\tilde{w}_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} + \sum_{n\neq i} \frac{\tilde{w}_n L_n}{\tilde{w}_j L_j} r_{ni,g}\rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} \right],$$

where $r_{ni,g} \equiv \frac{P_{ni,g}Q_{ni,g}}{\sum_\iota P_{n\iota,g}Q_{n\iota,g}}$ and $\rho_{n,g} \equiv \frac{\sum_\iota P_{n\iota,g}Q_{\iota,g}}{\sum_{\iota,s} P_{n\iota,s}Q_{\iota,s}}$ respectively denote the *good-specific* and *industry-wide* sales shares associated with origin n. Notice, $\omega_{ji,k}$ is a conditional elasticity that describes how the *producer* prices linked to economy i respond to a change in $Q_{ji,k}$, holding $\tilde{\mathbb{P}}_i$ and the entire vector of wage and income levels constant.[20] This elasticity encapsulates different economic forces under free and restricted entry, as we detail next.

Under restricted entry, producer prices from origin $j \in \mathbb{C}$ are fully determined by the (profit-adjusted) wage rate, $\tilde{w}_j$, and the aggregate profit margin, $\overline{\mu}_j$ (see Equation 6). Policy, thus, has two distinct effects on producer prices under restricted entry: One effect that channels through wages, $\mathbf{w}$; and another that channels through aggregate profit margins. To explain the latter, hold $\mathbf{w}$ constant: contracting the export supply of good ji,k with taxes will alter all producer prices associated with origin j through a change in origin j's aggregate profit margin, $\overline{\mu}_j$. The change in $\overline{\mu}_j$ derives from the fact that industries have differential markup margins, and that taxing good ji,k alters the industrial composition of output in origin $j \in \mathbb{C}$.

Under free entry, producer prices from origin $j \in \mathbb{C}$ are determined by the wage rate, $\tilde{w}_j$, and the *origin j–industry k*-specific scale of production. So, aside from wage-related effects, policy has a second effect on producer prices that channels through industry-level scale economies. To elaborate, consider an import tax on good ji,k (origin j–destination i–industry k). Such a tax contracts the supply of ji,k and the scale of production in *origin j–industry k*. Given Equation 6, this contraction in scale increases the entire vector of producer price indexes associated with *origin j–industry k*—all through additional firm entry.

In both cases, $\omega_{ji,k}$ describes how expanding or contracting good ji,k's export supply impacts country i's terms-of-trade via either profit-shifting or industry-level scale economies. Importantly, $\omega_{ji,k}$ can be characterized (to a first-order approximation) as a simple function of sales shares, scale elasticities, and Marshallian demand elasticities (see Appendix E):[21]

$$\omega_{ji,k} \approx \begin{cases} \dfrac{-\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1-\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}} \left[1 - \frac{\mu_k}{1+\mu_k}\frac{w_i L_i}{w_j L_j} \sum_{n\neq i} \frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{jn,k}} \varepsilon^{(jn,k)}_{in,k} \right] & \text{if entry is free} \\ \dfrac{\left(1-\frac{1+\overline{\mu}_j}{1+\mu_k}\right)\sum_g r_{ji,g}\rho_{j,g}}{1+\sum_g \sum_{\iota\neq i}\left[1+\left(1-\frac{1+\overline{\mu}_j}{1+\mu_g}\right) r_{j\iota,g}\rho_{j,g}\varepsilon_{j\iota,g}\right]} & \text{if entry is restricted} \end{cases}. \tag{10}$$

The above formulation for $\omega_{ji,k}$ is quite intuitive: Under restricted entry, $\omega_{ji,k}$ governs the relationship between export supply and the average markup paid on imports. Accordingly, $\omega_{ji,k}$ is non-zero only when industries exhibit differential markup levels. Otherwise, $\omega_{ji,k}$ collapses

[20] The conditional elasticity, $\omega_{ji,k}$, is strictly more distilled than the traditional notion of export supply elasticity (e.g., Dixit (1985)). It is, in particular, purged from general equilibrium wage-and-income effects, which we later show are redundant in the neighborhood of the optimum policy. This crucial feature makes our optimal policy formulas (that encapsulate $\omega_{ji,k}$) amenable to quantitative analysis—see Section 7 for details.

[21] The above approximation derives from Wu, Yin, Vosoughi, Studer, Cavallaro, and Dick's (2013) first-order approximated inverse of a diagonally-dominant matrix. Figure (4) in Appendix E illustrates the precision of this approximation. The same appendix also presents an exact (approximation-free) formulation for $\omega_{ji,k}$.

to zero as the average markup (or profit margin) paid on imports is constant and invariant to changes in export supply, *i.e.*, $\overline{\mu}_j = \mu_k = \mu \Longrightarrow \omega_{ji,k} = 0$. Under free entry, $\omega_{ji,k}$ regulates the terms-of-trade gains from policy that channel through scale economies. Accordingly, in the limit where industries operate based on constant-returns to scale, $\omega_{ji,k}$ once again collapses to zero—namely, $\lim_{\mu_k \to 0} \omega_{ji,k} = 0$.

Three-Step Dual Approach to Characterizing Optimal Policy. Our characterization of optimal policy employs the dual approach and is presented in Appendix E. Below, we provide a verbal summary of our approach, which involves three main steps.

First, we simplify Problem (P1) by reformulating it into a problem where country i's government chooses the vector of prices $\mathbb{P}_i = \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$ associated with its own economy. Country i's optimal tax/subsidy schedule $\mathbb{T}_i^* \equiv (\mathbf{t}_i^*, \mathbf{x}_i^*, \mathbf{s}_i^*)$ is then recovered as the wedge between the optimal price vector $\mathbb{P}_i^*$and producer prices.

Second, we derive the first-order conditions (F.O.C.) associated with country i's reformulated optimal policy problem. We use two technical tricks to overcome the complications related to general equilibrium analysis: First, we use the envelope conditions associated with optimal demand choices to net out redundant behavioral responses. Second, we identify additional welfare neutrality conditions specific to Problem (P1). Most importantly, we observe that terms in the F.O.C.s that account for general equilibrium wage and income effects are redundant in the neighborhood of the optimum. That is, we could specify the F.O.C.s associated with (P1) *as if* wages were constant and Marshallian demand functions were income-inelastic.[22]

Third, we combine the F.O.C.s and solve them as part of one system. In this process, we appeal to the tax neutrality result specified by Lemma 1 to eliminate redundant tax shifters, which are difficult to characterize. We then appeal to well-known properties of Marshallian demand functions (e.g., *Cournot aggregation* and *homogeneity of degree zero*) to establish that our system of F.O.C.s admits a unique solution.[23] Together, these steps lead us to simple sufficient statistics formulas for unilaterally optimal policies, as summarized by the following theorem.[24]

Theorem 1. *Country i's optimal policy is unique up to two uniform tax shifters* $1+\bar{s}_i$ *and* $1+\bar{t}_i \in \mathbb{R}_+$, *and is implicitly given by*

$$\begin{aligned}
&[\text{domestic subsidy}] && 1+s^*_{i,k} = (1+\mu_k)(1+\bar{s}_i)\\
&[\text{import tax}] && 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i)\\
&[\text{export subsidy}] && \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right);
\end{aligned}$$

where $\omega_{ji,k}$ *denotes the good ji, k's inverse supply elasticity as given by Equation 10, while* $\mathbf{E}_{ij} \sim \mathbf{E}_{ij}^{(ij)}$ *and* $\mathbf{E}_{ij}^{(-ij)}$ *denote matrixes of Marshallian demand elasticities as defined under (D1).*[25]

[22] Farrokhi and Lashkaripour (2021) streamline the dual approach developed in this paper, extending our result about the welfare neutrality of wages and income effects to settings with arbitrary inter-national externalities.

[23] We should perhaps emphasize an important nuance. It is possible that our model admits multiple optimal policy *equilibria*. Yet the optimal policy *formulas* are uniquely specified by Theorem 1 in each case.

[24] We later combine the formulas specified by Theorem 1 with micro-estimated parameter values to quantify the *ex-ante* gains from policy. In Appendix H, we test the accuracy and speed of our formulas by performing 150 numerical simulations in which the underlying model parameters are repeatedly sampled from a uniform distribution. The theoretical policy predictions are then compared to those obtained from numerical optimization.

[25] To be clear, $\mathbf{E}_{ij}^{(-ij)} = \left[\mathbf{E}_{ij}^{(nj)}\right]_{n\neq i}$ is a $K \times (N-1)K$ matrix and $\mathbf{1} \equiv \mathbf{1}_{(N-1)K\times 1}$ is a column vector of *ones*. Also,

The uniform tax shifters, $\bar{s}_i$, and $\bar{t}_i$ account for the multiplicity of optimal policy equilibria (as indicated by Lemma 1). These shifters can be assigned any arbitrary value, provided that $1+\bar{s}_i$ and $1+\bar{t}_i \in \mathbb{R}_+$. For instance, if we assign a sufficiently high value to $\bar{t}_i$ and $\bar{s}_i$, the optimal policy will involve import tariffs, export subsidies, and industrial subsidies. Conversely, if we assign a sufficiently low value to $\bar{t}_i$ and $\bar{s}_i$, the optimal policy will involve import subsidies, export taxes, and industrial production taxes.

Intuition Behind Optimal Tax Formulas. Theorem 1 states that country i's unilaterally optimal policy consists of (1) Pigouvian subsidies that restore marginal cost pricing in economy i; (2) import taxes/subsidies that exploit country i's collective import market power, delivering an optimal mark-down on the producer price of imported goods $P_{ji,k}$; and (3) export taxes/subsidies that exploit country i's collective export market power, charging the optimal national-level mark-up on the consumer price of exported goods $\tilde{P}_{ij,k}$.[26]

When gauging the scope of Theorem 1, note that Marshallian demand elasticities, $\varepsilon^{(ni,g)}_{ji,k}$, are fully-determined by expenditure shares, $\lambda_{ji,k}$, and σ_k. Likewise, the export supply elasticity, $\omega_{ji,k}$, is fully-determined by sales shares, $r_{ji,k}$, scale elasticities, μ_k, and Marshallian demand elasticities. As such, Theorem 1 characterizes optimal policy in terms of three sets of *sufficient statistics*: (i) observable shares, $r_{ij,k}$, and $\lambda_{ij,k}$, (ii) industry-level trade elasticities, $\sigma_k - 1$, and (iii) industry-level scale elasticities, μ_k. This particular feature of Theorem 1 greatly simplifies our quantitative analysis of optimal policy in Section 7.

A canonical special case of Theorem 1 is the multi-industry Armington case, in which $\mu_k = 0$ for all $k \in \mathbb{K}$. Under this special case, $\omega_{ji,k} = 0$ for all ji,k and optimal import tariffs are uniform, i.e., $t^*_{ji,k} = \bar{t}_i$ for all ji,k. This result can be understood as follows: Absent scale economies or profits, import tariffs cannot impact the producer price of imported goods on a good-by-good basis. At best, import taxes can induce a uniform reduction in import prices (per origin j) by deflating $\mathbf{w}_{-i}$ relative to w_i. This uniform reduction, though, can be perfectly mimicked with a uniform increase in export taxes per destination j. As such, optimal import taxes are either uniform or redundant by choice of $\bar{t}_i = 0$.

Theorem 1 has two additional implications worth highlighting. First, optimal tariffs and export subsidies are *misallocation-blind* but not necessarily blind to the overall magnitude of scale economies. To elaborate, holding the average scale elasticity constant but increasing the cross-industry dispersion in scale elasticities (i.e., the extent of misallocation) preserves $\mathbf{t}^*_{ji}$ and $\mathbf{x}^*_{ij}$. The intuition is that any gains from correcting misallocation from scale elasticity dispersion are fully-internalized by domestic subsidies. However, raising the average scale elasticity can modify $\mathbf{t}^*_{ji}$ and $\mathbf{x}^*_{ij}$ irrespective of the underlying degree of misallocation.[27]

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in the general case with asymmetric income elasticities of demand, $\mathbf{E}_{ij}$ should be replace with $\tilde{\mathbf{E}}_{ij} \equiv [\frac{e_{ij,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{ij,g}]_{g,k}$. Otherwise , the symmetry of the Slutsky matrix implies that $\frac{e_{ij,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{ij,g} = \varepsilon^{(ij,g)}_{ij,k}$, which implies that $\mathbf{E}_{ij} = \tilde{\mathbf{E}}_{ij}$.

[26] Our formula for $t^*_{ji,k}$ echoes the traditional optimal–tariffs–equal–foreign's–inverse–export–supply–elasticity result. Our formula, though, derives from a general equilibrium multi-country model, whereas traditional theories are often limited to partial equilibrium two-country settings. Another distinction is that $\omega_{jn,k}$ in our model is associated with a *backward falling* supply curve due to increasing-returns to scale. In traditional theories, $\omega_{jn,k}$ is associated with an upward-sloping supply curve resulting from quasi-fixed inputs. Our framework is, thus, consistent with a possibly negative $\omega_{jn,k}$, which conforms to recent evidence in Farrokhi and Soderbery (2020).

[27] Following the literature, we define misallocation as the log welfare distance to the efficient frontier ($\mathcal{L}_i$). In a

Second, the optimal export tax-cum-subsidy, $x^*_{ij,k}$, depends on the entire matrix of own- and cross-price demand elasticities associated with good ij,k. The explanation is that $x^*_{ij,k}$ (in Theorem 1) corresponds to the optimal markup of a multi-product monopolist. To better understand this point, assign $\bar{t}_i = 0$, in which case $x^*_{ij,k}$ represents a tax on good ij,k (rather than a subsidy). The optimal tax rate on ij,k is equal to the optimal mark-up on that good if country i's government was pricing its exports as a multi-product monopolist rather than an individual single-product firm. The government's optimal pricing decision, accordingly, internalizes the effect of raising $\tilde{P}_{ij,k}$ on its sales of other products in destination j.

Special Case with Cobb-Douglas Preferences. To gain deeper intuition about Theorem 1, consider a special case where preferences are Cobb-Douglas across industries. In that case, the formulas specified by Theorem 1 reduce to[28]

$$\begin{aligned}
&[\textit{domestic subsidy}] && 1+s^*_{i,k} = (1+\mu_k)(1+\bar{s}_i) \\
&[\textit{import tax}] && 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i) \\
&[\textit{export subsidy}] && 1+x^*_{ij,k} = \frac{(\sigma_k-1)\sum_{n\neq i}\left[(1+\omega_{ni,k})\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\lambda_{ij,k})}(1+\bar{t}_i),
\end{aligned} \tag{11}$$

A well-known special case of the above formula is the *single-industry*$\times$*two-country* formula in Gros (1987). To demonstrate this, drop the industry subscript k and reduce the global economy into two countries, i.e., $\mathbb{C} = \{i,j\}$. Noting that $1-\lambda_{ij} = \lambda_{jj}$ in the two-country case, we can deduce from the above formulas that

$$\frac{1+t^*_{ji}}{1+x^*_{ij}} = 1 + \frac{1}{(\sigma-1)\lambda_{jj}}.$$

By the Lerner symmetry, export and import taxes are equivalent in the single-industry model.[29] Hence, without loss of generality, we can set $x^*_{ij} = 0$ and arrive at the familiar-looking optimal tariff formula in Gros (1987), i.e., $t^*_{ji} = 1/(\sigma-1)\lambda_{jj}$.

The Cobb-Douglas case of Theorem 1 is also a strict generalization of the formula derived concurrently by Bartelme et al. (2019) for a small open economy with multiple sectors. Specifically, enforcing the ***small open economy*** assumption—i.e., setting $\omega_{ji,k} \approx \lambda_{ij,k} \approx 0$; $\lambda_{jj,k} \approx 1$—our optimal policy formulas in the Cobb-Douglas case reduce to:

$$1+s^*_{i,k} = 1+\mu_k; \qquad t^*_{ji,k} = 0; \qquad 1+x^*_{ij,k} = \frac{\sigma_k-1}{\sigma_k}. \tag{12}$$

closed economy with Cobb-Douglas-CES preferences, $\mathcal{L}_i = \mathbb{E}_{\rho_i}[\mu \log \mu] - \mathbb{E}_{\rho_i}[\mu]\log \mathbb{E}_{\rho_i}[\mu]$, where $\mathbb{E}_{\rho_i}[.]$ denotes the cross-industry mean weighted by the industry-level employment shares, $\rho_{i,k}$. If the scale elasticity is strictly positive but uniform across industries, then $\mathcal{L}_i = 0$. Our claim is that preserving the average scale elasticity, $\mathbb{E}_{\rho_i}[\mu]$, but raising the coefficient of variation, $\mathrm{CV}_{\rho_i}[\mu]$, intensifies misallocation but preserves $\mathbf{t}^*_{ji}$ and $\mathbf{x}^*_{ij}$.

[28] In the Cobb-Douglas case: (a) $\varepsilon^{(ij,k)}_{nj,k} = -\sigma_k \mathbb{1}_{n=j} + (\sigma_k-1)\lambda_{ij,k}$ and (b) $\varepsilon^{(ij,k)}_{nj,g} = 0$ if $g \neq k$. Plugging the expression for $\varepsilon_{ji,k}$ into Equation 6, the inverse of the export supply under restricted entry is given by

$$\omega_{ji,k} = \frac{\left(1-\frac{\bar{\mu}_j}{\mu_k}\right)\sum_g r_{ji,g}\rho_{j,g}}{1+\sum_{\iota\neq i}\left[1-\left(1-\frac{\bar{\mu}_j}{\mu_k}\right)r_{j\iota,k}\left(1+(\sigma_k-1)(1-\lambda_{j\iota,k})\right)\right]}.$$

The parameterization of $\omega_{ji,k}$ under free entry can be derived in a similar fashion.

[29] The Lerner symmetry is a special case of the equivalence result presented under Lemma 1. Also, note that the market equilibrium is efficient in the single industry Krugman model studied by Gros (1987). As such, the optimal industrial subsidy can be also normalized to zero, i.e., $s^*_i = 0$.

Second-Best: Unilaterally Optimal Import Tariffs and Export Subsidies

Suppose the government in $i \in \mathbb{C}$ cannot use domestic subsidies due to say institutional barriers or political pressures. It is optimal, in that case, to use trade taxes as a second-best policy to restore allocative efficiency in the domestic economy. In this section, we derive analytic formulas for *second-best* optimal trade taxes in such circumstances. Country i's optimal policy problem, in this case, includes an added constraint that $\mathbf{s}_i = \mathbf{0}$:

$$\max_{(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w})} \quad W_i\left(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}\right) \quad s.t. \begin{cases} (\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}) \in \mathbb{F} \\ \mathbf{s}_i = \mathbf{0} \end{cases} \qquad \text{(P2)}.$$

Using the dual approach discussed earlier, we analytically solve Problem (P2) and derive sufficient statistics formulas for second-best optimal trade taxes. The following theorem presents these formulas, with a formal proof provided in Appendix I.

Theorem 2. *Suppose industrial subsidies $\mathbf{s}_i$ are unavailable to the government: Second-best optimal trade taxes are unique up to a uniform tax shifter $\bar{t} \in \mathbb{R}_+$ and are implicitly given by:*

$$[\text{import tariff}] \qquad \mathbf{1} + \mathbf{t}_{ji}^{**} = (1 + \bar{t}_i)\left(\mathbf{1} + \mathbf{\Omega}_{ji}\right) \oslash \left(\mathbf{1} + \mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1 - \frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right)$$

$$[\text{export subsidy}] \qquad \mathbf{1} + \mathbf{x}_{ij}^{**} = -(1 + \bar{t}_i)\left(\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1} + \mathbf{\Omega}_{-ii}\right)\right) \odot \left[\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k,$$

where $\mathbf{\Omega}_{ji} = \left[\omega_{ji,k}\right]_k$ is a vector of inverse export supply elasticities (Equation 10); $\bar{\mu}_i$ denotes the output-weighted average markup in economy i (Equation 5); and $\mathbf{E}_{-ii}$, $\mathbf{E}_{-ii}^{(ii)}$, $\mathbf{E}_{ij}$, and $\mathbf{E}_{ij}^{(-ij)}$ are matrixes of Marshallian demand elasticities as defined under Definition (D1).[30]

Theorem 2 asserts that, when governments cannot use industrial subsidies, (i) the optimal export subsidy is adjusted to promote exports in high-returns-to-scale (high-μ) industries, and (ii) the optimal import tax is adjusted to restrict import competition in high-returns-to-scale (high-μ) industries. Intuitively, the government's objective when solving (P2) is to mimic Pigouvian industrial subsidies with trade taxes/subsidies. To reach this objective, import taxes and export subsidies should increase in high-returns-to-scale industries relative to the *first-best* benchmark. While these adjustments elevate domestic production in high-μ industries, they are insufficient for obtaining the unilaterally first-best allocation.

Special Case with Cobb-Douglas Preferences. We can invoke the Cobb-Douglas assumption to further elucidate the second-best tax formulas under Theorem 2. Under this assumption, there are zero cross-demand effects between industries and the optimal policy formulas specified by Theorem 2 can be simplified as follows:

[30]Letting N and K denote the number of countries and industries: $\mathbf{E}_{-ii} \sim \mathbf{E}_{-ii}^{(-ii)} = \left[\mathbf{E}_{ni}^{(ji)}\right]_{n\neq i, j\neq i}$ is a square $(N-1)K \times (N-1)K$ matrix, where $\mathbf{E}_{ni}^{(ji)} \equiv \left[\varepsilon_{ni,k}^{(ji,g)}\right]_{k,g}$ as defined under Definition (D1). Likewise, $\mathbf{E}_{ij}^{(-ij)} = \left[\mathbf{E}_{ij}^{(nj)}\right]_{n\neq i}$ and $\boldsymbol{E}_{-ii}^{(ii)} = \left[\boldsymbol{E}_{ni}^{(ii)}\right]_{n\neq i}$ are respectively $K \times (N-1)K$ and $(N-1)K \times K$ matrixes. In all the equations, $\mathbf{1} \equiv \mathbf{1}_{(N-1)K\times 1}$ is a columns vector of ones. Meanwhile, $\mathbf{\Omega}_{-ii} = \left[\omega_{ni,k}\right]_{n\neq i,k}$ is a $(N-1)K \times 1$ vector; and the operators $\odot$ and $\oslash$ denote element-wise multiplication and division.

$$\text{[import tariff]} \qquad 1+t^{**}_{ji,k} = \frac{1+(\sigma_k-1)\,\lambda_{ii,k}}{1+\frac{1+\overline{\mu}_i}{1+\mu_k}\,(\sigma_k-1)\,\lambda_{ii,k}}\,\left(1+t^{*}_{ji,k}\right)$$

$$\text{[export subsidy]} \qquad 1+x^{**}_{ij,k} = \frac{1+\mu_k}{1+\overline{\mu}_i}\,\left(1+x^{*}_{ij,k}\right),$$

where $1+t^{*}_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i)$ and $1+x^{*}_{ji,k} = \frac{(\sigma_k-1)\sum_{n\neq i}\left[(1+\omega_{ni,k})\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\lambda_{ij,k})}(1+\bar{t}_i)$ denote the first-best optimal rate (Equation 11). For a ***small open economy***, the formulas further reduce to

$$1+t^{**}_{ji,k} = \frac{1+(\sigma_k-1)\lambda_{ii,k}}{1+\frac{1+\overline{\mu}_i}{1+\mu_k}(\sigma_k-1)\lambda_{ii,k}}(1+\bar{t}_i); \qquad\qquad 1+x^{**}_{ij,k} = \frac{1+\mu_k}{1+\overline{\mu}_i}\left(\frac{\sigma_k-1}{\sigma_k}\right)(1+\bar{t}_i).$$

In summary, the above formulas indicate that second-best *import* taxes are higher in (1) industries with a greater-than-average markup, and (2) industries in which country i has a comparative advantage (i.e., high-$(\sigma_k-1)\lambda_{ii,k}$ industries). These two properties allow second-best import taxes to mimic Pigouvian subsidies to the best extent possible. Likewise, second-best *export* subsidies feature a *misallocation-correcting* component that favors industries with a higher-than-average scale elasticity or markup.

Importantly, if the markup or scale elasticity is uniform across industries (i.e., $\mu_k = \mu = \overline{\mu}_i$), the above formulas yield the *first-best* or purely ToT-improving tax rate—i.e., $t^{**}_{ji,k} = t^{*}_{ji,k}$ and $x^{**}_{ij,k} = x^{*}_{ij,k}$. The intuition is that the Krugman model *without* cross-industry markup heterogeneity is efficient; leaving no room for policy interventions to restore allocative efficiency.

Third-Best: Unilaterally Optimal Import Tariffs

Now suppose that, in addition to restriction on industrial subsidies, the use of export subsidies is also restricted. The government's optimal policy problem in this case features two additional constraints, $\mathbf{s}_i = \mathbf{x}_i = \mathbf{0}$:

$$\max_{(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w})} \quad W_i\left(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}\right) \quad s.t. \begin{cases}(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}) \in \mathbb{F} \\ \mathbf{s}_i = \mathbf{x}_i = \mathbf{0}\end{cases} \qquad \text{(P3)}.$$

Some variation of the above problem has been studied by an expansive literature on optimal tariffs. Though, nearly all existing studies are limited to partial equilibrium two-by-two models. Here, we use the same dual approach described earlier to analytically solve Problem (P3) within our multi-country, multi-industry general equilibrium framework. Our derivation, as before, yields simple sufficient statistics formulas for optimal third-best import taxes. The following theorem presents these formulas, with a formal proof provided in Appendix J.[31]

Theorem 3. *Suppose both industrial and export subsidies are unavailable to the government: Third-best optimal import taxes are uniquely given by:*

$$\mathbf{1}+\mathbf{t}^{***}_{ji} = (1+\bar{\tau}^{*}_i)\,(\mathbf{1}+\mathbf{\Omega}_{ji}) \oslash \left(\mathbf{1}+\mathbf{E}^{-1}_{-ii}\,\mathbf{E}^{(ii)}_{-ii}\,\left[1-\frac{1+\mu_k}{1+\overline{\mu}_i}\right]_k\right)$$

where $\bar{\tau}^{*}_i = \left[-\sum_{g,s}\sum_{j\neq i}\chi_{ij,g}\left(1+\varepsilon^{(ij,s)}_{ij,g}\right)\right]^{-1}$ *is a uniform tariff shifter that represents the elasticity of international demand for country i's labor (with* $\chi_{ij,g} \equiv P_{ij,g}Q_{ij,g}/\sum_{n\neq i}\mathbf{P}_{in}\cdot\mathbf{Q}_{in}$ *denoting export*

[31] In the special case where entry is restricted and countries are sufficiently small, the optimal tariff formula presented under Theorem 4 reduces to the formula used by Lashkaripour (2020b) to examine global tariff wars.

shares). $\overline{\mu}_i$ denotes the output-weighted average markup in economy i as described by Eq. 5; and $\mathbf{E}_{-ii}$ and $\mathbf{E}^{(ii)}_{-ii}$ are matrixes of Marshallian demand elasticities as defined under Definition (D1).

Unlike Theorems 1 and 2, the third-best optimal tariff schedule identified by Theorem 3 is unique. That is because the multiplicity implied by Lemma 1 no longer applies when both export and industrial subsidies are restricted to zero. Nevertheless, the third-best tariff specified by Theorem 3 differs from the second-best tariffs (in Theorem 2) by only a uniform tariff shifter, $1+\bar{\tau}_i^*$. So, barring the uniform component, $1+\bar{\tau}_i^*$, we can understand the above formula based on the same intuition provided under Theorem 2.

The uniform tariff component, $1+\bar{\tau}_i^*$, compensates for the unavailability of export tax-cum-subsidies to the government. By the Lerner symmetry, which is implicit in Lemma 1, import taxes can perfectly mimic a uniform export tax. This ability was previously redundant (under Theorems 1 and 2) because export taxes/subsidies were directly applicable, and there was no point in using other instruments to mimic them. But since export taxes are restricted under Problem (P3), it is optimal to uniformly raise all tariffs by a factor $1+\bar{\tau}_i^*$, using them as a second-best substitute for optimal export taxes/subsidies.

4 Tension between Allocative Efficiency and Terms-of-Trade

Following Theorems 2 and 3, second-best trade policies seek to strike a balance between (a) improving the terms-of-trade (ToT), which requires contracting exports in nationally-differentiated (low-σ) industries, and (b) correcting misallocation, which requires expanding output in high-returns-to-scale (high-μ) industries. Obtaining this balance becomes difficult if not impossible when $Cov\left(\sigma_k,\mu_k\right)<0$—which is the empirically-relevant case based on our forthcoming estimation. Consider, for instance, a country that is initially operating under Laissez-Faire. And suppose that due to political constraints, the government is unable to use industrial policy to correct misallocation. In that case, a trade policy intervention that improves the ToT by restricting exports in low-σ industries concurrently shrinks output in high-μ industries if $Cov\left(\sigma_k,\mu_k\right)<0$, thereby exacerbating misallocation.

To navigate this tension, a welfare-maximizing government must customize its 2nd-best trade policy in a way that curtails the ToT gains without necessarily correcting misallocation. These considerations all but erode the gains from 2nd-best trade policies and can even render them industry-blind—unable to beneficially correct inter-industry misallocation *or* manipulate industry-specific export market power. This is, for instance, the case in the canonical Krugman (1980) model where $\mu_k=1/\left(\sigma_k-1\right)$. Theorem 2 asserts that the 2nd-best trade policy for a small open economy in this particular setting consists of industry-blind or uniform import tariffs and export subsidies. Namely,

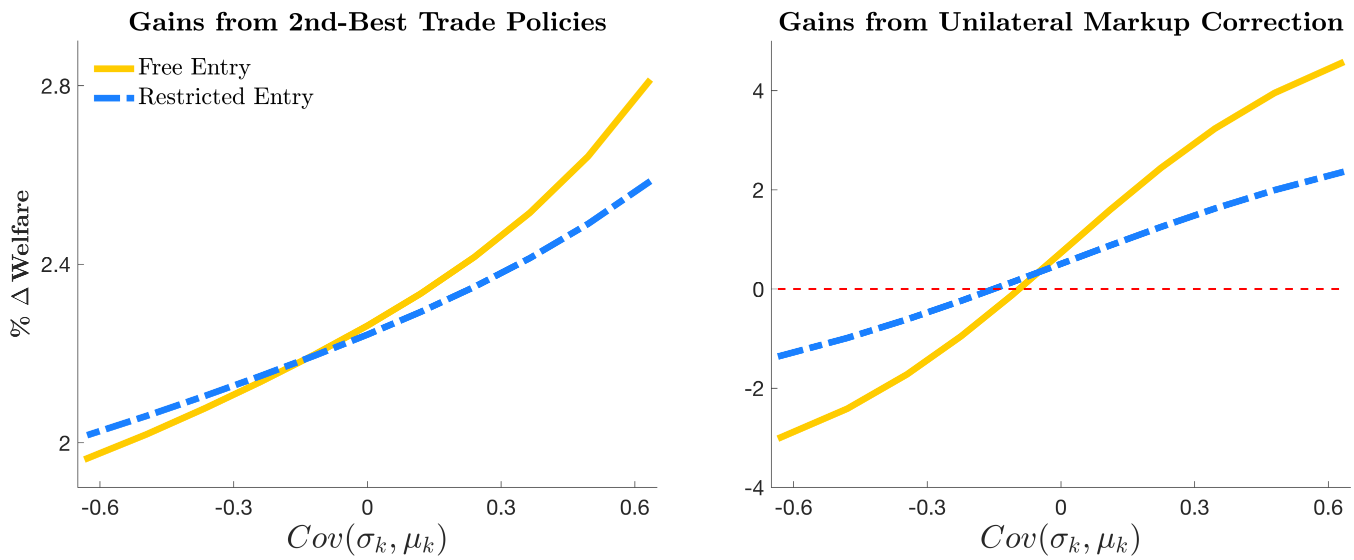
$$1+t^{**}_{ji,k}=1+\bar{t}_i;\qquad\qquad 1+x^{**}_{ij,k}=\left(1+\bar{t}_i\right)\left(1-\frac{1}{\overline{\sigma}_i}\right),$$

where $\bar{t}_i\in\mathbb{R}$ is an arbitrary tax shifter and $\overline{\sigma}_i=\left(\sum_k\rho_{i,k}/\sigma_k\right)^{-1}$ is the sales-weighted average trade elasticity facing country i. The optimal trade tax in each industry is evidently blind to misallocation (μ_k) or industry-specific export market power (σ_k)—reflecting the difficulty to reconcile these two policy considerations. All this policy choice can achieve is to improve country i's aggregate ToT by inflating its wage relative to the rest of the world. Proposition 1 summarizes these arguments, with a formal proof provided in Appendix L.

Proposition 1. *If industry-level trade and scale elasticities are negatively correlated ($Cov\left(\sigma_k, \mu_k\right) < 0$), trade policy interventions that seek to improve the terms-of-trade (relative to Laissez-Faire) exacerbate inter-industry misallocation. In the canonical case where $\mu_k = 1/\left(\sigma_k - 1\right)$, this tension forces a small open economy's optimal 2nd-best trade policy to be industry-blind—unable to beneficially correct misallocation or manipulate the ToT on an industry-by-industry basis.*

Figure 1 demonstrates how the tension between ToT and allocative efficiency erodes the potential gains from second-best trade policies. The left panel demonstrates that the gains from 2nd-best trade policies diminish rapidly as $Cov\left(\sigma_k, \mu_k\right)$ is artificially lowerred from positive to negative values. In each case, the trade elasticities are held constant, meaning that the scope for ToT gains remains that same. The only thing that changes is the rising tension between ToT and corrective gains from policy as $Cov\left(\sigma_k, \mu_k\right)$ becomes more negative.[32]

Figure 1: *Tension between ToT and allocative efficiency, when $Cov\left(\sigma_k, \mu_k\right) < 0$, can yield dire policy outcomes*



Note: This figure corresponds to a two-country and two-industry model with symmetric countries and Cobb-Douglas preferences across industries. The left panel report the gains from Second-best trade taxes, specified by Theorem 2. The trade elasticities in industries 1 and 2 are assigned values $\sigma_1 = 1.5$ and $\sigma_2 = 3$ and scale elasticities are adjusted to vary $Cov\left(\sigma_k, \mu_k\right)$. The right panel reports the welfare consequences of unilateral scale or markup correction. Industry-level scale elasticities are assigned values $\mu_1 = 0.5$ and $\mu_2 = 0.2$ and trade elasticities are adjusted to vary $Cov\left(\sigma_k, \mu_k\right)$.

Proposition 1 has a notable flip side: If $Cov\left(\sigma_k, \mu_k\right) < 0$, a unilateral adoption of Pigouvian subsidies will worsen the ToT, resulting in possibly adverse welfare consequences. This outcome is a basic manifestation of ***immiserizing growth*** (Bhagwati, 1958). This point is illustrated by the right panel in Figure 1. In each case, the degree of inter-industry misallocation is held fixed, but trade elasticities are adjusted to vary $Cov\left(\sigma_k, \mu_k\right)$ from negative to positive values. In regions where $Cov\left(\sigma_k, \mu_k\right) < 0$, a unilateral adoption of Pigouvian subsidies can diminish welfare considerably (i.e., cause *immiserizing growth*).

[32] This tension is distinct from the *targeting principle* (Bhagwati and Ramaswami (1963)), which applies irrespective of the sign of $Cov\left(\sigma_k, \mu_k\right)$. Indeed, 2nd-best trade taxes become more potent despite the targeting principle if $Cov(\sigma_k, \mu_k) > 0$; but we focus on the case where $Cov\left(\sigma_k, \mu_k\right) < 0$ as it aligns with our forthcoming estimation.

There is a simple intuition behind these *immiserizing growth* effects. If $Cov(\sigma_k, \mu_k) < 0$, Pigouvian subsidies (that restore marginal-cost-pricing) expand domestic output in high-σ industries. These are nationally-differentiated industries in which countries enjoy significant export market power. Raising output and, correspondingly, exports in these industries can worsen the ToT to the point of triggering immiserizing growth. As we discuss next, this tension can become a major obstacle for industrial policy implementation after countries commit to cooperative policies under a shallow trade agreement.

4.1 Avoiding Immiserizing Growth via Deep Trade Agreements

Immizerising growth can be a serious obstacle to industrial policy implementation in open economies. To convey this point, we adopt the common view that intentional negotiations involve two stages.[33] In the first-stage, governments negotiate over policy space to ensure each party restricts itself to the cooperative policy choice (Equation 9). In the second stage, governments negotiate a deeper agreement to ensure implementation. Each country, in this stage, has the choice to either implement their cooperative policy choice or withhold implementation.

Proposition 1 addressed the first stage of this game, making a strong case for cooperation. It formalized the lack of justification for non-cooperative trade taxes, even when economies are plagued with misallocation and governments are shortsighted and wary of political pressures. To elaborate, the conventional argument against non-cooperative trade taxes emphasizes the cost of retaliation. If trading partners retaliate, the ToT gains from trade taxation will more-than-evaporate and leave all parties in a prisoner's dilemma situation. Suppose, however, that governments are shortsighted and do not sufficiently fear retaliation. In that case, they may be tempted to use trade taxes to correct misallocation in domestic industries. This second-best choice may allow them to correct misallocation without triggering political backlash. Proposition 2, however, indicated that this justification is also weak provided that $Cov(\sigma_k, \mu_k) < 0$.

Now, suppose governments resolve the issue of cooperation with a shallow trade agreement. Under this arrangement, each government confines itself to the cooperative policy specified by Equation 9. Under perfect competition, the shallow agreement suffices to reach the global first-best. This need not be the case under market imperfections, where unilateral action can cause immiserizing growth when $Cov(\sigma_k, \mu_k) < 0$. Cooperative countries, in this case, are tangled in a coordination game over policy implementation. A unilateral implementation of cooperative industrial subsidies causes immizerising growth at home and benefits trading partners. Cooperative governments, therefore, have an incentive to free-ride on industrial policy implementation in the rest of the world, prompting a race to the bottom.

Table 2 illustrates the second-stage implementation game facing cooperative countries. The game involves two cooperative countries (i and j) that can take two actions: (1) implement Pigouvian subsidies that restore marginal cost-pricing in the domestic economy, *i.e.*, $\mathbf{s}_i = \boldsymbol{\mu}$, or (1) delay implementation and stick to business as usual, *i.e.*, $\mathbf{s}_i = \mathbf{0}$. The efficient outcome is the implementation of Pigouvian subsidies in both countries, which will boost welfare by 2.7%

[33]Modeling international cooperation as a two-stage game consisting of *enactment* and *implementation* stages is commonplace in the global governance literature (Shaffer and Pollack (2009)). In the trade and environmental policy literature, many studies treat international negotiations as multi-stage games where initial stages restrict policy choices and latter stages ensure implementation (e.g., Murdoch, Sandler, and Vijverberg (2003); Drazen and Limão (2008); Kosfeld, Okada, and Riedl (2009)).

across the board. But this outcome is not sustainable without formal coordination, because each country has an incentive to free-ride on the other country's implementation. The outcome of this game is a race to bottom, wherein no party is willing to correct misallocation in domestic industries without violating its commitments to cooperation (i.e., zero trade taxes).

Table 2: *The industrial policy implementation game facing cooperative governments when* $Cov\,(\sigma_k, \mu_k) < 0$

		Country j ($\%\Delta W_j$)	
		$\mathbf{s}_j = \mathbf{0}$	$\mathbf{s}_j = \boldsymbol{\mu}$
Country i ($\%\Delta W_i$)	$\mathbf{s}_i = \mathbf{0}$	(0% , 0%)	(3.7% , −1.2%)
	$\mathbf{s}_i = \boldsymbol{\mu}$	(−1.2% , 3.7%)	(2.7% , 2.7%)

Note: This figure corresponds to a two-country and two-industry model with symmetric countries and CES preferences across industries with substitution elasticity 1.2. Industry-level level trade elasticities are $\sigma_1 = 1.5$ and $\sigma_2 = 3$ and scale elasticities are $\mu_1 = 0.2$ and $\mu_2 = 0.5$, implying $Cov\,(\sigma_k, \mu_k) \approx -0.225$. The unique Nash equilibrium is a race to the bottom, wherein $\mathbf{s}_i = \mathbf{s}_j = \mathbf{0}$.

Countries can avoid a race to the bottom by supplementing their shallow agreement with a "deep agreement" that ensures reciprocity in industrial policy implementation. The following proposition (proven in Appendix L) formalizes this point and underscores the role of domestic policy coordination—the lack of which is often considered an impediment to growth among middle-income countries (Aiginger and Rodrik (2020)).

Proposition 2. [Markup Correction Paradox] *If* $Cov(\sigma_k, \mu_k) < 0$, *a unilateral implementation of scale- or markup-correcting industrial subsidies can trigger immizerising growth at home and deliver strong spillover gains to trading partners. Hence, without a deep agreement to ensures reciprocity in implementation, countries have an incentive to free-ride on other's corrective policies. The outcome of this game is a race to the bottom, wherein no party is willing to correct misallocation in domestic industries without violating its commitments to shallow cooperation (i.e., zero trade taxes).*

The above proposition resonates with the anecdote that governments pursuing industrial policy often pair it with various trade restrictions. The Chinese government, for instance, has paired its active industrial policy measures with hidden export taxes. These hidden export barriers are applied via partial value-added tax rebates and are designed to improve China's terms-of-trade (Garred (2018)). At the end, we must emphasize that a race to the bottom will not occur if $Cov(\sigma_k, \mu_k) \geq 0$. In that case, a unilateral adoption of corrective subsidies improves rather than deteriorates one's terms of trade. Implementing corrective subsidies, as a result, becomes a dominant strategy for cooperative countries. We nevertheless maintain our focus on the case where $Cov(\sigma_k, \mu_k) < 0$ because it aligns with our forthcoming estimation of scale and trade elasticities.

5 Extensions and Application to other Canonical Models

In this section, we first show that our theoretical results readily apply to two other canonical trade models. We then extend our baseline theoretical results to richer environments featuring input-output linkages and political economy pressures.

5.1 Application to Other Canonical Trade Models

The optimal policy formulas specified by Theorems 1-3 apply to two other canonical trade models. Though, parameters σ_k and μ_k in these formulas adopt different interpretations, which reflects the different micro-foundation underlying these frameworks.

The Eaton-Kortum model with Marshallian externalities. Consider a multi-industry Eaton and Kortum (2002) model where industry-level production is subject to agglomeration economies. Let ψ_k denote the constant agglomeration elasticity in industry k, and θ_k denote the Eaton-Kortum Fréchet shape parameter. Theorem 1 characterizes the optimal policy in this model under the following reinterpretation of parameters: $\mu_k^{\text{EK}} = \psi_k$ and $\sigma_k^{\text{EK}} = 1 + \theta_k$. The tension between the *ToT* and *allocative efficiency* (outlined by Proposition 1) extends to this model if $Cov(\psi_k, \theta_k) < 0$. The fact that our theory readily extends to the Eaton-Kortum model echos the isomorphism established in Kucheryavyy et al. (2016). Appendix C shows that this isomorphism is even more profound. The nested-CES import demand function implied by (A1), we demonstrate, may analogously arise from within-industry specialization á la Eaton-Kortum.

The Melitz-Pareto model. Consider a multi-industry Melitz (2003) model that features the same nested-CES demand function specified by (A1). Suppose the firm-level productivity distribution is Pareto in each industry with a shape parameter, ϑ_k. Appendix D establishes that the Melitz-Pareto model is isomorphic to our baseline Krugman model insofar as macro-level representation is concerned. Hence, Theorem 1 characterizes the optimal policy in the Melitz-Pareto model under the following reinterpretation of parameters: $\mu_k^{\text{Melitz}} = \frac{\gamma_k \vartheta_k}{(\gamma_k - 1)(\vartheta_k + 1) - \vartheta_k} - 1$ if entry is restricted and $\mu_k^{\text{Melitz}} = \frac{1}{\vartheta_k}$ if entry is free; and $\sigma_k^{\text{Melitz}} = 1 + \vartheta_k \left[1 + \vartheta_k \left(\frac{1}{\sigma_k - 1} - \frac{1}{\gamma_k - 1} \right) \right]^{-1}$. This mapping indicates that we need to estimate parameter ϑ_k, in addition to σ_k and γ_k, to quantify the gains from policy under firm-selection effects—a procedure we formally undertake and elaborate on in Section 7.

5.2 Extension #1: Accounting for Input-Output Networks

Suppose production employs both labor and intermediate inputs, which are distinguished from final goods by superscript $\mathcal{I}$. Cost minimization entails that the producer price of good ij,k (origin i–destination j–industry k) depends on (i) the wage rate in origin i and (ii) the price of all intermediate inputs, $\tilde{\mathbf{P}}_i^{\mathcal{I}} \equiv \{\tilde{P}_{nj,k}^{\mathcal{I}}\}$, available to firms in origin i. Namely,

$$P_{ij,k} = \bar{\rho}_{ij,k} C_{i,k}(w_i, \tilde{\mathbf{P}}_i^{\mathcal{I}}) \mathcal{Q}_{i,k}^{-\frac{\mu_k}{1+\mu_k}}, \tag{13}$$

where $C_{i,k}(.)$ is a homogeneous of degree one cost function w.r.t. w_i and $\tilde{\mathbf{P}}_i^{\mathcal{I}}$.[34] The dependence of $P_{ij,k}$ on *origin i–industry k*'s effective output, $\mathcal{Q}_{i,k} \equiv \sum_{j \in \mathbb{C}} \left[\bar{a}_{ij,k} Q_{ij,k} \right]$, accounts for scale economies under free entry. The formal definition of general equilibrium in the presence of input-output (IO) linkages is presented in Appendix K. The same appendix characterizes optimal policy using our previously-described dual approach, while appealing to additional

[34] Without loss of generality, we assume that good ji,k can be used as either an intermediate input or a final consumption good, with taxes being applied on a good irrespective of the intended final use, i.e., $\tilde{P}_{ij,k}^{\mathcal{I}} = \tilde{P}_{ij,k}$. This assumption is innocuous, because we can fragment every industry k into a final good version k' and an intermediate good version k''. Since we do not impose any restrictions on the number of industries, our theory extends to the case where differential taxes are imposed on fragments k' and k''.

supply-side envelope conditions. Our characterization indicates that optimal industrial subsidies and import taxes are IO-blind—i.e., they are described by the same formulas as in Theorem 1. The intuition is that after fixing the price of exported goods with export subsidies, import tariffs and industrial subsidies have no impact on prices in the rest of the world. For these policies to affect prices in the rest of the world, they need to propagate through re-exportation. But any possible gains the channel through re-exportation, will be already internalized by the optimal choice w.r.t. export subsidies. Consistent with this intuition, optimal export subsidies depend on the fraction of export value that is reimported via the global IO network. Putting the pieces together, country i's first-best optimal policy under IO linkages is given by:

Theorem 4. *Under IO linkages, the unilaterally first-best import tariffs and domestic subsidies are IO-blind; but 1st-best export subsidies exhibit an upward-adjustment that account for re-importation via the IO network. More formally, the unilaterally first-best policy schedule is given by*

$$\begin{aligned}
&\text{[domestic subsidy]} && 1+s^*_{i,k} = (1+\mu_k)\left(1+\bar{s}^{\mathscr{C}}_i\right)^{-1} \\
&\text{[import tax]} && 1+t^*_{ji,k} = (1+\omega_{ji,k})\left(1+\bar{t}_i\right)\left(1+\bar{s}^{\mathscr{C}}_i\right) \\
&\text{[export subsidy]} && \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right)+\boldsymbol{\Lambda}_{ij}(1+\bar{t}_i)(1+\bar{s}^{\mathscr{C}}_i)\right],
\end{aligned}$$

where elements of $\boldsymbol{\Lambda}_{ij} \equiv \left[\Lambda_{ij,k}\right]_k$ *correspond to the fraction of good* ij,k *that is reimported, and* $\bar{s}^{\mathscr{C}}_i$ *is an arbitrary tax shifter that assumes a positive value if the taxed item is a final good and zero otherwise.*[35]

The above theorem indicates that the tax-equivalent of export subsidies is relatively lower on intermediate inputs to mitigate tax-reimportation via the IO network. Moreover, there is uniform wedge between final and intermediate input taxes as represented by the final good tax-shifter, $\bar{s}^{\mathscr{C}}_i$.[36] This detail aside, the ToT-improving motive for policy still requires a contraction of exports in low-σ industries while the misallocation-correcting objective asks for an expansion of output in high-μ industries. So, unless intermediate input goods exhibit a systemically lower σ, the same tensions identified by Propositions 1 and 2 remain.[37]

5.3 Extension #2: Accounting for Political Economy Pressures

Suppose optimal policy choices internalize political economy pressures. To characterize optimal policy in these situations, we follow Ossa's (2014) adaptation of Grossman and Helpman (1994). Our baseline analysis, in particular, assumed that the government in country i maximizes $W_i \equiv V_i(w_i L_i + \mathcal{R}_i + \Pi_i, \tilde{\mathbf{P}}_i)$, where $\Pi_i \equiv \overline{\mu}_i w_i L_i$ denotes total profits. Now, the government maximizes a politically-weighted welfare function, $W_i \equiv V_i(w_i L_i + \mathcal{R}_i + \sum_k \pi_{i,k}\Pi_{i,k}, \tilde{\mathbf{P}}_i)$, where $\pi_{i,k}$ is the political economy weight assigned to industry k's profits (with $\sum_k \pi_{i,k}/K = 1$).

[35]If country i is a small open economy, $\Lambda_{ij,k} \approx \mathbf{0}$. Correspondingly, optimal policy formulas for a small open economy under IO linkages perfectly overlap with the baseline formulas specified under Equation 12.

[36]Theorem 4 indicates that uniform markup pricing is not a necessary condition for efficiency. Consider, for instance, a vertical production economy where goods are used as either final goods or intermediate inputs, but not both. The efficient policy, in this setup, must restore uniform markup pricing within input and final good segments but not across—see Antràs, Gutiérrez, Fort, and Tintelnot (2022) for further exploration of this issue.

[37]Theorem 4 offers insight into the structure of second-best import tariffs and export subsidies under IO linkages. Several papers, including Blanchard, Bown, and Johnson (2016), Beshkar and Lashkaripour (2020), Caliendo, Feenstra, Romalis, and Taylor (2021), and Antràs et al. (2022), examine in more detail how IO linkages impact 3rd-best import tariff choices. Beshkar and Lashkaripour (2020), moreover, adopt a special case of Theorem 4, with Cobb-Douglas production and no scale economies, to examine the cost of trade wars.

It follows trivially from Theorem 1 that the first-best policy in the political setup consists of the same trade tax/subsidy formulas but a politically-adjusted industrial subsidy rate. Namely,

$$1 + s^*_{i,k} = \left(1 + \mu^{\mathscr{P}}_{i,k}\right)\left(1 + \bar{s}_i\right),$$

where $\mu^{\mathscr{P}}_{i,k} = \frac{\mu_k}{\pi_{i,k} - (1-\pi_{i,k})\mu_k}$ is the *political economy-adjusted* markup of industry k. Considering the above formulas: if $Cov\left(\pi_{i,k}, \mu_k\right) < 0$, the optimal policy may tax high-μ industries to the detriment of social welfare. In that case, even if $Cov(\sigma_k, \mu_k) > 0$, the misallocation-correcting and ToT motives for trade taxation will clash. However, if $Cov\left(\pi_{i,k}, \mu_k\right) \geq 0$ Propositions 1 and 2 remains valid despite political economy pressures.

6 Estimating the Key Policy Parameters

Based on our theory, policy evaluation in open economies requires credible estimates for industry-level *trade elasticities*, σ_k, and industry-level *scale elasticities*, $\mu_k \sim \frac{1}{\gamma_k - 1}$. The former governs the degree of national-level market power, while the latter regulates the extent of love-for-variety. The trade literature has paid considerable attention to estimating σ_k, but less to estimating μ_k. The existing policy literature typically normalizes μ_k in one of two ways: (i) $\mu_k = \frac{1}{\sigma_k - 1}$ in imperfectly competitive models, and (ii) $\mu_k = 0$ in perfectly competitive models.[38] Both normalizations impose strong and artificial restrictions on $Cov\left(\mu_k, \sigma_k\right)$, which based on Propositions 1 and 2 can lead to possibly flawed policy predictions.

Against this backdrop, we seek to estimate σ_k and μ_k in a way that ascertains mutual consistency and gives us a credible evaluation of $Cov\left(\mu_k, \sigma_k\right)$. To this end, we propose a new methodology that simultaneously estimates σ_k and μ_k from the same data.[39] Our approach involves fitting the structural *firm-level* import demand function implied by A1 to the universe of Colombian import transactions from 2007–2013. We outline this approach below, starting with a description of the data used in our estimation.

Data Description. Our estimation uses data on import transactions from the Colombian Customs Office during 2007–2013.[40] The data include detailed information about each transaction, such as the Harmonized System 10-digit product category (HS10), country of origin, importing and exporting firm IDs, quantity, f.o.b. (free on board), and c.i.f. (customs, insurance, and freight) transaction values, freight, insurance, and value-added tax in US dollars. As a unique feature, our data reports the identities of all foreign firms exporting to Colombia. This feature allows us to define import varieties as firm-product combinations rather than country-product combinations, which is the standard approach. Table 6 (in the appendix) reports a summary of basic trade statistics in our data.[41] Working with firm-level data presents two challenges:

[38] See Ossa (2016) and Costinot and Rodríguez-Clare (2014) for a synthesis of the previous literature. Under restricted entry, μ_k denotes the firm-level markup, which can be alternatively estimated with firm-level production data (e.g., De Loecker and Warzynski (2012), De Loecker, Goldberg, Khandelwal, and Pavcnik (2016)).

[39] In the presence of firm-selection effects, our estimated parameters are necessary but not sufficient to pin down the trade and scale elasticities—see Appendixes D and X for details.

[40] The data is obtained from DATAMYNE, a company that specializes in documenting import and export transactions in Americas. For more detail, please see `www.datamyne.com`.

[41] Our estimation also employs data on monthly average exchange rates, which are taken from the Bank of Canada: `http://www.bankofcanada.ca/rates/exchange/monthly-average-lookup/`.

First, exporters are not identified by unique standardized IDs. Instead, they are identified by a name, a number, and an address. We handle this problem by standardizing the spelling and lengths of firms' names and using the information on firms' phone numbers (see Appendix M). Second, Colombia changed the HS10 classification for some products between 2007 and 2013. Fortunately, the Colombian Statistical Agency, DANE, keeps track of these changes. We utilize this information to concord the Colombian HS10 codes over time, using the guidelines in Pierce and Schott (2012).[42] Overall, changes in HS10 codes between 2007 and 2013 affect less than 0.1% of our data points.

6.1 Estimating Equation

Since we are focusing on one importer, we hereafter drop the importer's subscript i and add a year subscript t to account for the time dimension of our data. With this switch in notation, the demand facing firm ω located in country j and supplying product k in year t is given by:

$$q_{j,kt}(\omega) = \varphi_{j,kt}(\omega) \left(\frac{\tilde{p}_{j,kt}(\omega)}{\tilde{P}_{j,kt}} \right)^{-\gamma_k} \left(\frac{\tilde{P}_{j,kt}}{\tilde{P}_{kt}} \right)^{-\sigma_k} Q_{kt}, \tag{14}$$

Subscript k, in our theoretical model, designated industries. In our estimation, k denotes an HS10 product—the most disaggregated product classification in our data. The quadruplet "ωjkt" accordingly denotes a unique variety corresponding to *firm ω–country of origin j–HS10 product k–year t*. Let $\tilde{x}(\omega) \equiv \tilde{p}(\omega)q(\omega)$ denote gross sales. Rearranging Equation 14 yields the following log-linear import demand function facing individual varieties:

$$\ln \tilde{x}_{j,kt}(\omega) = (1-\sigma_k) \ln \tilde{p}_{j,kt}(\omega) + \left(1 - \frac{\sigma_k - 1}{\gamma_k - 1}\right) \ln \lambda_{j,kt}(\omega) + D_{kt} + \ln \varphi_{j,kt}(\omega), \tag{15}$$

where $D_{kt} \equiv \ln P_{kt}^{\sigma_k} Q_{kt}$ can be treated as a product-year fixed effect and $\lambda_{j,kt}(\omega)$ denotes the share of expenditure on firm ω *conditional* on buying good k from country of origin j,

$$\lambda_{j,kt}(\omega) \equiv \varphi_{j,kt}(\omega) \left(\frac{\tilde{p}_{j,kt}(\omega)}{\tilde{P}_{j,kt}} \right)^{1-\gamma_k} = \frac{\tilde{x}_{j,kt}(\omega)}{\sum_{\omega' \in \Omega_{j,kt}} \tilde{x}_{j,kt}(\omega)}.$$

We assume that $\varphi_{jkt}(\omega) = \bar{\varphi}_{j,k}(\omega) \times \varphi_{\omega jkt}$ can be decomposed into a time-invariant *firm-and-product*-specific quality component, $\bar{\varphi}_{j,k}(\omega)$, and a time-varying component $\varphi_{\omega jkt}$, that encompasses idiosyncratic variations in consumer taste, measurement errors, and/or omitted variables that account for dynamic demand optimization. To eliminate $\bar{\varphi}_{j,k}(\omega)$ from the estimating equation, we employ a first-difference estimator, which also drops observations pertaining to one-time exporters. We deem the first-difference estimator appropriate given the possibility that $\varphi_{\omega jkt}$'s are sequentially correlated. As a robustness check, we also report estimation results based on a two-ways fixed effects estimator in Appendix P.[43] Stated in terms of first-

[42] To preserve the industry identifier of the product codes, and in contrast to Pierce and Schott (2012), we try to minimize the number of the synthetic codes. The concordance data and do files are provided in the data appendix.

[43] Following Boehm, Levchenko, and Pandalai-Nayar (2020), the first-difference estimation offers a partial remedy for omitted variable bias and reverse causality. Both issues pose a serious challenge to traditional log-level estimations of import demand. Depending on the application, though, the first-difference estimator may not necessarily identify the desired long-run elasticity. As detailed in Appendix P, this limitation is less severe in our firm-level estimation—as we explicitly control for the extensive margin of trade. We illustrate this point formally in Appendix P by re-estimating Equation 15 in levels and comparing the estimation results to the baseline values.

differences, our estimating equation takes the following form

$$\Delta \ln \tilde{x}_{j,kt}(\omega) = (1-\sigma_k)\,\Delta \ln \tilde{p}_{j,kt}(\omega) + \left(1 - \frac{\sigma_k - 1}{\gamma_k - 1}\right)\Delta \ln \lambda_{j,kt}(\omega) + \Delta D_{kt} + \Delta \ln \varphi_{\omega jkt}, \quad (16)$$

where $\Delta \ln \varphi_{\omega jkt}$, roughly speaking, represents a variety-specific demand shock; and ΔD_{kt} is a product-year fixed effect.[44] Of the remaining variables, $\Delta \ln \tilde{p}_{j,kt}(\omega)$ and $\Delta \ln \tilde{x}_{j,kt}(\omega)$ are directly observable for each import variety. The change in the *within-national* market share, $\Delta \ln \lambda_{j,kt}(\omega)$, can be calculated using the universe of firm-level sales to Colombia.

Recovering Scale & Trade Elasticities from Demand Parameters. Equation 16 allows us to estimate demand parameters, σ_k and γ_k, from which we can recover the scale and trade elasticities as follows (see Section 2 for the underlying theoretical foundation):

$$\mu_k = \frac{1}{\gamma_k - 1} \sim \text{scale elasticity} \qquad\qquad \sigma_k \sim \text{trade elasticity}$$

The reason we can infer μ_k from demand parameters is that the scale elasticity in the generalized Krugman model reflects the extent of *love-for-variety*—the social benefits of which are not internalized by firms' entry decisions. In the Melitz-Pareto case, we also need estimates for the shape of the Pareto productivity distribution (in addition to γ_k and σ_k) to recover the scale elasticity—see Appendix O. Our demand-based estimation technique is of course unable to detect scale externalities unrelated to love-for-variety. These externalities can be estimated using supply-side techniques, which require strong assumptions on the variability of production inputs. We discuss the relative advantages of each technique in Appendix Q.[45]

Breaking the Sample into Broadly-Defined Industries. As noted earlier, k indexes an HS10 product category in Equation 16. To conduct our forthcoming quantitative analysis, we must estimate demand parameters for 14 broadly-defined industries based on the World Input-Output Database (WIOD) industry classification. Considering this, we pool all HS10 products belonging to the same WIOD industry $\mathcal{K}$ together, and estimate Equation 16 on this pooled sample assuming that σ_k and γ_k are uniform across products within the same industry (i.e., $\gamma_k = \gamma_{\mathcal{K}}$ and $\sigma_k = \sigma_{\mathcal{K}}$ for all $k \in \mathbb{K}_{\mathcal{K}}$). In principle, we can also estimate the import demand function separately for each HS10 product category to attain HS10-level elasticities. However, such elasticities will be of little use for quantitative policy analysis, as multi-country data on trade, production, and expenditure shares are scarce at such levels of disaggregation.

6.2 Identification Strategy

The identification challenge we face is that $\Delta \ln \tilde{p}_{j,kt}(\omega)$ and $\Delta \ln \lambda_{j,kt}(\omega)$ are endogenous variables that can covary with the demand shock, $\Delta \ln \varphi_{\omega jkt}$.[46] Traditional *country-level* import

[44] To handle outliers, we trims our sample to exclude observations that report a price change, $\Delta \ln \tilde{p}_{j,kt}(\omega)$, above the 99th percentile or below the 1st percentile of the HS10 product code k in year t.

[45] In the restricted entry case, our demand estimation identifies the firm-level markup in each industry. Following an old tradition in the industrial organization literature, we assume that market conduct is monopolistic competition and recover firm-level markups as $\mu_k = \frac{1}{\gamma_k - 1}$. Appendix V recover markups under alternative assumptions vis-à-vis market conduct.

[46] Another challenge is that unit price data may be contaminated with measurement errors, as they are averaged across many transactions. Following Berry (1994), this type of measurement error is fairly innocuous when dealing with log-linear demand functions. Furthermore, our instrumental variable approach will handle measurement errors, provided that lagged monthly sales patterns are uncorrelated with concurrent measurement errors.

demand estimations overcome a similar challenge by instrumenting for prices with plausibly exogenous tariff rates.[47] This strategy, however, does not suit our *firm-level* estimation, because tariffs discriminate by country-of-origin but not across firms from the same country.

We employ a shift-share research design to overcome our *firm-level* identification challenge. Our approach is rooted in two data observations: First, a typical variety is imported under multiple invoices spread across different months in a given year. As a matter of accounting, the annual-level price of a variety is the quantity-weighted average of monthly prices. Namely, $p_{j,kt}(\omega) = \sum_{m\in\mathcal{M}} \tilde{s}_{j,kt}(\omega;m) p_{j,kt}(\omega;m)$, where m denotes month and $\tilde{s}(\omega;m)$ and $p(\omega;m)$ denote the quantity share and price associated with month m. Second, the month m price of an imported variety (in Colombian Pesos) is equal to *markup-plus-taxes*$\times$*marginal input cost invoiced in local currency*$\times$*exchange rate in month m*. More formally, $p_{j,kt}(\omega;m) = \tau_{j,kt}(\omega) \times C_{j,kt}(\omega) \times \mathcal{E}_{jt}(m)$, where τ and C respectively denote markup-plus-tax and marginal cost, while $\mathcal{E}_{jt}(m)$ represents the exchange rate between origin j's currency and the Colombian Peso in month m of year t. To a first-order approximation, the change in variety-level annual prices in response to monthly exchange rate shock is, thus,

$$\Delta \ln \tilde{p}_{j,kt}(\omega) \approx \sum_{m\in\mathcal{M}} s_{j,kt}(\omega;m)\, \Delta \ln \mathcal{E}_{jt}(m),$$

where $\Delta \ln \mathcal{E}_{jt}(m)$ denotes the year-over-year change in origin j's exchange rate with the Colombian Peso in month m ; and $s_{j,kt}(\omega;m)$ is the share of month m in variety ωjkt's annual export sales to Colombia:

$$s_{j,kt}(\omega;m) = \frac{\tilde{x}_{j,kt}(\omega;m)}{\sum_{m'\in\mathcal{M}} \tilde{x}_{j,kt}(\omega;m')} \sim \text{share of month } m \text{ in annual export sales}$$

Capitalizing on the above observation, we construct our shift-share instrument as the inner product of *lagged* monthly export shares and monthly exchange rate shocks. Namely,

$$z_{j,kt}(\omega) = \sum_{m\in\mathcal{M}} s_{j,kt-1}(\omega, m)\, \Delta \ln \mathcal{E}_{jt}(m).$$

Stated verbally, $z_{j,kt}(\omega)$ measures exposure to exchange rate shocks at the *firm*$\times$*origin*$\times$*product*$\times$*year* level. The idea being that aggregate exchange rate shocks have differential effects on individual firms depending on the monthly composition of their prior export activity to Colombia. Encouragingly, the relevance of our shift-share instrument is supported by the strong and statistically significant correlation between z and $\Delta \ln \tilde{p}$. Appendix N illustrates this point using the example of two U.S.-based exporters.

Exclusion Restriction. Our instrument utilizes lagged export shares, which depend on lagged prices and a set of market-level indexes—namely, $s_{jkt-1}(\omega, m) = s(\tilde{p}_{jkt-1}(\omega, m); ...)$. Considering this, the exclusion restriction in our setup, $\mathbb{E}[z\, \Delta \ln \varphi] = 0$, rests on two conditions:

(C1) Prior price-setting decisions (and thus lagged export shares) are orthogonal to concurrent demand shocks: $\mathbb{E}\left[\tilde{p}_{j,kt-1}(\omega)\, \Delta \ln \varphi_{\omega jkt}\right] = 0$.

(C2) Monthly national-level exchange rate shocks are orthogonal to variety-level demand shocks: $\mathbb{E}\left[\Delta \ln \mathcal{E}_{jt}(m)\, \Delta \ln \varphi_{\omega jkt}\right] = 0$.

[47] A prominent example is Caliendo and Parro (2015) who use tariff data to identify the trade elasticity.

Since our sample features many firms and a finite number of months, Condition (C1) is sufficient for the consistency of our estimates (see Proposition 2.1 in Goldsmith-Pinkham, Sorkin, and Swift, 2020). In fact, our two-stage least square (2SLS) estimator is numerically equivalent to a generalized method of moments (GMM) estimator with lagged monthly export shares as instruments and a weight matrix constructed from monthly aggregate exchange rate shocks. Condition (C2), meanwhile, is more crucial for the finite sample properties of our estimator. Both conditions can in principle be violated if there are cross-inventory linkages or if individual export varieties account for a significant fraction of national exports to Colombia. We discuss and address these issues in Section 6.4.

Instruments for $\Delta \ln \lambda_{j,kt}(\omega)$. Following Khandelwal (2010), we construct two standard instruments for the annual variation in the within-national market shares: (i) annual changes in the total number of origin j firms serving the Colombian market in product category k, and (ii) changes in the total number of HS10 product categories actively served by firm ω in year t. These count measures will be correlated with $\Delta \ln \lambda_{j,kt}(\omega)$ but uncorrelated with $\Delta \ln \varphi_{\omega jkt}$ if variety-level entry and exit occurs prior to, or independent of, the demand shock realization of competing varieties. As noted by Khandelwal (2010), this assumption is widely-invoked when estimating discrete choice demands curves—see also Berry, Levinsohn, and Pakes (1995).[48]

6.3 Estimation Results

Table 3 reports our industry-level estimation results. We also report results corresponding to a pooled sample of all industries in Table 9 of the appendix. This table also compares the 2SLS and OLS estimates to ensure that our IV strategy operates in the expected direction. Our estimates point to a median trade elasticity of $\sigma - 1 = 3.9$ and a median scale elasticity of $\mu \approx 0.20$. Our pooled estimation yields a heteroskedasticity-robust Kleibergen-Paap Wald rk F-statistic of 259, rejecting the null of weak instruments given the Stock-Yogo critical values. A similar albeit weaker outcome emerges from the industry-level estimation.

The industry-level elasticities reported in Table 3 display considerable variation across industries. The estimated scale elasticity or markup margin is highest in the 'Electrical & Optical Equipment' ($\mu = 0.55$) and 'Petroleum' ($\mu = 1.2$) industries; both of which are associated with high R&D or fixed costs. The estimated scale elasticity is lowest in 'Agricultural & Mining' ($\mu = 0.14$) and 'Machinery' ($\mu = 0.12$) industries. Furthermore, with the exception of 'Agriculture & Mining,' we cannot reject the prevalence of scale economies.[49]

Importantly, our estimates indicate that $\frac{\sigma_k - 1}{\gamma_k - 1} \neq 1$ in nearly all industries. This finding rejects the arbitrary link often assumed between the firm-level and national-level degrees of market power in the literature. Our estimates also indicate that

$$Cov(\sigma_k, \mu_k) \approx -0.65,$$

which corroborates the innate tension between the *ToT-improving* and *misallocation-correcting*

[48] Border taxes tend to be a weak instrument for firm-level prices in our sample, but we include to comply with the past literature. These include applied ad-valorem tariffs and the Columbian value-added tax (VAT). We exclude the VAT component in the 'Transportation' and 'Petroleum' industries since the VAT in these industries discriminates by the delivery method and level of luxury—both of which may be correlated with $\Delta \ln \varphi_{\omega jkt}$.

[49] The finding that returns-to-scale are negligible in the agricultural sector aligns with a large body of evidence on the inverse farm-size productivity (IFSP) relationship—see Sen (1962) and subsequent references to IFSP.

objectives, which are the core of Propositions 1 and 2.[50]

Table 3: Industry-level estimation results

Sector	ISIC4 codes	Estimated Parameter $\sigma_k - 1$	$\frac{\sigma_k - 1}{\gamma_k - 1}$	μ_k	Obs.	Weak Ident. Test
Agriculture & Mining	100-1499	6.227	0.891	0.143	11,568	2.40
		(2.345)	(0.148)	(0.059)		
Food	1500-1699	2.303	0.905	0.393	19,615	6.27
		(0.765)	(0.046)	(0.132)		
Textiles, Leather & Footwear	1700-1999	3.359	0.753	0.224	125,120	66.65
		(0.353)	(0.022)	(0.024)		
Wood	2000-2099	3.896	0.891	0.229	5,872	1.41
		(1.855)	(0.195)	(0.120)		
Paper	2100-2299	2.646	0.848	0.320	37,376	3.23
		(1.106)	(0.061)	(0.136)		
Petroleum	2300-2399	0.636	0.776	1.220	3,973	2.83
		(0.464)	(0.119)	(0.909)		
Chemicals	2400-2499	3.966	0.921	0.232	133,142	38.01
		(0.403)	(0.025)	(0.024)		
Rubber & Plastic	2500-2599	5.157	0.721	0.140	106,398	7.16
		(1.176)	(0.062)	(0.034)		
Minerals	2600-2699	5.283	0.881	0.167	27,952	3.53
		(1.667)	(0.108)	(0.056)		
Basic & Fabricated Metals	2700-2899	3.004	0.627	0.209	153,102	20.39
		(0.484)	(0.030)	(0.035)		
Machinery	2900-3099	7.750	0.927	0.120	263,797	12.01
		(1.330)	(0.072)	(0.023)		
Electrical & Optical Equipment	3100-3399	1.235	0.682	0.552	257,775	26.27
		(0.323)	(0.017)	(0.145)		
Transport Equipment	3400-3599	2.805	0.363	0.129	85,920	5.50
		(0.834)	(0.036)	(0.041)		
N.E.C. & Recycling	3600-3800	6.169	0.938	0.152	70,264	11.57
		(1.012)	(0.090)	(0.029)		

Notes. Estimation results of Equation (16). Standard errors in parentheses. The estimation is conducted with HS10 product-year fixed effects. All standard errors are simultaneously clustered by product-year and by origin-product, which is akin to the correction proposed by Adao, Kolesár, and Morales (2019). The weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist, Imbens, and Rubin (1996).

6.4 Challenges to Identification

Our two conditions for identification, (C1) and (C2), can be contested under certain circumstances. Below, we discuss these issues and present additional evidence to address them.

Within-Cluster Correlation in Error Terms. Adao et al. (2019) show that identification based

[50]Our trade elasticity estimates are one of the few based on firm-level data. Traditional estimates of σ_k are typically based on country-level data (e.g., Simonovska and Waugh (2014); Caliendo and Parro (2015)).

on shift-share instruments exhibits an over-rejection problem if regression errors are cross-correlated. In the context of our estimation, this problem will arise if demand shocks are correlated across *firm-origin-product-year* varieties with a similar monthly export composition. We adopt a conservative two-way clustering of standard errors by *product-year* and *origin-product* to handle this issue. Clustering standard errors in this manner is akin to the correction proposed by Adao et al. (2019).

Dynamic Cross-Inventory Effects. Lagged inventory clearances can challenge our identifying assumptions on two fronts. First, firms' optimal pricing decisions may be forward-looking, violating Condition (C1). To address this concern, we reconstruct our shift-share instrument using four lags instead of one. If inventories clear in at most four years, we can deduce that pricing decisions do not internalize expected demand shocks beyond the four-year mark. Hence, $\mathbb{E}\left[\tilde{p}_{jkt-4}(\omega)\,\Delta\ln\varphi_{\omega jkt}\right]=0$, and the new instrument will satisfy the exclusion restriction. The trade-off is that we lose observations, as the instrument is constructible for only firms that continuously export in the 4 different years. The *top* panel of Figure 9 (in Appendix O) compares estimation results under this alternative instrument to the baseline results. The new estimation preserves the ordering and magnitude of our estimated elasticities. More importantly, it retains the negative correlation between σ_k and μ_k, which is the critical assumption underlying Propositions 1 and 2.

Second, with cross-inventory linkages, $\Delta\ln\varphi_{\omega jkt}$ may encompass omitted variables that reflect firms' dynamic inventory management decisions. One of these omitted variables is presumably the exchange rate. If so, $\mathbb{E}\left[\Delta\ln\mathcal{E}_{jt}(m)\,\Delta\ln\varphi_{\omega jkt}\right]\neq 0$, which violates Condition (C2). To address this concern, we reestimate Equation 16 while directly controlling for the annual change in the exchange rate, $\Delta\ln\mathcal{E}_{jt}$. Even if changes in inventory-related demand depend on the changes in the exchange rate, we can still assert that $\mathbb{E}\left[z_{j,kt}(\omega)\,\Delta\ln\varphi_{\omega jkt}\mid\Delta\ln\mathcal{E}_{jt}\right]=0$— i.e., the exclusions restriction is satisfied with the added control, $\Delta\ln\mathcal{E}_{jt}$. The *middle* panel of Figure 9 (in Appendix O) compares estimation results from this alternative specification to the baseline results. Reassuringly, The new estimation preserves the ordering and magnitude of our estimated elasticities and the negative correlation between σ_k and μ_k.

Export Varieties with Significant Market Share. Our identification can come under threat if individual varieties account for a significant fraction of a country's sales to Colombia. In such a case, variety-specific demand shocks can influence the bilateral exchange between the Colombian Peso and the origin country's currency, thereby violating Condition (C2). This concern, however, does not apply to our sample of exporters. The variety with the highest 99th percentile within-national market share accounts for only 0.1% of the origin country's total exports to Colombia. The variety with the highest 90th percentile within-national market share accounts for only 0.0008% of the origin country's total exports to Colombia.

One may remain concerned about large multi-product firms that export multiple product varieties to Colombia in a given year. Consider, for instance, a multi-product firm ω that exports goods k and g to Colombia in year t. If demand shocks are correlated across varieties supplied by this firm (i.e., $\mathbb{E}\left[\Delta\ln\varphi_{\omega jkt}\,\Delta\ln\varphi_{\omega jgt}\right]\neq 0$), Condition (C2) may be violated despite each variety's market share being infinitesimally small. To address this issue, we reestimate Equation 16 on a restricted sample that drops excessively large firms with a total within-national market share that exceeds 0.1%. The *bottom* panel of Figure 9 (in Appendix O) compares estimation results from the trimmed sample to the baseline results. Encouragingly, the ordering and magnitude of the estimated elasticities are preserved across industries. The new estimation also retains the negative correlation between σ_k and μ_k.

6.5 Plausibility of Estimates

We conclude this section, discussing the plausibility of our estimates. We do so by exploring their macro-level implications and comparing them to counterparts in the literature.

Plausibility from the Lens of Macro-Level Predictions. Our scale elasticity estimates can be evaluated based on their prediction about the income-size elasticity. As pointed out by Ramondo, Rodríguez-Clare, and Saborío-Rodríguez (2016), the factual relationship between real per capita income and population size (i.e., the income-size elasticity) is negative and statistically insignificant. Quantitative trade models featuring the normalization $\mu(\sigma-1)=\frac{\sigma-1}{\gamma-1}=1$, however, predict a strong and positive income-size elasticity that remains significant even after the introduction of domestic trade frictions. Ramondo et al. (2016) call this observation the income-size elasticity puzzle. Considering this puzzle, in Appendix R we compute the income-size elasticity implied by our estimated value of $\frac{\sigma-1}{\gamma-1}\approx 0.67$. Encouragingly, we find that our estimated value for $\frac{\sigma-1}{\gamma-1}$ completely resolves the aforementioned puzzle. In other words, our micro-estimated elasticities are consistent with the macro-level cross-national relationship between population size and real per capita income.

Comparison to Counterparts in the Literature. Reassuringly, our estimates align closely with well-known industry-level case studies. Take, for example, our elasticity estimates for the 'Petroleum' industry, which appear somewhat extreme. First, our estimate for σ_k aligns with the consensus in the Energy Economics literature that national-level demand for petroleum products is price-inelastic.[51] Second, our estimated μ_k for the 'Petroleum' industry closely resembles existing estimates in the Industrial Organization literature. Considine (2001), for instance, estimates $\mu\approx 1.15$ using detailed data on the U.S. petroleum industry. Moreover, our finding that the 'Petroleum' industry is the most scale-intensive industry is consistent with the finding in Antweiler and Trefler (2002), which is based on more aggregated data. Likewise, consider the 'Transportation' or auto industry, where our estimated $\mu_k=0.13$ implies an optimal markup of 13%. This estimate aligns with existing estimates from various industry-level studies. Recently, Coşar, Grieco, Li, and Tintelnot (2018) have estimated markups for the auto industry that range between roughly 6% to 13%. Previously, Berry et al. (1995) have estimated markups of around 20% in the U.S. auto industry using data from 1971-1990. Lastly, the scale elasticity can be alternatively estimated using supply-side techniques. These techniques can in principle detect scale externalities unrelated to *love-for-variety*—albeit under strong assumptions about the variability of production inputs. Supply-side estimation techniques, however, cannot separately identify the trade elasticity from the scale elasticity, and are thereby unable to determine $Cov\,(\sigma_k,\mu_k)$— see Appendix Q for further details.

[51] See Pesaran, Smith, and Akiyama (1998) for specific estimates and Fattouh (2007) for a survey of this literature.

7 Quantifying the Consequences of Trade and Industrial Policy

As a final step, we use our estimated values for μ_k and σ_k to quantify the gains from trade and industrial policy for a wide range of countries. Before outlining our quantitive approach, we describe the macro-level data used to discipline our quantitative model.

Trade, Production, and Tariff Data. We take macro-level data on domestic and international production and expenditure from the 2014 World Input-Output Database (WIOD, Timmer, Erumban, Gouma, Los, Temurshoev, de Vries, Arto, Genty, Neuwahl, Francois, et al. (2012)). This database spans 56 industries and 43 countries plus an aggregate of the rest of the world. The list of countries in the sample includes all 27 members of the European Union plus 16 other major economies—all of which are listed in Table 4. Following Costinot and Rodríguez-Clare (2014), we aggregate the 56 WIOD industries into 15 traded industries (for which we have estimated μ_k and σ_k) plus a service sector. Details for our industry aggregation are reported in Table 10 of the appendix. Our baseline analysis normalizes $\mu_k = 0$ and $\sigma_k = 11$ for all service-related industries. In Appendix X, however, we test the sensitivity of our results to alternative normalization choices. We also need to take a stance on applied tariffs and subsidies. We take data on applied tariffs, $t_{ji,k}$, from the UNCTAD-TRAINS database.[52] International data on domestic and export subsidies are not as widely available. Considering that these subsidy measures are generally prohibited by the WTO, we extrapolate that $x_{ij,k} \approx s_{i,k} \approx 0$.

7.1 Mapping Optimal Policy Formulas to Data

The sufficient statistics formulas provided by Theorems 1-3 let us compute the gains from *optimal* policy without appealing to numerical optimization. This feature is particularly advantageous as numerical optimization routines (like MATLAB's FMINCON) have well-known limitations when applied to non-linear models with many free-moving variables.[53] We present our optimization-free procedure focusing on first-best policies under free entry. Appendix S illustrates how a similar procedure can recover policy consequences in other scenarios.

To map our theory to data, we need to take a stance on the cross-industry utility aggregator. As is common in literature, we assume a Cobb-Douglas parameterization, $U_i(\mathbf{Q}_i) = \prod_k Q_{i,k}^{e_{i,k}}$. As explained earlier, we posses data on observable shares, national accounts, and applied taxes. We use $\mathbb{D} = \left\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\right\}_{j,i,k}$ to denote such data.[54] We have estimated the trade and scale elasticity across many industries, and use $\Theta = \{\sigma_k, \mu_k\}$ to denote our set of estimated parameters.

The logic behind our procedure is to (a) specify optimal tax/subsidy choices as a function of the equilibrium variables (e.g., expenditure shares), and (b) specify equilibrium variables as

[52] We closely follow Kucheryavyy et al. (2016) to clean the UNCTAD-TRAINS database and match it with WIOD. To make the data consistent with our theoretical model, we also purge it from trade imbalances following the procedure described in Costinot and Rodríguez-Clare (2014).

[53] Costinot and Rodríguez-Clare (2014) note that computing optimal policy via numerical optimization can become increasingly burdensome when dealing with many free-moving tax instruments. Their optimal tariff analysis is, therefore, limited to a uniform tariff applied to all industries (see P. 227 and the discussion following Figure 4.1)

[54] As explained in Section 2, under free entry, the number of firms operating in *origin n–industry k* can be expressed as $M_{i,k} = \bar{m}_{i,k}\rho_{i,k}$, where $\bar{m}_{i,k}$ is composed of parameters and variables that are invariant to policy. We can, therefore, use $\rho_{i,k}$ to track scale economies that channel through entry—as detailed under Equation 6.

a function of optimal tax/subsidy choices. We then jointly solve the system of equations that combine (a) and (b) to determine welfare outcomes under optimal policy choices. We use the exact hat-algebra notation to accomplish this task, whereby $\hat{z} = z^*/z$ denotes the change in a generic variable when moving from the factual value, z, to the counterfactual value under optimal policy, z^*.

As discussed under Theorem 1, country i's first-best policy schedule in the Cobb-Douglas case is described by the following set of formulas

$$1 + s^*_{i,k} = 1 + \mu_k; \qquad 1 + t^*_{ji,k} = 1 + \omega^*_{ji,k}; \qquad 1 + x^*_{ij,k} = \frac{(\sigma_k - 1)\sum_{n \neq i}\left[(1 + \omega^*_{ni,g})\lambda^*_{nj,k}\right]}{1 + (\sigma_k - 1)(1 - \lambda^*_{ij,k})};$$

where superscript "$*$" indicates that a variable is evaluated in the counterfactual optimal policy equilibrium. Using the hat-algebra notation and our expression for the good-specific supply elasticity, $\omega_{ji,k}$ (Equation 10), we can write the above formulas in changes as follows:[55]

$$\text{[optimal import tax]} \quad 1 + t^*_{ji,k} = \frac{-\frac{\mu_k}{1+\mu_k}\hat{r}_{ji,k} r_{ji,k} \Phi^*_{ji,k}}{1 - \frac{\mu_k}{1+\mu_k}\sum_{\iota \neq i}\left(\hat{r}_{j\iota,k} r_{j\iota,k}\left[1 + (\sigma_k - 1)(1 - \hat{\lambda}_{j\iota,k}\lambda_{j\iota,k})\right]\right)}$$

$$\text{[optimal export subsidy]} \quad 1 + x^*_{ij,k} = \frac{(\sigma_k - 1)\sum_{n \neq i}\left[(1 + t^*_{ni,g})\hat{\lambda}_{nj,k}\lambda_{nj,k}\right]}{1 + (\sigma_k - 1)(1 - \hat{\lambda}_{ij,k}\lambda_{ij,k})},$$

$$\text{[change in taxes]} \quad \widehat{1 + s_{i,k}} = \frac{1 + \mu_k}{1 + s_{i,k}}; \qquad \widehat{1 + t_{ji,k}} = \frac{1 + t^*_{ji,k}}{1 + t_{ji,k}}; \qquad \widehat{1 + x_{ij,k}} = \frac{1 + x^*_{ij,k}}{1 + x_{ij,k}}. \tag{17}$$

Since the rest of the world is passive in their use of taxes, $\widehat{1 + s_{n,k}} = \widehat{1 + t_{jn,k}} = \widehat{1 + x_{nj,k}} = 1$ for all $n \neq i$. To determine the change in expenditure shares, $\hat{\lambda}_{ji,k}$, we need to determine the change in consumer price indexes. Invoking the CES structure of within-industry demand, we can express the change in *market i–industry k*'s consumer price index as

$$\text{[price indexes]} \quad \hat{\tilde{P}}_{i,k} = \sum_{n \in \mathbb{C}}\left(\lambda_{ni,k}\left[\frac{\widehat{1 + t_{ni,k}}}{(\widehat{1 + x_{ni,k}})(\widehat{1 + s_{n,k}})}\hat{w}_n \hat{\rho}_{n,k}^{-\mu_k}\right]^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}. \tag{18}$$

Recall that $\rho_{n,k} = L_{n,k}/L_n$ denotes industry k's sales share in origin n, which—under free entry—is equal to the share of origin n's workers employed in that industry. The above formulation uses the fact that, by free entry, $\hat{M}_{i,k} = \hat{\rho}_{i,k}$. Given $\hat{\tilde{P}}_{i,k}$, we can calculate the change in expenditure and revenue shares as follows:

$$\text{[expenditure shares]} \quad \hat{\lambda}_{ji,k} = \left[\frac{\widehat{1 + t_{ji,k}}}{(\widehat{1 + x_{ji,k}})(\widehat{1 + s_{j,k}})}\hat{w}_j \hat{\rho}_{j,k}^{-\mu_k}\right]^{1-\sigma_k} \hat{\tilde{P}}_{i,k}^{\sigma_k - 1}$$

$$\text{[revenue shares]} \quad \hat{r}_{ji,k} = \left(\frac{\widehat{1 + x_{ji,k}}}{\widehat{1 + t_{ji,k}}}\hat{\lambda}_{ji,k}\hat{Y}_i\right)\left(\sum_{n \in \mathbb{C}}\frac{\widehat{1 + x_{jn,k}}}{\widehat{1 + t_{jn,k}}}\hat{\lambda}_{jn,k}\hat{Y}_n\right)^{-1}. \tag{19}$$

The change in the wage rate, $\hat{w}_i$, and industry-level sales shares, $\hat{\rho}_{i,k}$, are dictated by the labor

[55] The multiplier $\Phi^*_{ji,k} = 1 - \left(1 - \frac{1}{\mu_k}\right)(\sigma_k - 1)\sum_{\iota \neq i}\left[\frac{\hat{r}_{i\iota,k} r_{i\iota,k}}{\hat{r}_{ji,k} r_{ji,k}}\hat{\lambda}_{j\iota,k}\lambda_{j\iota,k}\right]\frac{\hat{\rho}_{i,k}\rho_{i,k}\hat{w}_i w_i L_i}{\hat{\rho}_{j,k}\rho_{j,k}\hat{w}_j w_j L_j}$ accounts for cross-demand effects in foreign markets—see Equation 10 from Section 3.

market clearing (LMC) condition, which ensures that industry-level sales match wage payments, industry by industry:

$$[\text{LMC}] \quad \hat{\rho}_{i,k}\rho_{i,k}\hat{w}_i w_i L_i = \sum_{j\in\mathbb{C}}\left[\frac{(1+x^*_{ij,k})(1+s^*_{i,k})}{1+t^*_{ij,k}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]; \qquad \sum_{k\in\mathbb{K}}\hat{\rho}_{i,k}\rho_{i,k} = 1. \quad (20)$$

The change in national expenditure, $\hat{Y}_i$, is governed by the balanced budget (BB) condition, which ensures that total expenditure matches total income from wage payments and tax revenues:

$$\begin{aligned}[\text{BB}] \quad \hat{Y}_i Y_i =& \hat{w}_i w_i L_i - \sum_{k\in\mathbb{K}}\left[s^*_{i,k}\hat{\lambda}_{ii,k}\lambda_{ii,k}e_{i,k}\hat{Y}_i Y_i\right] \\ &+ \sum_{j\neq i}\sum_{k\in\mathbb{K}}\left(\frac{t^*_{ji,k}}{1+t^*_{ji,k}}\lambda_{ji,k}\hat{\lambda}_{ji,k}e_{i,k}\hat{Y}_i Y_i + \frac{1-(1+x^*_{ij,k})(1+s^*_{i,k})}{1+t^*_{ij,k}}\lambda_{ij,k}\hat{\lambda}_{ij,k}e_{j,k}\hat{Y}_j Y_j\right). \quad (21)\end{aligned}$$

Equations 17-21 represent a system of $2N + NK + (2(N-1)+1)\,K$ independent equations and unknowns. The independent unknown variables are $\hat{w}_i$ (N unknowns), $\hat{Y}_i$ (N unknowns), $\hat{\rho}_{i,k}$ (NK unknowns), $\widehat{1+s_{i,k}}$ (K unknowns), $\widehat{1+t_{ji,k}}$ ($(N-1)K$ unknowns), and $\widehat{1+x_{ij,k}}$ ($(N-1)K$ unknowns). Solving the aforementioned system is possible with information on observable data points, $\mathbb{D}$, and estimated parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. Once we solve this system, the welfare consequences of country i's optimal policy are automatically determined. The following proposition outlines this result.[56]

Proposition 3. *Suppose we have data on observable shares, national accounts, and applied taxes, $\mathbb{D} = \left\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\right\}_{j,i,k}$, and information on structural parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. We can determine the economic consequences of country i's optimal policy by calculating $\mathbb{X} = \left\{\hat{Y}_i, \hat{w}_i, \hat{\rho}_{i,k}, \widehat{1+s_{i,k}}, \widehat{1+t_{ji,k}}, \widehat{1+x_{ij,k}}\right\}$ as the solution to the system of Equations 17-21. After solving for $\mathbb{X}$, we can fully determine the welfare consequence of country i's optimal policy as*

$$\hat{W}_i = \hat{Y}_i / \prod_{k\in\mathbb{K}} \hat{\tilde{P}}_{i,k}^{e_{i,k}}, \qquad (\forall n\in\mathbb{C})$$

where $\hat{\tilde{P}}_{i,k}$ is determined by Equation 18 as a function of $\mathbb{X}$ and data, $\mathbb{D}$.

To take stock, the optimization-free procedure described by Proposition 3 simplifies the task of computing the gains from *first-best* trade and industrial policies. We can use a similar procedure (based on Theorems 2 and 3) to compute the gains from second-best trade policies—see Appendix S. Without Proposition 3, we would have to rely on numerical optimization to recover country i's optimal policy.[57] As noted earlier, numerical optimization can become increasingly difficult-to-implement when dealing with many free-moving policy variables. Furthermore, in many instances, obtaining credible results from numerical optimization requires specialized commercial solvers like SNOPT or KNITRO. Propositions 3's optimization-free procedure allows us to bypass such complications, delivering notable gains in computational speed and accuracy.

[56]Under Proposition 3, the optimal policy specification uses our approximation for $\omega_{ji,k}$. In Appendix U, we examine the accuracy of our approximation and outline how our optimization-free approach can be alternatively conducted with an exact formula for $\omega_{ji,k}$.

[57]Such a problem is typically formulated as a Mathematical Programming with Equilibrium Constraints (MPEC) problem–see Ossa (2014) for further details.

7.2 The Consequences of Non-Cooperative Policies

Our first set of results elucidate policy consequences when governments act non-cooperatively. These results convey two basic points. First, non-cooperative trade taxes are ineffective at correcting misallocation in domestic industries—even without retaliation. Second, the cost of retaliation is sizable. Together these results point to little justification for non-cooperative trade taxation even in 2nd-best economies plagued with misallocation.

Table 4 reports the gains from optimal non-cooperative policies under free and restricted entry. The first three columns in each case report welfare gains assuming the rest of the world does not retaliate. The fourth column reports net welfare effects after retaliation. The 1st-best non-cooperative policy consists of Pigouvian subsidies, import tariffs, and export subsidies (Theorem 1); the 2nd-best consists of only import tariffs and export subsidies (Theorem 2); the 3rd-best consists of only import tariffs (Theorem 3).

We can draw two main conclusions from Table 4. First, trade taxes are a poor 2nd-best substitute for Pigouvian subsidies. Trade taxes can replicate only 1/3 of the welfare gains attainable under the 1st-best policy that combine Pigouvian subsidies with trade taxes.[58] Under free entry, the 1st-best non-cooperative policy increases welfare by 3.01% on average, whereas 2nd-best trade taxes/subsidies raise welfare by only 1.19%. 3rd-best import tariffs (not paired with export subsidies) are half as effective.

These results reflect the tension between the terms-of-trade and allocate efficiency emphasized by Proposition 1. Since our estimated scale and trade elasticities satisfy $Cov(\sigma_k, \mu_k) < 0$, correcting inter-industry misallocation with trade policy worsens the terms-of-trade—making it difficult for 2nd-best trade policies to strike a balance between these two policy targets. We elucidate this point further in Appendix V by artificially raising $Cov(\sigma_k, \mu_k)$ and recomputing the gains from policy. The results displayed in Figure 11 of the appendix point to a sharp rise in the efficacy of 2nd-best trade taxes as $Cov(\sigma_k, \mu_k)$ is artificially inflated.

Second, we find that retaliation more than wipes out the gains from non-cooperative taxation. The net effect of non-cooperative policies *after retaliation* is a welfare loss of 1.20% under free entry. Retaliation, in our calculation, occurs through the reciprocal adoption of optimal trade taxes by trading partners. As noted in Section 4, the cost of retaliation may not deter a short-sighted government from erecting trade taxes—at least when trade taxes are a less politically-controversial instrument for correcting misallocation. In such cases, our finding that trade policy is incapable of improving misallocation should serve as a deterrent.

7.3 The Gains from International Cooperation

Suppose governments recognize the danger of non-cooperation and limit themselves to the cooperative policy outlined in Section 3. As we show next, cooperative countries risk *immiserizing growth* if they take the lead in policy implementation. This issue can cause a race to the bottom in industrial policy implementation, but can be resolved via a deep agreement.

[58] This finding echoes the numerical result in Balistreri and Markusen (2009) that optimal tariffs yield smaller gains in the presence of positive firm-level markups.

Table 4: *The Gains from Non-Cooperative Policies and the Consequences of Retaliation*

	Restricted Entry				Free Entry			
Country	1st-best	2nd-best trade tax	3rd-best import tax	post retaliation	1st-best	2nd-best trade tax	3rd-best import tax	post retaliation
AUS	0.90%	0.21%	0.14%	-0.83%	2.31%	0.58%	0.35%	2.85%
AUT	1.31%	0.66%	0.45%	-1.67%	2.07%	1.11%	0.58%	-2.34%
BEL	1.31%	0.69%	0.50%	-3.07%	1.62%	0.90%	0.55%	-4.13%
BGR	2.01%	0.63%	0.52%	0.04%	5.48%	1.88%	0.83%	0.23%
BRA	1.87%	0.23%	0.17%	0.93%	4.32%	0.64%	0.36%	3.21%
CAN	1.72%	0.57%	0.45%	-1.10%	3.37%	1.19%	0.46%	0.34%
CHE	1.04%	0.56%	0.46%	-0.92%	1.27%	0.76%	0.53%	-1.26%
CHN	1.76%	0.27%	0.24%	1.43%	3.70%	0.39%	0.28%	2.90%
CYP	1.75%	0.68%	0.61%	-4.20%	5.47%	1.56%	1.43%	-10.88%
CZE	1.68%	0.88%	0.54%	-1.70%	2.86%	1.60%	0.73%	-2.13%
DEU	1.71%	0.78%	0.49%	-0.37%	2.74%	1.34%	0.65%	-0.10%
DNK	1.22%	0.58%	0.48%	-2.41%	1.94%	0.91%	0.49%	-4.78%
ESP	1.53%	0.52%	0.39%	-0.15%	2.53%	1.03%	0.49%	0.29%
EST	1.18%	0.62%	0.45%	-4.86%	2.69%	1.42%	0.56%	-7.90%
FIN	1.40%	0.54%	0.25%	-0.76%	2.00%	0.83%	0.47%	-1.08%
FRA	1.20%	0.45%	0.34%	-0.78%	2.49%	1.10%	0.50%	0.01%
GBR	1.09%	0.48%	0.42%	-0.65%	2.10%	1.03%	0.58%	0.17%
GRC	1.82%	0.59%	0.53%	-0.20%	2.58%	1.08%	0.68%	-0.04%
HRV	1.03%	0.55%	0.45%	-2.27%	1.79%	0.73%	0.52%	-2.99%
HUN	2.25%	1.06%	0.73%	-2.85%	4.08%	2.28%	0.96%	-3.33%
IDN	2.00%	0.36%	0.26%	-0.37%	4.81%	1.45%	0.50%	2.42%
IND	1.81%	0.35%	0.31%	1.60%	4.24%	1.10%	0.37%	2.82%
IRL	0.86%	0.68%	0.53%	-2.07%	1.49%	0.89%	0.40%	-3.32%
ITA	1.50%	0.46%	0.26%	0.36%	2.75%	0.95%	0.48%	0.80%
JPN	1.48%	0.32%	0.22%	0.31%	2.84%	0.75%	0.42%	1.55%
KOR	2.12%	0.65%	0.49%	0.14%	4.37%	1.62%	0.73%	1.25%
LTU	2.50%	0.93%	0.76%	-1.38%	3.54%	1.26%	0.85%	-2.47%
LUX	0.93%	0.81%	0.78%	-3.59%	0.84%	1.15%	1.01%	-4.90%
LVA	0.91%	0.54%	0.44%	-4.26%	1.31%	0.80%	0.46%	-4.85%
MEX	2.24%	0.60%	0.44%	-0.92%	4.96%	1.36%	0.76%	1.96%
MLT	1.35%	0.92%	0.82%	-3.75%	2.05%	1.39%	1.06%	-4.81%
NLD	1.35%	0.66%	0.54%	-3.44%	1.86%	0.98%	0.59%	-3.73%
NOR	1.19%	0.40%	0.27%	-0.74%	2.02%	0.81%	0.41%	-0.33%
POL	2.19%	0.76%	0.65%	-0.26%	4.03%	1.62%	0.81%	0.80%
PRT	2.04%	0.74%	0.65%	-0.36%	3.85%	1.72%	0.80%	0.73%
ROU	2.05%	0.77%	0.67%	-0.42%	3.97%	1.72%	0.98%	1.15%
RUS	2.39%	0.32%	0.27%	0.84%	5.25%	1.39%	0.37%	2.80%
SVK	2.01%	1.06%	0.79%	-2.17%	3.17%	2.09%	1.08%	-2.73%
SVN	1.42%	0.87%	0.67%	-3.09%	1.35%	1.20%	0.90%	-4.92%
SWE	1.21%	0.62%	0.45%	-0.95%	1.54%	0.77%	0.49%	-1.48%
TUR	1.43%	0.46%	0.32%	-1.52%	3.52%	1.34%	0.61%	-6.33%
TWN	2.18%	0.69%	0.56%	-0.94%	4.99%	1.85%	0.79%	0.81%
USA	1.53%	0.32%	0.27%	0.69%	3.04%	0.80%	0.30%	2.15%
Average	1.59%	0.60%	0.47%	-1.23%	3.01%	1.19%	0.63%	-1.20%

Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The *1st best* policy is characterized by Theorem 1; *2nd best trade taxes* are characterized by Theorem 2; and *3rd best import taxes* are characterized by Theorem 3. *Post retaliation* corresponds to a situation were the home country sets its 1st-best policies and the RoW retaliates.

7.3.1 The Immiserizing Growth Effects of Unilateral Industrial Policy

Suppose countries limit themselves to the cooperative policy choice consisting of zero trade taxes and (markup-correcting) Pigouvian subsidies. Industrial policy implementation, under this arrangement, can be obstructed by immiserizing growth effects (Proposition 2). In this section, we show that these effects are quantitatively important.

Table 5 reports the welfare consequences of unilateral and coordinated industrial policy implementation. The policy applied in each case is a set of Pigouvian subsidies that restore marginal cost-pricing in the local economy. Unilateral implementation corresponds to a scenario where the home country implements its industrial policy but trading partners do not reciprocate and stick to business as usual. Coordinated implementation corresponds to a reciprocal implementation of industrial policies worldwide.

Table 5: *Industrial Policy and Immiserizing Growth*

	Restricted Entry		Free Entry	
	Unilateral	Coordinated	Unilateral	Coordinated
Gains from *Corrective* Industrial Subsidies	-0.39%	1.65%	-2.78%	3.42%

Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The columns titled *unilateral* reports welfare gains when a country unilaterally adopts industrial subsidies that restore marginal cost pricing in the domestic economy. The columns titled *multilateral* reports welfare gains when all countries simultaneously adopt industrial subsidies that restore marginal cost pricing globally. The average gains are calculated as the simple average across all 43 countries in the WIOD sample. Country-level results are reported in Appendix W.

The results in Table 5 confirm the strong immiserizing growth effects of unilateral scale correction. Real income in the average country drops by more than 2.7% if corrective industrial policies are implemented unilaterally. By comparison, welfare increases by more than 3.4% under coordinated or reciprocal implementation. These results suggest that we may be witnessing a *race to the bottom* in industrial policy implementation—without a deep agreement to ensures reciprocity in implementation. As things stand, cooperative countries two two choices: (i) immplement scale correction and risk immiserizing growth, or (ii) violate their commitments to cooperation by pairing corrective subsidies with trade restrictions.

It is worth emphasizing that the immiserizing growth effects reported in Table 5 stem from the tension between misallocation and the terms of trade (ToT). Given that our estimated scale and trade elasticities satisfy $Cov(\sigma_k, \mu_k) < 0$, restoring allocate efficiency with Pigouvian subsidies worsens one's ToT, to the point of causing immiserizing growth. We confirm this point in Appendix V by artificially raising $Cov(\sigma_k, \mu_k)$ and recomputing the gains from unilateral scale (or markup) correction. As $Cov(\sigma_k, \mu_k)$ is artificially inflated relative to its estimated value, immiserizing growth effects fade and are even reversed (see Figure 12 of the appendix).

7.3.2 The Gains from Deep vs. Shallow Cooperation

Recall from Section 4 that we can model international cooperation as a two-stage process:

i. The first stage involves a *shallow agreement* that disciplines non-cooperative trade taxes helping countries avert a full-fledged trade war.

ii. The second stage involves a *deep agreement* that ensures reciprocity in industrial policy implementation, helping countries avoid a race to the bottom.

Figure 2 reports the welfare gains associated with each stage. The blue bars correspond to the welfare gains brought by the existing nexus of shallow agreements. These gains are computed relative to a counterfactual equilibrium where all countries adopt their non-cooperative trade taxes. The blue bars correspond to the prospective-but-unrealized welfare gains from deep cooperation. These gains are computed as the welfare gains associated with a universal implementation of (markup-correcting) industrial policies.[59] As discussed earlier, a *deep agreement* is necessary to uncover these welfare gains.

The welfare gains from shallow cooperation are on average 3.2%. That is, the average country is poised to lose 3.2% of its real income if trade taxes are counterfactually raised to their non-cooperative level everywhere. The existing nexus of shallow agreements have already materialized these gains. The prospective gains from deep cooperation are 1.6% under restricted entry. That is, if countries can agree to a reciprocal implementation of markup-correcting industrial policies, they can boost their real income by an additional 1.6%. Similar but larger welfare gains will occur under free entry.

Our estimated gains from shallow cooperation relate to the theoretical arguments in Bagwell and Staiger (2001, 2004). As explained earlier, shallow cooperation is sufficient for global efficiency if (i) $Cov\left(\mu_k, \sigma_k\right) \geq 0$, or (ii) governments play a one-shot game where they simultaneously choose and implement their best-policy response with the belief that others do the same. Our micro-level estimation rejected the former condition. Exhaustive literature on the history of international policy coordination disputes the latter condition. If we suspect that governments move sequentially in the implementation stage and are not convinced about reciprocity in policy implementation, then deep cooperation is necessary for global efficiency.

7.3.3 A Stronger Case for International Cooperation

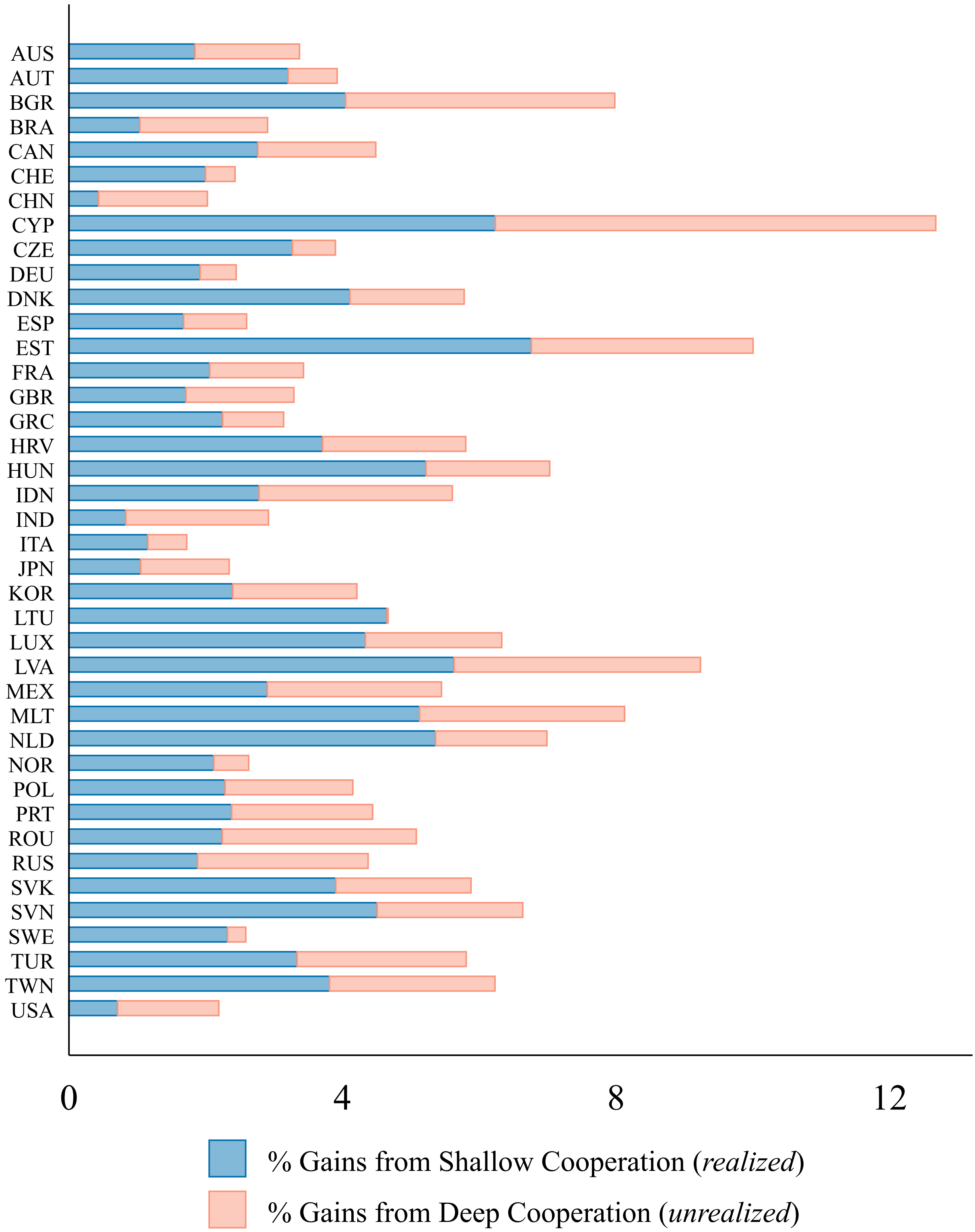
The standard argument for international cooperation recognizes that governments can reap short-term gains if they adopt non-cooperative trade taxes. But these short-term gains will turn into losses if trading partners retaliate. The standard argument may, thus, fail to deter a short-sighted government from taking the non-cooperative route. After all, from the lens of traditional theories, the only way to reap short-term welfare gains (relative to the status quo) is to adopt non-cooperative trade restrictions and hope for delayed retaliation.

Our quantitative analysis unveils a stronger augment for international cooperation. Starting from the status quo, a government seeking welfare improvements has two options: (1) engage in a coordinated industrial policy effort, or (2) raise non-cooperative trade taxes to reap short-lived ToT benefits. We find that the former option not only delivers sustainable welfare gains but strictly dominates the latter option even before we adjust for the cost of retaliation.

This point is illustrated in Figure (3). The *x-axis* corresponds to the unrealized gains from deep cooperation. The *y-axis* corresponds to the maximal short-term gains from (non-cooperative) unilateral policies, which transpire before retaliation. For most economies, the unrealized

[59] The gains from deep cooperation can be computed with the aid of the optimal policy formulas specified under Equation 9 and the logic presented earlier under Section (7.1).

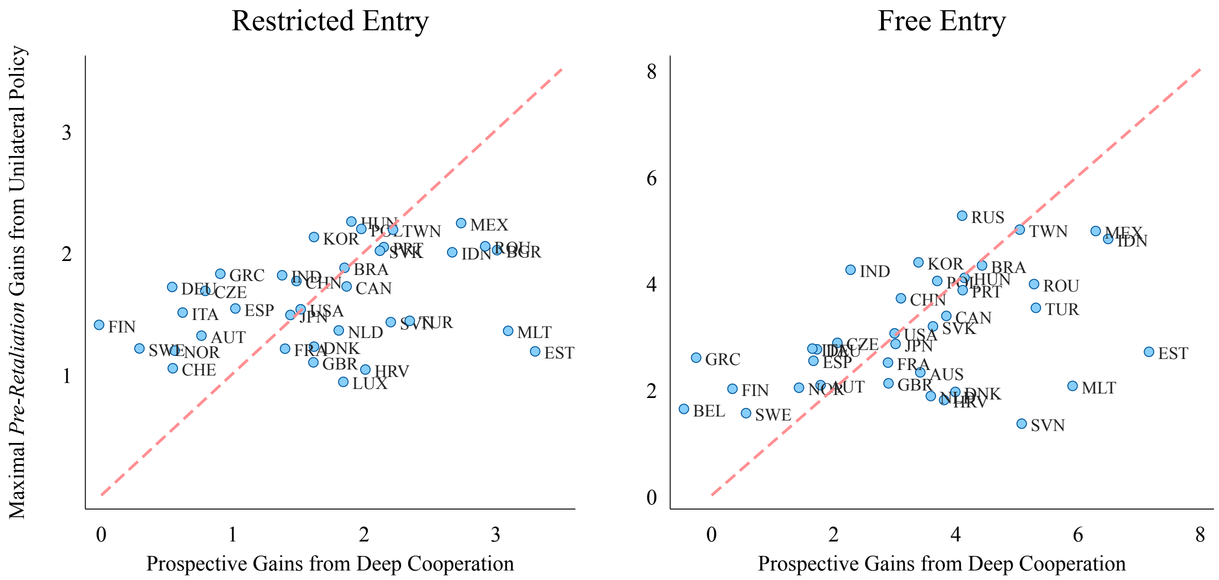
Figure 2: The welfare gains from deep and shallow cooperation



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). Welfare effects are computed under restricted entry. The gains from *deep cooperation* correspond to welfare gains when moving from the status quo to the globally efficient equilibrium in which all countries coordinate their corrective Pigouvian subsides. The gains from *shallow cooperation* correspond to the avoided welfare losses when moving from the status quo to an non-cooperative equilibrium where all countries adopt Nash trade taxes and subsidies.

gains from deep cooperation dominate even the short-term gains from unilateral policy interventions. This finding indicates that the spillover gains from corrective policies in the rest of the world exceed the short-term ToT gains from non-cooperative taxes—echoing our earlier claim that the ToT gains from policy are limited in scope, even before retaliation.[60]

Figure 3: *Deep cooperation vs. unilateral policy interventions*



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The gains from 1st best non-cooperative policy are the gains when each country implements the policy characterized by Theorem 1 and the rest of the world is passive. The gains from global cooperation correspond to a scenario where all countries forgo trade taxation and apply industrial subsidies that restore marginal cost pricing.

7.4 Sensitivity Analysis

In Appendix X we recalculate the gains from policy under several alternative specifications. First, we recompute the gains assuming that the data-generating process is a *Melitz-Pareto* model. Second, we recompute the gains based on alternative values for σ_k and μ_k, which are estimated via a two-ways fixed effects estimation (as reported in Appendix P). Lastly, we recompute the gains from policy under a more conservative set of values assigned to μ_k and σ_k in services. In all cases, trade policy turns out to be a poor second-best instrument for resorting allocative efficiency. Another noteworthy observation is that accounting for firm-selection effects à la Melitz (2003), magnifies the gains from (first-best) optimal policies. However, these greater gains are primarily driven by the larger misallocation-correcting gains. If anything, second-best trade taxes/subsidies are even less effective at replicating the firs-best policy gains in the presence of firm-selection into export markets.

What parameter values would imply larger gains from policy? We analyze this question in Appendix Y, noting that the gains from optimal policy are increasing in two statistics: (i) the

[60] Interestingly, the gains from deep cooperation favor small countries that have a comparative disadvantage in high-returns-to-scale (or high-profit) industries, *e.g.*, Estonia, Malta, and Slovenia. The intuition is that these countries depend relatively more on imported varieties in high-returns-to-scale industries and under deep cooperation, these industries are subsidized across the globe.

cross-industry variance of the scale elasticities, $\text{Var}(\log \mu_k)$, and (ii) the average of the (inverse) trade elasticities, $\mathbb{E}\left[\frac{1}{\sigma_k-1}\right]$. In Appendix Y, we adjust our estimated parameter values to artificially increase both of theses statistics, and recompute the gains from policy under these artificial parameter values. The results are reported in Figure 17 of the same appendix. They reveal that the gains from optimal policy nearly double for all countries if we artificially increase $\text{Var}(\log \mu_k)$ by a factor of about three. The policy gains for different countries, however, exhibit different degrees of sensitivity to an artificial increase in $\mathbb{E}\left[\frac{1}{\sigma_k-1}\right]$—with the gains for larger countries like the U.S. or China being noticeably less sensitive. The intuition is that $\text{Var}(\log \mu_k)$ governs the gains from correcting misallocation, whereas $\mathbb{E}\left[\frac{1}{\sigma_k-1}\right]$ regulates the extent to which countries can improve their ToT. For large countries, where trade accounts for a smaller fraction of the GDP, there is less scope for raising real GDP via ToT improvements—hence, the lower sensitivity of policy gains to $\mathbb{E}\left[\frac{1}{\sigma_k-1}\right]$.

8 Concluding Remarks

For centuries, scale economies have served as a justification for controversial trade and industrial policy practices. Yet we know surprisingly little about the actual empirics of trade and industrial policy in increasing returns to scale industries. Against this backdrop, we took a preliminary step toward identifying the force of industry-level scale economies using micro-level trade data. Our estimates indicated that trade restrictions are a poor second-best policy for correcting misallocation from industry-level scale economies. Unilateral industrial policy can be equally ineffective, as it triggers immiserizing growth in most countries. However, industrial policies coordinated via deep agreements deliver welfare gains that are more transformative than any unilateral policy intervention.

We used our micro-estimated scale elasticities to uncover a range of macro-level policy implications, but our estimates have an even broader reach. Three issues, in particular, deserve closer attention. First, our scale elasticity estimates can help disentangle the relative contribution of scale economies and Ricardian comparative advantage to patterns of international specialization. This is an old topic of interest for which our empirical understanding is surprisingly limited.

Second, our estimates can shed fresh light on the puzzlingly large income gap between advanced and emerging economies. Economists have always hypothesized that a fraction of this income gap stems from asymmetric specialization across low- and high returns-to-scale industries. An empirical assessment of this hypothesis is impeded by a lack of comprehensive estimates for industry-level scale elasticities. Our micro-level estimates pave the way for an empirical exploration in this direction.

Finally, we are the first to empirically document the negative cross-industry correlation between trade elasticities and firm-level markups. This relationship is crucial for policy evaluation in open economies, as it creates tension between the terms-of-trade and allocative efficiency. Our theoretical model is purposely agnostic about the direction and origins of this relationship. We leave it to future research to explore this particular matter more profoundly.

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Theoretical Appendix

A The Redundancy of Consumption Taxes

Without loss of generality suppose country $i \in \mathbb{C}$ imposes a full set of tax instruments, while the rest of the world is passive. Now, consider any arbitrary combination of taxes (indexed by A) that includes (i) industrial (or domestic production) subsidies, $s^A_{i,k}$, (ii) domestic consumption taxes, $\tau^A_{i,k}$, (iii) import taxes, $t^A_{ji,k}$, and (iv) export subsidies, $x^A_{ij,k}$. This set of tax instruments –which includes consumption taxes– produces the following wedges between producer and consumer price indexes for various product varieties:

$$\tilde{P}^A_{ii,k} = \frac{1+\tau^A_{i,k}}{1+s^A_{i,k}} P_{ii,k}; \qquad \tilde{P}^A_{ji,k} = (1+t^A_{ji,k})(1+\tau^A_{i,k}) P_{ji,k}; \qquad \tilde{P}^A_{ij,k} = \frac{1}{(1+x^A_{ij,k})(1+s^A_{i,k})} P_{ij,k}; \quad (j \neq i)$$

Our claim here is that the same wedges can be replicated without appealing to consumption taxes, $\tau_{i,k}$. This claim can be established by considering an alternative tax schedule, B, which excludes consumption taxes (i.e., $1+\tau^B_{i,k} = 0$), but includes the following set of production subsidies, export subsidies, and import taxes:

$$1+s^B_{i,k} = \frac{1+s^A_{i,k}}{1+\bar{\tau}^A_{i,k}}; \qquad 1+t^B_{ji,k} = \left(1+t^A_{ji,k}\right)\left(1+\tau^A_{i,k}\right); \qquad 1+x^B_{ij,k} = (1+x^A_{ij,k})(1+\tau^A_{i,k})$$

It is straightforward to see that schedule B can reproduce the same wedge between producer and consumer prices as the original schedule A (i.e., $\tilde{\mathbf{P}}^A = \tilde{\mathbf{P}}^B$). In particular,

$$\begin{aligned}
\tilde{P}^B_{ii,k} &= \frac{1}{1+s^B_{i,k}} P_{ii,k} = \frac{1+\tau^A_{i,k}}{1+s^A_{i,k}} P_{ii,k} = \tilde{P}^A_{ii} \\
\tilde{P}^B_{ji,k} &= (1+t^B_{ji,k}) P_{ji,k} = (1+t^A_{ji,k})(1+\tau^A_{i,k}) P_{ji,k} = \tilde{P}^A_{ji,k} \\
\tilde{P}^B_{ij,k} &= \frac{1}{(1+x^B_{ij,k})(1+s^B_{i,k})} P_{ij,k} = \frac{1}{(1+x^A_{ij,k})(1+s^A_{i,k})} P_{ij,k} = \tilde{P}^A_{ij,k}.
\end{aligned}$$

It also follows trivially that $\tilde{P}^B_{nj,k} = P_{nj,k} = \tilde{P}^A_{nj,k}$ if $n, j \neq i$, because the rest of the world does not impose taxes.[61] This equivalence indicates that consumption taxes are redundant if the government has access to the other three sets of instruments. Note that the same can be said about production subsidies. More specifically, the effect of industry-level production subsidies can be perfectly replicated with a combination of consumption taxes, import taxes, and export subsidies. However, due to *product differentiation*, if two (of the $2(N-1)+2$) tax instruments are restricted, the replication argument fails. That is, if both production subsidies and consumption taxes are restricted, export subsidies and import taxes cannot fully replicate their effect.

[61]Note that the rest of the world imposing or not imposing taxes, does not matter for the redundancy of consumption taxes. The above argument can be easily extrapolated to the case where all countries impose arbitrary taxes.

B Proof of Lemma 1

Consider two policy-wage combinations, $\mathbf{T} = (\mathbf{s}, \mathbf{t}, \mathbf{x}; \mathbf{w})$, and $\mathbf{T}' = (\mathbf{s}', \mathbf{t}', \mathbf{x}'; \mathbf{w}')$, that differ in uniform shifters a and $\tilde{a} \in \mathbb{R}_+$:

$$\begin{cases} \mathbf{1} + \mathbf{x}'_i = a\left(\mathbf{1} + \mathbf{x}_i\right) & \mathbf{1} + \mathbf{x}'_{-i} = \mathbf{1} + \mathbf{x}_{-i} \\ \mathbf{1} + \mathbf{t}'_i = a\left(\mathbf{1} + \mathbf{t}_i\right) & \mathbf{1} + \mathbf{t}'_{-i} = \mathbf{1} + \mathbf{t}_{-i} \\ \mathbf{1} + \mathbf{s}'_i = \left(\mathbf{1} + \mathbf{s}_i\right)/\tilde{a} & \mathbf{1} + \mathbf{s}'_{-i} = \mathbf{1} + \mathbf{s}_{-i} \\ w'_i = \left(a/\tilde{a}\right) w_i & \mathbf{w}'_{-i} = \mathbf{w}_{-i} \end{cases}.$$

Our goal is to prove that (i) if $\mathbf{T} \in \mathbb{F}$ then $\mathbf{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ for all $n \in \mathbb{C}$. To prove these claims, we appeal to two intermediate lemmas. The first lemma establishes the following: Suppose equilibrium quantities are identical under policy-wage vectors $\mathbf{T}$ and $\mathbf{T}'$ (*i.e.*, $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k). Then, the implied nominal income and price levels under $\mathbf{T}$ and $\mathbf{T}'$ are the same up to a scale. The second lemma is a standard result from consumer theory: It indicates the nominal income and price levels implied by the first lemma confirm the original assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Below, we state and prove the first of these lemmas for any $a \in \mathbb{R}_+$.

Lemma 2. *$Q_{jn,k}(\boldsymbol{T}') = Q_{jn,k}(\boldsymbol{T})$ for all jn, k* $\Longrightarrow \begin{cases} \tilde{\mathbf{P}}_i\left(\mathbf{T}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbf{T}\right); & \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}\right) \\ Y_i\left(\mathbf{T}'\right) = aY_i\left(\mathbf{T}\right); & \mathbf{Y}_{-i}\left(\mathbf{T}'\right) = \mathbf{Y}_{-i}\left(\mathbf{T}\right) \end{cases}$

Proof. Our goal is to compute nominal income and consumer prices under $\mathbf{T}$ and $\mathbf{T}'$ starting from the assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. We start our proof with nominal prices: To simplify the notation, define $\delta_{jn,k}(\mathbf{T}) \equiv \bar{\rho}_{jn,k} \mathcal{Q}_{j,k}(\mathbf{T})^{-\frac{\mu_k}{1+\mu_k}}$. Note that –by assumption– $\delta_{jn,k}(\mathbf{T}) = \delta_{jn,k}(\mathbf{T}') = \bar{\delta}_{jn,k}$. First, consider the price of a typical good ji, k imported by i from origin $j \neq i$. Using Equations 6 and 7, the consumer price of ji, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows:

$$\tilde{P}_{ji,k}(\mathbf{T}') = \bar{\delta}_{ji,k} \frac{1 + t'_{ji,k}}{(1 + x'_{ji,k})(1 + s'_{j,k})} w'_j = \bar{\delta}_{ji,k} \frac{a(1 + t_{ji,k})}{(1 + x_{ji,k})(1 + s_{j,k})}\, w_j = a\tilde{P}_{ji,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + t'_{ji,k} = a(1 + t_{ji,k})$, while $w'_j = w_j$, $x'_{ji,k} = x_{ji,k}$, and $s'_{j,k} = s_{j,k}$ (since $w_j \in \mathbf{w}_{-i}$, $x_{ji,k} \in \mathbf{x}_{-i}$, and $s_{j,k} \in \mathbf{s}_{-i}$). Second, consider a typical good ii, k that is produced and consumed locally in country i. The consumer price of ii, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows

$$\tilde{P}_{ii,k}(\mathbf{T}') = \bar{\delta}_{ii,k} \frac{1}{1 + s'_{i,k}} w'_i = \bar{\delta}_{ii,k} \frac{1}{\frac{1}{\tilde{a}}(1 + s_{i,k})} \times \frac{a}{\tilde{a}} w_i = a\tilde{P}_{ii,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + s'_{i,k} = (1 + s_{i,k})/\tilde{a}$ and $w'_i = aw_i/\tilde{a}$. Third, consider the price of a typical good ij, k export by i to destination market $j \neq i$. The consumer price of ij, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows:

$$\tilde{P}_{ij,k}(\mathbf{T}') = \bar{\delta}_{ij,k} \frac{1 + t'_{ij,k}}{(1 + x'_{ij,k})(1 + s'_{i,k})} w'_i = \bar{\delta}_{ij,k} \frac{1 + t_{ij,k}}{a(1 + x'_{ij,k}) \times \frac{1}{\tilde{a}}(1 + s'_{i,k})} \times \frac{a}{\tilde{a}} w_i = \tilde{P}_{ij,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + x'_{ij,k} = a(1 + x_{ij,k})$, $1 + s_{i,k} = (1 + s'_{i,k})/\tilde{a}$, and $w'_i = aw_i/\tilde{a}$; while $t'_{ij,k} = t_{ji,k}$ since $t_{ji,k} \in \mathbf{t}_{-i}$. Lastly, is follows trivially that $\tilde{P}_{jn,k}(\mathbf{T}') = \tilde{P}_{jn,k}(\mathbf{T})$ if j and $n \neq i$. Considering that $\tilde{\mathbf{P}}_i = \left\{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\right\}$, the above equations establish that

$$\tilde{\mathbf{P}}_i\left(\mathbf{T}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbf{T}\right), \qquad \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}\right).$$

Next, we turn to our claim about nominal income levels. To simplify the presentation, we hereafter use $X \equiv X(\mathbf{T})$ and $X' \equiv X(\mathbf{T}')$ to denote the value of a generic variable X under policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$. Keeping in mind this choice of notation, country i's nominal income under $\mathbf{T}'$, i.e., $Y_i' \equiv Y_i(\mathbf{T}')$ is given by:

$$
\begin{aligned}
Y_i' &= w_i' L_i + \sum_k \left[\left(\frac{1}{1+s_{i,k}'} - 1 \right) P_{ii,k}' Q_{ii,k}' \right] + \sum_k \sum_{j \neq i} \left(\frac{t_{ji,k}'}{(1+x_{ji,k}')(1+s_{j,k}')} P_{ji,k}' Q_{ji,k}' + \left[\frac{1}{(1+x_{ij,k}')(1+s_{i,k}')} - 1 \right] P_{ij,k}' Q_{ij,k}' \right) \\
&= w_i' L_i + \sum_k \left[\left(1 - [1+s_{i,k}'] \right) \tilde{P}_{ii,k}' Q_{ii,k}' \right] + \sum_k \sum_{j \neq i} \left(\left(1 - \frac{1}{1+t_{ji,k}'} \right) \tilde{P}_{ji,k}' Q_{ji,k}' + \left[\frac{1}{1+t_{ij,k}'} - \frac{(1+x_{ij,k}')(1+s_{i,k}')}{1+t_{ij,k}'} \right] \tilde{P}_{ij,k}' Q_{ij,k}' \right).
\end{aligned}
$$

Note that, by assumption, policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$ exhibit the same output schedule, i.e., $Q_{ii,k}' = Q_{ii,k}$, $Q_{ji,k}' = Q_{ji,k}$, and $Q_{ij,k}' = Q_{ij,k}$. Also, recall that ($\mathbf{T}$ and $\mathbf{T}'$ are constructed such that) $1 + t_{ji,k}' = a(1+t_{ji,k})$, $1 + x_{ij,k}' = a(1+x_{ij,k})$, $1 + s_{i,k} = (1+s_{i,k}')/\tilde{a}$, and $w_i' = a w_i/\tilde{a}$, $t_{ij,k}' = t_{ji,k}$. Considering these relationships and plugging our earlier result that (i) $\tilde{P}_{ii,k} = a P_{ii,k}$, (ii) $P_{ji,k}' = a \tilde{P}_{ji,k}$, and (iii) $\tilde{P}_{ij,k}' = \tilde{P}_{ij,k}$ into the above equation, yields the following expression for Y_i' :

$$
\begin{aligned}
Y_i' = \frac{a}{\tilde{a}} w_i L_i + \sum_k \left[\left(1 - \frac{1}{\tilde{a}} (1+s_{i,k}) \right) a \tilde{P}_{ii,k} Q_{ii,k} \right] \\
+ \sum_{j,k} \left[\left(1 - \frac{1}{a(1+t_{ji,k})} \right) a \tilde{P}_{ji,k} Q_{ji,k} + \left[\frac{1}{1+t_{ij,k}} - \frac{a(1+x_{ij,k}) \times \frac{1}{\tilde{a}}(1+s_{i,k})}{1+t_{ij,k}} \right] \tilde{P}_{ij,k} Q_{ij,k} \right].
\end{aligned}
$$

Appealing to the balanced trade condition, $\sum_k \sum_{j \neq i} \left(\frac{1}{1+t_{ji,k}} \tilde{P}_{ji,k} Q_{ji,k} - \frac{1}{1+t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) = 0$, and observing that $(1+s_{i,k}) \tilde{P}_{ii,k} = P_{ii,k}$ and $\frac{(1+x_{ij,k})(1+s_{i,k})}{1+t_{ij,k}} \tilde{P}_{ij,k} = P_{ij,k}$, the above equation reduces to

$$
Y_i' = \frac{a}{\tilde{a}} w_i L_i + a \sum_k \left[\tilde{P}_{ii,k} Q_{ii,k} + \sum_{j \neq i} \tilde{P}_{ji,k} Q_{ji,k} \right] - \frac{a}{\tilde{a}} \sum_k \left[P_{ii,k} Q_{ii,k} + \sum_{j \neq i} P_{ij,k} Q_{ij,k} \right].
$$

Invoking the labor market clearing condition, $w_i L_i - \sum_k \sum_n P_{ijn,k} Q_{in,k} = 0$, the above equation further simplifies as follows

$$
Y_i' = a \sum_k \left[\tilde{P}_{ii,k} Q_{ii,k} + \sum_{j \neq i} \tilde{P}_{ji,k} Q_{ji,k} \right] = a \left[w_i L_i + \mathcal{R}_i \right] = a Y_i,
$$

where $\mathcal{R}_i \equiv \mathcal{R}_i(\mathbf{T})$ denotes country i's tax revenues under $\mathbf{T}$. To bel clear, the third line, in the above equation, follows from country i's balanced budget condition (i.e., total expenditure = total income). Turning to the rest of the world: The fact that $Y_n(\mathbf{T}') = Y_n(\mathbf{T})$ for all $n \neq i$ follows trivially from a similar line of arguments–hence, establishing our claim about nominal income levels:

$$
Y_i \left(\mathbf{T}' \right) = a Y_i \left(\mathbf{T} \right); \qquad \mathbf{Y}_{-i} \left(\mathbf{T}' \right) = \mathbf{Y}_{-i} \left(\mathbf{T} \right)
$$

□

Lemma 2 (proved above) starts from the assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Our next lemma indicates that this assumption is validated by the nominal income and price levels implied by $\mathbf{T}$ and $\mathbf{T}'$. Below, we state this lemma noting that it follows trivially from the Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, being homogeneous of degree zero.

Lemma 3. $\forall a \in \mathbb{R}_+$: $\begin{cases} \tilde{\mathbf{P}}_i \left(\mathbf{T}' \right) = a \tilde{\mathbf{P}}_i \left(\mathbf{T} \right) \\ Y_i(\mathbf{T}') = a Y_i(\mathbf{T}) \end{cases} \implies Q_{ji,k}(\mathbf{T}') = Q_{ji,k}(\mathbf{T})$ *for all* ji, k

Together, Lemmas 3 and 2 establish that equilibrium quantities should be indeed identical under policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$—i.e., $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Hence, if $\mathbf{T} \in \mathbb{F}$ it follows immediately that (i) $\mathbf{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ for all $n \in \mathbb{C}$, which is the claim of Lemma 1.

C Nested-Eaton and Kortum (2002) Framework

Here we show that the nested CES import demand function specified by Assumption (A1), can also arise from within-product specialization à la Eaton and Kortum (2002). To this end, suppose that each industry k is comprised of a continuum of homogenous goods indexed by ν. The sub-utility of the representative consumer in country i with respect to industry k is a log-linear aggregator across the continuum of goods in that industry:

$$Q_{i,k} = \int_0^1 \ln \tilde{q}_{i,k}(\nu) d\nu$$

As in our main model, country j hosts $\overline{M}_{j,k}$ firms indexed by ω, with $\Omega_{j,k}$ denotes the set of all firms serving industry k from country j.[62] Each firm ω supplies good ν to market i at the following *quality-adjusted* price:

$$\tilde{p}_{ji,k}(\nu; \omega) = \tilde{p}_{ji,k}(\omega) / \varphi(\nu; \omega),$$

where $\tilde{p}_{ji,k}(\omega)$ is a nominal price (driven by production costs) that applies to all goods supplied by firm ω in industry k, while the quality component, $\varphi(\nu; \omega)$, is good$\times$firm-specific. Suppose for any given good ν, firm-specific qualities are drawn independently from the following nested Fréchet joint distribution:

$$F_k(\boldsymbol{\varphi}(\boldsymbol{\nu})) = \exp\left[-\sum_{i=1}^{N} \left(\sum_{\omega \in \Omega_{i,k}} \varphi(\nu; \omega)^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k}} \right],$$

The above distribution generalizes the basic Fréchet distribution in Eaton and Kortum (2002). In particular, it relaxes the restriction that productivities are perfectly correlated across firms within the same country. Instead, it allows for sub-national productivity differentiation and also for the degrees of cross- and sub-national productivity differentiations (ϑ_k and θ_k, respectively) to diverge. A special case of the distribution where $\vartheta_k \longrightarrow \infty$ corresponds to the standard Eaton and Kortum (2002) specification.

The above distribution also has deep theoretical roots. The Fisher–Tippett–Gnedenko theorem states that if ideas are drawn from a (normalized) distribution, in the limit the distribution of the best draw takes the form of a general extreme value (GEV) distribution, which includes the above Fréchet distribution as a special case. A special application of this result can be found in Kortum (1997) who develops an idea-based growth model where the limit distribution of productivities is Fréchet, with $\varphi_{\omega,k}$ reflecting the stock of technological knowledge accumulated by firms ω in category k.

Given the vector of effective prices, the representative consumer in county i (who is endowed with income Y_i) maximizes their real consumption of each good, $\tilde{q}_{i,k}(\nu) = e_{i,k} Y_i / \tilde{p}_{i,k}(\nu)$, by choosing $\tilde{p}_{i,k}(\nu) = \min_\omega \{\tilde{p}_{ji,k}(\omega)\}$. That being the case, the consumer's discrete choice problem for each good ν can be expressed as:

$$\min_\omega \ \tilde{p}_{ji,k}(\omega) / z(\nu; \omega) \sim \max_\omega \ \ln z(\nu; \omega) - \ln \tilde{p}_{ji,k}(\omega).$$

[62] The implicit assumption here is that entry is restricted, so that $\overline{M}_{j,k}$ is exogenous.

To determine the share of goods for which firm ω is the most competitive supplier, we can invoke the theorem of "General Extreme Value." Specifically, define $G(\tilde{\boldsymbol{p}}_i)$ as follows

$$G_k(\tilde{\boldsymbol{p}}_i) = \sum_{j=1}^{N} \left(\sum_{\omega \in \Omega_{j,k}} \exp(-\vartheta_k \ln \tilde{p}_{ji,k}(\omega)) \right)^{\frac{\theta_k}{\vartheta_k}} = \sum_{j=1}^{N} \left(\sum_{\omega \in \Omega_{j,k}} \tilde{p}_{ji,k}(\omega)^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k}}.$$

Note that $G_k(.)$ is a continuous and differentiable function of vector $\tilde{\boldsymbol{p}}_i \equiv \{\tilde{p}_{ji,k}(\omega)\}$ and has the following properties:

i. $G_k(.) \geq 0$;

ii. $G_k(.)$ is a homogeneous function of rank θ_k: $G_k(\rho \tilde{\mathbf{p}}_i) = \rho^{\theta_k} G_k(\tilde{\mathbf{p}}_i)$ for any $\rho \geq 0$;

iii. $\lim_{\tilde{p}_{ji,k}(\omega) \to \infty} G_k(\tilde{\mathbf{p}}_i) = \infty$, $\forall \omega$;

iv. the m'th partial derivative of $G_k(.)$ with respect to a generic combination of m variables $\tilde{p}_{ji,k}(\omega)$, is non-negative if m is odd and non-positive if m is even.

Manski and McFadden (1981) prove that if $G_k(.)$ satisfies the above conditions, and $\varphi(\nu;\omega)$'s are drawn from distribution,

$$F_k(\boldsymbol{\varphi}(\nu)) = \exp\left(-G_k(e^{-\ln \boldsymbol{\varphi}})\right) = \exp\left(-\sum_{j=1}^{N} \left(\sum_{\omega \in \Omega_{j,k}} \varphi(\nu;\omega)^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k}} \right),$$

then the probability of choosing variety ω (from origin j in industry k) is given by

$$\pi_{ji,k}(\omega) = \frac{\left(\frac{\tilde{p}_{ji,k}(\omega)}{\theta_k} \right) \frac{\partial G_k(\tilde{\mathbf{p}}_i)}{\partial p_{ji,k}(\omega)}}{G_k(\tilde{\mathbf{p}}_i)} = \frac{\tilde{p}_{ji,k}(\omega)\, \tilde{p}_{ji,k}(\omega)^{\vartheta_k - 1} \left(\sum_{\omega' \in \Omega_{j,k}} \tilde{p}_{ji,k}(\omega')^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k} - 1}}{\sum_{n=1}^{N} \left(\sum_{\omega' \in \Omega_{j,k}} \tilde{p}_{ji,k}(\omega')^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k}}}.$$

Rearranging the above equation yields the following expression for probability shares,

$$\pi_{ji,k}(\omega) = \left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}} \right)^{-\vartheta_k} \left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}} \right)^{-\theta_k},$$

where $\tilde{P}_{ji,k} \equiv \left[\sum_{\omega' \in \Omega_{j,k}} \tilde{p}_{ji,k}(\omega')^{-\vartheta_k} \right]^{-1/\vartheta_k}$ and $\tilde{P}_{i,k} \equiv \left[\sum \tilde{P}_{ji,k}^{-\theta_k} \right]^{-\frac{1}{\theta_k}}$. Given that the probability shares coincide with the share of goods sourced from firm ω, total sales of firm ω to market i, in industry k can be calculated as:

$$\tilde{p}_{ji,k}(\omega) q_{ji,k}(\omega) = \tilde{p}_{ji,k}(\omega) \frac{\pi_{ji,k}(\omega) e_{i,k} Y_i}{\tilde{p}_{ji,k}(\omega)} = \left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}} \right)^{-\vartheta_k} \left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}} \right)^{-\theta_k} e_{i,k} Y_i$$

which is identical to the nested-CES function specified by Assumption (A1), with corresponding substitution parameters $\gamma_k - 1 = \vartheta_k$ and $\sigma_k - 1 = \theta_k$.

D Firm-Selection under Melitz-Pareto

In this appendix, we outline the isomorphism between our baseline model and one that admits selection effects. In doing so, we borrow heavily from Kucheryavyy et al. (2016) (KLR, hereafter). We rely on three key assumptions, hereafter:

i. Within-industry demand is governed by the same nested-CES utility function presented under Assumption (A1). As in the baseline mode, σ_k and γ_k respectively denote the upper- and lower-tier elasticities of substitution.

ii. The firm-level productivity distribution, $G_{i,k}(z)$, is Pareto with shape parameter, ϑ_k.

iii. The fixed "marketing" cost is paid in terms of labor in the destination market.

iv. Taxes are applied before the markup, and operate as a cost-shifter.

Following KLR, we also assume that cross-industry utility aggregator is Cobb-Douglas, with $e_{i,k}$ denoting the constant share of country i's expenditure on industry k. Following the derivation in KLR, we can define the effective supply of production labor in country i as

$$\widetilde{L}_i = \left[1 - \sum_k e_{i,k}\left(\frac{\vartheta_k - \gamma_k + 1}{\vartheta_k \gamma_k}\right)\right] L_i.$$

The labor market clearing condition is, accordingly, given by $\sum w_i L_{i,k} = w_i \widetilde{L}_i$. With regards to aggregate markup levels, we can appeal to the well-known result that the profit margin in each industry is constant and given by the following expression:

$$\text{mark-up} \sim \frac{\sum_n P_{in,k} Q_{in,k}}{w_i L_{i,k}} = \frac{\gamma_k \vartheta_k}{(\gamma_k - 1)(\vartheta_k + 1) - \vartheta_k}.$$

With regards to aggregate demand functions, we can follow the derivation in Appendix B.2 of KLR to express demand for national-level variety ji,k as

$$Q_{ji,k} = \left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\sigma_k^{\text{Melitz}}} Q_{i,k},$$

where $\sigma_k^{\text{Melitz}} \equiv 1 + \vartheta_k\left[1 + \vartheta_k\left(\frac{1}{\sigma_k - 1} - \frac{1}{\gamma_k - 1}\right)\right]^{-1}$ denotes the trade elasticity under firm-selection. Moreover, we can show that national-level producer price indexes are given by the following formulation:

$$P_{ij,k}^{\text{Melitz}} = \begin{cases} \bar{\varrho}_{ij,k} w_i & \text{if entry is restricted} \\ \bar{\varrho}'_{ij,k} w_i Q_{i,k}^{-\frac{\vartheta_k}{1+\vartheta_k}} & \text{if entry is free} \end{cases},$$

where $\bar{\varrho}_{ij,k}$ and $\bar{\varrho}'_{ij,k}$ are composed of structural parameters that are invariant to policy–this includes ϑ_k that regulates firm selection.[63] Abstracting from taxes, $\tilde{P}_{i,k} = \left(\sum P_{ji,k}^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}$ is the CES industry-level consumer price index that shows up in indirect utility $V_i(.)$. Referring to our earlier result about constant markup margins, aggregate profits in country i given by

$$\Pi_i^{\text{Melitz}} = \begin{cases} \sum_k \sum_j \left(\frac{\frac{\gamma_k \vartheta_k}{(\gamma_k-1)(\vartheta_k+1)-\vartheta_k}}{1 + \frac{\gamma_k \vartheta_k}{(\gamma_k-1)(\vartheta_k+1)-\vartheta_k}} P_{ij,k} Q_{ij,k}\right) & \text{if entry is restricted} \\ 0 & \text{if entry is free} \end{cases}.$$

To fixe ideas, recall that we used μ_k to denote both (1) the scale elasticity under free entry, and (2) the profit margin under restricted entry in the baseline model. This overlapping choice of notation was motivated by the observation that in the generalized Krugman model, the scale elasticity (under

[63]Unlike $\tilde{P}_{i,k}$, the national-level indexes, $\tilde{P}_{ji,k}$, are not the same as the CES price indexes defined in the main text, but this is not problematic from the point of the isomorphism result we are seeking.

free entry) and the profit margin (under restricted entry) are identical and equal to $\mu_k = \frac{1}{\gamma_k - 1}$. This equivalence, though, was not used to derive any of our theorems. Instead, it was only invoked to simplify the presentation of our theorems. Evidently, under the Melitz-Pareto model the equivalence between the scale elasticity and the profit margin crumbles. Taking note of this nuance, the Melitz-Pareto model is isomorphic to our baseline model with the following reinterpretation of parameters:

$$1 + \mu_k^{\text{Melitz}} = \begin{cases} 1 + \frac{1}{\vartheta_k} & \text{if entry is free} \\ \frac{\gamma_k \vartheta_k}{(\gamma_k - 1)(\vartheta_k + 1) - \vartheta_k} & \text{if entry is restricted} \end{cases}; \qquad \sigma_k^{\text{Melitz}} = 1 + \frac{\vartheta_k}{1 + \vartheta_k \left(\frac{1}{\sigma_k - 1} - \frac{1}{\gamma_k - 1} \right)}.$$

The Marshallian demand elasticities in the Melitz-Pareto model are accordingly given by the following equations as a function σ_k^{Melitz} and expenditure shares:

$$\varepsilon_{ji,k}^{(ji,k)} = -1 - (\sigma_k^{\text{Melitz}} - 1) \left(1 - \lambda_{ji,k}\right); \qquad \varepsilon_{ji,k}^{(ji,k)} = \sigma_k^{\text{Melitz}} \lambda_{ji,g}$$

In the above expressions, γ_k and σ_k can be taken directly from our firm-level demand estimation. Doing so, identifies the Melitz-Pareto model's key parameters up to a Pareto shape parameter, ϑ_k. To obtain an estimate for ϑ_k, we can estimate the trade elasticity, $\sigma_k^{\text{Melitz}} - 1$, using macro-level trade data and standard techniques from the literature. Given the estimated trade elasticities, we can simply recover ϑ_k by plugging our micro-level estimates for γ_k and σ_k into the expression for σ_k^{Melitz}.

D.1 The Case where Taxes are Applied After Markups

Our derivation, above, assumed that taxes are applied before the markup, and act as a cost shifter. Below, we discuss how relaxing this assumption may affect the arguments listed above. To this end, we focus on the spacial case where preferences are non-nested. Namely,

$$\text{non-nested preferences} \sim \sigma_k = \gamma_k, \qquad \forall k \in \mathbb{K}.$$

Following the Online Appendix 5 in Costinot and Rodríguez-Clare (2014), the trade elasticity in the Melitz-Pareto model with non-nested preferences is described by the following formulation:

$$\sigma_k^{\text{Melitz}} = \begin{cases} 1 + \vartheta_k & \text{tax applied before markup} \\ \frac{\sigma}{\sigma - 1} \vartheta & \text{tax applied after markup} \end{cases}.$$

Appealing to the above formulation, we can show that *Theorem 1* nests, as a special case, the optimal tariff formula derived by Demidova and Rodriguez-Clare (2009) for a small open economy in a *single-industry×two-country* Melitz-Pareto model. To demonstrate this, drop the industry subscript k and reduce the global economy into two countries, i.e., $\mathbb{C} = \{i, j\}$. Noting that $1 - \lambda_{ij} = \lambda_{jj}$ in the two-country case, we can deduce from the above formulation and Theorem 1 that

$$\frac{1 + t_{ji}^*}{1 + x_{ij}^*} = 1 + \frac{1}{(\sigma^{\text{Melitz}} - 1)\lambda_{jj}} = \frac{1}{(\frac{\sigma}{\sigma - 1}\vartheta - 1)\lambda_{jj}}.$$

By the Lerner symmetry, export and import taxes are equivalent in the single-industry model.[64] Hence, without loss of generality, we can set $x_{ij}^* = 0$. Moreover, if country i is a small open economy, then $\lambda_{jj} \approx 1$. Combining these two observations, we can arrive at the familiar-looking optimal tariff for-

[64] The Lerner symmetry is a special case of the equivalence result presented under Lemma 1. Also, note that the market equilibrium is efficient in the single industry Krugman model studied by Gros (1987). As such, the optimal industrial subsidy can be normalized to zero, i.e., $s_i^* = 0$.

mula in Demidova and Rodriguez-Clare (2009):

$$t_{ji}^{*} = \frac{\frac{\sigma-1}{\sigma}}{\vartheta - \frac{\sigma-1}{\sigma}} \sim \text{ small open economy w/ one traded sector.}$$

E Proof of Theorem 1

Our proof proceeds in five steps. The first four steps characterize the optimal tax/subsidy schedule for country $i \in \mathbb{C}$ under *free entry*. The last step demonstrates that this characterization can be extrapolated to the case with *restricted entry*.

Step #1: Express Equilibrium Variables as function of $\tilde{\mathbb{P}}_i$ and w

Our goal is to characterize optimal policy for country $i \in \mathbb{C}$ assuming the rest of the world is passive in their use of taxes: $\mathbf{t}_{-i} = \mathbf{x}_{-i} = \mathbf{s}_{-i} = \mathbf{0}$. To simplify the proof, we reformulate country i's optimal policy problem as one where the government chooses the optimal consumer prices (rather than the actual taxes) associated with its economy. By construction, country i's optimal tax schedule can be recovered from its optimal consumer-to-producer price ratios. The first step in reformulating the optimal policy problem is to express equilibrium variables (e.g., $Q_{ji,k}$, Y_i, etc.) as a function of (1) the vector of consumer prices associated with economy i, $\tilde{\mathbb{P}}_i \equiv \left\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\right\}$, where

$$\tilde{\mathbf{P}}_{ii} \equiv \{P_{ii,k}\}_k; \qquad \tilde{\mathbf{P}}_{ji} \equiv \left\{P_{ji,k}\right\}_{j\neq i,k}; \qquad \tilde{\mathbf{P}}_{ij} \equiv \left\{P_{ij,k}\right\}_{j\neq i,k} \tag{22}$$

and (2) the vector of national-level wage rates across the world,

$$\mathbf{w} = \{w_1, ..., w_N\}.$$

The following lemma shows that our desired formulation of equilibrium variables follows from (a) treating $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$ as given, and (b) solving a system that satisfies all equilibrium conditions excluding the labor market clearing condition.

Lemma 4. *All equilibrium outcomes (excluding $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$) can be uniquely determined as a function of $\tilde{\mathbb{P}}_i \equiv \left\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\right\}$, and $\mathbf{w}$.*

Proof. As noted above, the proof follows from solving all equilibrium conditions excluding the equilibrium expression for consumer prices, $\tilde{P}_{ji,k}$ (which are encompassed by $\tilde{\mathbb{P}}_i$), and the country-specific balanced trade conditions (which pin down $\mathbf{w}$).[65] Stated formally, we need to solve the following system treating $\tilde{\mathbb{P}}_i$, and $\mathbf{w}$ as given:

$$\begin{aligned}
&[\text{optimal pricing}] && P_{jn,k} = \bar{\rho}_{ji,k} w_j \left[\sum_i \bar{a}_{ji,k} Q_{ji,k}\right]^{-\frac{\mu_k}{1+\mu_k}} \\
&[\text{optimal consumption}] && Q_{jn,k} = \mathcal{D}_{jn,k}(Y_n, \tilde{\mathbf{P}}_{1n}, ...\tilde{\mathbf{P}}_{Nn}) \\
&[\text{RoW imposes zero taxes}] && \tilde{P}_{jn,k} = P_{jn,k} \quad (\tilde{P}_{jn,k} \notin \tilde{\mathbb{P}}_i); \qquad Y_n = w_n L_n \quad (n \neq i) \\
&[\text{Balanced Budget in } i] && Y_i = w_i L_i + \left(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}\right) \cdot \mathbf{Q}_{ii} + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right) \cdot \mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right) \cdot \mathbf{Q}_{ji},
\end{aligned}$$

where "$\cdot$" denotes the inner product operator for equal-sized vectors (i.e., $\mathbf{a} \cdot \mathbf{b} = \sum_n a_n b_n$). Since there is a unique equilibrium, the above system is exactly identified in that it uniquely determines $P_{jn,k}$, $Q_{jn,k}$, and Y_n as a function of $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$. □

[65] Note that by Walras' law, the balanced trade condition is equivalent to the labor market clearing condition in each country.

Following Lemma 4, we can express total income in country i, Y_i, as well as the entire demand schedule in that country as follows:

$$Y_i \equiv Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}); \qquad Q_{ji,k} \equiv Q_{ji,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) = \mathcal{D}_{ji,k}\left(Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}), \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\right).$$

Recall that $\mathcal{D}_{ji,k}(.)$ denotes the Marshallian demand function facing variety ji,k. Taking note of the above representation, our main objective is to reformulate country i's policy problem as one where the government chooses $\tilde{\mathbb{P}}_i$ (as opposed to directly choosing tax rates). This reformulation, though, needs to take into account that $\mathbf{w}$ is an equilibrium outcome that implicitly depends on the choice of $\tilde{\mathbb{P}}_i$. To track this constraint, we define the $(\tilde{\mathbb{P}}_i; \mathbf{w})$ combinations that are feasible as follows.

Definition 1. *A policy-wage combination $(\tilde{\mathbb{P}}_i; \boldsymbol{w})$ is **feasible** iff given $\tilde{\mathbb{P}}_i$, the vector of wages, $\boldsymbol{w}$, satisfies the balanced trade condition in every country $n \in \mathbb{C}$. In particular,*

$$(\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P \quad \iff \quad \begin{cases} \sum_{j \neq n} \sum_{k \in \mathbb{K}} \left[P_{jn,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) Q_{jn,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) - P_{nj,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) Q_{nj,k}(\tilde{\mathbb{P}}_i; \mathbf{w})\right] = 0 & \text{if } n \neq i \\ \sum_{j \neq n} \sum_{k=1}^{K} \left[P_{ji,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) Q_{jn,k}(\tilde{\mathbb{P}}_i; \mathbf{w}) - \tilde{P}_{ij,k} Q_{nj,k}(\tilde{\mathbb{P}}_i; \mathbf{w})\right] = 0 & \text{if } n = i \end{cases}.$$

To elaborate on the above definition: The balanced trade condition for a generic country $n \in \mathbb{C}$ can be expresses as $\sum_{j \neq n,k} \left[\frac{1}{1+t_{jn,k}} \tilde{P}_{nj,k} Q_{jn,k} - \frac{1}{1+t_{nj,k}} \tilde{P}_{nj,k} Q_{nj,k}\right]$. The expression for the balanced trade condition, above, follows from the assumption that only country i imposes taxes and the rest of the world is passive. We should emphasize one more time that by Walras' law the satisfaction of the balanced trade condition is analogous to the satisfaction of the labor market clearing condition in each country. Relatedly, take note of the equivalence between $\mathbb{F}_P$ and $\mathbb{F}$–with the latter being defined in the main text under Definition (D2). Taking note of these implicit details, we now proceed to reformulate the optimal policy problem (P1).

Step #2: Reformulate the Optimal Tariff Problem

Before proceeding with the second step of the proof, we formally present our notation for partial derivatives. We will rely heavily on this choice of notation, especially in the subsequent steps of the proof where we derive the first-order conditions.

Notation [*Partial Derivative*] *Let $f(x_1, x_2)$ be a function of two variables, where $x_2 = g(x_1)$ is possibly an implicit function of x_1. We henceforth use*

$$\left(\frac{\partial f(x_1, x_2)}{\partial x_1}\right)_{x_2} = \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1}$$

to denote the derivative of $f(.)$ w.r.t. x_1, treating $x_2 = \bar{x}_2$ as a constant.[66]

Moving on with Step 2, recall the original formulation of the optimal policy problem (P1) from Section 2:

$$\max_{\mathbb{T}_i} \; W_i(\mathbb{T}_i; \text{w}) \qquad s.t. \;\; (\mathbb{T}_i; \text{w}) \in \mathbb{F} \qquad \text{(P1)}$$

In the above formulation, $\mathbb{T}_i \equiv (\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i)$ denotes country i's vector of taxes and $\mathbb{F}$ is defined according to Definition (D2, Section 2) and analogously to $\mathbb{F}_P$. Our next intermediate result shows that Problem

[66]Based on the above notation and the chain rule, the full derivative of $f(.)$ w.r.t. x_1 is given by

$$\frac{\mathrm{d}f(x_1, x_2)}{\mathrm{d}x_1} = \left(\frac{\partial f(x_1, x_2)}{\partial x_1}\right)_{x_2} + \left(\frac{\partial f(x_1, x_2)}{\partial x_2}\right)_{x_2} \frac{\mathrm{d}g(x_1)}{\mathrm{d}x_1}$$

(P1) can be alternatively cast as one where the government chooses the optimal vector of consumer prices $\tilde{\mathbb{P}}_i$ associated with its economy. After determining $\tilde{\mathbb{P}}_i$, the optimal tax vectors, $\mathbf{t}_i^*$, $\mathbf{x}_i^*$, and $\mathbf{s}_i^*$ can be automatically recovered from the optimal consumer-to-producer price ratios.

Lemma 5. *Country i's vector of optimal taxes, $\{\boldsymbol{t}_i^*, \boldsymbol{x}_i^*, \boldsymbol{s}_i^*\}$, can be determined by solving the following problem instead of (P1):*

$$\max_{\tilde{\mathbb{P}}_i} \; W_i(\tilde{\mathbb{P}}_i; \mathrm{w}) \equiv V_i(Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \quad s.t. \; \begin{cases} (\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P \\ \mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}, \end{cases} \qquad (\widetilde{\text{P1}}),$$

where $\overline{\mathbf{w}}_{-i}$ denotes the vector wages in the rest of the world under the status quo.

Proof. The proof consists of two parts. First, we can verify that there is a one-to-one correspondence between the optimal choice w.r.t. $\tilde{\mathbb{P}}_i \equiv \left\{\tilde{\mathbf{P}}_{ii}^*, \tilde{\mathbf{P}}_{ji}^*, \tilde{\mathbf{P}}_{ij}^*\right\}$ and $\mathbb{T}_i^* \equiv \{\mathbf{t}_i^*, \mathbf{x}_i^*, \mathbf{s}_i^*\}$. More specifically, given information on $\tilde{\mathbb{P}}_i$ (and the accompanying wage vector $\mathbf{w}^*$), we can uniquely recover the optimal tax/subsidy rates using the following set of equations:

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}(\tilde{\mathbb{P}}_i, \mathbf{w}^*)}; \qquad 1 + x_{ij,k}^* = \frac{P_{ji,k}(\tilde{\mathbb{P}}_i, \mathbf{w}^*) / \tilde{P}_{ji,k}^*}{P_{ii,k}(\tilde{\mathbb{P}}_i, \mathbf{w}^*) / \tilde{P}_{ii,k}^*}; \qquad 1 + s_{i,k}^* = \frac{P_{ii,k}(\tilde{\mathbb{P}}_i, \mathbf{w}^*)}{\tilde{P}_{ii,k}^*}.$$

The correspondence presented above, indicates an equivalence between choosing $\tilde{\mathbb{P}}_i$ versus choosing $\mathbb{T}_i$ directly. That is,

$$\max_{\tilde{\mathbb{P}}_i} \; W_i(\tilde{\mathbb{P}}_i; \mathrm{w}) \;\; s.t. \; (\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P \; \sim \; \max_{\mathbb{T}_i} \; W_i(\mathbb{T}_i; \mathrm{w}) \;\; s.t. \; (\mathbb{T}_i; \mathbf{w}) \in \mathbb{F}.$$

Second, we must rationalize the constraint on foreign wages, $\mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}$. This constraint in the two-country case ($N = 2$) follows directly from Walras' law. Beyond that, it follows from Walras' law *and* the existence of cooperative buffers, which preserve relative wages in the rest of the world—see Appendix G for details.[67] □

Step #3. Deriving and Simplifying the System of First-Order Conditions

This step derives and solves the system of first-order necessary conditions (F.O.C.s) associated with Problem $\widetilde{\text{P1}}$. This system of F.O.C.s can be formally expressed as follows:

$$\nabla_{\tilde{P}} W_i(\tilde{\mathbb{P}}_i; \mathbf{w}) + \nabla_{\mathbf{w}} W_i \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\tilde{P}}\right)_{(\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P} = 0, \qquad \forall \tilde{P} \in \tilde{\mathbb{P}}_i.$$

where recall that $\tilde{\mathbb{P}}_i = \left\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{ji}\right\}$ includes all consumer price variables associated with economy i. To elaborate the right-hand side of the above equation consists of two terms, as implied by the chain rule: The first term accounts for the change in welfare holding $\mathbf{w}$ fixed. The second term account for the change in $\mathbf{w}$ w.r.t. $\tilde{P} \in \tilde{\mathbb{P}}_i$ in order to satisfy feasibility.

[67] As noted in Appendix G, the constraint $\mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}$, holds in some canonical special cases of our framework irrespective of cooperative wage buffers in the RoW. One can also show that the constraint $\mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}$ is non-binding *at the optimum* if trade is bilaterally balanced. In particular, specify country i's welfare as $W_i\left(\tilde{\mathbb{P}}_i, w_i, \mathbf{w}_{-i}\right) = V_i\left(w_i L_i + \mathcal{R}_i\left(\tilde{\mathbb{P}}_i, w_i, \mathbf{w}_{-i}\right), \tilde{\mathbf{P}}_i\right)$, where recall that $\tilde{\mathbf{P}}_i \subset \tilde{\mathbb{P}}_i$. Taking partial derivatives w.r.t. $w_n \in \mathbf{w}_{-i}$ and noting that $Y_n = w_n \bar{L}_n$, yields $\frac{\partial W_i\left(\tilde{\mathbb{P}}_i, w_i, \mathbf{w}_{-i}\right)}{\partial w_n} = \frac{\partial \mathcal{R}_i\left(\tilde{\mathbb{P}}_i, w_i, \mathbf{w}_{-i}\right)}{\partial w_n} - \sum_k \left[P_{ji,k} Q_{ji,k}\right] + \sum_k \left[\left(\tilde{P}_{ij,k} - P_{ij,k}\right) Q_{ij,k} \eta_{ij,k}\right] + \sum_{n \neq i} \sum_{k,g} \left[\left(\tilde{P}_{in,k} - P_{in,k}\right) Q_{in,k} \varepsilon_{in,k}^{(jn,g)}\right]$. Now, suppose the gross trade matrix is bilaterally balanced, $\sum_k \left[P_{ji,k} Q_{ji,k} - \tilde{P}_{ij,k} Q_{ij,k}\right]$. Then, one can invoke the optimality condition w.r.t. $\tilde{\mathbf{P}}_{in}$ (Equation 42) and appeal to the Slutsky symmetry, $\varepsilon_{in,k}^{(jn,g)} = e_{jn,g} \varepsilon_{jn,g}^{(in,k)} / e_{in,k}$ and the demand function's homogeneity of degree zero property $\varepsilon_{ij,k}^{(ij,k)} + = \eta_{ij,k} + \sum_{n,g \neq i,k} e_{ij,k}^{(nj,g)}$ to get $\frac{\partial W_i(.)}{\partial w_n} \mid_{\tilde{\mathbb{P}}_i = \tilde{\mathbb{P}}_i^*} = 0$.

Our characterization of optimal policy employs the dual approach, the presentation of which relies heavily on Marshallian demand elasticities. So, for future reference, we formally define these elasticities below.

Notation [*Marshallian Demand Elasticities*] *Let $Q_{ji,k} \equiv \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$ denote the Marshallian demand function facing variety ji,k. This demand function is characterized by the following set of demand elasticities:*

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}_i)}{\partial \ln \tilde{P}_{ni,g}} \sim \textit{price elasticity}$$

$$\eta_{ji,k} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}_i)}{\partial \ln Y_i} \sim \textit{income elasticity,}$$

where $\tilde{\mathbf{P}}_i = \{\tilde{\mathbf{P}}_{1i}, \tilde{\mathbf{P}}_{2i}, ..., \tilde{\mathbf{P}}_{Ni}\}$ corresponds to the entire of vector of consumer prices in market i. Also, recall from the main text that $V(Y_i, \tilde{\mathbf{P}}_i)$ denotes the indirect utility associated with the Marshallian demand function, $\mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$.

In what follows, we appeal the above definition to characterize the first-order condition w.r.t. each element of $\tilde{\mathbb{P}}_i$. We start with country i's import prices, $\tilde{\mathbf{P}}_{ji}$, and then proceed to domestic and export price instruments, $\tilde{\mathbf{P}}_{ii}$, and $\tilde{\mathbf{P}}_{ij}$.

***Step 3.A*: Deriving the F.O.C. w.r.t. $P_{ji,k} \in \tilde{\mathbb{P}}_i$.**

Consider the price of import variety ji,k, supplied by *origin j–industry k* (where $j \neq i$). To present the first-order necessary condition (F.O.C.) w.r.t. the price of ji,k, we use $\mathbb{P}_{-ji,k}$ to denote all elements of $\tilde{\mathbb{P}}_i$ excluding $\tilde{P}_{ji,k}$:

$$\mathbb{P}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\} \sim \text{entire policy vector excluding } \tilde{P}_{ji,k}$$

Next, recall that $W_i(\tilde{\mathbb{P}}_i; \mathbf{w}) \equiv V_i(Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}), \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji})$ where income, $Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = \bar{w}_i L_i + \mathcal{R}_i(\tilde{\mathbb{P}}_i; \mathbf{w})$, is dictated by the balanced budget condition. Applying the chain rule to $W_i(\tilde{\mathbb{P}}_i; \mathbf{w})$, the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ (holding the remaining elements of $\tilde{\mathbb{P}}_i$ constant) can be stated as follows:[68]

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \overbrace{\frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i} \left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}}^{\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0 \tag{23}$$

The first term on the right-hand side of the above equation accounts for the direct welfare effects of a change in the price of good ji,k (holding Y_i and $\tilde{\mathbb{P}}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\}$ constant). The second term accounts for welfare effects that channel through revenue-generation (holding $\mathbf{w}$ and $\tilde{\mathbb{P}}_{-ji,k}$ constant).

[68]We can alternatively formulate the above optimization problem using the method of Lagrange multipliers, and by appealing to Lagrange sufficiency theorem. In that case the objective function can be formulated as follows:

$$\max_{\tilde{\mathbb{P}}_i, Y_i} \mathcal{L}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = V_i(Y_i, \tilde{\mathbf{P}}_i) + \lambda_y \left(Y_i - \bar{w}_i L_i - \mathcal{R}_i(\tilde{\mathbb{P}}_i; \mathbf{w})\right).$$

The F.O.C.with respect to Y_i entails that $\lambda_Y = \frac{\partial V_i(.)}{\partial Y_i}$. Hence, the F.O.C. with respect to $\tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$ can be expressed as

$$\frac{\mathrm{d}\mathcal{L}_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\mathrm{d} \ln \tilde{P}_{ji,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \lambda_y \left(\frac{\partial(\bar{w}_i L_i + \mathcal{R}_i(\tilde{\mathbb{P}}_i; \mathbf{w}))}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} + \left(\frac{\partial \mathcal{L}_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0,$$

which is equivalent to the F.O.C. expressed above.

The last term accounts for general equilibrium wage effects. Below, we characterize each of these elements one-by-one.

The term accounting for direct price effects can be simplified by appealing to Roy's identity, $\frac{\partial V_i/\tilde{P}_{ji,k}}{\partial V_i/\partial Y_i} = -Q_{ji,k}$, which indicates that

$$\text{[Roy's identity]} \qquad \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q_{ji,k} \left(\frac{\partial V_i}{\partial Y_i} \right). \tag{24}$$

To characterize $\left(\partial Y_i(\tilde{\mathbb{P}}_i; \mathbf{w})/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}$, note that total income in country i (which dictates total expenditure) is the sum of wage payments plus tax revenues:[69]

$$Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = w_i L_i + \underbrace{\sum_{n \neq i} \left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}) \cdot \mathbf{Q}_{ni} \right]}_{\text{import tax revenues}} + \underbrace{(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}) \cdot \mathbf{Q}_{ii}}_{\text{production tax revenues}} + \underbrace{\sum_{n \neq i} \left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}) \cdot \mathbf{Q}_{in} \right]}_{\text{export tax revenues}},$$

Holding w and $\mathbb{P}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\}$ fixed, $\tilde{P}_{ji,k}$ has no effect on wage payments: $\left(\partial \left(w_i L_i\right) / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} = 0$. The effect of $\tilde{P}_{ji,k}$ on import tax revenues can be unpacked as follows:

$$\begin{aligned}
\left(\frac{\partial \sum_{n \neq i} \left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}) \cdot \mathbf{Q}_{ni} \right]}{\partial \ln \tilde{P}_{ji,k}} \right) &= \tilde{P}_{ji,k} Q_{ji,k} + \sum_g \sum_{n \neq i} \left[\left(\tilde{P}_{ni,g} - P_{ni,g} \right) Q_{ni,g} \left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \right] \\
&- \sum_g \sum_{n \neq i} \left[P_{ni,g} Q_{ni,g} \left[\sum_{j \neq i} \frac{P_{ji,g} Q_{ji,g}}{P_{ni,g} Q_{ni,g}} \left(\frac{\partial \ln P_{ji,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} + \sum_{\ell \in \mathbb{C}} \frac{P_{i\ell,g} Q_{i\ell,g}}{P_{ni,g} Q_{ni,g}} \left(\frac{\partial \ln P_{i\ell,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} \right] \left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \right].
\end{aligned} \tag{25}$$

The first term in the above expression accounts for the direct, arithmetic effect of $\tilde{P}_{ji,k}$ on import tax revenues. The second term accounts for the change in revenue due to the change in country i's import demand schedule as a result of changing $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$. The change in demand can itself be decomposed into two components:

$$\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} = \underbrace{\frac{\partial \ln \mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln \tilde{P}_{ji,k}}}_{\text{price effect}} + \underbrace{\frac{\partial \ln \mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln Y_i} \left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}}_{\text{income effect}} = \varepsilon_{ni,g}^{(ji,k)} + \eta_{ni,g} \left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}, \tag{26}$$

where $\varepsilon_{ni,g}^{(ji,k)}$ and $\eta_{ni,g}$ denote the Marshallian price and income elasticities of demand. The presence of $\left(\partial \ln Y_i / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}$ in the above expression, manifests the circular nature of our general equilibrium setup. We will not unpack this term for now. Instead, we show later that income effects sum up to zero at the optimum.

The last term in Equation 25, accounts for scale effects: Noting that $P_{ni,g} = \bar{\varrho}_{ni,g} w_n \left[\sum_\iota \tau_{n\iota,g} Q_{n\iota,g} \right]^{-\frac{\mu_g}{1+\mu_g}}$, a change in the export supply of good ni, g (due to a change in $\tilde{P}_{ji,k}$) alters the scale of production in *origin n–industry g* and the producer prices associated with that location. Due to cross demand effects, this change also impacts the producer price of domestic suppliers as well as foreign suppliers outside of origin n.[70] To keep track of the general equilibrium scale effects, we use $\omega_{ni,g}$ to denote (the inverse

[69] The operator "$\cdot$" denotes the inner product of two equal-sized vectors. Also, since we are focused on the free entry case, for now, the profit-adjusted wage rate is equal to the actual (unadjusted) wage rate, i.e., $\tilde{w}_i = w_i$.

[70] To give an example, the producer price of goods supplied by country i in industry g ($P_{ij,g}$) respond to a reduction in

of) good ni, g's export supply elasticity:

$$\omega_{ni,g} \equiv \sum_{\ell \in \mathbb{C}} \left[\frac{P_{i\ell,g} Q_{i\ell,g}}{P_{ni,g} Q_{ni,g}} \left(\frac{\partial \ln P_{i\ell,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} \right] + \sum_{j \neq i} \left[\frac{P_{ji,g} Q_{ji,g}}{P_{ni,g} Q_{ni,g}} \left(\frac{\partial \ln P_{ji,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} \right]$$

$$= \frac{1}{r_{ni,g} \rho_{n,g}} \sum_{g} \left[\frac{\bar{w}_i L_i}{\bar{w}_n L_n} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} + \sum_{j \neq i} \frac{\bar{w}_j L_j}{\bar{w}_n L_n} r_{ji,g} \rho_{j,g} \left(\frac{\partial \ln P_{ji,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} \right] \sim \text{export supply elasticity} \tag{27}$$

The second line in the above definition derives from the fact that $\left(\frac{\partial \ln P_{i\ell,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} = \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i}$ for all $\ell \in \mathbb{C}$ (as the price of origin i's good sold to different locations differ in only a constant iceberg cost shifter) and that sales shares for each origin $n \in \mathbb{C}$ are defined as follows:

$$r_{ni,g} \equiv \frac{P_{ni,g} Q_{ni,g}}{\sum_{\iota \in \mathbb{C}} \left(P_{n\iota,g} Q_{n\iota,g} \right)} \sim \text{good-specific sales share}; \qquad \rho_{n,g} = \frac{\sum_{\iota \in \mathbb{C}} \left(P_{n\iota,g} Q_{n\iota,g} \right)}{\bar{w}_n L_n} \sim \text{industry-wide sales share.}$$

For now, we do not unpack the supply elasticity, $\omega_{ni,g}$. We relegate this task instead to Step #4 of the proof, where we solve our full system of F.O.C.s. Using the above definition for $\omega_{ni,g}$, we can simplify Equation 25 as follows:

$$\left(\frac{\partial \sum_{n \neq i} \left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}) \cdot \mathbf{Q}_{ni} \right]}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k} Q_{ji,k} + \sum_{g} \sum_{n \neq i} \left[\left(\tilde{P}_{ni,g} - [1 + \omega_{ni,g}] P_{ni,g} \right) Q_{ni,g} \left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \right]. \tag{28}$$

Moving on, the effect of a change in $\tilde{P}_{ji,k}$ on country i's production and export tax revenues can be unpacked as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}} \left\{ (\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}) \cdot \mathbf{Q}_{ii,g} + \sum_{n \neq i} \left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}) \cdot \mathbf{Q}_{in} \right] \right\} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}$$

$$= \sum_{g} \left[\left(\tilde{P}_{ii,g} - P_{ii,g} \right) Q_{ii,g} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \right] - \sum_{g} \sum_{n} \left[P_{in,g} Q_{in,g} \left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w}, \tilde{\mathbb{P}}_i} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \right]. \tag{29}$$

The first term in the above equation accounts for revenue effects that channel through a change in the demand for domestic varieties (i.e., ii, g). The second term accounts for scale effects—i.e., a change in $Q_{ii,g}$ alters the scale of production in origin i–industry k, and the producer prices associated with country i in all export markets. To simplify the term accounting for scale effects, we invoke the following observation, which follows from the *Free Entry* condition:[71]

$Q_{ni,g}$ through the following chain of effects:

$$Q_{ni,g} \downarrow \xrightarrow{\text{scale effects } (n,g)} P_{n\ell,g} \uparrow \xrightarrow{\text{cross-demand effects } (\ell \neq i)} Q_{i\ell,g} \uparrow \xrightarrow{\text{scale effects } (i,g)} P_{ij,g} \downarrow$$

Stated verbally, a reduction in $Q_{ni,g}$ lowers the producer price of *origin n–industry g* goods in all markets including $\ell \neq i$. Since consumer prices in location $\ell \neq i$ are not regulated by policy, an increase in $P_{n\ell,g}$ is fully passed onto consumer prices (provided that $n \neq i$), leading to a increase in $Q_{i\ell,g}$ through cross-substitution (or cross-demand) effects. The increase in $Q_{i\ell,g}$, in turn, lowers the producer price of goods supplied by *origin i–industry g* to all markets.

[71] In particular, note that $P_{in,g} = \tau_{in,g} P_{ii,g}$, where by Free Entry, $P_{ii,g} = \bar{\rho}_{ii,g} w_i \mathcal{Q}_{i,g}^{-\frac{\mu_g}{1+\mu_g}}$, with $\mathcal{Q}_{i,g} = \sum_n \bar{a}_{in,g} Q_{in,g}$ denoting country i's effective output in industry g. Hence, holding $\mathbf{w}$ and $\tilde{\mathbb{P}}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\}$ constant, we can show that

$$\sum_{n} \left[\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{ij,g}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \frac{r_{in,g}}{r_{ij,g}} \right] = \sum_{n} \left[\left(\frac{\partial \ln P_{ii,g}}{\partial \ln \mathcal{Q}_{i,g}} \frac{\partial \ln \mathcal{Q}_{i,g}}{\partial \ln Q_{ij,g}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} \frac{r_{in,g}}{r_{ij,g}} \right] = \frac{\partial \ln P_{ii,g}}{\partial \ln \mathcal{Q}_{i,g}} = -\frac{\mu_g}{1 + \mu_g}, \tag{30}$$

where the second line follows from the fact that $\partial \ln \mathcal{Q}_{i,g} / \partial \ln Q_{ij,g} = r_{ij,g}$, by definition.

$$\sum_n \left[P_{in,g} Q_{in,g} \left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \right] = \sum_n \left[\frac{P_{in,g} Q_{in,g}}{P_{ii,g} Q_{ii,g}} \left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \right] P_{ii,g} Q_{ii,g} = \tag{31}$$

$$= \sum_n \left[\frac{r_{in,g}}{r_{ii,g}} \left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \right] P_{ii,g} Q_{ii,g} = -\frac{\mu_g}{1+\mu_g} P_{ii,g} Q_{ii,g},$$

The last line in the above equation follows from the fact that *(a)* $\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} = -\frac{\mu_g}{1+\mu_g} r_{ii,g}$, and *(b)* $\sum_n r_{in,g} = 1$. We can plug the above equation back into Equation 29 to simplify it as follows:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}} \left\{ \sum_n \left[\left(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in} \right) \cdot \mathbf{Q}_{in} \right] \right\} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \sum_g \left[\left(\tilde{P}_{ii,g} - \left[1 - \frac{\mu_g}{1+\mu_g} \right] P_{ii,g} \right) Q_{ii,g} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \right]. \tag{32}$$

Note that $\left(\partial \ln Q_{ii,g} / \partial \ln \tilde{P}_{ji,k} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses price and income effects as indicated by Equation 26. Combining Equations 28 and 32, we can express the sum of all tax revenue-related effects as

$$\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k} Q_{ji,k} + \sum_g \sum_{n \neq i} \left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}] P_{ni,g} \right) Q_{ni,g} \varepsilon^{(ji,k)}_{ni,g} \right]$$
$$+ \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1}{1+\mu_g} P_{ii,g} \right) Q_{ii,g} \varepsilon^{(ji,k)}_{ii,g} \right] + \Delta_i(\tilde{\mathbb{P}}_i, \mathbf{w}) \left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}}. \tag{33}$$

The uniform term $\Delta_i(\tilde{\mathbb{P}}_i)$ regulates the net force of (circular) general equilibrium income effects. It correspondingly depends on the Marshallian income elasticities of demand:

$$\Delta_i(\tilde{\mathbb{P}}_i; \mathbf{w}) \equiv \sum_g \sum_{n \neq i} \left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}] P_{ni,g} \right) Q_{ni,g} \eta_{ni,g} \right] + \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1}{1+\mu_g} P_{ii,g} \right) Q_{ii,g} \eta_{ii,g} \right]. \tag{34}$$

To characterize the general equilibrium wage effects in the F.O.C. (i.e., the last term on the right-hand side of Equation 23), we invoke our earlier result under Lemma 5: By the targeting principle $\mathbf{w}_{-i}$ is welfare neutral at the optimum (i.e., $\tilde{\mathbb{P}}_i = \tilde{\mathbb{P}}_i$), which implies that

$$\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \mathbf{w}} \right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right)_{\mathbb{P}_{-ji,k}} = \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial w_i} \right)_{\mathbf{w}_{-i},\tilde{\mathbb{P}}_i} \left(\frac{\mathrm{d} w_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}.$$

That is, we can characterize the term that encompasses wage effects, treating $\mathbf{w}_{-i}$ as given. Accordingly, the term $\left(\mathrm{d} w_i / \mathrm{d} \ln \tilde{P}_{ji,k} \right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}$ can be calculated by applying the Implicit Function Theorem to country i's balanced trade condition,[72]

$$[\text{Balanced Trade}] \qquad \mathrm{T}_i\left(\tilde{\mathbb{P}}_i, \mathbf{w} \right) \equiv \sum_{n \neq i} \left[\mathbf{P}_{ni}(\tilde{\mathbb{P}}_i; \mathbf{w}) \cdot \mathbf{Q}_{ni,g}(\tilde{\mathbb{P}}_i; \mathbf{w}) - \tilde{\mathbf{P}}_{ni} \cdot \mathbf{Q}_{ni,g}(\tilde{\mathbb{P}}_i; \mathbf{w}) \right],$$

while treating $\mathbf{w}_{-i} = \bar{\mathbf{w}}_{-i}$ as if it were given. This step yields the following equation

$$\left(\frac{\mathrm{d} \ln w_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}} = -\left(\frac{\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i, \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \Big/ \left(\frac{\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i, \mathbf{w})}{\partial \ln w_i} \right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}$$
$$= \frac{-\sum_{n \neq i} \left[(\mathbf{P}_{ni} \odot \mathbf{Q}_{ni}) \cdot \left(\frac{\partial \ln \mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + (\mathbf{P}_{ni} \odot \mathbf{Q}_{ni}) \cdot \left(\mathbf{\Omega}_{ni} \odot \frac{\partial \ln \mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \right]}{\left(\frac{\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i, \mathbf{w})}{\partial \ln w_i} \right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}}. \tag{35}$$

[72] To be clear about the notation, we can write country i's balanced trade condition without appealing to the inner product operator as follows: $\mathrm{T}_i\left(\tilde{\mathbb{P}}_i, \mathbf{w} \right) \equiv \sum_g \sum_{n \neq i} \left(P_{ni,g}(\tilde{\mathbb{P}}_i, \mathbf{w}) Q_{ni,g}(\tilde{\mathbb{P}}_i, \mathbf{w}) - \tilde{P}_{in,g} Q_{in,g}(\tilde{\mathbb{P}}_i, \mathbf{w}) \right) = 0.$

where $\mathbf{\Omega}_{ni} \equiv \{\omega_{ni,k}\}_k$ is a vector composed of export supply elasticities (as defined under Equation 27) and $\odot$ denotes the element-wise product of two equal-sized vectors (i.e., $\mathbf{a} \odot \mathbf{b} = [a_n b_n]_n$). The second line in the above equation follows from the fact that $\left(\partial \ln Q_{in,g}(\tilde{\mathbb{P}}_i, \mathbf{w}) / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} = 0$ if $n \neq i$. That is, if we fix the vector of wages, $\mathbf{w}$, the choice of $\tilde{P}_{ji,k}$ has no effect on the demand schedule in the rest of the world. In other words, the only way the effect of $\tilde{P}_{ji,k}$ transmits to foreign markets is through its effect on $\mathbf{w}$. Now, define the importer-wide term, $\bar{\tau}_i$, as follows:

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial w_i}\right)_{\bar{\mathbf{w}}_{-i}, \tilde{\mathbb{P}}_i} \left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i, \mathbf{w}) / \partial \ln w_i\right)_{\bar{\mathbf{w}}_{-i}, \tilde{\mathbb{P}}_i}}. \tag{36}$$

Importantly, note that $\bar{\tau}_i$ does not feature an industry-specific subscript. Combining Equation 35 with the expression for $\bar{\tau}_i$, we can summarize the wage effects in the F.O.C. (Equation 23) as follows

$$\begin{aligned}\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} &= -\sum_g \sum_{n \neq i} \left[[1+\omega_{ni,g}]\bar{\tau}_i P_{ni,g} Q_{ni,g} \varepsilon_{ni,g}^{(ji,k)}\right] \\ &\quad - \sum_g \sum_{n \neq i} \left[[1+\omega_{ni,g}]\bar{\tau}_i P_{ni,g} Q_{ni,g} \eta_{ni,g}\right] \left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}. \end{aligned} \tag{37}$$

Finally, plugging Equations 24, 33, and 37 back into the F.O.C. (Equation 23); yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$:

$$\begin{aligned}[\text{FOC w.r.t. } \tilde{P}_{ji,k}] \qquad & \sum_{n \neq i} \sum_g \left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}} - (1+\bar{\tau}_i)(1+\omega_{ni,g})\right) P_{ni,g} Q_{ni,g} \varepsilon_{ni,g}^{(ji,k)}\right] \\ & + \sum_g \left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}} - \frac{1}{1+\mu_g}\right) P_{ii,g} Q_{ii,g} \varepsilon_{ii,g}^{(ji,k)}\right] + \widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w}) \left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} = 0. \end{aligned} \tag{38}$$

The uniform term $\widetilde{\Delta}_i(.)$ is defined analogously to $\Delta_i(.)$, but adjusts for the interaction of general equilibrium wage and income effects:

$$\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w}) \equiv \sum_g \sum_{n \neq i} \left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}} - (1+\omega_{ni,g})(1+\bar{\tau}_i)\right) P_{ni,g} Q_{ni,g} \eta_{ni,g}\right] + \sum_g \left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}} - \frac{1}{1+\mu_g}\right) P_{ii,g} Q_{ii,g} \eta_{ii,g}\right]. \tag{39}$$

Before moving forward, a remark on the uniform term $\bar{\tau}_i$ is in order. We do not unpack this term because the multiplicity of country i's optimal tax schedule (per Lemma 1) will render the exact value assigned to $\bar{\tau}_i$ as redundant. We will elaborate more on this point when we combine the F.O.C.s *w.r.t.* all tax instruments in step #4 of the proof.

***Step 3.B*: Deriving the F.O.C. w.r.t. $P_{ii,k} \in \tilde{\mathbb{P}}_i$.**

Next, we derive the F.O.C. w.r.t. to a locally produced and locally consumer variety ii, k. Recall that the objective function can is given by $W_i = V_i(Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}), \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji})$. The F.O.C. w.r.t. $\tilde{P}_{ii,k}$, holding the remaining elements of $\tilde{\mathbb{P}}_i$ (namely, $\mathbb{P}_{-ii,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ii,k}\}$) constant, can be stated as

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ii,k}}\right)_{\mathbb{P}_{-ii,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ii,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i} \left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ii,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ii,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ii,k}}\right)_{\mathbb{P}_{-ii,k}} = 0. \tag{40}$$

Each element of the right-hand side can be characterized in a manner identical to *Step 3.A*. Specifically, the first term can be simplified using Roy's identity. The second term, which accounts for revenue-raising effects can be characterized using cross-demand elasticities w.r.t. $\tilde{P}_{ii,k}$ instead of $\tilde{P}_{ji,k}$. The same goes for the last term accounting for general equilibrium wage effects. Repeating the derivations in *Step 3.A*, the F.O.C. characterized by Equation 40 can be unpacked as follows:

$$[\text{FOC w.r.t. } \tilde{P}_{ii,k}] \qquad \sum_{n\neq i}\sum_{g}\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ii,k)}\right]$$
$$+\sum_{g}\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ii,k)}\right]+\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ii,k}}\right)_{\mathbf{w},\mathbb{P}_{-ii,k}}=0, \tag{41}$$

where the uniform terms, $\widetilde{\Delta}_i(.)$, and $\bar{\tau}_i$, have the same definition as that introduced under Equations 39 and 36.

***Step 3.C*: Deriving the F.O.C. w.r.t. $P_{ij,k}\in\tilde{\mathbb{P}}_i$.**

Finally, we derive the F.O.C. w.r.t. to export variety ij,k, which is sold to destination $j\neq i$ in industry k. Note again that the objective function is given by $W_i=V_i(Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}),\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji})$. The F.O.C. w.r.t. $\tilde{P}_{ij,k}$, holding the remaining elements of $\tilde{\mathbb{P}}_i$ (namely, $\mathbb{P}_{-ij,k}\equiv\tilde{\mathbb{P}}_i-\{\tilde{P}_{ij,k}\}$) constant, can be stated as

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ij,k}}+\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}+\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=0. \tag{42}$$

The first term as before accounts for the direct effect of a price change on consumer surplus. This term is trivially equal to zero in this case, since $\tilde{P}_{ij,k}\notin\tilde{\mathbf{P}}_i$. That is, since ij,k is not part of the domestic consumption bundle, raising its price has no direct effect on consumer surplus in country i:

$$\tilde{P}_{ij,k}\notin\tilde{\mathbf{P}}_i \implies \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ij,k}}=0. \tag{43}$$

The second term in Equation 42 accounts for the revenue-raising effects of a change in $\tilde{P}_{ij,k}\in\tilde{\mathbb{P}}_i$. To unpack this term note that total income (or expenditure) in country i is dictated by the sum of wage payments and tax revenues:

$$Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})=w_iL_i+\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot\mathbf{Q}_{ni}\right]+(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii})\cdot\mathbf{Q}_{ii}+\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right],$$

Hence, holding wages $\mathbf{w}$ constant, the change in country i's income amounts to the change in import, domestic, and export tax revenues. The effect on import tax revenues can be unpacked as follows:

$$\left(\frac{\partial\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot\mathbf{Q}_{ni}\right]}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\sum_{g}\sum_{n\neq i}\left[(\tilde{P}_{ni,g}-P_{ni,g})\,Q_{ni,g}\left(\frac{\partial\ln Q_{ni,g}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\omega_{nj,g}\left(\frac{\partial\ln Q_{nj,g}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]. \tag{44}$$

where $\omega_{nj,g}$ is the export supply elasticity as defined by 27. The first term on the right-hand side accounts for general equilibrium income effects: Specifically, a change in $\tilde{P}_{ij,k}$ can raise country i's income Y_i through higher tax revenues, and alter the entire demand schedule, $Q_{ni,g}=\mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_i,Y_i)$, in the local market. The second term accounts for scale effects: To elaborate, a change in $\tilde{P}_{ij,k}$ distorts origin i's export supply schedule in market $j \in \mathbb{C}$. This change alters the scale of production and the producer prices associated with *origin n–industry g* that serves market j (this includes $P_{ni,g}$ which is associated with economy i). It also changes the scale of production and producer prices from foreign suppliers through cross-demand effects. These changes in international producer prices, impacts country i's terms-of-trade by changing its import tax revenues. Also, note that since the rest of the world (including country j) is passive in terms of taxation, their income is pinned to their wage rate and vector $\mathbf{w}$. Hence, $\left(\partial Y_j / \partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0$, which implies that $\left(\partial \ln Q_{nj,g} / \partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \partial \ln \mathcal{D}_{nj,g}(\bar{Y}_j, \tilde{\mathbf{P}}_j)/\partial \ln \tilde{P}_{ij,k} = \varepsilon^{(ij,k)}_{nj,g}$. Likewise, since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$, its only effect on the demand schedule in the local market i is through general equilibrium income effects. Putting these results together, we can posit that

$$\left(\frac{\partial \ln Q_{n\iota,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \begin{cases} \varepsilon^{(ij,k)}_{nj,g} & \text{if } \iota = j \\ \eta_{ni,g}\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} & \text{if } \iota = i \end{cases}.$$

Considering the above expressions and noting our earlier definition for $\omega_{ni,g}$ under Equation 27, Equation 44 can be simplified as

$$\begin{aligned}\left(\frac{\partial \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot\mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} &= -\sum_g\sum_{n\neq i}\left[\omega_{nj,g}P_{ni,g}Q_{ni,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} \\ &= -\sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}.\end{aligned} \tag{45}$$

The last line in the above equation follows from (1) the definition of ω, which entails that $\omega_{nj,g}r_{ni,g} = \omega_{ni,g}r_{nj,g}$, and (2) the fact that $r_{ni,g}/r_{nj,g} = P_{ni,g}Q_{ni,g}/P_{nj,g}Q_{nj,g}$, since the markup is uniform across output sold to different destinations in the same industry.

The effect of a change in $\tilde{P}_{ij,k}$ on country i's production and export tax revenues can be unpacked as follows:[73]

$$\begin{aligned}\left(\frac{\partial}{\partial \ln \tilde{P}_{ij,k}}\sum_n\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right]\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} &= \sum_g\left[\left(\tilde{P}_{ii,g}-P_{ii,g}\right)Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right] \\ + \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g}-P_{ij,g}\right)Q_{ij,g}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right] &+ \sum_g\sum_n\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right].\end{aligned} \tag{46}$$

The first term (in the first line) account for the effect on domestic tax revenues that channel through general equilibrium income effects. The second term on the right-hand side ($\tilde{P}_{ij,k}Q_{ij,k}$) accounts for the direct, arithmetic effect of $\tilde{P}_{ij,k}$ on export tax revenues. The third term account for revenue effects that channel through a change in the demand for all varieties sold to destination j (i.e., ij, g). The last term accounts for scale effects—i.e., a change in $Q_{ij,g}$ alters the scale of production in *origin i–industry g*, and modifies all the producer prices associated with that industry. As noted in *Step 3.A*, the last term in

[73] To be clear, $\sum_n\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right] = (\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii})\cdot\mathbf{Q}_{ii,g} + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right]$ denotes the sum of domestic and export tax revenues.

Equation 46 can be simplified using the Free Entry condition, which entails that (See Equation 31):

$$\sum_{n\in\mathbb{C}}\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial\ln Q_{ij,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]=-\frac{\mu_g}{1+\mu_g}P_{ij,g}Q_{ij,g},$$

Also, recall from our earlier discussion that since country $j\neq i$ collects no tax revenues by assumption, $\left(\partial Y_j/\partial\ln\tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0$, which implies that $\left(\partial\ln Q_{nj,g}/\partial\ln\tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\partial\ln\mathcal{D}_{nj,g}(\bar{Y}_j,\tilde{\mathbf{P}}_j)/\partial\ln\tilde{P}_{ij,k}=\varepsilon^{(ij,k)}_{nj,g}$. Plugging these expressions back into Equation 46 simplifies it as

$$\left(\frac{\partial}{\partial\ln\tilde{P}_{ij,k}}\sum_n\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right]\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\tilde{P}_{ij,k}Q_{ij,k}+\sum_g\left[\left(\tilde{P}_{ij,g}-[1-\frac{\mu_g}{1+\mu_g}]P_{ij,g}\right)Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g}-\frac{1}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\eta_{ii,g}\right]\left(\frac{\partial\ln Y_i}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{47}$$

Combining Equations 45 and 47, we can express the sum of tax revenue-related effects as

$$\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\tilde{P}_{ij,k}Q_{ij,k}+\sum_g\left[\left(\tilde{P}_{ij,g}-[1-\frac{\mu_g}{1+\mu_g}]P_{ij,g}\right)Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right]+\Delta_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}, \tag{48}$$

where $\Delta_i(.)$ encompasses the terms accounting for circular income effects and is given by Equation 34. No we turn to characterizing the general equilibrium wage effects in the F.O.C.—namely, the last term on the right-hand side of Equation 23. To this end, we invoke our observation based on the *targeting principle* (as stated under Lemma 5) that $\left(\frac{\partial W_i(.)}{\partial\mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{d\mathbf{w}}{d\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=\left(\frac{\partial W_i(.)}{\partial w_i}\right)_{\mathbf{w}_{-i},\tilde{\mathbb{P}}_i}\left(\frac{dw_i}{d\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ij,k}}$. The term $\left(\frac{dw_i}{d\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}$ can be calculated by applying the Implicit Function Theorem to country i's balanced trade condition,

$$\text{[Balanced Trade]}\qquad \mathrm{T}_i\left(\tilde{\mathbb{P}}_i,\mathbf{w}\right)\equiv\sum_{n\neq i}\left[\mathbf{P}_{ni}(\tilde{\mathbb{P}}_i;\mathbf{w})\cdot\mathbf{Q}_{ni,g}(\tilde{\mathbb{P}}_i;\mathbf{w})-\tilde{\mathbf{P}}_{ni}\cdot\mathbf{Q}_{ni,g}(\tilde{\mathbb{P}}_i;\mathbf{w})\right],$$

while treating $\mathbf{w}_{-i}=\bar{\mathbf{w}}_{-i}$ as given. This application yields the following equation (*Notation*: $\mathbf{\Omega}_{nj}\equiv\{\omega_{nj,k}\}_k$ is a vector composed of export supply elasticities, while $\odot$ and $\cdot$ denotes the element-wise and inner products of two equal-sized vectors):

$$\left(\frac{d\ln w_i}{d\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ij,k}}=-\left(\frac{\partial\mathrm{T}_i(\tilde{\mathbb{P}}_i,\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}/\left(\frac{\partial\mathrm{T}_i(\tilde{\mathbb{P}}_i,\mathbf{w})}{\partial\ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}$$
$$=\frac{-\tilde{P}_{ij,k}Q_{ij,k}-\left(\tilde{\mathbf{P}}_{ij}\odot\mathbf{Q}_{ij}\right)\cdot\left(\frac{\partial\ln\mathbf{Q}_{ij}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}+\sum_{n\neq i}\left[\left(\mathbf{P}_{ni}\odot\mathbf{Q}_{ni}\right)\cdot\left(\frac{\partial\ln\mathbf{Q}_{ni}}{\partial\ln\tilde{P}_{ij,k}}+\mathbf{\Omega}_{nj}\odot\frac{\partial\ln\mathbf{Q}_{nj}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]}{\left(\frac{\partial\mathrm{T}_i(\tilde{\mathbb{P}}_i,\mathbf{w})}{\partial\ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}} \tag{49}$$

The numerator in the second line of the above equation is composed of three terms: The first term accounts for the arithmetic effect of $\tilde{P}_{ji,k}$ on country i's trade balance. The second term account for own- and cross-price effects that are specific to market j—the market to which good ij,k is being exported. The last term accounts for scale effects: Specifically, a change in $\tilde{P}_{ij,k}$ interacts with the *balanced trade* *condition* by modifying the producer of a generic good ni,g imported from *origin i–industry g*. As before, define the uniform importer-wide term, $\bar{\tau}_i$, as follows

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial w_i}\right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}\left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i,\mathbf{w})/\partial \ln w_i\right)_{\bar{\mathbf{w}}_{-i},\tilde{\mathbb{P}}_i}}. \tag{50}$$

Combining Equation 49 with the expression for $\bar{\tau}_i$, we can summarize the wage effects in the F.O.C. (Equation 23) as follows:

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i\tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]-\sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{51}$$

Finally, plugging Equations 43, 48, and 51 back into the F.O.C. (Equation 42); and dividing all the expressions by $(1+\bar{\tau}_i)$ yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ij,k}\in\tilde{\mathbb{P}}_i$:

$$[\text{FOC w.r.t. } \tilde{P}_{ij,k}]\qquad \tilde{P}_{ij,k}Q_{ij,k}+\sum_g\left[\left(1-\frac{1}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]+\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i,\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0, \tag{52}$$

where $\widetilde{\Delta}_i(.)$ is defined as in Equation 39. Also, we are not unpacking the term $\bar{\tau}_i$, for the same reasons discussed under *Step 3.A*.

Step #4: Solving the System of F.O.C.s and Establishing Uniqueness

To determine the optimal tax schedule we need to collect the each of first order conditions and simultaneously solve them under one system. For the ease of reference, we will restate the F.O.C. w.r.t. to each element of $\tilde{\mathbb{P}}_i$ below. Following Equations 38 and 41, the F.O.C. w.r.t. $\tilde{P}_{\ell i,k}\in\tilde{\mathbb{P}}_i$ (where $\ell=i$ or $\ell=j\neq i$) is given by the following equation:

$$(1)\quad \sum_{n\neq i}\sum_g\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)e_{ni,g}\varepsilon_{ni,g}^{(\ell i,k)}\right]+$$
$$\sum_g\left[\left(1-\frac{1}{1+\mu_g}\frac{P_{ii,g}}{\tilde{P}_{ii,g}}\right)e_{ii,g}\varepsilon_{ii,g}^{(\ell i,k)}\right]+\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial \ln Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{\ell i,k}}\right)_{\mathbf{w},\mathbb{P}_{-\ell i,k}}=0.$$

where $e_{ni,g}=\tilde{P}_{ni,g}Q_{ni,g}/Y_i$ denotes the (unconditional) expenditure share on good ni,g. Likewise, dividing Equation 52 by $\tilde{P}_{ij,k}Q_{ij,k}$, the F.O.C. w.r.t. export price $\tilde{P}_{ij,k}\in\tilde{\mathbb{P}}_i$ is given by the following equation:

$$(2)\quad 1+\sum_g\left[\left(1-\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_{n\neq i}\sum_g\left[\omega_{ni,g}\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right]+\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i,\mathbf{w})\frac{Y_i}{Y_j}\left(\frac{\partial \ln Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0.$$

To set the stage for what follows, let us emphasize four points:

(1) In accordance with the tax-neutrality result presented under Lemma 1, the optimal policy schedule is unique only up-to two arbitrary tax shifters.[74] That is, there are multiple optimal policy schedules that are welfare-equivalent but differ in the average level assigned to domestic and trade taxes—we come back to this point when finalizing our optimal policy formulas.

(2) The system of F.O.C.s labeled (1) can be solved independently of (2) to recover the optimal export- and import-side price wedges.

(3) The trivial solution to system (1) satisfies $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = 0$. Moreover, we can invoke Lemma 1 to show that if there exists an optimal policy schedule for which $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) \neq 0$, that policy choice is welfare-equivalent to another that satisfies $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = 0$—see Appendix E.1 for an formal proof. These two observations together affirm that we can identify the full set of (welfare-equivalent) optimal policy schedules by setting $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = 0$ in the F.O.C.s. This particular proposition can be alternatively stated as an envelope-type result: If the government is afforded sufficient policy instruments, the system of F.O.C.s can be derived and solved *as if* the Marshallian demand functions were income inelastic.

(4) We focus on interior solutions that do not assign a prohibitive price to any good (i.e., $e_{ni,g} > 0$, $\forall\, ni, g$). Since prohibitive prices exclude goods from the system of F.O.C.s., one may worry that a non-interior solution that prohibits some goods but satisfies the necessary first-order conditions w.r.t. the other goods is optimal. Appendix E.2 rules out the optimality of prohibitive taxes/prices by appealing to the Inada conditions, which is standard in the literature.

All in all, these points indicate that System (1) can be solved independent of (2) and by restricting attention to interior solutions that satisfy $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i; \mathbf{w}) = 0$. Doing so leads us to a unique trivial solution from which we can infer the remaining optimal tax schedules—all of which deliver the same welfare outcome. To establish this claim, set $\widetilde{\Delta}_i(.) = 0$ and express System (1) in matrix notation as follows:

$$\underbrace{\begin{bmatrix} e_{1i,1}\varepsilon^{(1i,1)}_{1i,1} & \cdots & e_{Ni,}\varepsilon^{(1i,1)}_{Ni,1} & \cdots & e_{1i,K}\varepsilon^{(1i,1)}_{1i,K} & \cdots & e_{Ni,K}\varepsilon^{(1i,1)}_{Ni,K} \\ \vdots & & \ddots & \ddots & & & \vdots \\ e_{1i,1}\varepsilon^{(Ni,K)}_{1i,1} & \cdots & e_{Ni,}\varepsilon^{(Ni,K)}_{Ni,1} & \cdots & e_{1i,K}\varepsilon^{(Ni,K)}_{1i,K} & \cdots & e_{Ni,K}\varepsilon^{(Ni,K)}_{Ni,K} \end{bmatrix}}_{\widetilde{\mathbf{E}}_i} \begin{bmatrix} 1 - (1+\omega_{1i,k})(1+\bar{\tau}_i)\frac{P_{1i,1}}{\tilde{P}^{\star}_{1i,k}} \\ \vdots \\ 1 - \frac{1}{1+\mu_k}\frac{P_{ii,k}}{\tilde{P}^{\star}_{ii,k}} \\ \vdots \\ 1 - (1+\omega_{Ni,k})(1+\bar{\tau}_i)\frac{P_{Ni,k}}{\tilde{P}^{\star}_{Ni,k}} \end{bmatrix}_k = \mathbf{0}.$$

To prove that the above equation exhibits a unique, trivial solution it suffices to show that the expenditure-adjusted elasticity matrix, $\mathbf{E}_i = \left[e_{ji,k}\varepsilon^{(ni,g)}_{ji,k}\right]_{jk,ng}$ is non-singular. The following intermediate lemma establishes this result using the primitive properties of Marshallian demand functions.

Lemma 6. *The $NK \times NK$ matrix $\widetilde{\mathbf{E}}_i \equiv \left[e_{ji,k}\varepsilon^{(ni,g)}_{ji,k}\right]_{jk,ng}$ is non-singular.*

Proof. We can appeal to Proposition 2.E.2 in Mas-Colell, Whinston, Green, et al. (1995), which indicates that the Marshallian demand function satisfies $e_{ji,k} = \mid e_{ji,k}\varepsilon^{(ji,k)}_{ji,k} \mid - \sum_{n,g\neq j,k} \mid e_{ni,g}\varepsilon^{(ji,k)}_{ni,g} \mid$—a property often referred to as Cournot aggregation. Since $e_{ji,k} > 0$ (as we have ruled out prohibitive prices), Cournot aggregation ensures the matrix $\widetilde{\mathbf{E}}_i$ is strictly diagonally dominant. The Lèvy-Desplanques

[74] To be clear the pseudo-uniqueness of the *optimal policy formula* is different from the uniqueness of the *optimal policy equilibrium*. Establishing the latter is a daunting task well beyond the scope of this paper (see Kucheryavyy et al. (2016)).

Theorem (Horn and Johnson (2012)), accordingly, ensures that $\widetilde{\mathbf{E}}_i$ is non-singular. The lower bound on $\det(\widetilde{\mathbf{E}}_i)$ follows trivially from Gerschgorin's circle theorem. Specifically, following Ostrowski (1952),

$$|\det\left(\widetilde{\mathbf{E}}_i\right)| \geq \prod_{j\in\mathbb{C}}\prod_{k\in\mathbb{K}}\left(\left|e_{ji,k}\varepsilon_{ji,k}^{(ji,k)}\right| - \sum_{(n,g)\neq(j,k)}\left|e_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right|\right) = \prod_{j\in\mathbb{C}}\prod_{k\in\mathbb{K}} e_{ji,k} > 0.$$

□

Appealing to above lemma, it is immediate that the unique solution to the above matrix equation is indeed the trivial solution, given by:

$$\frac{\tilde{P}^*_{ji,k}}{P_{ji,1}} = (1+\omega_{ji,k})(1+\bar{\tau}_i); \qquad \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}} = \frac{1}{1+\mu_k}. \tag{53}$$

It is straightforward to check that the above solution constitutes a global maximum by contradiction. To present the logic: Since $\lim_{\tilde{\mathbf{P}}_i\to\infty} W_i(\tilde{\mathbb{P}}_i,\mathbf{w}) \to 0$, the above solution identifies a vector of consumer prices at home, $\tilde{\mathbf{P}}^*_i \in \tilde{\mathbb{P}}_i$, that yields a strictly higher welfare level than prohibitive prices. As such, $\tilde{\mathbb{P}}_i$ cannot constitute a global minimum. Lastly, it is straightforward to see that if the domestic price elements in $\tilde{\mathbb{P}}_i$ satisfy 53, then

$$\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\frac{\tilde{P}^*_{ni,g}}{P_{ni,g}} - (1+\omega_{ni,g})(1+\bar{\tau}_i)\right)P_{ni,g}Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(\frac{\tilde{P}^*_{ii,g}}{P_{ii,g}} - \frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right] = 0.$$

That is, the term accounting for general equilibrium income effects amounts to zero in the neighborhood of the optimum, *as if* demand functions were income inelastic (i.e., $\eta_{ni,g} = \eta_{ii,g} = 0$) Capitalizing on this result, we can proceed to solving System (2), knowing that $\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i,\mathbf{w}) = 0$. To this end, let us economize on the notation by defining $\mathscr{X}$ as follows:

$$\mathscr{X}_{ij,k} \equiv \frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}.$$

Invoking this minor switch of notation, the F.O.C. specified by System (2) implies the following optimality condition:

$$1+\sum_g\left[(1-\mathscr{X}_{ij,g})\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}}\right] - \sum_{n\neq i}\sum_g\left[\omega_{ni,g}\frac{e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}}\right] = 0. \tag{54}$$

To simplify the above expression we will appeal to the Cournot aggregation property–a well-known primitive property of Marshallian demand as discussed earlier (see Mas-Colell et al. (1995)):

$$[\text{Cournot aggregation}] \qquad 1+\sum_g\left[\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right] = -\sum_{n\neq i}\sum_g\left[\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right].$$

Next, combine the above expression with Equation 54, while noting that by Slutsky's equation $\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)} = \varepsilon_{ij,k}^{(nj,g)}$ if $\eta_{ni,g} = 1$ for all ni, g. Performing these steps yields the following:

$$-\sum_g\left[\mathscr{X}_{ij,g}\varepsilon_{ij,k}^{(ij,g)}\right] - \sum_{n\neq i}\sum_g\left[(1+\omega_{ni,g})\varepsilon_{ij,k}^{(nj,g)}\right] = 0 \qquad \forall(ij,k).$$

We can rewrite the above equation in matrix algebra as follows:

$$-\mathbf{E}_{ij}\mathbf{X}_{ij} - \mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K}+\mathbf{\Omega}_i\right) = 0, \tag{55}$$

where $\mathbf{X}_{ij} \equiv \left[\mathscr{X}_{ij,k}\right]_k$ is a $K\times 1$ vector. The $K\times K$ matrix $\mathbf{E}_{ij} \sim \mathbf{E}_{ij}^{(ij)} \equiv \left[\varepsilon_{ij,k}^{(ij,g)}\right]$ encompasses the own- and cross-price elasticities between the different varieties sold by origin i to market j—see Definition

(D1). Analogously, $\mathbf{E}_{ij}^{(-ij)} \equiv \left[\varepsilon_{ij,k}^{(nj,g)}\right]_{k,n\neq i,g}$ is a $K \times (N-1)K$ matrix summarizing the cross-price elasticity of market j's demand between varieties sold by origin i and all other (non-i) origin countries. $\boldsymbol{\Omega}_i \equiv \left[\omega_{ni,g}\right]_{n,g}$ is a $(N-1)K \times 1$ vector of all *import good-specific* inverse supply elasticities. To invert the above system we need to establish that $\mathbf{E}_{ij}$ is non-singular, which is done under the following lemma.

Lemma 7. *The $K \times K$ matrix $\mathbf{E}_{ij} \equiv \left[\varepsilon_{ij,k}^{(ij,g)}\right]_{k,g}$ is non-singular.*

Proof. The proof proceeds similar to Lemma 6: The Marshallian demand function's homogeneity of degree zero implies that $\mid \varepsilon_{ij,k}^{(ij,k)} \mid = \eta_{ij,k} + \sum_{n,g\neq i,k} \mid \varepsilon_{ij,k}^{(nj,g)} \mid$. Based on this property, since $\eta_{ij,k} > 0$, the matrix $\mathbf{E}_{ij}$ is strictly diagonally dominant. The Lèvy-Desplanques Theorem (Horn and Johnson (2012)), therefore, ensures that $\mathbf{E}_{ij}$ is non-singular. $\square$

Following the above lemma we can invert the system specified by Equation 55 to obtain the optimal level of $\mathbf{X}_{ij} = \left[\mathscr{X}_{ij,k}\right]_k$:

$$\mathbf{X}_{ij}^* = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)} \left(\mathbf{1}_{(N-1)K} + \boldsymbol{\Omega}_i\right). \tag{56}$$

Next, there remains the task of recovering the optimal tax/subsidy rates from the optimal price wedges implies by Equations 53 and 56. Noting the following relationship between taxes/subsidies and price wedges,

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}}{P_{ji,k}}; \qquad 1 + s_{i,k}^* = \frac{P_{ii,k}}{\tilde{P}_{ii,k}^*}; \qquad 1 + x_{ij,k} = \frac{P_{ij,k}/\tilde{P}_{ij,k}^*}{P_{ii,k}/\tilde{P}_{ii,k}^*};$$

country i's unilaterally optimal tax schedule can be expressed as follows:

$$\begin{aligned} &[\text{domestic subsidy}] \quad 1 + s_{i,k}^* = 1 + \mu_k \\ &[\text{import tariff}] \quad 1 + t_{ji,k}^* = (1 + \omega_{ji,k})(1 + \bar{\tau}_i) \\ &[\text{export subsidy}] \quad \mathbf{1} + \mathbf{x}_{ij}^* = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)} \left(\mathbf{1} + \mathbf{t}_i^*\right). \end{aligned} \tag{57}$$

The last step is to invoke the multiplicity of optimal tax schedules as indicated by Lemma 1. Doing so indicates that the uniform term $\bar{\tau}_i$ is redundant and need not be unpacked. To elaborate, Lemma 1 indicates that any policy schedule that includes an import tax equal to $(1 + \bar{t}_i \in \mathbb{R}_+)$

$$1 + t_{ji,k} = (1 + \omega_{ji,k})(1 + \bar{\tau}_i)(1 + \bar{t}_i)$$

is also optimal, since it delivers an identical level of welfare to the original optimal policy schedule specified by 57. As such, the exact value assigned to $\bar{\tau}_i$ is redundant for a welfare standpoint. This is why we did not unpack the term $\bar{\tau}_i$ earlier in Step #3. Lemma 1 indicates that there is another dimension of multiplicity, whereby any uniform shift in domestic production subsidies (paired with a proportional adjustment to w_i) preserves the equilibrium. Considering these points, the optimal policy schedule (after accounting for all dimensions of multiplicity) is given by:

$$\begin{aligned} &[\text{domestic subsidy}] \quad 1 + s_{i,k}^* = (1 + \mu_k)(1 + \bar{s}_i) \\ &[\text{import tariff}] \quad 1 + t_{ji,k}^* = (1 + \omega_{ji,k})(1 + \bar{t}_i) \\ &[\text{export subsidy}] \quad \mathbf{1} + \mathbf{x}_{ij}^* = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)} \left(\mathbf{1} + \mathbf{t}_i^*\right), \end{aligned}$$

where $1 + \bar{s}_i = 1 + \bar{t}_i \in \mathbb{R}_+$ are arbitrary tax shifters. What remains is a formal characterization of the good-specific supply elasticity, $\omega_{ji,k}$, which is presented below.

Characterizing the (Inverse) Export Supply Elasticity, $\omega_{ji,k}$. To fix ideas, it is helpful to repeat the definition of the export supply elasticity presented earlier:

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_g \left[\frac{\bar{w}_i L_i}{\bar{w}_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} + \sum_{n \neq i} \frac{\bar{w}_n L_n}{\bar{w}_j L_j} r_{ni,g} \rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \right], \tag{58}$$

where $r_{ni,g} = P_{ni,g}Q_{ni,g} / \sum_{\iota \in \mathbb{C}} (P_{n\iota,g}Q_{n\iota,g})$ and $\rho_{n,g} = \sum_{\iota \in \mathbb{C}} (P_{n\iota,g}Q_{n\iota,g}) / \bar{w}_n L_n$ respectively denote the good ni, g-specific and industry-wide sales shares associated with origin $n \in \mathbb{C}$. Also, note that the producer price of good ni, g under free entry is given by $P_{ni,g} = \tau_{ni,g} P_{nn,g}$, where

$$P_{nn,g} = \bar{\varrho}_{nn,g} w_n \sum_{\iota \in \mathbb{C}} \left[\tau_{n\iota,g} Q_{n\iota,g} \right]^{-\frac{\mu_g}{1+\mu_g}} \qquad \forall (n, g)$$

To characterize $\omega_{ji,k}$, we need to characterize $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i} = \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i}$ for each origin n–industry g. To this end we can apply the Implicit Function Theorem to the following function:

$$F_{ni,g}(Q_{1i,g}, ..., Q_{Ni,g}, P_{11,g}, ..., P_{NN,g}) = P_{nn,g} - \bar{\varrho}_{nn,g} w_n \left[\tau_{ni,g} Q_{ni,g} + \sum_{\ell \neq i} \tau_{n\ell,g} Q_{n\ell,g} (\underbrace{\boldsymbol{\tau}_{-i\ell} \odot \mathbf{P}_{-i}}_{\tilde{\mathbf{P}}_{-i\ell}}) \right]^{-\frac{\mu_g}{1+\mu_g}} = 0.$$

where $\boldsymbol{\tau}_{-in} \odot \mathbf{P}_{-i} \sim \{\tau_{jn,g} P_{jj,g}\}_{j \neq i, g}$ denotes the vector of consumer prices in market $n \neq i$ from all origins aside from i. The above function implicitly characterizes the producer prices in each origin j–industry g as a function of export supply levels to market i (i.e., $Q_{1i,g}, ..., Q_{Ni,g}$). Importantly, the above function treats both $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$ as given, as all elements of $\tilde{\mathbb{P}}_i$ are chosen directly the by the government in i. Accordingly, the function $Q_{n\iota,g}(.)$ on the right-hand side derives from the Marshallian demand function,

$$Q_{jn,g}(\underbrace{\boldsymbol{\tau}_{-in} \odot \mathbf{P}_{-i}}_{\tilde{\mathbf{P}}_{-in}}) = \mathcal{D}_{n\iota,g}(\tilde{\mathbf{P}}_{-in}, \overline{\tilde{\mathbf{P}}_{in}}, \underbrace{\overline{w_n L_n}}_{Y_n}),$$

treating $\overline{\tilde{\mathbf{P}}_{in}} \in \tilde{\mathbb{P}}_i$ and $w_n \in \mathbf{w}$ as given. This function accounts for the fact that any change in the producer price of varieties associated with *origin n–industry g* will affect the consumer prices and the demand schedule in all market excluding i. The reason is that prices in international markets (excluding i) are not directly pinned down by the choice, $\tilde{\mathbb{P}}_i$. For the sake of presentation, abstract from cross-industry demand effects. Applying the Implicit Function Theorem to the system of equations specified by $F_{ni,k}(\,.\,)$, yields the following:

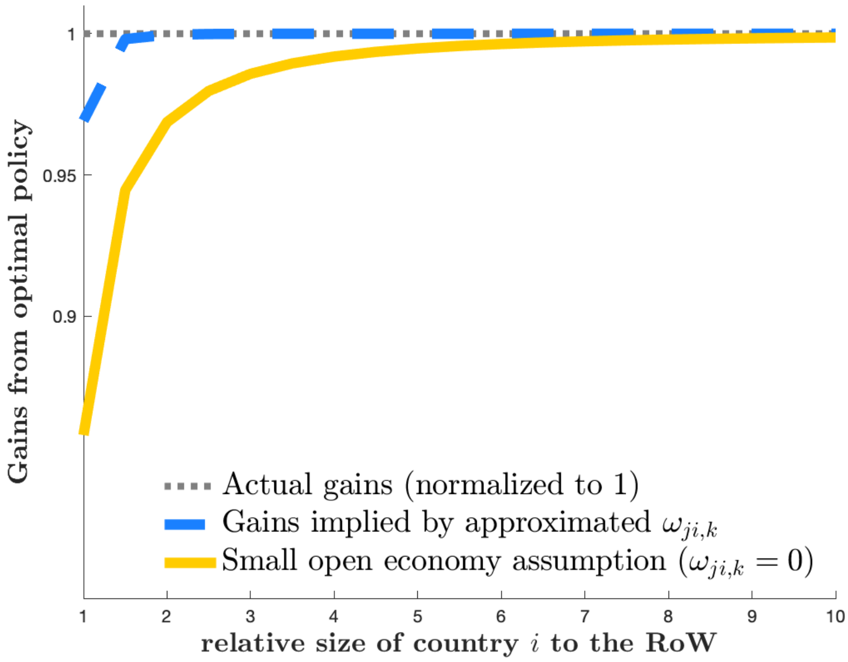
$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \end{bmatrix} = - \underbrace{\begin{bmatrix} \frac{\partial F_{1i,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{1i,k}(.)}{\partial \ln P_{NN,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{Ni,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{Ni,k}(.)}{\partial \ln P_{NN,k}} \end{bmatrix}^{-1}}_{\mathbf{A}_i} \begin{bmatrix} \frac{\partial F_{1i,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{1i,k}(.)}{\partial \ln Q_{Ni,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{Ni,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{Ni,k}(.)}{\partial \ln Q_{Ni,k}} \end{bmatrix}. \tag{59}$$

The elements of the matrixes on the right-hand side of the above equation are given by

$$\frac{\partial F_{ni,k}(\,.\,)}{\partial \ln P_{jj,k}} = \mathbb{1}_{j=n} + \mathbb{1}_{j \neq i} \times \frac{\mu_k}{1+\mu_k} \sum_{\iota \neq i} r_{n\iota,k} \varepsilon_{n\iota,k}^{(j\iota,k)}; \qquad \frac{\partial F_{ni,k}(\,.\,)}{\partial \ln Q_{ji,k}} = \mathbb{1}_{j=n} \frac{\mu_k}{1+\mu_k} r_{ji,k}.$$

Notice that the off-diagonal elements of $\mathbf{A}_i$ are near-zero (i.e., $r_{n\iota,k} \varepsilon_{n\iota,k}^{(j\iota,k)} \propto r_{n\iota,k} \lambda_{j\iota,k} \approx 0$ if $n \neq j \neq \iota$). So, we can apply the method proposed by Wu et al. (2013) to characterize $\mathbf{A}_i^{-1}$ to a first-order approximation around $r_{j\iota,k} \approx \lambda_{j\iota,k} \approx 0$ (for $j \neq \iota$). This procedure is detailed in Appendix E.3 and

Figure 4: The efficacy of the approximated $\omega_{ji,k}$ at predicting gains from policy



Note: the above simulation is based on a two country–two industry model with the following specifications: (2) $\sigma_1 = \sigma_2 = 5$, (2) $\mu_1 = 0.25$ and $\mu_2 = 0.5\mu_1$; (3) expenditure shares are assigned the following values $\lambda_{21,1} = 0.6$, $\lambda_{12,1} = 0.25/\delta$, $\lambda_{21,2} = 0.25$; $\lambda_{12,2} = 0.4/\rho$ where ρ is relative size.

yields the following expression based on the matrix Equation 59:

$$\left(\frac{\partial \ln P_{nn,k}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \approx \begin{cases} \frac{-\frac{\mu_k}{1+\mu_k} r_{ni,k}}{1+\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{n\iota,k}\varepsilon_{n\iota,k}^{(n\iota,k)}} & n = j \\ \frac{\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1+\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{n\iota,k}\varepsilon_{n\iota,k}^{(n\iota,k)}} \left(\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{n\iota,k}\varepsilon_{n\iota,k}^{(j\iota,k)}\right) & n \neq j \end{cases}$$

Plugging the above expression back into the definition specified by Equation 58, while noting that $r_{ni,k} \times r_{ji,k} \approx 0$ if $j \neq i$ and $n \neq i$, yields the following approximation for the export supply elasticity:

$$\omega_{ji,k} \approx \frac{-\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1+\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}} \left[1 - \frac{\mu_k}{1+\mu_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{ji,k}}\varepsilon_{in,k}^{(jn,k)}\right].$$

For the sake of clarity, note that $w_i = \tilde{w}_i$ under free entry—so, we can replace w_i with $\tilde{w}_i$ everywhere in the above approximation. Figure 4 illustrates the goodness of our approximated $\omega_{ji,k}$ using a rather conservative numerical example. We simulate a two-country×two-industry economy in which trade is relatively open and the tax-imposing country is relatively large compared to the rest of the world. We compute the actual gains from optimal policy for the tax-imposing country i, and compare them to gains implied by (1) our approximated $\omega_{ji,k}$ as well (2) the small open economy approximation, $\omega_{ji,k} \approx 0$. Evidently, our approximated value for $\omega_{ji,k}$ yields indistinguishable results relative to approximation-free benchmark.[75]

[75]To be clear, the above approximation is only intended for the quantitative applications. It should not be viewed as a limitation of our theory. The optimal tax formula derived earlier in combination with Equation 59 deliver an exact theoretical specification for the first-best optimal policy schedule.

Step #5. Extending the Derivation to the Restrict Entry Case

Equipped with a full characterization of optimal policy under free entry, we now switch attention to the case of restricted entry. The main difference between the two cases is in how producer prices vary with export supply: Under restricted entry, holding $\mathbf{w} = \{\tilde{w}_n\}$ fixed, contacting the export supply of good ni, g affects the producer prices associated with origin n through a uniform reduction in the average markup $\overline{\mu}_n$. Namely,

$$P_{ni,g} = \bar{\varrho}'_{ni,g}\frac{1+\mu_k}{1+\overline{\mu}_n}\tilde{w}_n \quad \Longrightarrow \quad \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} = -\left(\frac{\partial \ln(1+\overline{\mu}_n)}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i},$$

where economy n's (endogenously-determined) average profit margin is given by

$$1+\overline{\mu}_n = \frac{\sum_{\iota\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[P_{n\iota,k}Q_{n\iota,k}\right]}{\sum_{\iota\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1}{1+\mu_k}P_{n\iota,k}Q_{n\iota,k}\right]}.$$

Another difference is that non-tax-revenue income in country i is the sum of wage payments plus profits. Stated formally, total income in country i can be specified as follows (*notation*: the operator "$\cdot$" denotes the inner product of two equal-sized vectors):

$$Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}) = \underbrace{(1+\overline{\mu}_i)w_iL_i}_{\tilde{w}_iL_i} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}\right] + \left(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii}\right)\cdot\mathbf{Q}_{ii} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in}\right)\cdot\mathbf{Q}_{in}\right], \tag{60}$$

In the above formulation, $\tilde{w}_iL_i = (1+\overline{\mu}_i)w_iL_i$, stands for the sum of wage payments plus profits.

With the background information provided above, we can recycle our earlier derivations from the free entry case to characterize the F.O.C. w.r.t. each price instrument in $\tilde{\mathbb{P}}_i$.

First-Order Condition w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k} \in \tilde{\mathbb{P}}_i$. To fix ideas, recall from Step #3 of the proof that the F.O.C. w.r.t. $\tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$ (where possibly $j = i$) is given by

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0. \tag{61}$$

As before, $\mathbb{P}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\}$ denotes the vector of country i's price instruments excluding $\tilde{P}_{ji,k}$. Each term on the right-hand can be unpacked as in the free entry case, with one difference: holding $\mathbf{w}$ constant, a change in good ji, k's export supply affects the entire vector of prices from origin j. Specifically, noting that $P_{ji,g} = \bar{\varrho}'_{ji,g}\frac{1+\mu_g}{1+\overline{\mu}_j}w_j$, indicates that

$$\left(\frac{\partial\ln P_{ji,g}}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} = -\left(\frac{\partial\ln(1+\overline{\mu}_j)}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \qquad \forall g\in\mathbb{K}.$$

Noting this distinction, we now repeat the steps present earlier to unpack each term on the right-hand side of Equation 61. By Roy's identity, the first term on the right-hand side can be unpacked as follows:

$$\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} = -\tilde{P}_{ji,k}Q_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Recall that the second term on the right-hand side of Equation 61 accounts for the revenue-raising effects of policy. Specifically, taking note of Equation 60, the effect on import tax revenues can be unpacked as follows:

$$\left(\frac{\partial \sum_{n\neq i}\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k}+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]$$
$$-\sum_{g\in\mathbb{K}}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\sum_{s\in\mathbb{K}}\left[\sum_{j\neq i}\frac{P_{ji,s}Q_{ji,s}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial P_{ji,s}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}+\sum_{\ell\in\mathbb{C}}\frac{P_{i\ell,s}Q_{i\ell,s}}{P_{i\ell,g}Q_{i\ell,g}}\left(\frac{\partial P_{i\ell,s}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{62}$$

As in the free entry case, $\left(\partial \ln Q_{ni,g}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses demand adjustments that channel through both price and income effects—see Equation 26. We can simplify the last term on the right-hand side of above equation, by appealing to our definition of the export supply elasticity:

$$\begin{aligned}\omega_{ni,g} &\equiv \sum_{\ell\in\mathbb{C}}\left[\frac{P_{i\ell,g}Q_{i\ell,g}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial \ln P_{i\ell,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]+\sum_{j\neq i}\left[\frac{P_{ji,g}Q_{ji,g}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial \ln P_{ji,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]\\ &= \frac{1}{r_{ni,g}\rho_{n,g}}\sum_{g}\left[\frac{\bar{w}_i L_i}{\bar{w}_n L_n}\rho_{i,g}\left(\frac{\partial \ln\left(1+\overline{\mu}_i\right)}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}+\sum_{j\neq i}\frac{\bar{w}_j L_j}{\bar{w}_n L_n}r_{ji,g}\rho_{j,g}\left(\frac{\partial \ln\left(1+\overline{\mu}_n\right)}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]\end{aligned} \tag{63}$$

The second line indicates our focus on the restricted entry case, wherein $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} = \left(\frac{\partial \ln(1+\overline{\mu}_n)}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$ for all g. That is, holding $\mathbf{w}$ constant, producer prices from each origin change equal-proportionally across all industries with the aggregate profit margin, $1+\overline{\mu}_i$. Plugging the above expression back into Equation 62 yields the following expression that summarizes the (conditional) effect of policy on import tax revenues:

$$\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k}+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{64}$$

The effect of policy on export and domestic tax revenues can be unpacked as in Equation 29, which was derived earlier for the free entry case. To simplify this equation under restricted entry, we can use the following observation:

$$\begin{aligned}\sum_{n}\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right] &= -\sum_{s\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left[\frac{P_{in,s}Q_{in,s}}{P_{ii,g}Q_{ii,g}}\left(\frac{\partial \ln(1+\overline{\mu}_i)}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right]P_{ii,g}Q_{ii,g} =\\ &= \sum_{s\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left[\frac{r_{in,s}\rho_{i,s}}{r_{ii,g}\rho_{i,g}}\left(\frac{\overline{\mu}_i-\mu_g}{1+\mu_g}r_{ii,g}\rho_{i,g}\right)\right]P_{ii,g}Q_{ii,g} = -\left(1-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g},\end{aligned}$$

To explain, the second line on the above equation follows from that fact that all prices associated with economy i are included in the set $\tilde{\mathbb{P}}_i$. So, holding $\tilde{\mathbb{P}}_i$ and wages $\mathbf{w}$ constant, the policy-induced change in $Q_{ii,g}$ has only a direct arithmetic effect on country i's aggregate profit margin, i.e., $\left(\frac{\partial \ln(1+\overline{\mu}_i)}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} = \frac{\overline{\mu}_i-\mu_g}{1+\mu_g}r_{ii,g}\rho_{i,g}$.[76] Plugging the above equation back into Equation 29 yields the following equation de-

[76]Note that this argument does not extend to the aggregate profit margin in other countries. Changing the export supply of say good ji,k with policy has a circular effect on origin j's profit margin, $\overline{\mu}_j$, which occurs because the prices associated with economy $j\neq i$ are not pegged to $\tilde{\mathbb{P}}_i$. Specifically, a change in $Q_{ji,k}$ affects the entire vector of origin j's prices outside of market i. This change in prices affects the industrial composition of origin j's output and $\overline{\mu}_j$ in a circular fashion.

scribing the (conditional) effects of policy on export and domestic tax revenues:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\sum_n \left[\left(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in}\right)\cdot \mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \sum_g \left[\left(\tilde{P}_{ii,g}-\frac{1+\overline{\mu}_i}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{65}$$

Recall that $\left(\partial \ln Q_{ni,g}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$, in the above equations, encompasses price- and income-related demand adjustments—see Equation 26. Taking note of this detail, we can combine Equations 64 and 65 to arrive at the following expression that summarizes the (conditional) effect of raising $\tilde{P}_{ji,k}$ on country i's tax revenues:

$$\begin{aligned}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} =& \tilde{P}_{ji,k}Q_{ji,k}+\sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+\sum_g\left[\left(\tilde{P}_{ii,g}-\frac{1+\overline{\mu}_i}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right]+\Delta_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}},\end{aligned}$$

where $\Delta_i(.)$, as before, encapsulated the circular income effects. The expression for $\Delta_i(.)$ is specified analogously to Equation 34 with two amendments: (1) $\omega_{ni,g}$ is redefined according to 63; and (2) $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.[77] Next, we unpack the last term on the right-hand side of Equation 61, which accounts for general equilibrium wage effects. Repeating the steps presented for the free entry case, while noting the differences discussed above, yields the following:

$$\begin{aligned}\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} =& -\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\left(1+\omega_{ni,g}\right)P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right], \\ &-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\left(1+\omega_{ni,g}\right)P_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\end{aligned}$$

where $\bar{\tau}_i$ is given by 36. Note that the above expression differs from the analogous expression derived under free entry in the economic forces that regulate export supply elasticity, $\omega_{ni,g}$. Under restricted entry, the export supply elasticity governs the change in aggregate profit margins in response to distortions to export supply. Combining the various terms on the right-hand side of Equation 61, yields the following simplified representation of the F.O.C. w.r.t. $\tilde{P}_{ji,k}\in\tilde{\mathbb{P}}_i$ under restricted entry:

$$\begin{aligned}&\sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)\right)P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+\sum_g\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right]+\widetilde{\Delta}_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=0.\end{aligned}$$

The uniform term $\widetilde{\Delta}_i(.)$ is described by Equation 39, but with $\omega_{ni,g}$ redefined according to 63 and $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.

[77] To be more specific, $\Delta_i(.)$ is described by the following equation:

$$\Delta_i(\tilde{\mathbb{P}}_i;\mathbf{w})\equiv\sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]+\sum_g\left[\left(\tilde{P}_{ii,g}-\frac{1+\overline{\mu}_i}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\eta_{ii,g}\right],$$

where $\overline{\mu}_i>0$ and $\omega_{ni,g}$ is given by Equation 63 for the case of restricted entry.

First-Order Condition w.r.t. $\tilde{P}_{ij,k}$ ($j \neq i$). Now consider the F.O.C. w.r.t. the price of a generic export good ij,k (where $j \neq i$). Recall from Step #3 that the F.O.C. w.r.t. $\tilde{P}_{ij,k} \in \tilde{\mathbb{P}}_i$ is given by

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \tag{66}$$

where $\mathbb{P}_{-ij,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ij,k}\}$ denotes the vector of country i's price instruments excluding $\tilde{P}_{ij,k}$. Building on our previous discussion, each term on the right-hand side is characterized by the same formulas derived in Step #3, with two qualification: (1) The formulation assigned to $\omega_{ni,g}$ should be revised to account for restricted entry (see Equation 10), (2) all equations should be adjusted to admit a non-zero $\overline{\mu}_j$, as is required by restricted entry (see Equation 5).

Without repeating all the details from Step 3, we can unpack the terms on the right-hand side of Equation 66 as follows: Since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$ is not part of the domestic consumer price index, $\partial V_i(Y_i,\tilde{\mathbf{P}}_i)/\partial \ln \tilde{P}_{ji,k} = 0$. The second-term on the right-hand side of Equation 66 is given by:

$$\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k}Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1-\frac{\mu_g}{1+\mu_g}]P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$- \sum_g \sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] + \Delta_i(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}},$$

where $\omega_{ni,g}$ is defined as in Equation 63, while $\Delta_i(.)$ is given by Equation 34, with the necessary adjustments described earlier.The last term on the right-hand side of Equation 66 , which accounts for general equilibrium wage effects, can be unpacked as

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i\tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] - \sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}.$$

where $\bar{\tau}_i$ is given by 36. To be clear, the above formula differs from the one derived under free entry in only how $\omega_{ni,g}$ is defined—see Equation 63. Combining the various terms on the right-hand side of Equation 66, yields the following simplified representation of the F.O.C. w.r.t. $\tilde{P}_{ij,k} \in \tilde{\mathbb{P}}_i$:

$$\tilde{P}_{ij,k}Q_{ij,k} + \sum_{g\in\mathbb{K}}\left[\left(1-\frac{1+\overline{\mu}_i}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$\sum_{g\in\mathbb{K}}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] + \widetilde{\Delta}_i(\tilde{\mathbb{P}}_i,\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0,$$

The uniform term $\widetilde{\Delta}_i(.)$ is described by Equation 39, but with $\omega_{ni,g}$ redefined according to Equation 63 and $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.

Solving the system of F.O.C. Given the tight correspondence between the F.O.C.s derived under the restricted and free entry cases, we can repeat the arguments as in step #4 to solve the system of F.O.C.s and establish the uniqueness of the resulting solution. Doing so yield the following formula for optimal taxes/subsidies under restricted entry:

$$\begin{aligned} &[\text{domestic subsidy}] \quad 1+s_{i,k}^{*}=(1+\mu_k)/(1+\overline{\mu}_i) \\ &[\text{import tariff}] \quad 1+t_{ji,k}^{*}=(1+\omega_{ji,k})(1+\overline{\tau}_i) \\ &[\text{export subsidy}] \quad \mathbf{1}+\mathbf{x}_{ij}^{*}=-\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}_i^{*}\right). \end{aligned}$$

Recall from Lemma 1 that there are two degrees of multiplicity associated with optimal policy schedule. As a result, we need not to unpack the uniform terms $\overline{\tau}_i$ and $\overline{\mu}_i$. Instead, for any arbitrary choice of tax shifters $1+\bar{s}_i$ and $1+\bar{t}_i \in \mathbb{R}_+$, the following tax/subsidy schedule represents an optimal solution:

$$\begin{aligned} &[\text{domestic subsidy}] \quad 1+s_{i,k}^{*}=(1+\mu_k)(1+\bar{s}_i) \\ &[\text{import tariff}] \quad 1+t_{ji,k}^{*}=(1+\omega_{ji,k})(1+\bar{t}_i) \\ &[\text{export subsidy}] \quad \mathbf{1}+\mathbf{x}_{ij}^{*}=-\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}_i^{*}\right). \end{aligned}$$

The above formula is identical to that derived under free entry, with one qualification. The (inverse) export supply elasticity $\omega_{ji,k}$ has a different interpretation under restricted entry, and is given by 63. So, to conclude the proof, we characterize $\omega_{ji,k}$ under restricted entry next.

Characterizing the (Inverse) Export Supply Elasticity. Following Equation 63, the inverse of the export supply elasticity under restricted entry is defined as

$$\omega_{ji,k}=\frac{-1}{r_{ji,k}\rho_{j,k}}\left[\frac{\tilde{w}_iL_i}{\tilde{w}_jL_j}\left(\frac{\partial\ln(1+\overline{\mu}_i)}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}+\sum_{n\neq i}\sum_{g}\left(\frac{\tilde{w}_nL_n}{\tilde{w}_jL_j}r_{ni,g}\rho_{n,g}\right)\left(\frac{\partial\ln(1+\overline{\mu}_n)}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}\right], \tag{67}$$

where the second line follows from the fact that $P_{ni,s}=\bar{\varrho}'_{ni,s}\frac{1+\mu_s}{1+\overline{\mu}_n}\tilde{w}_n$, which implies that $\left(\frac{\partial\ln P_{ni,s}}{\partial\ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}=-\left(\frac{\partial\ln(1+\overline{\mu}_n)}{\partial\ln Q_{ni,g}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$. To unpack the above equation, note that (for a given $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$) the aggregate profit margin implicitly solves the following equation:

$$F_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})=(1+\overline{\mu}_n)-\underbrace{\frac{\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni}+\sum_{\iota\neq i}\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\boldsymbol{\mu}}_{-i})}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni}+\sum_{\iota\neq i}\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\boldsymbol{\mu}}_{-i})}}_{g_{ni}(\overline{\mu}_n,\mathbf{Q}_{ni})}=0.$$

As before, $\odot$ and $\cdot$ respectively denote the *inner* and *element-wise* products of equal-sized vectors (i.e., $\mathbf{a}\cdot\mathbf{b}=\sum_n a_nb_n$ and $\mathbf{a}\odot\mathbf{b}=[a_nb_n]_n$), while with a slight abuse of notation, $\frac{1}{1+\boldsymbol{\mu}}\equiv\left[\frac{1}{1+\mu_k}\right]_k$. The vector $\mathbf{Q}_{ni}$ represents the export supply of goods from origin $n\neq i$ to market i (which is fully determined by $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$). Outside of market i, consumer prices are not directly pegged to $\tilde{\mathbb{P}}_i$. So, holding $\tilde{w}_n\in\mathbf{w}$ and $\tilde{\mathbf{P}}_{i\iota}\in\tilde{\mathbb{P}}_i$ constant, a change in $\overline{\mu}_i$ affects the producer and consumer price of goods supplied by origin n to any market $\iota\neq i$. Accordingly, $\mathbf{Q}_{n\iota}(\overline{\boldsymbol{\mu}}_{-i})\equiv\left\{Q_{n\iota,k}(\overline{\boldsymbol{\mu}}_{-i})\right\}_k$ in the above equation is determined by the Marshallian demand function (treating $\tilde{w}_n\in\mathbf{w}$ and $\tilde{\mathbf{P}}_{i\iota}\in\tilde{\mathbb{P}}_i$ as given):

$$Q_{n\iota,k}(\overline{\boldsymbol{\mu}}_{-i})=\mathcal{D}_{n\iota,k}(\overline{\tilde{w}_\iota L_\iota},\overline{\tilde{\mathbf{P}}_{i\iota}},\tilde{\mathbf{P}}_{-i\iota}(\overline{\boldsymbol{\mu}}_{-i}))$$

Taking note of this detail, we can compute $\left(\partial\ln(1+\overline{\mu}_n)/\partial\ln Q_{ni,g}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$ by applying the Implicit Function Theorem to the system of equations specified by $F_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})$. Namely,

$$\begin{bmatrix}\frac{\partial\ln(1+\overline{\mu}_1)}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial(1+\overline{\mu}_1)}{\partial\ln\mathbf{Q}_{Ni}}\\ \vdots & \ddots & \vdots\\ \frac{\partial\ln(1+\overline{\mu}_N)}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial(1+\overline{\mu}_N)}{\partial\ln\mathbf{Q}_{Ni}}\end{bmatrix}=-\begin{bmatrix}\frac{\partial F_{1i}(.)}{\partial\ln(1+\overline{\mu}_1)} & \cdots & \frac{\partial F_{1i}(.)}{\partial\ln(1+\overline{\mu}_N)}\\ \vdots & \ddots & \vdots\\ \frac{\partial F_{Ni}(.)}{\partial\ln(1+\overline{\mu}_1)} & \cdots & \frac{\partial F_{Ni}(.)}{\partial\ln(1+\overline{\mu}_N)}\end{bmatrix}^{-1}\begin{bmatrix}\frac{\partial F_{1i}}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial F_{1i}}{\partial\ln\mathbf{Q}_{Ni}}\\ \vdots & \ddots & \vdots\\ \frac{\partial F_{Ni}}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial F_{Ni}}{\partial\ln\mathbf{Q}_{Ni}}\end{bmatrix}. \tag{68}$$

Next, we characterize the elements of the matrixes on the right-hand side of the above equation. Considering that $F_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni}) = (1+\overline{\mu}_n) - g_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})$, we can unpack the elements of $\left[\frac{\partial F_{ni}(.)}{\partial \ln(1+\overline{\mu}_j)}\right]_{n,j}$ as follows. Using vector algebra we can show that if $n \neq i$, then

$$\frac{\partial g_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})}{\partial \ln(1+\overline{\mu}_n)} = \underbrace{\frac{-\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} - \sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{-(1+\overline{\mu}_n)} + \underbrace{\frac{-\sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\odot\varepsilon_{n\iota}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{-(1+\overline{\mu}_n)\sum_\iota[\mathbf{r}_{n\iota}\cdot\varepsilon_{n\iota}]}$$

$$-(1+\overline{\mu}_n)\left[\underbrace{\frac{-\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} - \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{-1} - \underbrace{\frac{\sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\odot\varepsilon_{n\iota}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{(1+\overline{\mu}_n)\sum_\iota\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{r}_{n\iota}\cdot\varepsilon_{n\iota}\right]}\right].$$

where $\boldsymbol{\varepsilon}_{ni} \equiv \left[\varepsilon_{ni,g}^{(ni,g)}\right]_g$ is a $K\times 1$ vector of own-price elasticities of demand. $\mathbf{r}_{ni} \equiv \left[r_{ni,g}\rho_{n,g}\right]_g$ is a $K\times 1$ vector of sales shares. The above derivation appeals to the definition of sales shares, whereby $r_{ni,k}\rho_{n,k} = \frac{P_{ni,k}Q_{ni,k}}{\sum_j\sum_g P_{nj,g}Q_{nj,g}}$. Likewise, for any n and $\ell\neq i$, we can

$$\frac{\partial g_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})}{\partial \ln(1+\overline{\mu}_\ell)} = \frac{-\sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\odot\varepsilon_{n\iota}^{(\ell\iota)}\right] + (1+\overline{\mu}_n)\sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\odot\varepsilon_{n\iota}^{(\ell\iota)}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}.$$

Combining the above two equations we can characterizes each element of the matrix $\left[\frac{\partial F_{ni}(.)}{\partial \ln(1+\overline{\mu}_\ell)}\right]_{n,\ell}$ as follows:

$$\frac{\partial F_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})}{\partial \ln(1+\overline{\mu}_\ell)} = (1+\overline{\mu}_n)\left[\mathbb{1}_{\ell=n} + \mathbb{1}_{\ell\neq i}\sum_k\sum_{\iota\neq i}\left[\left(1-\frac{1+\overline{\mu}_n}{1+\mu_k}\right)r_{n\iota,k}\rho_{n,k}\varepsilon_{n\iota,k}^{(\ell\iota,k)}\right]\right]$$

The elements of the matrix $\left[\frac{\partial F_{ni}}{\partial \ln \mathbf{Q}_{\ell i}}\right]_{n,\ell}$ can be unpacked with a similar logic. Specifically, if $n\neq\ell$ then $\frac{\partial F_{ni}}{\partial \ln \mathbf{Q}_{\ell i}} = 0$. Otherwise, for any $n\in\mathbb{C}$ we can derive the following expression:

$$\frac{\partial g_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})}{\partial \ln Q_{ni,k}} = \underbrace{\frac{P_{ni,k}Q_{ni,k}}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{(1+\overline{\mu}_n)r_{ni,k}\rho_{n,k}} - \underbrace{\frac{(1+\overline{\mu}_n)\frac{1}{1+\mu_k}P_{ni,k}Q_{ni,k}}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\overline{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\overline{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\overline{\mu}_n)\right]}}_{(1+\overline{\mu}_n)\frac{1+\overline{\mu}_n}{1+\mu_k}r_{ni,k}\rho_{n,k}},$$

which, in turn, characterizes every element of matrix $\left[\frac{\partial F_{ni}}{\partial \ln \mathbf{Q}_{\ell i}}\right]_{n,\ell}$ as follows:

$$\frac{\partial F_{ni}(\overline{\boldsymbol{\mu}},\mathbf{Q}_{ni})}{\partial \ln Q_{\ell i,k}} = \mathbb{1}_{\ell=n}(1+\overline{\mu}_n)\left[\left(1-\frac{1+\overline{\mu}_n}{1+\mu_k}\right)r_{ni,k}\rho_{n,k}\right].$$

As in the free entry case, the off-diagonal elements of $\tilde{\mathbf{A}}_i \equiv \left[\frac{\partial F_{ni}(.)}{\partial \ln(1+\overline{\mu}_j)}\right]_{n,j}$ are near zero. So, we can once again invoke the first-order approximation proposed by Wu et al. (2013) to characterize $\tilde{\mathbf{A}}_i^{-1}$. Doing so and plugging the implied values of $\frac{\partial \ln(1+\overline{\mu}_n)}{\partial \ln \mathbf{Q}_{ji}}$ back into Equation 67, implies the following approximation for the export supply elasticity under restricted entry:

$$\omega_{ni,g} \approx \frac{-\left(1-\frac{1+\overline{\mu}_n}{1+\mu_g}\right)\sum_k r_{ni,k}\rho_{n,k}}{1+\sum_k\sum_{\iota\neq i}\left[1+\left(1-\frac{1+\overline{\mu}_n}{1+\mu_k}\right)r_{n\iota,k}\rho_{n,k}\varepsilon_{n\iota,k}\right]}.$$

E.1 Redundancy of Solutions for which $\Delta_i \neq 0$

To finalize the proof, we appeal to the multiplicity of optimal taxes to show the following: If there exists an optimal tax vector for which $\widetilde{\Delta}_i \neq 0$, that tax vector can be recovered from an equivalent optimal tax vector that satisfies $\widetilde{\Delta}_i = 0$. This can be shown building on two intermediate points: *First*, following Lemma 1, if $\mathbb{T}_i^* = \left\{t^*_{ji,k}, x^*_{ij,k}, s^*_{i,k}\right\}$ is an optimal policy choice, then policy $\mathbb{T}_i^{\star}(a) = \left\{t^{\star}_{ji,k}(a), x^{\star}_{ij,k}(a), s^{\star}_{ii,k}(a)\right\}$ is also optimal for any $a \in \mathbb{R}$, where

$$1+t^{\star}_{ji,k}(a) = \left(1+t^*_{ji,k}\right)(1+a)^{-1}; \qquad 1+x^{\star}_{ij,k}(a) = \left(1+x^*_{ij,k}\right)(1+a); \qquad 1+s^{\star}_{i,k}(a) = \left(1+s^*_{i,k}\right)(1+a)^{-1}.$$

Second, the aggregate term $\widetilde{\Delta}_i$, specified below in terms of taxes, appears identically in the F.O.C.s associated with every policy instrument:

$$\widetilde{\Delta}_i = \sum_g \sum_{n\neq i} \left[\left(1 - \frac{(1+\omega_{ni,g})(1+\bar{\tau}_i)}{1+t_{ni,g}}\right) e_{ni,g}\eta_{ni,g}\right] + \sum_g \left[\left(1 - \frac{1+s_{i,g}}{1+\mu_g}\right) e_{ii,g}\eta_{ii,g}\right].$$

Suppose there exists an optimal policy choice, $\mathbb{T}_i^*$ for which $\widetilde{\Delta}_i^* \neq 0$. Analogously, let $\widetilde{\Delta}_i^{\star}(a)$ denote the aggregate term collecting income effects under the equivalent policy choice, $\mathbb{T}_i^{\star}(a)$—with $\widetilde{\Delta}_i^* = \widetilde{\Delta}_i^{\star}(1)$, by construction. In particular,

$$\widetilde{\Delta}_i^{\star}(a) = \sum_g \sum_{n\neq i} \left[\left(1 - \frac{(1+\omega_{ni,g})(1+\bar{\tau}_i)}{(1+a)\left(1+t^*_{ni,g}\right)}\right) e_{ni,g}\eta_{ni,g}\right] + \sum_g \left[\left(1 - \frac{1+s_{i,g}}{(1+\mu_g)(1+a)}\right) e_{ii,g}\eta_{ii,g}\right],$$

where all equilibrium variables are evaluated at $\mathbb{T}_i^{\star}(a)$. Based on Lemma 1, all variables in the above equation (e.g., $e_{ni,g}$, $\omega_{ni,g}$, $\eta_{ni,g}$, etc.) are independent of a—since varying a preserves real equilibrium outcomes based on Lemma 1. Considering this, it should be the case that

$$\lim_{a\to -1} \widetilde{\Delta}_i^{\star}(a) < 0; \qquad \lim_{a\to\infty} \widetilde{\Delta}_i^{\star}(a) > 0.$$

So, following the Intermediate Value Theorem, there exists an $a \in (-1, \infty)$ such that $\widetilde{\Delta}_i^{\star}(a) = 0$. That is, if we suspect there to be an optimal policy $\mathbb{T}_i^*$ satisfying $\widetilde{\Delta}_i^* \neq 0$, that policy can be recovered by re-scaling an optimal policy, $\mathbb{T}_i^{\star}(a)$, that is welfare-equivalent to $\mathbb{T}_i^*$but satisfies $\widetilde{\Delta}_i^{\star}(a) = 0$.

E.2 Non-Optimality of Prohibitive Taxes

Since prohibitive taxes exclude goods from the system of F.O.C.s, we must prove that a tax schedule that prohibits say good ji,k but satisfies the F.O.C.s w.r.t. all other goods is not optimal. We prove this point separately for taxes applied to domestically-consumed goods and taxes applied to export goods.

Prohibitive tax on domestically-consumed good ji,k—We first provide a generic proof starting from the first principles to communicate the logic behind the non-optimality of prohibitive taxes. Then, we provide an alternative proof invoking the system of F.O.C.s derived earlier. To articulate our generic proof, suppose without loss of generality that good ii,k is the good not subjected to a prohibitive tax. Utility maximization entails that $\partial U_i(\mathbf{Q}_i)/\partial Q_{ji,k} = \lambda_i^{\mathcal{L}} \tilde{P}_{ji,k}$ for all ji,k, where $\lambda_i^{\mathcal{L}}$ is the Lagrange multiplier associated with the representative consumer's budget constraint ($\tilde{\mathbf{P}}_i \cdot \mathbf{Q}_i \leq Y_i$). Assuming that $U_i(.)$ satisfies the Inada conditions, utility maximization implies that the marginal utility associated with good ji,k at the prohibitive price is infinitely large; and so is the marginal rate of substitution between goods ji,k and ii,k:

$$\lim_{\frac{\tilde{P}_{ji,k}}{\tilde{P}_{ii,k}}\to\infty}\frac{\partial U_i/\partial Q_{ji,k}}{\partial U_i/\partial Q_{ii,k}}=\infty.$$

Let $F_k\left(\mathbf{Q}_i;\tilde{\mathbf{P}}_i,\tilde{\mathbf{P}}_{-i}\right)=0$ denote the transformation frontier for country i, which reflects country i's production possibility frontier (PPF) augmented for its ability to transform exports to imports subject to balanced trade. Following Dixit and Norman (1980), the relative marginal rate of transformation, $\frac{\partial F_i/\partial Q_{ii,k}}{\partial F_i/Q_{ji,k}}$, is finite if the utility and production functions satisfy the Inada conditions and $Q_{ii,k}$ is strictly positive—which is the case since good ii,k is not subjected to a prohibitive price. As a result,

$$\frac{\tilde{P}_{ji,k}}{\tilde{P}_{ii,k}}\to\infty\quad\Longrightarrow\quad\frac{\partial U_i/\partial Q_{ji,k}}{\partial U_i/\partial Q_{ii,k}}>\frac{\partial F_i/\partial Q_{ii,k}}{\partial F_i/Q_{ji,k}},$$

indicating that (when $\tilde{P}_{ji,k}=\infty$, $Q_{ji,k}=0$) the marginal rate of substitution between $Q_{ji,k}$ and $Q_{ii,k}$ exceeds the marginal rate of transformation. Hence, welfare ($U_i\sim W_i$) can be improved by increasing the consumption of good ji,k relative to ii,k, which entails lowering the price of ji,k from its prohibitive level. Prohibitive taxes, as such, cannot be optimal unless the scale or substitution elasticities are unbounded. This result echoes the *Grinols-Wong* theorem that a piecemeal reduction in prohibitive tariffs is welfare improving (see Feenstra (2015, P. 198)). The logic is that a prohibitively-taxed good exhibits an infinitely large marginal utility. So, the gains from restoring its consumption dominate the possible efficiency loss from cross-substitution and a lower scale-of-production on other goods.

The above point can be alternatively proven by appealing to the F.O.C.s specified by Equation 38. Suppose all prices other than $\tilde{P}_{ji,k}$ are set to their non-prohibitive optimal level. let $\underline{\mathbb{P}}_i^*=\left\{P_{ji,k},\mathbb{P}^*_{-ji,k}\right\}$ denote the policy vector representing this choice of prices. Following Equation 38, the marginal welfare effects of adjusting $\tilde{P}_{ji,k}$ in the neighborhood of $\underline{\mathbb{P}}_i^*$ is

$$\frac{\partial W_i}{\partial\ln\tilde{P}_{ji,k}}\mid_{\underline{\mathbb{P}}_i^*}=\left(\frac{\tilde{P}_{ji,k}}{P_{ji,k}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\left[\varepsilon_{ji,k}^{(ji,k)}+\eta_{ji,k}\left(\frac{\partial Y_i}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right].$$

Notice that $\varepsilon_{ji,k}^{(ji,k)}<0$ and $\eta_{ji,k}>0$, since the demand function is assumed to be well-behaved. Also, the tax revenues from good ji,k (namely,$T_{ji,k}$) approach zero *from above* as $\tilde{P}_{ji,k}\to\infty$. Hence, for sufficiently large values of $\tilde{P}_{ji,k}$, it must be the case that

$$\left(\frac{\partial\ln Y_i}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=\left(\frac{\partial\ln T_{ji,k}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}<0.$$

The above equation, correspondingly, implies that $\frac{\partial W_i}{\partial\ln\tilde{P}_{ji,k}}\mid_{\underline{\mathbb{P}}_i^*}$ is negative when the tax rate on good ji,k is sufficiently large (or nearly-prohibitive):

$$\frac{\tilde{P}_{ji,k}}{P_{ji,k}}\gg(1+\bar{\tau}_i)(1+\omega_{ni,g})\implies\frac{\partial W_i}{\partial\ln\tilde{P}_{ji,k}}\mid_{\underline{\mathbb{P}}_i^*}<0.$$

The above result reveals that lowering $\tilde{P}_{ji,k}/P_{ji,k}$ starting from a prohibitive price/tax rate will improve welfare (W_i)—asserting that a prohibitive tax, which excludes good ji,k from the system of F.O.C.s, cannot be optimal. The same logic can be applied to show that a prohibitive tax on two-or-more goods is not optimal either.

Prohibitive tax on export good ij,k—The price of export good, ij,k, does not explicitly enter the representative consumer's indirect utility function, $V_i\left(Y_i,\tilde{\mathbf{P}}_i\right)$. So, given the government's choice w.r.t. $\tilde{\mathbf{P}}_i\subset\tilde{\mathbb{P}}_i$, the choice of $\tilde{P}_{ij,k}\in\tilde{\mathbb{P}}_i$influences welfare solely through its effect on tax revenues, which contribute to income, Y_i. Under a prohibitive export tax rate, i.e., $\tilde{P}_{ij,k} = \infty$, the export tax revenues associated with good ij,k are trivially zero, i.e., $T_{ij,k} = 0$. Lowering $\tilde{P}_{ij,k}$ from its prohibitive level elevates $T_{ij,k}$ to a positive value and, thus, raises total tax revenues, T_i.[78] Lowering $\tilde{P}_{ij,k}$ from its prohibitive level, thus, raises income and thereby welfare given that $\partial V_i(.)/\partial Y_i > 0$. This chain of arguments asserts that prohibitive export taxes cannot be optimal since they yield the lowest possible tax revenue—resonating with the conventional Laffer curve argument. The same point can be demonstrated using the F.O.C.s specified by Equation 52. In particular, suppose all prices other than $\tilde{P}_{ij,k}$ are set to their non-prohibitive optimal level. Equation 52 indicates that the marginal welfare effect of lowering $\tilde{P}_{ij,k}$ is strictly positive *if* the initial value assigned to $\tilde{P}_{ij,k}$ is arbitrarily large.

E.3 Approximate Export Supply Elasticity: *Derivation Details*

This appendix provides a detailed derivation of our approximate export supply elasticity formula. Our derivation, recall, relies on the approximate matrix inversion technique developed by Wu et al. (2013). Note that the inverse export supply elasticity (when country i is the tax-imposing authority) is

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_{g\in\mathbb{K}} \left[\frac{w_i L_i}{w_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} + \sum_{n\neq i} \frac{w_n L_n}{w_j L_j} r_{ni,g}\rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \right], \tag{69}$$

where each of the price derivative $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$ can be characterized using the following system:

$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \end{bmatrix} = -\mathbf{A}_i^{-1}\mathbf{C}_i, \tag{70}$$

where, from Appendix X, $\mathbf{A}_i$ and $\mathbf{C}_i$ are square matrixes whose elements are

$$(\mathbf{A}_i)_{\iota,\jmath} = \mathbb{1}_{\jmath=\iota} + \mathbb{1}_{\jmath\neq i}\frac{\mu_k}{1+\mu_k}\sum_{n\neq i}\left[r_{\iota n,k}\varepsilon_{\iota n,k}^{(\jmath n,k)}\right]; \qquad (\mathbf{C}_i)_{\iota,\jmath} = \mathbb{1}_{\jmath=\iota}\frac{\mu_k}{1+\mu_k}r_{\iota i,k}.$$

Our goal is to apply Wu et al.'s (2013) approach to derive a first-order approximation for $\mathbf{A}_i$, which is then used to compute $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$ and $\omega_{ji,k}$. For this, decompose $\mathbf{A}_i$ into its diagonal, $\mathbf{D}_i$, and off-diagonal, $\mathbf{E}_i$, such that

$$\mathbf{A}_i = \mathbf{D}_i + \mathbf{E}_i.$$

The elements of the diagonal matrix associated with $\mathbf{A}_i$ are given by

$$(\mathbf{D}_i)_{\iota,\iota} = 1 + \mathbb{1}_{\iota\neq i}\frac{\mu_k}{1+\mu_k}\sum_{n\neq i}\left[r_{\iota n,k}\varepsilon_{\iota n,k}\right].$$

The elements of the off-diagonal matrix associated with $\mathbf{A}_i$ are, correspondingly,

$$(\mathbf{E}_i)_{\iota,\jmath} = \begin{cases} 0 & \text{if } (\iota = i) \vee (\iota = \jmath) \\ \frac{\mu_k}{1+\mu_k}\sum_{n\neq i}\left(r_{\iota n,k}\varepsilon_{\iota n,k}^{(\jmath n,k)}\right) & \text{if } (\iota \neq i) \wedge (\iota \neq \jmath) \end{cases}.$$

Following Wu et al. (2013), if the off-diagonal elements of $\mathbf{A}_i$ are small, we can appeal to the Neumann Series to approximate the inverse of $\mathbf{A}_1$as

$$\mathbf{A}_i^{-1} \approx \mathbf{D}_i^{-1} - \mathbf{D}_i^{-1}\mathbf{E}_i\mathbf{D}_i^{-1}.$$

[78]Beyond the Cobb-Douglas case, lowering the price of good ij,k can alter the revenue raised from other goods through cross-demand effects. But total revenue always increase, in response, given Cournot aggregation.

Based on the above equation, each element of the inverse of $\mathbf{A}_i$ can be written (approximately) in closed-form in terms of the diagonal and off-diagonal elements of $\mathbf{A}_i$:

$$\mathbf{A}_i^{-1} \approx \begin{bmatrix} \frac{1}{(\mathbf{D}_i)_{11}}\left(1-\frac{(\mathbf{E}_i)_{11}}{(\mathbf{D}_i)_{11}}\right) & \frac{-(\mathbf{E}_i)_{12}}{(\mathbf{D}_i)_{11}(\mathbf{D}_i)_{22}} & \cdots & \frac{-(\mathbf{E}_i)_{1N}}{(\mathbf{D}_i)_{11}(\mathbf{D}_i)_{NN}} \\ \frac{-(\mathbf{E}_i)_{21}}{(\mathbf{D}_i)_{22}(\mathbf{D}_i)_{11}} & \frac{1}{(\mathbf{D}_i)_{22}}\left(1-\frac{(\mathbf{E}_i)_{22}}{(\mathbf{D}_i)_{22}}\right) & \cdots & \frac{-(\mathbf{E}_i)_{2N}}{(\mathbf{D}_i)_{22}(\mathbf{D}_i)_{NN}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-(\mathbf{E}_i)_{N1}}{(\mathbf{D}_i)_{NN}(\mathbf{D}_i)_{11}} & \frac{-(\mathbf{E}_i)_{N2}}{(\mathbf{D}_i)_{NN}(\mathbf{D}_i)_{22}} & \cdots & \frac{1}{(\mathbf{D}_i)_{NN}}\left(1-\frac{(\mathbf{E}_i)_{NN}}{(\mathbf{D}_i)_{NN}}\right) \end{bmatrix}.$$

Invoking the above approximation, Equation 70 yields the following approximation for $\left(\frac{\partial \ln P_{\iota,k}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i}$:

$$\left(\frac{\partial \ln P_{\iota,k}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\tilde{\mathbb{P}}_i} \approx \begin{cases} \dfrac{\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1+\mathbb{1}_{\iota\neq i}\times\frac{\mu_k}{1+\mu_k}\sum_{n\neq i}(r_{\iota n,k}\varepsilon_{\iota n,k})} & \text{if } \iota = j \\ \dfrac{\frac{\mu_k}{1+\mu_k}\sum_{n\neq i}\left[r_{\iota n,k}\varepsilon_{\iota n,k}^{(jn,k)}\right]\left(\frac{\mu_k}{1+\mu_k} r_{ji,k}\right)}{\left(1+\frac{\mu_k}{1+\mu_k}\sum_{n\neq i} r_{jn,k}\varepsilon_{jn,k}\right)\left(1+\mathbb{1}_{\iota\neq i}\times\frac{\mu_k}{1+\mu_k}\sum_{n\neq i} r_{\iota n,k}\varepsilon_{\iota n,k}\right)} & \text{if } \iota \neq j \end{cases}$$

Plugging the price derivatives specified above into Equation 69, while noting that $r_{ni,k} \times r_{ji,k} \approx 0$ if $j \neq i$ and $n \neq i$, yields our approximation for the export supply elasticity:

$$\omega_{ji,k} \approx \frac{-\frac{\mu_k}{1+\mu_k} r_{ji,k}}{1+\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}}\left[1-\frac{\mu_k}{1+\mu_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{ji,k}}\varepsilon_{in,k}^{(jn,k)}\right].$$

E.4 Unilaterally Optimal Policy Net of ToT Considerations

Suppose we redo the entire proof under one restriction: Country i treats the entire vector of international prices as given. This includes (a) all consumer prices, $\tilde{\mathbf{P}}_{-i} = \{\tilde{P}_{nj,k}\}_{j\neq i}$, unassociated with domestic consumers and (b) all producer prices, $\mathbf{P}_{-i} \equiv \{P_{nj,k}\}_{n\neq i}$, unassociated with domestic firms. The idea here is that government presumes that cannot manipulate the consumer, $\tilde{P}_{ij,k}$, of export goods. Nor can it influence the producer price, $P_{ji,k}$, of import (or non-imported foreign) goods. Going back to the derivations presented above, we can solve this new terms-of-trade-blind problem by discarding the F.O.C.s relating to export prices (which amounts to setting $\tilde{P}_{ij,k} = P_{ij,k}$), and setting the inverse export supply elasticity to zero everywhere. Performing these alterations yields the following formula for optimal taxes:

$$\begin{aligned} &[\text{domestic subsidy}] && 1+s_{i,k}^* = (1+\mu_k)/(1+\overline{\mu}_i) \\ &[\text{import tariff}] && 1+t_{ji,k}^* = 1+\bar{\tau}_i \\ &[\text{export subsidy}] && \mathbf{1}+\mathbf{x}_{ij}^* = 1+\bar{\tau}_i. \end{aligned}$$

Normalizing the tax-shifters to zero (i.e., $\bar{\tau}_i = \overline{\mu}_i = 0$) yields the cooperative optimal tax structures consisting of zero trade taxes and Pigouvian industrial subsidies. So, if welfare-maximizing governments were (i) blind to terms-of-trade gains from policy and (ii) granted a complete vector of domestic policy instruments, they would adopt the cooperative policy choice. This point can be restated as follows: When welfare-maximizing governments have sufficient policy instruments at their disposal, their non-cooperative choice only inflicts a terms-of-trade externality on partners. So, the sole purpose of *shallow trade* agreements is to remedy the terms-of-trade externality. This result is a strict generalization of Bagwell and Staiger (2001, 2004). Notice a *shallow agreement* cannot resolve the problem of policy implementation, which is highlighted in Section 4 and quantified in Section 7.

F Efficient Policy from a Global Standpoint

The central planner seeks to maximize a weighted sum of national-level welfare using two sets of policy instruments. (1) A vector of good-specific taxes which grants them the ability to set consumers prices, $\tilde{\mathbf{P}}$, in every location. (2) A vector of inter-country lump-sum transfers, which enables them to control the share of each country's income from global income. Accordingly, the planner's choice of inter-country transfers is summarized by $\boldsymbol{\alpha} = \{\alpha_i\}_i$, where α_i denotes country i's share from global income ($Y = \sum_{i,n,k} \tilde{P}_{ni,k} Q_{ni,k}$) after transfers.

Recall that country i's welfare is summarized by the indirect utility function, $W_i \sim V_i\left(\tilde{Y}_i, \tilde{\mathbf{P}}_i\right)$, where country i's income under the planner's choice of policy is $\tilde{Y}_i = \alpha_i Y$. Considering this choice of notation, the global planner's problem can be formulated as

$$\max_{\tilde{\mathbf{P}},\boldsymbol{\alpha}} \sum_i \delta_i \ln \underbrace{V_i\left(\alpha_i Y, \tilde{\mathbf{P}}_i\right)}_{W_i},$$

where δ_i denotes the Pareto weight assigned to country i. Notice that Y_i (the equilibrium income raised by country i) does not explicitly appear in the objective function since the planner can obtain any desired vector of national-level incomes, $\{\tilde{Y}_i\}_i \sim \{\alpha_i Y\}_i$, with an appropriate choice of transfers subject to $\sum_i \tilde{Y}_i = Y$, where Y is the sum of equilibrium wage payments and tax revenues across all countries.[79] Namely,

$$Y\left(\mathbb{P};\mathbf{w},Y\right) = \sum_n w_n L_n + \sum_{i,n}\sum_k \left(\tilde{P}_{ni,k} - P_{ni,k}\right) Q_{ni,k}\left(\mathbb{P};Y\right),$$

where $\mathbb{P} \equiv \left\{\tilde{\mathbf{P}},\boldsymbol{\alpha}\right\}$ denotes the complete set of policy instruments available to the central planner and $Q_{ni,k}\left(\mathbb{P};Y\right) = \mathcal{D}_{ni,k}\left(\tilde{\mathbf{P}}_i, \alpha_i Y\right)$ with $\tilde{\mathbf{P}}_i \subset \tilde{\mathbf{P}}$.[80] Following the logic presented in Section 2, we can specify the planner's objective function, $W\left(\mathbb{P};\mathbf{w},Y\right) = \sum_i \delta_i \ln W_i\left(\mathbb{P};\mathbf{w},Y\right)$, as an explicit function of policy, $\mathbb{P}$, wages, $\mathbf{w}$, and global income, Y—noting that $\mathbf{w}$ and Y are feasible if they satisfy equilibrium conditions given $\mathbb{P}$. The first-order condition w.r.t. price instrument, $\tilde{P}_{ji,g} \in \tilde{\mathbf{P}} \subset \mathbb{P}$, can be written as

$$\frac{\partial W\left(\mathbb{P};\mathbf{w},Y\right)}{\partial \ln \tilde{P}_{ji,g}} = \sum_n \left[\delta_n \left(\frac{\partial \ln V_n(.)}{\partial \ln \tilde{Y}_n}\right) \frac{\partial \ln Y}{\partial \ln \tilde{P}_{ji,g}}\right] + \delta_i \frac{\partial \ln V_i(.)}{\partial \ln \tilde{P}_{ji,g}} = 0. \tag{71}$$

where the right-hand side uses the fact that $\tilde{Y}_n = \alpha_n Y$, implying that $\frac{\partial \ln \tilde{Y}_n}{\partial \ln \tilde{P}_{ji,g}} = \frac{\partial \ln Y}{\partial \ln \tilde{P}_{ji,g}}$ since α_n is a policy choice. Borrowing from our earlier derivation leading to the proof of Theorem 1, we can specify the change in global income in response to $\tilde{P}_{ji,k}$ as

$$\begin{aligned}
\frac{\partial \ln Y\left(\mathbb{P};\mathbf{w},Y\right)}{\partial \ln \tilde{P}_{ji,g}} &= \frac{1}{Y}\left\{\frac{\partial Y(.)}{\partial \ln \tilde{P}_{ji,g}} + \sum_n \left[\frac{\partial Y(.)}{\partial \ln w_n}\frac{\mathrm{d}\ln w_n}{\mathrm{d}\ln \tilde{P}_{ji,g}}\right] + \frac{\partial Y(.)}{\partial \ln Y}\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln \tilde{P}_{ji,g}}\right\} \\
&= \frac{1}{Y}\left\{\tilde{P}_{ji,g} Q_{ji,g} + \sum_n \sum_k \left(\left[\tilde{P}_{ni,k} - \frac{1}{1+\mu_k} P_{ni,k}\right] Q_{ni,k} \varepsilon_{ni,k}^{ji,k}\right)\right. \\
&\left. + \sum_n \left(\left[w_n L_n - \sum_{\iota,k} P_{n\iota,k} Q_{n\iota,k}\right] \frac{\mathrm{d}\ln w_n}{\mathrm{d}\ln \tilde{P}_{ji,g}}\right) + \sum_{n,\iota}\sum_k \left(\left[\tilde{P}_{ni,k} - \frac{1}{1+\mu_k} P_{ni,k}\right] Q_{ni,k}\eta_{ni,k}\right) \frac{\mathrm{d}\ln Y}{\mathrm{d}\ln \tilde{P}_{ji,g}}\right\},
\end{aligned} \tag{72}$$

where notice that the first term in the second line is zero given the labor market clearing condition,

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[79] To put it differently, the planner's problem (as we specify it) separates the issue of restoring production efficiency from inter-national redistribution. The former objective is attained with the proper choice of $\tilde{\mathbf{P}}$; the latter is attained with the proper choice of $\boldsymbol{\alpha}$—a point we discuss more later.

[80] The notation $Y = Y\left(\mathbb{P};\mathbf{w},Y\right)$ makes explicit the circular nature of income effects in general equilibrium.

$w_n L_n = \sum_{\iota,k} P_{n\iota,k} Q_{n\iota,k}$. Appealing to Roy's identity we can formulate the mechanical consumption loss from raising $\tilde{P}_{ji,k}$ as

$$\delta_i \frac{\partial \ln V_i\left(.\right)}{\partial \ln \tilde{P}_{ji,g}} = -\frac{\delta_i}{\tilde{Y}_i} \tilde{P}_{ji,g} Q_{ji,g} \frac{\partial \ln V_i\left(.\right)}{\partial \ln Y_i} \mid_{Y_i = \tilde{Y}_i}, \tag{73}$$

where note that $\tilde{Y}_i = \alpha_i Y$ by the definition of α_i, which is the country i's optimal share of global income given lump-sum transfers. Combining Equations 71-73, and dividing by the final expression by $\sum_n \delta_n \frac{\partial \ln V_n}{\partial \ln Y_n} > 0$ yields the following F.O.C. with respect to price instrument $\tilde{P}_{ji,k}$:

$$\left(1 - \frac{1}{\alpha_i} \frac{\delta_i \frac{\partial \ln V_i}{\partial \ln Y_i}}{\sum_n \delta_n \frac{\partial \ln V_n}{\partial \ln Y_n}}\right) \tilde{P}_{ji,k} Q_{ji,k} + \sum_n \sum_k \left(\left[1 - \frac{1}{1+\mu_k} \frac{P_{ni,k}}{\tilde{P}_{ni,k}}\right] \tilde{P}_{ni,k} Q_{ni,k} \varepsilon_{ni,k}^{(ji,k)}\right)$$
$$+ \sum_{n,\iota} \sum_k \left(\left[1 - \frac{1}{1+\mu_k} \frac{P_{n\iota,k}}{\tilde{P}_{n\iota,k}}\right] \tilde{P}_{n\iota,k} Q_{n\iota,k} \eta_{n\iota,k}\right) \frac{\mathrm{d} \ln Y}{\mathrm{d} \ln \tilde{P}_{ji,g}} = 0. \tag{74}$$

Optimal Policy Implementation—Next, we specify the taxes that deliver the optimal consumer-to-producer price wedges. We also specify the optimal income shares, which implicitly determine the optimal international lump-sum transfers. The trivial solution to Equation 74 involves price wedges equal to $\tilde{P}_{ni,k}/P_{ni,k} = 1/\left(1+\mu_k\right)$ and income shares that ensure the term in the first parenthesis to zero. The optimal price wedges, notice, are trade blind, indicating the optimal price wedges can be implemented with production subsidies alone. Appealing to this basic point, the production tax and transfers that satisfy the system of F.O.C.s consist of zero trade taxes, production subsidies that are proportional to the industry-level scale elasticities, and lump-sum transfers such that each country's share of global income reflects its Pareto weight and marginal utility from income. Stated formally,

$$1 + s_{i,k}^{\star} = \frac{P_{ni,k}^{\star}}{\tilde{P}_{ni,k}^{\star}} = 1 + \mu_k; \qquad x_{ij,k}^{\star} = t_{ji,k}^{\star} = 0; \qquad \alpha_i^{\star} = \frac{\delta_i \frac{\partial \ln V_i}{\partial \ln Y_i}}{\sum_n \delta_n \frac{\partial \ln V_n}{\partial \ln Y_n}}.$$

The Logic Behind Efficient Policy Formulas—The notion of optimal policy in our framework (as in much of the trade policy literature) is formulated to deliver the first-best outcome from the planner's standpoint. The planner, in particular, is afforded sufficient policy instruments to achieve both production efficiency and their desired level of redistribution. Production subsidies that restore marginal cost pricing are used to achieve production efficiency, while efficient lump-sum transfers are used to attain redistributive objectives based on Pareto weights. To elucidate this point, suppose lump-sum transfers where unavailable. Then, implementing the efficient tax schedule $\mathbb{T}^{\star} = \left(\mathbf{t}^{\star}, \mathbf{x}^{\star}, \mathbf{s}^{\star}\right)$ without transfers would deliver a Kaldor-Hicks (Kaldor (1939); Hicks (1939)) improvement but *not* necessarily a Pareto improvement. Still, the resulting equilibrium would be Hicks-optimal and, therefore, Pareto efficient. To ensure Pareto improvements (relative to Laissez-Faire) without lump-sum transfers, the optimal policy must also include non-zero trade taxes that redistribute the welfare gains from restoring marginal cost pricing across countries. But when efficient lump-sum transfers are available, the planner avoids redistribution via trade taxes as they undermine production efficiency.

Efficient Policy vs. Cooperative Tariffs—It is important to distinguish between efficient policies and cooperative tariffs of the sort examined by Ossa (2014) and Lashkaripour (2020b). Efficient policies deliver the global planner's first-best outcome. Cooperative tariffs, on the other hand, maximize global welfare in second-best scenarios, where efficient production subsidies and transfers are unavailable. More formally, cooperative tariffs are given by

$$\mathbf{t}^{\star\star} = \arg\max \sum_i \delta_i \log W_i\left(\mathbf{t}\right) \qquad s.t. \begin{cases} \mathbf{s} = 0 \\ \alpha_{i,k} = Y_i\left(\mathbf{t}\right)/Y\left(\mathbf{t}\right) \end{cases}$$

So, $\mathbf{t}^{\star\star}$, by design, mimics the first-best (or efficient) subsidies and transfers to deliver the second-best.[81] In other words, cooperative seek to improve allocative efficiency by restricting trade (and thus global output) in low-μ industries. They also seek to redistribute inter-nationally, taking into account the Pareto weights in the planner's objective function or the bargaining weights in the Nash bargaining formulation of the same problem.[82]

Efficient Policy Under Political Economy Considerations—Our baseline model and the efficient policies implied by it abstract from political economy considerations. What if governments assign different political weights to say profits collected from different industries? In that case, efficient production subsidies should take into account not only the industry-level scale elasticity (or markup) but also its political weight. We can, moreover, refer to the discussion in Section 5 to specify the politically-adjusted efficient policy. Recall, in particular, that the political economy model is isomorphic to an augmented version of our baseline model wherein markups are politically-adjusted and given by $\mu^{\mathscr{P}}_{i,k} = \frac{\mu_k}{\pi_{i,k}-(1-\pi_{i,k})\mu_k}$, where $\pi_{i,k}$ is the political economy weight assigned by the planner to profits collected from industry k in origin i. The efficient production subsidy, accordingly, becomes $s^{\star}_{i,k} = \mu^{\mathscr{P}}_{i,k}$.

G Internal Cooperation within the Rest of the World

When characterizing country i's unilaterally optimal policy, we treat the rest of the world as internally cooperative. Our notion of cooperation is based on the WTO's core principles: *reciprocity* and *non-discrimination* (see Bagwell and Staiger (2004)). The principle of reciprocity entails that cooperative countries maintain the balance of market access concessions internally. In our model, where labor is the sole factor of production, any change in relative market access is equivalent to a change in relative wages (see Footnote 84). Hence, to maintain the balance of concessions, cooperative countries must adopt policy buffers that neutralize relative wage disruptions among each other. Otherwise, the subset of countries whose relative wage improves in response to country i's policy reap terms-of-trade (or market access) gains at the expense of others whose relative wage deteriorates.

To formalize these arguments, we first specify the change in country n's welfare in response to country i's policy, $\{\mathrm{d}\ln\left(\mathbf{1}+\mathbf{x}_i\right), \mathrm{d}\ln\left(\mathbf{1}+\mathbf{t}_i\right), \mathrm{d}\ln\left(\mathbf{1}+\mathbf{s}_i\right)\}$. Suppose consumer preferences in country $n \neq i$ are homothetic. Appealing to Roy's identity, the welfare impacts of country i's policy shock on country n's welfare, can be expresses as $\mathrm{d}\ln W_n = \mathrm{d}\ln Y_n - \sum_j \lambda_{jn,k} e_{n,k} \mathrm{d}\ln \tilde{P}_{jn,k}$. Next, we characterize $\mathrm{d}\ln Y_n$ focusing on restricted entry for expositional purposes. Nominal income in country i is the sum of wage income adjusted for profit payments—namely,$Y_n = \left(1+\overline{\mu}_n\right) w_n L_n$, where $\overline{\mu}_n = \sum_k \mu_k \rho_{n,k}$ is

[81]As we argue shortly, cooperative tariffs also internalize political economy pressures if any. Put together, these points echo Ossa' (2016) verbal argument that second-best cooperative tariffs pursue three objectives: First, they seek to improve allocative efficiency by mimicking efficient production subsidies. Second, they seek to redistribute welfare inter-nationally based on Pareto or bargaining weights. Third, they seek to promote politically-organized industries. Though, they are not the first-best instrument for reaching either objective.

[82]Correspondingly, if the baseline economy is efficient, there exists a set of Pareto/bargaining weights (δ) for which $\mathbf{t}^{\star\star} = 0$ —see e.g., the analytic formula for $\mathbf{t}^{\star\star}$ in Lashkaripour (2020b).

the employment-weighted average markup in country n.[83] Taking full derivatives from the expression for Y_n, yields

$$\mathrm{d}\ln Y_n = \mathrm{d}\ln\left(\sum_k (1+\mu_k)\,\rho_{n,k}\right) + \mathrm{d}\ln w_n = \sum_k \left[\rho_{n,k}\cdot\left(\frac{1+\mu_k}{1+\overline{\mu}_n}\right)\mathrm{d}\ln\rho_{n,k}\right] + \mathrm{dln} w_n,$$

where $\mathrm{d}\ln\rho_{n,k}$ and $\mathrm{d}\ln w_n$ respectively denote the change in country n's employments shares and wage rate in response to country i's tax policy. To economize on the notation let $\mathbb{E}_\rho[.]$ and $Cov_\rho(.)$ denote cross-industry mean and covariance operators with weights, $\{\rho_{i,k}\}$. As a matter of accounting, $\sum_k \rho_{n,k}\mathrm{d}\ln\rho_{n,k} \sim \mathbb{E}_\rho[\mathrm{dln}\,\rho_{n,k}] = 0$, indicating that the first term in the last line of the above equation can be specified as

$$\sum_k \left[\rho_{n,k}\cdot\left(\frac{1+\mu_k}{1+\overline{\mu}_n}\right)\mathrm{d}\ln\rho_{n,k}\right] \sim Cov_\rho\left(\frac{1+\mu_k}{1+\overline{\mu}_n}, \mathrm{dln}\,\rho_{n,k}\right).$$

Next, we specify the welfare effects due to changes in consumer prices. The change in good-specific consumers prices for goods originating from $j \neq i$ is determined by the underlying wage change, i.e., $\mathrm{d}\ln\tilde{P}_{jn,k} = \mathrm{d}\ln w_j$ for all $j \neq i$. The change in consumers prices for goods originating from country i is the sum of the direct tax change and the indirect wage effects, i.e. $\mathrm{d}\ln\tilde{P}_{in,k} = \mathrm{d}\ln w_i + \mathrm{d}\ln(1+x_{in,k})$. Putting the pieces together, we can write the change in country n's welfare in response to country i's non-cooperative policy as

$$\begin{aligned}\mathrm{d}\ln W_n = {} & Cov_\rho\left(\frac{1+\mu_k}{1+\overline{\mu}_n}, \mathrm{dln}\,\rho_{n,k}\right) + (1-\lambda_{ii})\,\mathrm{dln} w_n \\ & - \sum_{j\neq i,n}\sum_k \left(\lambda_{jn,k}e_{n,k}\mathrm{dln}\, w_j\right) - \sum_k \left(\lambda_{in,k}e_{n,k}\left[\mathrm{d}\ln w_i + \mathrm{d}\ln(1+x_{in,k})\right]\right).\end{aligned}$$

We can rearrange the above equation and decompose the various welfare terms as

$$\begin{aligned}\mathrm{d}\ln W_n \;=\; & -\overbrace{\left(\lambda_{in}\mathrm{d}\ln(w_i/w_n) + \sum_k \lambda_{ni,k}e_{n,k}\mathrm{d}\ln(1+x_{in,k})\right)}^{\text{Terms-of-Trade vis-a-vis Country } i} \\ & + \underbrace{Cov_\rho\left(\frac{1+\mu_k}{1+\overline{\mu}_n}, \mathrm{dln}\,\rho_{n,k}\right)}_{\text{Allocative Efficiency}} - \underbrace{\sum_{j\neq i}\left[\lambda_{jn}\mathrm{dln}\left(w_j/w_n\right)\right]}_{\text{Terms-of-Trade vis-a-vis RoW}},\end{aligned} \tag{75}$$

where $\lambda_{jn} \equiv \sum_k \lambda_{ji,k}e_{n,k}$ denotes aggregate expenditure shares.[84] Following Baqaee and Farhi (2017), *Allocative Efficiency* effects are defined as the welfare change net of Hulten (1978) in response to policy shocks that do not raise revenues for closed-economy n. The remaining terms are, by design, terms of trade (ToT) effects. Theses can be divided into changes in ToT vis-à-vis country i and changes to ToT vis-à-vis the rest of the world, with which country n maintains cooperation.

Extraterritorial Terms of Trade Effects—Following Equation 75, country i's non-cooperative policy can disrupt country n's ToT and balance of concessions vis-à-vis countries other than i. Consider

[83] Recall for Section 2, that $\overline{\mu}_n \equiv \frac{\sum_{k,j}\frac{\mu_k}{1+\mu_k}P_{nj,k}Q_{nj,k}}{\sum_{k,j}\frac{1}{1+\mu_k}P_{nj,k}Q_{nj,k}}$. Noting that $w_n L_{n,k} = \frac{1}{1+\mu_k}P_{nj,k}Q_{nj,k}$, we can rewrite $\overline{\mu}_n$ as

$$\overline{\mu}_n = \frac{\sum_{k,j}\frac{\mu_k}{1+\mu_k}P_{nj,k}Q_{nj,k}}{\sum_{k,j}\frac{1}{1+\mu_k}P_{nj,k}Q_{nj,k}} = \sum_{k,j}\mu_k\frac{L_{n,k}}{L_n} \sim \sum_{k,j}\mu_k\rho_{n,k},$$

where $\rho_{n,k} \equiv L_{n,k}/L_n$ is the employment share assocaited with industry k in country n.

[84] We can re-formulate the above decomposition in the spirit of Arkolakis, Costinot, and Rodriguez-Clare (2012), to clarify

country $j \neq i$ who is cooperative with country n. If $\mathrm{d}\ln\left(w_j/w_n\right) < 0$, country n's ToT improves relative to j in response to country i's policy. Or stated differently, the bilateral balance of market access concessions tilts in favor of country n, which violates *reciprocity*. We call these *"Extraterritorial Terms of Trade Effects,"* as they disrupt the ToT and balance of concessions between countries in the rest of the world. To restore *reciprocity*—one of WTO's core principles—the rest of the world must exert wage buffers that neutralize the *extraterritorial ToT effects* associated with country i's policy.

Neutralizing Extraterritorial ToT Effects with Cooperative Wage Buffers—The rest of the world can institute cooperative wage buffers to neutralize the extraterritorial ToT effects associated with country i's policy, ensuring that $\Delta\ln\left(w_j/w_n\right) = 0$ for all $n, j \neq i$. Ideally, these policies must satisfy WTO's *non-discrimination* principle and be efficient, which effectively rules out trade tax measures. A policy option that satisfies these requirements is a wage tax-cum-subsidy that is either revenue-neutral or financed via an efficient lump-sum tax on residents of all countries outside of i. To elaborate, let $\underline{w}_n^*$ denote the wage rate in country n after the implementation of country i's optimal policy, $\tilde{\mathbb{P}}_i$, *if* no policy buffers were in place. The country-specific wage subsidy, $\boldsymbol{\tau} \equiv \{\tau_n^w\}_{n\neq i}$, is allotted such that $\tau_n^w/\tau_j^w < 1$ if $\underline{w}_n^*/\underline{w}_j^* < w_n/w_j$—to the point that the post-subsidy effective relative wage rates $\left(w_n^*\left(\boldsymbol{\tau}\right)/w_j^*\left(\boldsymbol{\tau}\right)\right)$ remains equal to their status-quo level. Namely, $w_n^*/w_j^* = w_n/w_j$ for all $n, j \neq i$.

Treating the RoW as Internally Cooperative vs. Merely Passive—Treating the rest of the world as cooperative or passive is, after all, a theoretical formality since for all practical purposes, country i's good-specific taxes have little-to-no effect on aggregate relative wages in the rest of the world. We demonstrate this point numerically in Appendix H using multiple simulations of our model—including some where country i is large. But even in theory, one can envision many settings in which the rest of the world being cooperative or passive is immaterial. Let us provide one such example. Suppose there is a traded homogeneous sector, k_0, operating under constant-returns to scale technologies (i.e., $\sigma_{k0} \approx \gamma_{k0} \to \infty$). Moreover, assume that sector k_0 has a strictly positive employment share, i.e., $\rho_{n,k_0} > 0$, in every country n (with the possible exception of i). Assuming that $e_{n,k0}$ is sufficiently large such that $\hat{\rho}_{n,k_0} \neq 0$ in response to country i's policy, ensures that $w_n/w_{n'}$ remains constant for all $n, n' \neq i$—even if the rest of the world is passive.[85] Notice that this is a strictly weaker version of the common assumption adopted by Fajgelbaum, Grossman, and Helpman (2011) and Ossa (2011), among others. In particular, these studies assume that neither country i's nor the rest of the world's employment in the homogeneous sector reduces to zero in response to country i's policy (i.e., $\hat{\rho}_{i,k_0} \neq 0$ and $\hat{\rho}_{n,k_0} \neq 0$, $\forall n \neq i$). Our example, in contrast, only requires that the rest of the world's employment in the homogeneous sector does not collapse to zero.

that market access is fully-determined by relative wages. In particular, appealing to the CES demand system whereby $\mathrm{d}\ln\left(\tilde{P}_{jn,k}/\tilde{P}_{nn,k}\right) = \frac{1}{1-\sigma_k}\mathrm{d}\ln\left(\lambda_{jn,k}/\lambda_{nn,k}\right)$, we can alternatively express the welfare effects of an external shock to economy n as

$$\begin{aligned}
\mathrm{d}\ln W_n &= \mathrm{d}\ln Y_n - \sum_k\left[e_{i,k}\mathrm{d}\ln\tilde{P}_{nn,k}\right] + \sum_{j,k}\left[\frac{e_{i,k}}{1-\sigma_k}\lambda_{jn,k}\mathrm{d}\ln\left(\frac{\lambda_{jn,k}}{\lambda_{nn,k}}\right)\right] \\
&= Cov_\rho\left(\frac{1+\mu_k}{1+\overline{\mu}_n}, \mathrm{d}\ln\rho_{n,k}\right) + \sum_k\left[\frac{e_{i,k}}{1-\sigma_k}\mathrm{d}\ln\lambda_{ii,k}\right],
\end{aligned}$$

where the last line follows from the adding up constraint, $\sum_j\lambda_{jn,k}\mathrm{d}\ln\lambda_{ji,k} = 0$. Notice that $\lambda_{ii,k}$, by definition, summarizes an open economy's market access. Comparing the above representation to Equation 75 indicates that the change in market access can be alternatively summarized by changes to relative wages.

[85]To elaborate, the price of the homogeneous good k_0 must be equalized across origins. Let a_{n,k_0} denote the constant unit labor requirement for producing good k_0 in origin n. Price equalization entails that $w_n/w_{n'} = a_{n',k_0}/a_{n,k_0}$, which is constant.

H Numerical Examination of Optimal Policy Formulas

This appendix illustrates the accuracy and speed of our theoretical optimal policy formulas by benchmarking against results obtained from numerical optimization. We, more specifically, demonstrate two points. First, our formulas often outperform numerical optimization as they identify an optimal policy schedule that is strictly superior to that specified by standard numerical optimization routines. The improved accuracy is especially notable when analyzing a global economy with many countries. Second, our theoretical formulas are orders of magnitude faster than numerical optimization at detecting optimal policy.

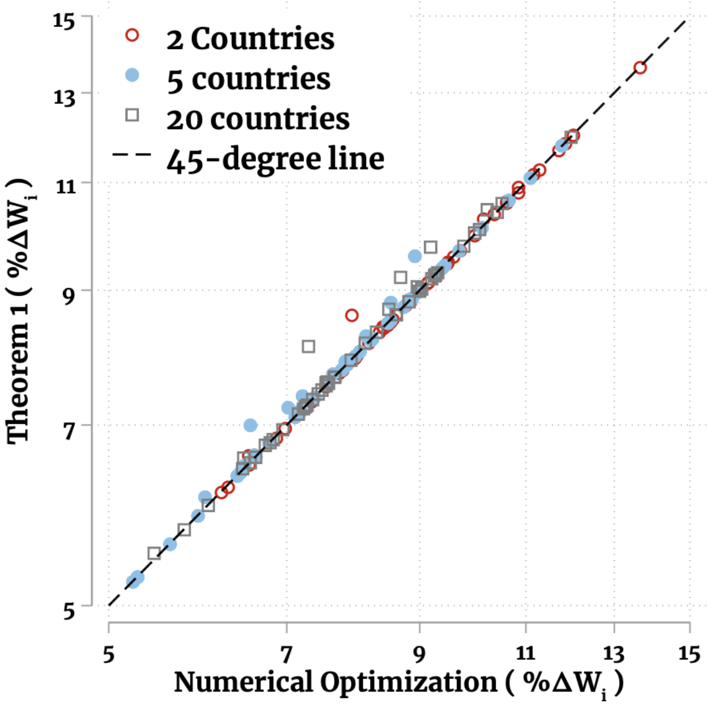
We must underscore two points to set the stage for our numerical analysis. First, throughout this section, we use our approximate formula for the inverse export supply elasticity. Second, we treat the rest of the world as passive rather than internally cooperative. As such, our numerical analysis reveals two additional points. First, that our approximation of the export supply elasticity exhibits great numerical precision. Second, treating the rest of the world as internally cooperative vs. passive is virtually inconsequential. Since individual policy instruments have negligible impacts on relative wages in the rest of the world, our optimal policy formulas retain accuracy even if the rest of the world is passive—even if the tax-imposing country is relatively large.

Details of Numerical Simulation—We examine three hypothetical economies with $N = 2$, 5, and 20 countries, each containing $S = 10$ industries. We assume that preferences across industries are Cobb-Douglas, with $e_{i,k}$ denoting the Cobb-Douglas weight on industry k in country i. To compute the optimal policy, we need to assign values to the following vector of parameters/endowments, $\Theta = \{\mu_k, \sigma_k, e_{i,k}, L_i\}_{i,k}$. The information relating to other parameters is implicit in the value assigned to the matrix of bilateral expenditure shares and national income levels, $\mathbf{X} = \{\lambda_{ij,k}, Y_i\}_{i,j,k}$. We normalize $Y_i = 100$ for all countries and randomly draw the remaining parameters/variables from a uniform distribution using the RAND function in MATLAB. We repeat this 50-time for each case, resulting in 150 simulations of the global economy under randomly-selected parameters. For each choice of parameters, we numerically solve for the optimal policy equilibrium using (a) our theoretical formulas relying on the optimization-free approach described under Proposition 3 in Section 7 and (b) using numerical optimization relying on the MPEC approach described in Ossa (2014). The latter is the standard approach when theoretical formulas are unavailable, so we benchmark our formulas' numerical accuracy and speed against it. Our implementation of MPEC, as in Ossa (2014), uses MATLAB's standard optimization routine, FMINCON.

Accuracy of Theoretical Formulas

Figure 5 compares the welfare gains implied by our optimal policy formulas to those obtained from numerical optimization (MPEC). Each dot corresponds to one of our 150 simulations. A dot lying on the 45-degree line in Figure 5 indicates that our theoretical formulas identify the same optimal policy schedule numerical optimization. Dots lying above the 45-degree line correspond to simulations where our theoretical formulas (Theorem 1) outperform numerical optimization (MPEC)—that is, they identify an optimal policy schedule that strictly dominates in terms of implied welfare gains, which is the policy objective. The reverse is true for dots below the 45-degree line. Keep in mind that the simulations in Figure 5 use our approximation of the export supply elasticity and treat the rest

Figure 5: Gains from optimal policy: theoretical formula vs. numerical optimization



Note: This figure reports results from 150 simulations in which parameters are randomly sampled for three cases of our model—namely, $N = 2, 5, 10$, with $K = 10$. The y-axis reports the pre-cent welfare gains predicted by our optimal policy formulas (Theorem 1) in a simulated model. The x-axis reports reports the welfare gains obtained from numerical optimization conducted using MATLAB's FMINCON routine.

of the world as passive—each of which can possibly compromise the performance of our theoretical formulas relative to numerical optimization.

Figure 5 reveals that the prediction of our theory is virtually identical to numerical optimization in most cases. On several occasions, our theoretical formulas outperform numerical optimization by a non-trivial margin. These are more frequent when we simulate a global economy consisting of more countries. We summarize this point more clearly in Figure 6. We divide simulation outcomes into three categories:

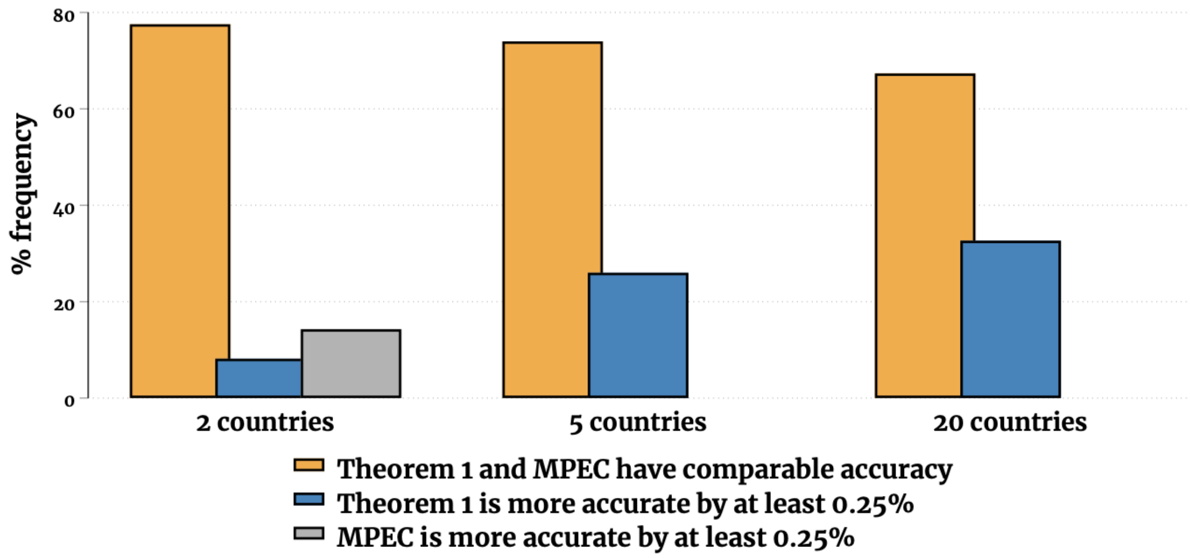
i. [*Orange*] Simulations where our theoretical formulas and numerical optimization (MPEC) predicted comparable gains from optimal policy, i.e., $\frac{\left|\Delta W_i^{\text{theory}} - \Delta W_i^{\text{MPEC}}\right|}{\min\left\{\Delta W_i^{\text{theory}}, \Delta W_i^{\text{MPEC}}\right\}} \leq 0.0025$.

ii. [*Blue*] Simulations where our theoretical formulas outperform numerical optimization (MPEC) by at least 0.25%, i.e., $\Delta W_i^{\text{theory}} > 1.0025 \times \Delta W_i^{\text{MPEC}}$.

iii. [*Grey*] Simulations where numerical optimization (MPEC) outperforms our theoretical formulas by at least 0.25%, i.e., $\Delta W_i^{\text{MPEC}} > 1.0025 \times \Delta W_i^{\text{theory}}$.

A clear takeaway is that –for all practical purposes– our theoretical formulas either deliver comparable accuracy or outperform numerical optimizations. Numerical optimization exhibits great accuracy when dealing with only two countries. With 20 countries, however, our theoretical formulas outperform numerical optimization by at least 0.25% in more than thirty percent of the simulations. This

Figure 6: *Performance of theoretical formulas vs. numerical optimization (MPEC)*

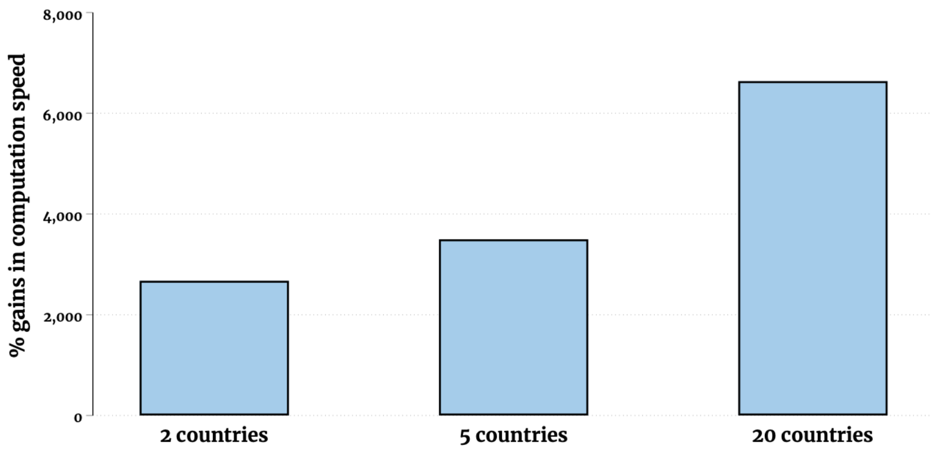


Note: This figure summarizes results from 150 simulations with randomly-sampled parameters. The yellow bars represent the frequency of simulations where our optimal policy formulas and MPEC (numerical optimization) predict welfare gains that are within 0.25% of one another in terms of magnitude. The blue bars represent the frequency of simulations where our optimal policy formulas predict welfare gains that at least 0.25% greater than those implied by numerical optimization (MPEC). The grey bars represents the frequency of simulations where our optimal policy formulas predict welfare gains that at least 0.25% greater than those implied by numerical optimization (MPEC).

improvement is noteworthy in practice, as we are often interested in cases where country i implements trade policy in relation to tens if not hundreds of trading partners. In these cases, numerical optimization must identify an optimal vector of policies consisting of hundreds and thousands of free-moving policy instruments—which can compromise accuracy depending on the properties of the underlying objective function.

Why does numerical optimization become less accurate with many countries?—Figure 5 reveals that, when dealing with many countries, our theoretical formulas occasionally outperform numerical optimization by a non-trivial margin. The reason is that with many countries and free-moving tax instruments, numerical optimization may detect a near-prohibitive good-specific tax rate that is non-optimal but artificially satisfies the first-order conditions to a good approximation. Even though numerical optimization identifies the appropriate policy vis-à-vis most goods in these cases, it fails with respect to one or more goods for which it converges to a high and non-optimal tax rate. One can perhaps navigate this pitfall by setting bounds on feasible tax choices, but it is unclear what these bounds should be without theory. Relatedly, Figure 6 suggests that our *approximated* optimal policy formulas occasionally underperform numerical optimization when dealing with two countries. This is a mere reflection of our export-supply-elasticity-approximation error, which can be non-trivial when country i is excessively large relative to the rest of the world. To elaborate, our simulation assigns the same size to all countries. Correspondingly, country i is similar in size to the entire rest of the world in the simulation

Figure 7: Improvement in computation speed from using theoretical formulas



Note: The figure compares the per-cent increase in computation speed when using our optimal policy formulas over numerical optimization. Each bar represents the average increase over 50 simulations with randomly-sampled parameters.

with two countries—thereby, the possibly large approximation error.[86]

The RoW being Internally Cooperative or Passive is Immaterial—When performing numerical optimization, we purposely treat the rest of the world as passive—i.e., we do not restrict relative wages to remain constant in the rest of the world. Yet our optimal policy formulas (which treat the rest of the world as internally cooperative) deliver predictions that are virtually identical to those obtained from numerical optimization (which treats the rest of the world as passive)—even though the tax-imposing country is large relative to the rest of the world in our simulations. These outcomes all but corroborate our previous assertion that the rest of the world being passive vs. cooperative is a theoretical formality and virtually immaterial from a quantitative standpoint.

Computational Speed of Theoretical Formulas

Figure 7 reveals that our theoretical formulas deliver orders of magnitude improvements in computation speed relative to numerical optimization (MPEC). Let t^{theory} denote the time it takes to compute the optimal policy equilibrium with the aid of our theoretical formulas. Correspondingly, let t^{MPEC} denote the computational time required to run numerical optimization. The y-axis in Figure 7 corresponds to $100 \times \left(t^{\text{MPEC}}/t^{\text{thoery}}\right)$, which is the per-cent improvement in computation speed when using our theoretical formulas over numerical optimization. Our theory delivers a more than 20-fold improvement in speed with two countries, and a more than 60-fold improvement with 20 countries. The gains will be, accordingly, greater in real-world scenarios involving many countries (like those examined in Section 7). The improvement in computation speed is especially crucial when determining the Nash equilibrium of a non-cooperative policy game, wherein each country's optimal policy must be solved iteratively as a function of others' policies. We perform such an analysis in Section 7, where it takes us a few minutes to identify the Nash equilibrium versus many hours if we had relied on numerical optimization.

[86]To be clear, in the two-country case, the rest of the world being passive (rather than internally cooperative) is is irrelevant and our approximation of the export supply elasticity is the only source of numerical error.

I Proof of Theorem 2

The proof of Theorem 2 has the same basic foundation as Theorem 1. We reformulate the optimal policy problem, expressing equilibrium variables (e.g., $Q_{ji,k}$, Y_i, etc.) as a function of (1) the vector of consumer prices associated with economy i, excluding $\tilde{\mathbf{P}}_{ii}$, i.e., $\tilde{\mathbb{P}}_i \equiv \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$;[87] and (2) the vector of national-level wage rates all over the world, $\mathbf{w} = \{w_1, ..., w_N\}$. To implement this reformulation of equilibrium variables, we need to solve the following system treating $\tilde{\mathbb{P}}_i$, and $\mathbf{w}$ as given:

$$\begin{aligned}
&\text{[optimal pricing]} && P_{jn,k} = \bar{\rho}_{ji,k} w_j \\
&\text{[optimal consumption]} && Q_{jn,k} = \mathcal{D}_{jn,k}(Y_n, \tilde{\mathbf{P}}_{1n}, ...\tilde{\mathbf{P}}_{Nn}) \\
&\text{[RoW imposes zero taxes]} && \tilde{P}_{jn,k} = P_{jn,k} \quad (\tilde{P}_{jn,k} \notin \tilde{\mathbb{P}}_i); \qquad Y_n = \overbrace{(1+\overline{\mu}_n) w_n L_n}^{w_n L_n + \Pi_n} \quad (n \neq i) \\
&\text{[Balanced Budget in } i\text{]} && Y_i = (1+\overline{\mu}_i) w_i L_i + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right) \cdot \mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right) \cdot \mathbf{Q}_{ji} \\
&\text{[avg. profit margin in } j\text{]} && 1 + \overline{\mu}_j = \frac{\sum_{n \in \mathbb{C}} \left[\mathbf{P}_{jn} \cdot \mathbf{Q}_{jn}\right]}{\sum_{n \in \mathbb{C}} \left[\mathbf{P}_{jn} \cdot \left(\mathbf{Q}_{jn} \oslash (\mathbf{1} + \boldsymbol{\mu})\right)\right]}
\end{aligned}$$

where "$\cdot$" denotes the inner product operator for vectors of equal size. "$\oslash$" denotes element-wise division of equal-sized vectors, with $\boldsymbol{\mu} \equiv \{\mu_k\}_k$. Since there is a unique equilibrium, the above system is exactly identified in that it uniquely determines $P_{jn,k}(\tilde{\mathbb{P}}_i; \mathbf{w})$, $Q_{jn,k}(\tilde{\mathbb{P}}_i; \mathbf{w})$, $Y_n(\tilde{\mathbb{P}}_i; \mathbf{w})$, and $\overline{\mu}_i(\tilde{\mathbb{P}}_i; \mathbf{w})$ as a function of $\tilde{\mathbb{P}}_i$ and $\mathbf{w}$. Appealing to the above reformulation of the equilibrium, we can reformulate the original optimal policy problem (P2) as follows.

Lemma 8. *Country i's vector of second-best trade taxes, $\{\boldsymbol{t}_i^{**}, \boldsymbol{x}_i^{**}\}$, can be determined by solving the following problem:*

$$\max_{\tilde{\mathbb{P}}_i} \; W_i(\tilde{\mathbb{P}}_i; \mathbf{w}) \equiv V_i(Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \quad s.t. \; (\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P \qquad (\widetilde{\text{P2}}),$$

where the feasibility constraint is satisfied if, given $\tilde{\mathbb{P}}_i$, the wage vector $\boldsymbol{w}$ satisfies balanced trade in each country:

$$(\tilde{\mathbb{P}}_i; \boldsymbol{w}) \in \mathbb{F}_P \iff \begin{cases} \sum_{j \neq n} \sum_{k \in \mathbb{K}} \left[P_{jn,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w}) Q_{jn,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w}) - P_{nj,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w}) Q_{nj,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w})\right] = 0 & \text{if } n \neq i \\ \sum_{j \neq n} \sum_{k=1}^{K} \left[P_{ji,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w}) Q_{jn,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w}) - \tilde{P}_{ij,k} Q_{nj,k}(\tilde{\mathbb{P}}_i; \boldsymbol{w})\right] = 0 & \text{if } n = i \end{cases}.$$

The system of F.O.C.'s underlying Problem $(\widetilde{\text{P2}})$ can be expressed as follows:

$$\nabla_{\tilde{P}} W_i(\tilde{\mathbb{P}}_i; \mathbf{w}) + \nabla_{\mathbf{w}} W_i \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\tilde{P}}\right)_{(\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P} = \mathbf{0}, \qquad \forall \tilde{P} \in \tilde{\mathbb{P}}_i = \left\{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\right\}.$$

In what follows we characterize and simplify the system of F.O.C., building heavily on the results presented in Appendix E.

Deriving the First-Order Condition w.r.t. $\tilde{\mathbf{P}}_{ji}$

Consider the consumer price index $\tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$ associated with a good imported by i from *origin j–industry k.* The F.O.C. w.r.t. this price instrument can be stated as follows:

[87] Recall that vectors $\tilde{\mathbf{P}}_{ji} \equiv \left\{\tilde{P}_{ji,k}\right\}_{j \neq i,k}$ and $\tilde{\mathbf{P}}_{ij} \equiv \left\{\tilde{P}_{ij,k}\right\}_{j \neq i,k}$ encompass only the export/import prices linked to economy i.

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0, \tag{76}$$

where $\mathbb{P}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \left\{\tilde{P}_{ji,k}\right\}$ denotes the vector of price instruments excluding $\tilde{P}_{ji,k}$. The above equation is similar to what we characterized in Appendix E under restricted entry, with two distinctions: First, country i's government does not control the price of domestically produced and domestically consumed varieties, i.e., $\tilde{\mathbf{P}}_{ii} \notin \tilde{\mathbb{P}}_i$. Second, country i's income does not include domestic tax revenues:

$$Y_i = (1+\overline{\mu}_i)w_i L_i + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right)\cdot \mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right)\cdot \mathbf{Q}_{ji}.$$

Taking note of these two differences, we can build on the derivation in Appendix E to simplify Equation 76. By Roy's identity, the first term on the right-hand side of Equation 76 can be stated as

$$\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k}Q_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Without repeating the derivations, the second term on the right-hand side of Equation 76 reduces to

$$\begin{aligned}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} =& \tilde{P}_{ji,k}Q_{ji,k} + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] \\ & \sum_g\left[\left(1-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \Delta_i'(\tilde{\mathbb{P}}_i,\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\end{aligned}$$

where $\Delta_i'(\tilde{\mathbb{P}}_i;\mathbf{w})$ is a uniform term (without industry subscripts) and is given by

$$\Delta_i'(\tilde{\mathbb{P}}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(1-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right]. \tag{77}$$

To be clear, the above expressions can be derived by repeating the steps in Appendix E, while dropping domestic tax revenues from the expression for income Y_i. Likewise, the third term on the right-hand side of Equation 76 can be stated as

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = -\sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\bar{\tau}_i P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] - \sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\bar{\tau}_i P_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-j}}$$

where $\bar{\tau}_i$ is given by 88. Combining the above equations the F.O.C. specified by Equation 76 can be simplified as

$$\begin{aligned}&\sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}} - (1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] \\ &+ \sum_g\left[\left(1-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \widetilde{\Delta}_i'(\tilde{\mathbb{P}}_i,\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0,\end{aligned} \tag{78}$$

where $\widetilde{\Delta}_i'(\tilde{\mathbb{P}}_i;\mathbf{w})$ is specified analogously to $\Delta_i'(\tilde{\mathbb{P}}_i,\mathbf{w})$, but features an adjustment for general equilibrium wage effects:

$$\widetilde{\Delta}_i'(\tilde{\mathbb{P}}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\bar{\tau}_i)(1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(1-\frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right]. \tag{79}$$

Deriving the First-Order Condition w.r.t. $\tilde{\mathbf{P}}_{ij}$

Now, consider the consumer price index $\tilde{P}_{ij,k} \in \tilde{\mathbb{P}}_i$ associated with a good exported by i from *destination j–industry k*. The F.O.C. w.r.t. this price instrument can be stated as follows:

$$\left(\frac{\mathrm{d}W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \tag{80}$$

where $\mathbb{P}_{-ij,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ij,k}\}$ denotes the vector of price instruments excluding $\tilde{P}_{ij,k}$. As with the previous subsection, The above equation is similar to what we characterized in Appendix E, with two distinctions: First, country i's government does not control the price of domestically produced and domestically consumed varieties, i.e., $\tilde{\mathbf{P}}_{ii} \notin \tilde{\mathbb{P}}_i$. Second, country i's income does not include domestic tax revenues. Noting these two distinctions, we can borrow from the derivation in Appendix E to simplify Equation 80.

Namely, since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$ is not part of the domestic consumer price index in i, $\partial V_i(Y_i,\tilde{\mathbf{P}}_i)/\partial \ln \tilde{P}_{ji,k} = 0$. So, the first term on the right-hand side of Equation 80 collapses to zero. Without repeating the derivations from Appendix E, the second term on the right-hand side of Equation 80 reduces to

$$\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - \frac{1+\overline{\mu}_i}{1+\mu_g}P_{ij,g}\right)Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right] - \sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \Delta_i'(\tilde{\mathbb{P}}_i;\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}$$

where $\Delta_i'(\tilde{\mathbb{P}}_i;\mathbf{w})$ is a uniform term without industry subscripts, as defined by Equation 77. To elaborate, the above expression can be derived by repeating the steps in Appendix E, while dropping domestic tax revenues from the expression for income Y_i. Likewise, the third term on the right-hand side of Equation 80 can be stated as

$$\left(\frac{\partial W_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i\tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] - \sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}},$$

where $\bar{\tau}_i$ is given by 88. Combining the above equations the F.O.C. specified by Equation 80 can be simplified as

$$\tilde{P}_{ij,k}Q_{ij,k} + \sum_{g\in\mathbb{K}}\left[\left(1 - \frac{1+\overline{\mu}_i}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$\sum_{g\in\mathbb{K}}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \widetilde{\Delta}_i'(\tilde{\mathbb{P}}_i,\mathbf{w})\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0, \tag{81}$$

where $\widetilde{\Delta}_i'(\tilde{\mathbb{P}}_i;\mathbf{w})$ is given by Equation 79.

Solving the System of First-Order Conditions

First, note that we can solve the system specified by Equation 78 independent of 81. To solve the system of Equations 78, we can rely on the intermediate observation that if

$$\left(\mathbf{1} - \frac{1+\overline{\mu}_i}{\mathbf{1}+\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\odot\mathbf{Q}_{ii}\cdot\boldsymbol{\varepsilon}^{(ji,k)}_{ii} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - (1+\bar{\tau}_i)(\mathbf{1}+\boldsymbol{\Omega}_{ni})\odot\mathbf{P}_{ni}\right)\odot\mathbf{Q}_{ni}\cdot\boldsymbol{\varepsilon}^{(ji,k)}_{ni}\right] = 0, \tag{82}$$

then, to a first-order approximation around $\mu_k \approx \overline{\mu}_i$, $\widetilde{\Delta}_i'(\boldsymbol{\mu}) \approx 0$. So, the optimal choice of $\tilde{\mathbf{P}}_{ji}^{**}$ (and the implied tariff vector) can be determined by solving Equation 82 instead of 78.[88] Before moving forward, though, let us clarify the vector notation used to express Equation 82. The vector operators "$\cdot$" and "$\odot$" are respectively the inner product and element-wise product operators. The $K \times 1$ vector $\frac{1+\overline{\mu}_i}{1+\boldsymbol{\mu}} = \left[\frac{1+\overline{\mu}_i}{1+\mu_k}\right]_k$ is composed of industry-level The $K \times 1$ vectors $\tilde{\mathbf{P}}_{ni} = \left\{\tilde{P}_{ni,k}\right\}_k$ and $\mathbf{Q}_{ni} = \left\{Q_{ni,k}\right\}_k$ encompass the consumer price and quantity associated with all of country i's import goods for origin $n \neq i$. Analogously, $\boldsymbol{\varepsilon}_{ni}^{(ji,k)} = \left\{\varepsilon_{ni,g}^{(ji,k)}\right\}_g$ encompasses the elasticity of demand for each the goods imported from n w.r.t. the price of ji,k.

We simplify Equation 82 in three steps: First, by noting that $\tilde{\mathbf{P}}_{ii} = \mathbf{P}_{ii}$ and appealing to Cournot's aggregation, $\sum_{j\in\mathbb{C}}\left[\tilde{\mathbf{P}}_{ji} \odot \mathbf{Q}_{ji} \cdot \boldsymbol{\varepsilon}_{ji}^{(ji,k)}\right] = -\tilde{P}_{ji,k}Q_{ji,k}$, we can rewrite Equation 82 as

$$\frac{1+\overline{\mu}_i}{1+\boldsymbol{\mu}} \odot \tilde{\mathbf{P}}_{ii} \odot \mathbf{Q}_{ii} \cdot \boldsymbol{\varepsilon}_{ii}^{(ji,k)} + (1+\bar{\tau}_i)\sum_{n\neq i}\left[(\mathbf{1}+\boldsymbol{\Omega}_{ni}) \odot \mathbf{P}_{ni} \odot \mathbf{Q}_{ni} \cdot \boldsymbol{\varepsilon}_{ni}^{(ji,k)}\right] + \tilde{P}_{ji,k}Q_{ji,k} = 0. \tag{83}$$

Second, we invoke the *Slutsky Equation*,[89] to rewrite the first two term in the above equation. Specifically, taking note that

$$\eta_{ii,g} = \eta_{ji,k} = 1 \quad \xrightarrow{\text{Slutsky Equation}} \quad \tilde{P}_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)} = \tilde{P}_{ji,k}Q_{ji,k}\varepsilon_{ji,k}^{(ni,g)}.$$

We can reduces the F.O.C. described under Equation 83 to

$$1+\sum_g\left[\frac{1+\mu_g}{1+\overline{\mu}_i}\varepsilon_{ji,k}^{(ii,g)}\right] + (1+\bar{\tau}_i)\sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\varepsilon_{ji,k}^{(ni,g)}\right] = 0. \tag{84}$$

Lastly, we use the Marshallian demand function's *homogeneity of degree zero* property, whereby $\eta_{ji,k} + \sum_{J,g}\varepsilon_{ji,k}^{(ji,g)} = 1 + \sum_{J,g}\varepsilon_{ji,k}^{(ji,g)} = 0$. Invoking this property we rewrite Equation 84 as follows

$$\sum_g\left[\left(1-\frac{1+\mu_g}{1+\overline{\mu}_i}\right)\boldsymbol{\varepsilon}_{ji,k}^{(ii,g)}\right] + \sum_g\sum_{n\neq i}\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)\boldsymbol{\varepsilon}_{ji,k}^{(ni,g)}\right] = 0.$$

The above equation, which should hold for all $ji,k \neq ii,k$ specifies a system of FOCS that can be expressed in matrix no notation as

$$\underbrace{\begin{bmatrix}\varepsilon_{1i,1}^{(ii,1)} & \cdots & \varepsilon_{Ni,K}^{(ii,1)}\\ \vdots & \ddots & \vdots\\ \varepsilon_{1i,1}^{(ii,K)} & \cdots & \varepsilon_{Ni,K}^{(ii,K)}\end{bmatrix}}_{\mathbf{E}_{-ii}^{(ii)}}\begin{bmatrix}1-\frac{\mu_1}{\overline{\mu}_i}\\ \vdots\\ 1-\frac{\mu_K}{\overline{\mu}_i}\end{bmatrix} + \underbrace{\begin{bmatrix}\varepsilon_{1i,1}^{(1i,1)} & \cdots & \varepsilon_{i-1i,k}^{(1i,1)} & \varepsilon_{i+1i,k}^{(1i,1)} & \cdots & \varepsilon_{Ni,K}^{(1i,1)}\\ \vdots & \ddots & & & \ddots & \vdots\\ \varepsilon_{1i,1}^{(Ni,K)} & \cdots & \varepsilon_{i-1i,k}^{(Ni,K)} & \varepsilon_{i+1i,k}^{(Ni,K)} & \cdots & \varepsilon_{Ni,K}^{(Ni,K)}\end{bmatrix}}_{\mathbf{E}_{-ii}}\begin{bmatrix}1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{1i,1}}{\tilde{P}_{1i,1}}\\ \vdots\\ 1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{Ni,K}}{\tilde{P}_{Ni,K}}\end{bmatrix} = \mathbf{0}. \tag{85}$$

Following the proof of Lemma 7 from Appendix E, we can easily show the matrix $\mathbf{E}_{-ii}^{(ii)}$ is invertible. We can, thus, invert the system specified by Equation 85 to produce the following formula for optimal import price wedges:

$$\left[(1+\bar{\tau}_i)(1+\omega_{ji,k})\frac{P_{ji,k}}{\tilde{P}_{ji,k}^{**}}\right]_{j,k} = \mathbf{1} + \mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\overline{\mu}_i}\right]_k, \tag{86}$$

[88] Note that Equation 82 is essentially 78 with $\widetilde{\Delta}_i'(.)$ set to zero.

[89] Recalling that $e_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}/Y_i$ denotes the share of expenditure on ji,k, the Slutsky equation can be formally stated as

$$\text{[Slutsky equation]} \qquad e_{ii,g}\varepsilon_{ii,g}^{(ji,k)} + e_{ji,k}e_{ii,g}\eta_{ii,g} = e_{ji,k}\varepsilon_{ji,k}^{(ii,g)} + e_{ii,g}e_{ji,k}\eta_{ji,k}.$$

where, to be clear, $\mathbf{E}_{-ii} \equiv \left[\mathbf{E}_{ni}^{(ji)}\right]_{j,n\neq i}$ and $\tilde{\mathbf{E}}_{-ii}^{(ii)} \equiv \left[\mathbf{E}_{ni}^{(ii)}\right]_{n\neq i}$ are respectively $(N-1)K \times (N-1)K$ and $(N-1)K \times K$ matrixes of demand elasticities. Note that the optimal choice w.r.t. $\tilde{\mathbf{P}}_{ji}$, ensures that $\widetilde{\Delta}_i'(.) \approx 0$. Hence, the system of F.O.C. specified by Equation 81, transforms to the exact same system we solved in Appendix E. Without repeating the details of our prior derivation, the optimal export price wedges are given by

$$\left[\frac{P_{ij,k}}{\tilde{P}_{ij,k}^{**}}(1+\bar{\tau}_i)^{-1}\right]_{j,k} = \mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K} + \mathbf{\Omega}_{-ii}\right), \tag{87}$$

where $\mathbf{1}_{(N-1)K}$ is a $N(K-1)\times 1$ column vector of ones; $\mathbf{\Omega}_{-ii} = \left[\Omega_{ni,g}\right]_{n\neq i,g}$ is a $N(K-1)\times 1$ vector of (inverse) export supply elasticities; and $\mathbf{E}_{ij}^{(-ij)}$ and $\mathbf{E}_{ij}$ have the same description as in Appendix E. The "$**$" notation is used to highlight the fact that we are solving for second-best price wedges. Next, we can recover the optimal (second-best) import tax and export subsidy rates from the optimal (second-best) price wedges implies by Equations 86 and 87. Specifically, noting the following relationships,

$$1+t_{ji,k}^{**} = \frac{\tilde{P}_{ji,k}^{**}}{P_{ji,k}}; \qquad\qquad 1+x_{ij,k}^{**} = \frac{P_{ij,k}}{\tilde{P}_{ij,k}^{**}};$$

country i's unilaterally second-best trade tax schedule can be expressed as follows:

$$\text{[import tariff]} \qquad \mathbf{1}+\mathbf{t}_{ij}^{**} = (1+\bar{\tau}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right)\oslash\left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right)$$

$$\text{[export subsidy]} \qquad \mathbf{1}+\mathbf{x}_{ij}^{**} = -(1+\bar{\tau}_i)\left(\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}(\mathbf{1}+\mathbf{\Omega}_{-ii})\right)\odot\left[\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k.$$

To conclude the proof we can invoke the multiplicity of the optimal trade tax schedules (Lemma 1). As in Theorem 1, this feature indicates that the value assigned to $\bar{\tau}_i$ is redundant. In particular, following Lemma 1, we can multiply $(1+\bar{\tau}_i)$ in the above equation with any non-negative tax shifter $1+\bar{t}_i \in \mathbb{R}_+$, and maintain optimality. That being the case, the exact value assigned to $\bar{\tau}_i$ is redundant and the following describes all possible optimal tax schedules:aa

$$\text{[import tariff]} \qquad \mathbf{1}+\mathbf{t}_{ij}^{**} = (1+\bar{t}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right)\oslash\left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right)$$

$$\text{[export subsidy]} \qquad \mathbf{1}+\mathbf{x}_{ij}^{**} = -(1+\bar{t}_i)\left(\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}(\mathbf{1}+\mathbf{\Omega}_{-ii})\right)\odot\left[\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k.$$

J Proof of Theorem 3

Theorem 3 concerns the second-best case where the government in i can choose only $\tilde{\mathbf{P}}_{ji}$, which is the vector of import prices (i.e., $\tilde{\mathbb{P}}_i = \{\tilde{\mathbf{P}}_{ji}\}$). To prove this theorem we capitalize on two results from Appendix I: First, the F.O.C. derived w.r.t. $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_{ji}$ does not change with the unavailability of $\tilde{\mathbf{P}}_{ij}$ from the government's policy set $\tilde{\mathbb{P}}_i$. Hence, the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ is described by Equation 78 even if $\tilde{P}_{ij,k} \notin \tilde{\mathbb{P}}_i$. Second, recall from Appendix that we were able to solve the system specified by 78 independent of the F.O.C. w.r.t. $\tilde{\mathbf{P}}_{ij}$. Invoking these two observations, the formula for optimal tariff in the case studied by Theorem 3 is given by 86:

$$1+\mathbf{t}_{ji}^{***} = (1+\bar{\tau}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right)\oslash\left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right).$$

Unlike Theorem 2, through, $\bar{\tau}_i$ is no longer redundant. Since export taxes (or equivalently $\tilde{\mathbf{P}}_{ij}$) are excluded from the government's policy set, we can no longer invoke the multiplicity implied by Lemma

1. Instead, we have to formally characterize, $\bar{\tau}_i$, starting from its definition:

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(.)}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} \left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\tilde{\mathbb{P}}_i,\mathbf{w})/\partial \ln w_i\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}}. \tag{88}$$

Also, recall that $W_i(\tilde{\mathbb{P}}_i;\mathbf{w}) = V_i(Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}),\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji})$, where $\tilde{\mathbf{P}}_{ji} \sim \tilde{\mathbf{P}}_{-ii} \equiv \{\tilde{P}_{ji,k}\}_{j\neq i,k}$ while income equals wage payments, plus profits, plus import tax revenues: $Y_i = (1+\overline{\mu}_i)w_iL_i + (\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji})\cdot\mathbf{Q}_{ji}$. Borrowing from the results in Appendixes E and I, the numerator in Equation 88 can be unpacked as follows:

$$\begin{aligned}
\left(\frac{\partial W_i(.)}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} \left(\frac{\partial V_i}{\partial Y_i}\right)^{-1} &= \left(\frac{\partial Y_i}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} + \left(\frac{\partial V_i}{\partial Y_i}\right)^{-1} \frac{\partial V_i(Y_{i,},\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{\mathbf{P}}_{ii}} \cdot \frac{\partial \ln \tilde{\mathbf{P}}_{ii}}{\partial \ln w_i} \\
&= \overline{\mu}_i w_i L_i + \left(\frac{\partial \overline{\mu}_i}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} w_i L_i \; + \; (\tilde{\mathbf{P}}_{-ii} - \mathbf{P}_{-ii}) \cdot \left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} - \mathbf{P}_{ii}\cdot\mathbf{Q}_{ii} \\
&= \sum_{n\neq i} [\mathbf{P}_{in}\cdot\mathbf{Q}_{in}] + \left(\mathbf{1} - \frac{\overline{\mu}_i}{\boldsymbol{\mu}}\right) \odot \mathbf{P}_{ii} \cdot \left(\frac{\partial \mathbf{Q}_{ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} + (\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{-ii}) \cdot \left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}.
\end{aligned} \tag{89}$$

To be clear about the notation, $\frac{\overline{\mu}_i}{\boldsymbol{\mu}} \equiv \left[\frac{\overline{\mu}_i}{\mu_k}\right]_k$, while $\odot$ and $\cdot$ respectively denote *inner* and *element-wise* products of equal-sized vectors, i.e., $\mathbf{a}\cdot\mathbf{b} = \sum_n a_n b_n$ and $\mathbf{a}\odot\mathbf{b} = [a_n b_n]_n$. Next, we move on to characterizing the denominator of Equation 88. Noting that $\mathrm{T}(\tilde{\mathbb{P}}_i,\mathbf{w}) \equiv \sum_{j\neq i}\left[\mathbf{P}_{ji}\cdot\mathbf{Q}_{ji} - \mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}\right]$, we can borrow from the results in Appendixes E and I to unpack the aforementioned term as follows:

$$\left(\frac{\partial \mathrm{T}_i(.)}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} = \left(\frac{\partial}{\partial \ln w_i}\sum_{j\neq i}\left[\mathbf{P}_{ji}\cdot\mathbf{Q}_{ji} - \mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}\right]\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} = \mathbf{P}_{-ii}\cdot\left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} - \sum_{j\neq i}\left[\left(\frac{\partial \mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}\right]. \tag{90}$$

Plugging Equations 89 and 90 back into the expression for $\bar{\tau}_i$ yields the following:

$$\bar{\tau}_i = \frac{\sum_{n\neq i}[\mathbf{P}_{in}\cdot\mathbf{Q}_{in}] + \left(\mathbf{1} - \frac{\overline{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial \mathbf{Q}_{ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} + (\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{-ii})\cdot\left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}}{\mathbf{P}_{-ii}\left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} - \sum_{j\neq i}\left[\left(\frac{\partial \mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}\right]}. \tag{91}$$

We can further simplify the above expression by invoking the F.O.C. described by Equation 83. This equation indicates that the following relationship ought to hold at the optimum $\tilde{\mathbb{P}}_i = \tilde{\mathbb{P}}_i^{***}$:

$$\sum_{j\neq i}\sum_k\left[\left(\mathbf{1} - \frac{\overline{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial \mathbf{Q}_{ii}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\tilde{\mathbb{P}}_i^{***},\mathbf{w}_{-i}} + (\tilde{\mathbf{P}}_{-ii} - (1+\bar{\tau}_i)\mathbf{P}_{-ii})\cdot\left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\tilde{\mathbb{P}}_i^{***},\mathbf{w}_{-i}}\right] = 0.$$

Now, we will rearrange and simplify the above relationship in such a way that will help us simply Equation 91. To this, we invoke the property that the Marshallain demand function is homogeneous of degree zero. Combining this property with the fact that $\frac{\partial \ln Y_i}{\partial \ln w_i} \approx \frac{\partial \ln \tilde{P}_{ii,k}}{\partial \ln w_i} = 1$, we can simplify the above as follows:

$$\left(\mathbf{1} - \frac{\overline{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial \mathbf{Q}_{ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} + (\tilde{\mathbf{P}}_{ii} - (1+\bar{\tau}_i)\mathbf{P}_{-ii})\cdot\left(\frac{\partial \mathbf{Q}_{-ii}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}} = 0.$$

Using the above equation, we can cancel out the mirroring expressions in the numerator and denominator of Equation 91. Doing so reduces and simplifies the expression for $\bar{\tau}_i$ to the following:

$$\bar{\tau}_i = \frac{-\sum_{n\neq i}(\mathbf{P}_{in}\cdot\mathbf{Q}_{in})}{\sum_{j\neq i}\left[\left(\frac{\partial \mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial \ln w_i}\right)_{\tilde{\mathbb{P}}_i,\mathbf{w}_{-i}}\right]} = \frac{-1}{\sum_{j\neq i}\left[\mathbf{X}_{ij}\cdot(\mathbf{I}_K + \mathbf{E}_{ij})\,\mathbf{1}_K\right]}. \tag{92}$$

The $K\times 1$ vector $\mathbf{X}_{ij} = [\chi_{ij,k}]_k$ is compose of export shares, which are defined as $\chi_{ij,k} \equiv \frac{P_{ij,k}Q_{ij,k}}{\sum_{n\neq i}\mathbf{P}_{in}\cdot\mathbf{Q}_{in}}$. To provide some intuition, the denominator of the above equation corresponds to the elasticity of international demand for origin i's labor. As such, $\bar{\tau}_i$ can be interpreted as country i's optimal markup on its wage rate in international (non-i) markets.

K Optimal Policy under IO Linkages (Theorem 4)

We first present a formal description of equilibrium under input-output (IO) linkages. We use the $\mathcal{C}$ superscript to denote final consumption goods and the $\mathcal{I}$ superscript to denote intermediate inputs. To given an example: $Q^{\mathcal{C}}_{ji,k}$ denotes the quantity of a "final" goods associated with *origin j–destination i–industry k*, while $Q^{\mathcal{I}}_{ji,k}$ denotes the quantity of an "intermediate" goods associated with origin j–destination i–industry k. Without loss of generality, we assume that good ji,k exhibits the same price irrespective of whether it is used as a final good or an intermediate input good: $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{C}}_{ji,k} = \tilde{P}^{\mathcal{I}}_{ji,k}$.

On the production side, we impose no restrictions on how intermediate inputs are aggregated in the production process. We, however, assume that the share of labor in production is constant and equal to $1 - \bar{\alpha}_{i,k}$ for each *origin i–industry k*. To track the demand for inputs, we use $\mathcal{Y}_{i,k}$ to denote the gross revenue associated with *origin i–industry k*. Correspondingly, $\bar{\alpha}_{i,k}\mathcal{Y}_{i,k}$ denotes *origin i–industry k*'s total expenditure on intermediate inputs.

Marshallian Demand under IO Linkages

We suppose that overall demand for good ji,k, which is the sum of final good demand based on utility maximization and input demand based on cost minimization, is given by the following demand function

$$Q_{ji,k} = Q^{\mathcal{I}}_{ji,k} + Q^{\mathcal{C}}_{ji,k} = \mathcal{D}_{ji,k}(E_i, \tilde{\mathbf{P}}_i),$$

where $E_i = Y_i + \sum_g \bar{\alpha}_{i,g}\mathcal{Y}_{i,g}$ denotes market i's total expenditure on final and intermediate input goods. To make the notation consistent with our previous derivations, we use $\varepsilon^{(ni,g)}_{ji,k}$ and $\eta_{ji,k}$ to denote the price and income elasticities associated with the IO-augmented Marshallian demand function $\mathcal{D}_{ji,k}(E_i, \tilde{\mathbf{P}}_i)$.

General Equilibrium under IO Linkages

As in the baseline model, we express all equilibrium outcomes (except for wages) as a function of global taxes ($\mathbf{x}$, $\mathbf{t}$, and $\mathbf{s}$), treating wages $\mathbf{w} \equiv \{w_i\}_i$ as given. This formulation derives from solving a system that imposes all equilibrium conditions aside from the labor market clearing conditions. We formally outline this formulation below.

Notation. For a given vector of taxes and wages $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$, *equilibrium outcomes* $Y_i(\mathbf{T})$, $\mathcal{Y}_{i,k}(\boldsymbol{T})$, $P_{ji,k}(\mathbf{T})$, $\tilde{P}_{ji,k}(\mathbf{T})$, $Q_{ji,k}(\mathbf{T})$ *are determined such that* (i) *producer prices are characterized by 13;* (ii) *consumer prices are given by 7;* (iii) *Consumption and input demand choices are given by* $\mathcal{D}_{ji,k}(E_i, \tilde{\mathbf{P}}_i)$, *where* $E_i = Y_i + \sum_g \bar{\alpha}_{i,g}\mathcal{Y}_{i,g}$; (iv) *net income (which dictates total final good expenditure by country i) equals wage payments plus tax revenues:* $Y_i = w_i L_i + \mathcal{R}_i$,[90] *where* $\mathcal{R}_i$ *are described by 8 and* (v) *gross industry-level revenues are given by* $\mathcal{Y}_{i,g} = \sum_n P_{in,k} Q_{in,k}$.

As in the baseline model, $\mathbf{w}$ is itself an equilibrium outcome. So, a vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ is feasible insofar as $\mathbf{w}$ is the equilibrium wage, consistent with $\mathbf{t}$, $\mathbf{x}$, and $\mathbf{s}$. So, to fix ideas we define the set of feasible *policy–wage* vectors as follows.

[90] *Note that net profits are equal top zero (i.e.,* $\Pi_i = 0$*) as we are focusing on the case of free entry.*

Definition (D2-IO). The set of feasible policy–wage vectors, $\mathbb{F}$*, consists of any vector* $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ *where* $\mathbf{w}$ *satisfies the labor market clearing condition in every country, given* $\mathbf{t}$*,* $\mathbf{x}$*, and* $\mathbf{s}$*:*

$$\mathbb{F} = \left\{ \mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}) \;\middle|\; w_i L_i = \sum_j \sum_k Qz_{ij,k}(\mathbf{T}) - \sum_j \sum_k P^{\mathcal{I}}_{ji,k}(\mathbf{T}) Q^{\mathcal{I}}_{ji,k}(\mathbf{T}); \quad \forall i \in \mathbb{C} \right\}.$$

Before moving on to the proof, two important details are in order: First, we can easily verify that the labor market clearing condition specified by Definition D2-IO is equivalent to the balanced trade condition. Second, under IO linkages, the choice w.r.t. taxes (or equivalently $\tilde{\mathbb{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$) may affect the entire vector of producer prices, $\{P_{nj,k}\}$, through its effect on input prices. To track these IO-related effects, let $\alpha^{j,g}_{i,k}$ denotes the (possibly variable) cost share of intermediate inputs from *origin* $j\times$ *industry* g used in the output of *origin* $i\times$ *industry* k. By Shepherd's Lemma, the direct effect of raising input price $\tilde{P}^{\mathcal{I}}_{ji,g}$ on the producer price $P_{ij,k}$ can be expressed as follows:

$$\text{[Shepherd's Lemma]} \qquad \left(\frac{\partial \ln P_{ij,k}(\tilde{\mathbb{P}}_i; \mathbf{w})}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,g}} \right)_{\mathbb{P}_{-ji,g}, \mathbf{w}} = \alpha^{j,g}_{i,k}. \quad \forall (\jmath, j, i \in \mathbb{C}); \;\; \forall (g, k \in \mathbb{K}).$$

We use the Shepherd's Lemma in combination with our dual approach (from Appendix E) to characterize the optimal policy schedule for each country i. Recall that the optimal policy problem in our dual approach is reformulated as

$$\max_{\tilde{\mathbb{P}}_i} \; W_i(\tilde{\mathbb{P}}_i; \mathrm{w}) \equiv V_i(Y_i(\tilde{\mathbb{P}}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \quad s.t. \;\; (\tilde{\mathbb{P}}_i; \mathbf{w}) \in \mathbb{F}_P,$$

where $\tilde{\mathbb{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$ denotes the vector of consumer prices directly linked to economy i. The feasible set $\mathbb{F}_P$ is defined analogously to $\mathbb{F}$. Below, we derive and solve the system of F.O.C. associated with the above problem, building on the results introduced earlier under Appendix E.

Tax Neutrality under IO Linkages

Our baseline characterization of optimal policy relied on the tax neutrality result presented under Lemma 1. An analogous (but slightly different) neutrality result holds under IO linkages. To present this result, we use operator$\mathscr{C}(.)$, which configures a uniform tax-shifter depending on whether the taxed item is used for final consumption or intermediate input use. In particular, for an arbitrary tax-shifter, $\tilde{a} \in \mathbb{R}_+$, $\mathscr{C}(\tilde{a}) = \tilde{a}$ if the taxed item is a final good and $\mathscr{C}(\tilde{a}) = 1$ otherwise.

Lemma 9. *[Tax Neutrality under IO Linkages] For any a and* $\tilde{a} \in \mathbb{R}_+$ (i) *if* $\mathbf{T} = (\mathbf{1}+\mathbf{t}_i, \mathbf{t}_{-i}, \mathbf{1}+\mathbf{x}_i, \mathbf{x}_{-i}, \mathbf{1}+\mathbf{s}_i, \mathbf{s}_{-i}; \dot{w}_i, \mathbf{w}_{-i}) \in \mathbb{F}$*, then* $\boldsymbol{T}' = (a(\mathbf{1}+\boldsymbol{t}_i)/\mathscr{C}(\tilde{a}), \boldsymbol{t}_{-i}, a(\mathbf{1}+\boldsymbol{x}_i)/\mathscr{C}(\tilde{a}), \boldsymbol{x}_{-i}, (\mathbf{1}+\boldsymbol{s}_i)\mathscr{C}(\tilde{a}), \boldsymbol{s}_{-i}; a\dot{w}_i, \boldsymbol{w}_{-i}) \in \mathbb{F}$*. Moreover,* (ii) *welfare is preserved under* $\mathbf{T}$ *and* $\mathbf{T}'$*:* $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ *for all* $n \in \mathbb{C}$*.*

The above lemma is akin to Lemma 1, but differs in one basic detail. The neutrality of uniform trade tax adjustments (i.e., the Lerner Symmetry) holds in the IO model without qualification. The neutrality of uniform domestic tax adjustments holds the consumption side but not on the production side. More specifically, a uniform increase in consumption taxes is welfare-neutral in the IO model. Accordingly, the tax adjustments that apply via $\mathscr{C}(\tilde{a})$ are constructed to mimic a uniform consumption tax hike. With the above background, we are now ready to derive and solve the system of F.O.C.s that determine optimal policy under IO linkages.

Step #1: Deriving the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k} \in \tilde{\mathbb{P}}_i$

First, we derive the F.O.C. w.r.t. to import variety ji,k, supplied by origin j–industry k. Given that $W_i = V_i(Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}), \tilde{\mathbf{P}}^{\mathcal{C}}_{ii}, \tilde{\mathbf{P}}^{\mathcal{C}}_{ji})$, the F.O.C. w.r.t. $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k} \sim \tilde{P}^{\mathcal{C}}_{ji,k}$, holding $\mathbb{P}_{-ji,k} \equiv \tilde{\mathbb{P}}_i - \{\tilde{P}_{ji,k}\}$ constant, can be stated as

$$\left(\frac{\partial W_i}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}^{\mathcal{C}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\tilde{\mathbb{P}}_i,\mathbf{w})}{\partial \mathbf{w}}\right)_{\tilde{\mathbb{P}}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0. \tag{93}$$

The right-hand side of the above equation can be characterized similar to Appendix E, with two distinctions: First, total demand for good ji,k is the sum of consumption plus input demand: $Q_{ji,k} = Q^{\mathcal{C}}_{ji,k} + Q^{\mathcal{I}}_{ji,k}$. So, we have to distinguish between welfare effects that channel through consumption and those that channel through input demand. Second, we need to account for the effect of a change in input price $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k}$ on the producer prices associated with economy i. To this end, we can invoke Shepherd's Lemma, which implies that

$$\left(\frac{\partial \ln P_{ij,k}(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,g}}\right)_{\mathbb{P}_{-ji,g},\mathbf{w}} = \alpha^{j,g}_{i,k}. \qquad \forall \jmath, j, i \in \mathbb{C};\ \ g,k \in \mathbb{K}.$$

Considering the above caveats, we can proceed as in Appendix E. By Roy's identity, the first term on the right-hand side of the F.O.C. (Equation 93) can be stated as

$$\frac{\partial V_i(Y_i, \tilde{\mathbf{P}}^{\mathcal{C}}_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q^{\mathcal{C}}_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Next, consider the second term on the right-hand side of Equation 93, which accounts for income effects. Recall that total income in country i equals the sum of wage payments plus import, production and export tax revenues:

$$Y_i(\tilde{\mathbb{P}}_i;\mathbf{w}) = w_i L_i + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni})\cdot \mathbf{Q}_{ni}\right] + (\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii})\cdot \mathbf{Q}_{ii} + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right].$$

The effect of $\tilde{P}_{ji,k}$ on import tax revenues can be derived and express exactly as in Appendix E:

$$\left(\frac{\partial \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni})\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k} + \sum_g \sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right) Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right] \tag{94}$$

The logic is that holding the vector of wages $\mathbf{w}$ and country i's export prices $\tilde{\mathbf{P}}_{ij} \in \tilde{\mathbb{P}}_i$ fixed, a change in $\tilde{P}_{ji,k}$ has not effect on the producer price of imports $\mathbf{P}_{ji}$ through the input-output network.

The effect of a change in $\tilde{P}_{ji,k}$ on country i's production and export tax revenues can be formulated as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii})\cdot \mathbf{Q}_{ii,g} + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$$

$$= \sum_g\left[\left(\tilde{P}_{ii,g} - P_{ii,g}\right) Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right] + \sum_g\sum_n\left[P_{in,g}Q_{in,g}\left[\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_i}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial \ln P_{in,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]\right], \tag{95}$$

The above expression differs from Equation 96 (in Appendix E) in the last term on the second line.

This term accounts for the effect of raising input price $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k}$ on the producer prices associated with economy i. As explained above, we can appeal to Shephard's lemma to simplify this extra term as

$$\sum_{g\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left[P_{in,g}Q_{in,g}\left(\frac{\partial \ln P_{in,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,k}}\right)_{\mathbb{P}_{-ji,g},\mathbf{w}}\right] = -\sum_{g\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left(Q_{in,g}P_{in,g}\alpha^{j,k}_{i,g}\right) = -\tilde{P}_{ji,k}Q^{\mathcal{I}}_{ji,k}.$$

Plugging the above expression back into Equation 95 and redoing the derivations covered in Appendix E, yields the following expression for the effect of $\tilde{P}_{ji,k}$ on country i's production and export tax revenues:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\sum_n\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = -\tilde{P}_{ji,k}Q^{\mathcal{I}}_{ji,k} + \sum_g\left[\left(\tilde{P}_{ii,g}-\left[1-\frac{\mu_g}{1+\mu_g}\right]\right)P_{ii,g}Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{96}$$

where recall that $\left(\partial \ln Q_{ii,g}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses price and income effects à la Equation 26 in Appendix E. Combining Equations 94 and 96, and noting that $\tilde{P}_{ji,k}Q_{ji,k} - \tilde{P}_{ji,k}Q^{\mathcal{I}}_{ji,k} = \tilde{P}_{ji,k}Q^{\mathcal{C}}_{ji,k}$ yields the following expression that summarizes all the revenue-related welfare effects in the F.O.C.:

$$\begin{aligned}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} &= \tilde{P}_{ji,k}Q^{\mathcal{C}}_{ji,k} + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+ \sum_g\left[\left(\tilde{P}_{ii,g}-\frac{1}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right] + \Delta_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}.\end{aligned} \tag{97}$$

The uniform term $\Delta_i(.)$ accounts for circular income effects and is given by Equation 34 in Appendix E. Finally, the last term on the right-hand side of Equation 93, which accounts for general equilibrium wage effects, can be specified in the same exact way as in Appendix E:

$$\left(\frac{\partial W_i(\mathbb{P}_i,\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{d\mathbf{w}}{d\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = -\bar{\tau}_i\sum_g\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right],$$

where $\bar{\tau}_i$ is given by Equation 36 in Appendix E. Combining the above expressions, the F.O.C. specified by Equation 93 reduced to

$$\begin{aligned}[\text{FOC w.r.t. } \tilde{P}_{ji,k}] \qquad &\sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+\sum_g\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right] + \widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0,\end{aligned} \tag{98}$$

where $\widetilde{\Delta}_i(.)$ is given by Equation 39 in Appendix E. Note that the above equation has an identical representation to the F.O.C. in the baseline model. The intuition is that holding country i's export prices $\tilde{\mathbf{P}}_{ij} \in \mathbb{P}_i$ fixed, the choice w.r.t. $\tilde{P}_{ji,k}$ has no first-order effect on country i's terms-of-trade channels through the input-output network. If good ji,k is used as an input in export good in,g, any possible terms-of-trade gains from taxing $\tilde{P}_{ji,k}$ will be internalized by the optimal choice w.r.t. $\tilde{P}_{in,g}$. Furthermore, it is easy to check that Equation 98 characterizes the F.O.C. w.r.t. $\tilde{P}_{ii,k} \in \mathbb{P}_i$ as long as we replace ji,k with ii,k everywhere in that equation. Finally, as in Appendix E, we do not unpack the uniform term $\bar{\tau}_i$ because the multiplicity of country i's optimal tax schedule will render the exact value assigned to $\bar{\tau}_i$ redundant.

***Step* #2: Deriving the F.O.C. *w.r.t.* $P_{ij,k} \in \tilde{\mathbb{P}}_i$**

Consider export variety ij,k, which is sold to destination $j \neq i$ in industry k. Noting that $W_i = V_i(Y_i(\mathbb{P}_i;\mathbf{w}), \tilde{\mathbf{P}}_{ii}^{\mathcal{C}}, \tilde{\mathbf{P}}_{ji}^{\mathcal{C}})$, the F.O.C. w.r.t. $\tilde{P}_{ij,k}$, holding $\mathbb{P}_{-ij,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ij,k}\}$ constant, can be stated as

$$\left(\frac{\partial W_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\mathbb{P}_i,\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{d\mathbf{w}}{d \ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \tag{99}$$

The first term as before accounts for direct price effects. This term is trivially equal to zero since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$. That is, since ij,k is not part of the domestic consumption bundle, raising its price has no direct effect on consumer surplus in country i:

$$\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i \implies \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} = 0. \tag{100}$$

The second term in Equation 99 accounts for welfare effects that channel through tax revenues. Specifically, Holding wages $\mathbf{w}$ fixed, the change in country i's income amounts to the change in import, domestic, and export tax revenues. The effect on import tax revenues can be expressed as follows:

$$\begin{aligned}\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}\right)\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = & \sum_g \sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - P_{ni,g}\right) Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right] \\ & - \sum_g \sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left[\left(\frac{\partial P_{ni,g}}{\partial \ln Q_{nj,g}}\right)_{\mathbf{w},\mathbb{P}_i}\left(\frac{\partial \ln Q_{nj,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial \ln P_{ni,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]\right].\end{aligned} \tag{101}$$

The above equation differs from Equation 44 in Appendix E in only the last term on the second line. This term accounts for that fact that raising the price of input good ij,k can affect the entire vector of producer prices in the rest of the world through input-output networks. Given Shephard's lemma we can simplify this term by noting that

$$\Lambda_{ij,k} \equiv \sum_{n\neq i}\sum_{g\in\mathbb{K}}\left(P_{ni,g}Q_{ni,g}\left(\frac{\partial P_{ni,g}}{\partial \tilde{P}^{\mathcal{I}}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right)/\tilde{P}_{ij,k}Q_{ij,k}$$

denotes the share of the export value associated with good ij,k that is reimported back into economy i. Plugging the above expression back into 101 and repeating the derivation performed in Appendix E, yields the following:

$$\begin{aligned}\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}\right)\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = & -\Lambda_{ij,k}\tilde{P}_{ij,k}Q_{ij,k} - \sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] \\ & + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln E_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}.\end{aligned} \tag{102}$$

Repeating the derivation in Appendix E, the effect of a change in $\tilde{P}_{ij,k}$ on country i's production and export tax revenues can be formulated as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ij,k}} \sum_n \left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}) \cdot \mathbf{Q}_{in} \right] \right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \frac{\mu_g}{1+\mu_g}] P_{ij,g} \right) Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)} \right]$$
$$+ \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1}{1+\mu_g} P_{ii,g} \right) Q_{ii,g} \eta_{ii,g} \right] \left(\frac{\partial \ln E_i}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{103}$$

To be clear, holding $\tilde{P}_{ji,k} \in \mathbb{P}_i$ fixed, a change in $\tilde{P}_{ij,k}$ has no effect on the input price faced by firm located in i. That is, $\left(\partial P_{ni,g} / \partial \tilde{P}^{\mathcal{I}}_{ij,k} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0$. This point explains why the above expression is rather identical to that derived in Appendix E. Combining Equations 102 and 99, we can express the sum of all tax-revenue-related terms as

$$\left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = (1 - \Lambda_{ij,k}) \, \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \frac{\mu_g}{1+\mu_g}] P_{ij,g} \right) Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)} \right]$$
$$- \sum_g \sum_{n \neq i} \left[\omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)} \right] + \Delta_i(\mathbb{P}_i, \mathbf{w}) \left(\frac{\partial E_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}}, \tag{104}$$

where $\Delta_i(.)$ encompasses the terms accounting for circular income effects and is given by Equation 34. Taking note of the already-discussed distinctions between the present and baseline models and repeating the derivations performed earlier in Appendix E, the last term in right-hand side of Equation 99) can be formulated as

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}} \right)_{\mathbb{P}_i} \cdot \left(\frac{d \mathbf{w}}{d \ln \tilde{P}_{ij,k}} \right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i (1 - \Lambda_{ij,k}) \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\bar{\tau}_i \tilde{P}_{ij,g} Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)} \right]$$
$$- \sum_g \sum_{n \neq i} \left[\bar{\tau}_i \omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)} \right] - \sum_g \sum_{n \neq i} \left[[1 + \omega_{ni,g}] \bar{\tau}_i P_{ni,g} Q_{ni,g} \eta_{ni,g} \right] \left(\frac{\partial E_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{105}$$

Finally, plugging Equations 100, 104, and 105 back into the F.O.C. (Equation 99); and dividing by $(1 + \bar{\tau}_i)$ yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ij,k}$:

$$[\text{FOC w.r.t. } \tilde{P}_{ij,k}] \qquad (1 - \Lambda_{ij,k}) \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(1 - \frac{1}{(1+\bar{\tau}_i)(1+\mu_g)} \frac{P_{ij,g}}{\tilde{P}_{ij,g}} \right) \tilde{P}_{ij,g} Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)} \right]$$
$$- \sum_g \sum_{n \neq i} \left[\omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)} \right] + \widetilde{\Delta}_i(\mathbb{P}_i, \mathbf{w}) \left(\frac{\partial E_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} \right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0, \tag{106}$$

where $\widetilde{\Delta}_i(.)$ is defined as in Equation 39. Also, we are not unpacking the term $\bar{\tau}_i$, for the same reasons discussed earlier.

Step #3: Solving the System of F.O.C.s and Establishing Uniqueness

To determine the optimal tax schedule we need to collect the system of first order conditions and simultaneously solve them under one system. For the ease of reference, we will restate the F.O.C. w.r.t. to each element of $\mathbb{P}_i$ below. Following Equation 98, the F.O.C. w.r.t. $\tilde{P}_{\ell i,k}$ (where $\ell = i$ or $\ell = j \neq i$), can be expressed as

$$(1)\quad \sum_{n\neq i}\sum_{g}\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)e_{ni,g}\varepsilon_{ni,g}^{(\ell i,k)}\right]+$$

$$\sum_{g}\left[\left(1-\frac{1}{1+\mu_g}\frac{P_{ii,g}}{\tilde{P}_{ii,g}}\right)e_{ii,g}\varepsilon_{ii,g}^{(\ell i,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial\ln E_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{\ell i,k}}\right)_{\mathbf{w},\mathbb{P}_{-\ell i,k}}=0.$$

where $e_{ni,g}=\tilde{P}_{ni,g}Q_{ni,g}/Y_i$ denotes the expenditure share on good ni,g. Following Equation 106, the F.O.C. w.r.t. export price $\tilde{P}_{ij,k}$ is given by

$$(2)\quad 1-\Lambda_{ij,k}+\sum_{g}\left[\left(1-\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right]$$

$$-\sum_{n\neq i}\sum_{g}\left[\omega_{ni,g}\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\frac{E_i}{E_j}\left(\frac{\partial\ln E_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0.$$

First, note that the system of F.O.C.s (1) Appealing to above lemma, it immediately follows that the unique solution to the above equation is the trivial solution given by:

$$\frac{\tilde{P}^*_{ji,k}}{P_{ji,1}}=(1+\omega_{ji,k})(1+\bar{\tau}_i);\qquad \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}}=\frac{1}{1+\mu_g}. \tag{107}$$

With the aid of the above result, we can proceed to solving System (2), knowing that $\widetilde{\Delta}_i(\mathbb{P}^*_i,\mathbf{w})=0$. To this end, let us economize on the notation by defining

$$\chi_{ij,k}\equiv\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}.$$

Appealing to this choice of notation the F.O.C. specified by System (2) implies the following optimality condition:

$$1-\Lambda_{ij,k}+\sum_{g}\left[(1-\chi_{ij,g})\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}}\right]-\sum_{n\neq i}\sum_{g}\left[\omega_{ni,g}\frac{e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}}\right]=0. \tag{108}$$

To simplify the above expression we will use a well-know result from consumer theory, namely, the Cournot aggregation, which implies:

$$[\text{Cournot aggregation}]\qquad 1+\sum_{g}\left[\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right]=-\sum_{n\neq i}\sum_{g}\left[\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right].$$

Combining the above expression with Equation 108 and noting that by Slutsky's equation $\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}=\varepsilon_{ij,k}^{(nj,g)}$ (if $\eta_{ni,g}=1$ for all ni,g), yields the following:

$$-\sum_{g}\left[\chi_{ij,g}\varepsilon_{ij,k}^{(ij,g)}\right]-\sum_{n\neq i}\sum_{g}\left[(1+\omega_{ni,g})\varepsilon_{ij,k}^{(nj,g)}\right]=0\qquad\forall(ij,k).$$

We can formulate the above equation in matrix algebra as

$$-\mathbf{E}_{ij}\mathbf{X}_{ij}-\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K}+\mathbf{\Omega}_i\right)-\mathbf{\Lambda}_{ij}=0, \tag{109}$$

where $\mathbf{X}_{ij}\equiv\left[\chi_{ij,k}\right]_k$ and $\mathbf{\Lambda}_{ij}\equiv\left[\Lambda_{ij,k}\right]_k$ are $K\times 1$ vectors. The $K\times K$ matrix $\mathbf{E}_{ij}\sim\mathbf{E}_{ij}^{(ij)}\equiv\left[\varepsilon_{ij,k}^{(ij,g)}\right]$ encompasses the own- and cross-price elasticities between the different varieties sold by origin i to market j. Analogously, $\mathbf{E}_{ij}^{(-ij)}\equiv\left[\varepsilon_{ij,k}^{(nj,g)}\right]_{k,n\neq i,g}$ is a $K\times(N-1)K$ matrix of cross-price elasticities between varieties sold by i and by all other origin countries in market j. $\mathbf{\Omega}_i\equiv\left[\omega_{ni,g}\right]_{n,g}$ is a $(N-1)K\times 1$ vector of inverse export supply elasticities associated with domestic market i. To invert the system specified by Equation 109 we can use our result (from Appendix E) that $\mathbf{E}_{ij}$ is non-singular, which yields the following formulation for $\mathbf{X}^*_{ij} = \left[\chi^*_{ij,k}\right]_k$:

$$\mathbf{X}^*_{ij} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K} + \boldsymbol{\Omega}_i\right) + \boldsymbol{\Lambda}_{ij}\right]. \tag{110}$$

Now, we can recover the optimal tax/subsidy rates from the optimal price wedges implies by Equations 107 and 110. Specifically, noting that

$$1 + t^*_{ji,k} = \frac{\tilde{P}_{ji,k}}{P_{ji,k}}; \qquad 1 + s^*_{i,k} = \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}}; \qquad 1 + x_{ij,k} = \frac{\tilde{P}^*_{ij,k}/P_{ij,k}}{\tilde{P}^*_{ii,k}/P_{ii,k}};$$

country i's unilaterally optimal tax schedule can be expressed as follows:

$$\begin{aligned}
&\text{[domestic subsidy]} \quad 1 + s^*_{i,k} = 1 + \mu_k \\
&\text{[import tariff]} \quad 1 + t^*_{ji,k} = (1 + \omega_{ji,k})(1 + \bar{\tau}_i) \\
&\text{[export subsidy]} \quad \mathbf{1} + \mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1} + \boldsymbol{\Omega}_i\right) + \boldsymbol{\Lambda}_{ij}\right](1 + \bar{\tau}_i).
\end{aligned}$$

The last step, is to invoke the multiplicity of optimal tax schedules provided by Lemma 9. Given the multiplicity of optimal import tax and export subsidies, the uniform trade tax-shifter, $\bar{\tau}_i$, is redundant. Following Lemma 1, any tax schedule that satisfies $1 + \tilde{t}^*_{ji,k} = \left(1 + t^*_{ji,k}\right)\frac{1+\bar{t}_i}{1+\bar{\tau}_i}$ and $1 + \tilde{x}^*_{ij,k} = \left(1 + x^*_{ij,k}\right)\frac{1+\bar{t}_i}{1+\bar{\tau}_i}$, and where $1 + \bar{t}_i \in \mathbb{R}_+$, is also optimal. As such the exact value assigned to $\bar{\tau}_i$ is redundant. This explains why we did not unpack the term $\bar{\tau}_i$ in Step #3. There is also another dimension of multiplicity whereby any uniform shift in final good production subsidies (paired with a proportional adjustments final good import tariffs and export subsidies) preserves the equilibrium. Accounting for both dimensions of multiplicity, the optimal policy schedule is given by:

$$\begin{aligned}
&\text{[domestic subsidy]} \quad 1 + s^*_{i,k} = (1 + \mu_k)\left(1 + \bar{s}_i^{\mathscr{C}}\right)^{-1} \\
&\text{[import tax]} \quad 1 + t^*_{ji,k} = (1 + \omega_{ji,k})(1 + \bar{t}_i)\left(1 + \bar{s}_i^{\mathscr{C}}\right) \\
&\text{[export subsidy]} \quad \mathbf{1} + \mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1} + \mathbf{t}^*_i\right) + \boldsymbol{\Lambda}_{ij}(1 + \bar{t}_i)(1 + \bar{s}_i^{\mathscr{C}})\right],
\end{aligned}$$

where $\bar{s}_i^{\mathscr{C}}$ is an arbitrary tax shifter that assumes a positive value if the taxed item is a final good and zero otherwise. Also, recall that the elements of $\boldsymbol{\Lambda}_{ij} \equiv \left[\Lambda_{ij,k}\right]_k$ correspond to the fraction of good ij,k that is reimported via the IO network.

L Proof of Propositions 1 and 2

This appendix provides a formal proof for Propositions 1 and 2. The former asserts that, if $Cov(\sigma_k, \mu_k) < 0$, piecemeal trade policy interventions that seek to improve the terms-of-trade (relative to Laissez-Faire) worsen misallocation and vice versa. The latter asserts that, if $Cov(\sigma_k, \mu_k) < 0$, a unilateral implementation of efficient industrial subsidies in country i (without reciprocity by partners) can harm welfare through adverse terms-of-trade effects.

Proposition 1. Suppose country i is initially in an equilibrium where the government has implemented a uniform (possibly zero) import tariff or export tax. If existing taxes are zero then the economy is essentially operating under Laissez-Faire. Our goal is to prove the following: If $Cov(\mu_k, \sigma_k) < 0$, then any adjustment to trade policy that seeks to improve allocative efficiency (relative to the baseline equilibrium) worsens country i's terms of trade (ToT) and *vice versa*. Since a uniform import tariff is equivalent to a uniform export tax by the Lerner symmetry, we can without loss of generality focus on the case where the initial trade policy consists of a (possibly zero) uniform export tax. We first present our proof for the case of restricted entry, but extend it later to account for free entry. To economize on the notation, we hereafter use $1+\tilde{x} \sim (1+x)^{-1}$ to denote the export tax counterpart of export subsidy, x.[91]

Welfare Accounting under Piecemeal Policy Change—As an intermediate step, we first characterize the change in welfare in response to a piecemeal trade policy change, decomposing the welfare impacts into changes in allocative efficiency and terms of trade. When preferences are homothetic, the change in country i's welfare in response to an adjustment to export taxes, $\{\mathrm{d}\ln(1+\tilde{x}_{i,k})\}_k$, is the sum of corresponding income and price effects.[92] Namely,

$$\mathrm{d}\ln W_i = \mathrm{d}\ln Y_i - \sum_k\sum_n \lambda_{ni,k} e_{i,k} \mathrm{d}\ln\tilde{P}_{ni,k}. \tag{111}$$

To formalize the tension between allocative efficiency and terms of trade succinctly, suppose that country i is sufficiently small so that its piecemeal trade policy reform has a negligible impact on relative wages and labor allocations in the rest of the world. In that case, $\mathrm{d}\ln\tilde{P}_{ii,k} = \mathrm{d}\ln w_i$ and $\mathrm{d}\ln\tilde{P}_{ni,k} \approx 0$ for all $n \neq i$. Under theses assumptions, the price effects in Equation 111 reduce to $\sum_k\sum_n \lambda_{ni,k} e_{i,k} \mathrm{d}\ln\tilde{P}_{ni,k} = \lambda_{ii}\mathrm{d}\ln w_i$, where $\lambda_{ii} \equiv \sum_k \lambda_{ii,k} e_{i,k}$ denotes the aggregate domestic expenditure share in country i. Next we characterize income effects, $\mathrm{d}\ln Y_i$. For this, note that nominal income in country i is the sum of wage income, profits, and net revenues associated with export tax. In particular,

$$Y_i = \sum_k \left[(1+\mu_k)\,\rho_{i,k}\right] w_i L_i + \sum_{n\neq i}\sum_k \left[\tilde{x}_{i,k} P_{in,k} Q_{in,k}\right],$$

where $\tilde{x}_{i,k}$ is the export tax on industry k goods, which is uniform in the baseline equilibrium (i.e., $\tilde{x}_{i,k} = \bar{x}_i$) but is subsequently adjusted to improve allocative efficiency. Taking full derivatives of the above expression yields

$$\begin{aligned}\mathrm{d}\ln Y_i = &\left(1-\pi_i^{\mathscr{X}}\right)\left[\mathrm{d}\ln\left(\sum_k (1+\mu_k)\,\rho_{i,k}\right) + \mathrm{d}\ln w_i\right] \\ &+ \pi_i^{\mathscr{X}} \sum_{n\neq i}\sum_k \left[\frac{\bar{x}_i P_{in,k} Q_{in,k}}{\sum_{n',k'} \bar{x}_i P_{in',k'} Q_{in',k'}}\left(\frac{\partial \ln P_{in,k}}{\partial \ln w_i}\mathrm{d}\ln w_i + \frac{\partial \ln Q_{in,k}}{\partial \ln \tilde{P}_{in,k}}\mathrm{d}\ln(1+\tilde{x}_{i,k})\right)\right],\end{aligned}$$

where $\pi_i^{\mathscr{X}} \equiv \sum_{n\neq i}\sum_k \left[\tilde{x}_{i,k} P_{in,k} Q_{in,k}\right]/Y_i$ denotes the share of export tax revenues in total revenues. One can immediately verify that the change in the (employment-weighted) aggregate profit margin is

$$\mathrm{d}\ln\left(\sum_k (1+\mu_k)\,\rho_{i,k}\right) = \sum_k \left[\rho_{i,k}\cdot\left(\frac{1+\mu_k}{1+\overline{\mu}_i}\right)\mathrm{d}\ln\rho_{i,k}\right],$$

where $\overline{\mu}_i \equiv \sum_k [\mu_k \rho_{i,k}]$, recall, is our short-hand notation for the aggregate profit margin in country i. Notice that $\partial\ln P_{in,k}/\partial\ln w_i = 1$ under restricted entry and $\partial \ln Q_{in,k}/\partial\ln\tilde{P}_{in,k} = -\sigma_k$, since country i is a small open economy. Invoking these points, we can simplify the expression for $\mathrm{d}\ln Y_i$ as

$$\mathrm{d}\ln Y_i = \mathrm{d}\ln w_i + \left(1-\pi_i^{\mathscr{X}}\right)\sum_k\left[\rho_{i,k}\cdot\left(\frac{1+\mu_k}{1+\overline{\mu}_i}\right)\mathrm{d}\ln\rho_{i,k}\right] - \pi_i^{\mathscr{X}}\sum_k\left[\chi_{i,k}\cdot\sigma_k\,\mathrm{d}\ln(1+\tilde{x}_{i,k})\right], \tag{112}$$

where $\chi_{i,k} \equiv P_{in,k}Q_{in,k}/\sum_{n',k'}\left[P_{in',k'}Q_{in',k'}\right]$ denotes the share of industry k goods in country i's export

[91] Note that a positive export tax is akin to an negative export subsidy and vice versa—i.e., $x<0 \implies \tilde{x}>0$.

[92] To be clear, we assume that the baseline export tax policy does not discriminate between destination markets, with $x_{i,k}$ denoting the export tax applied to all export good in industry k.

revenues. To economize on the notation we henceforth use $\mathbb{E}_{\omega}[.]$ and $Cov_{\omega}(.)$ to denote the cross-industry mean and covariance with weights, $\{\omega_{i,k}\}_k$, that satisfy $\sum_k \omega_{i,k} = 1$. Considering this choice of notation, suppose the piecemeal trade policy reform is mean-preserving—i.e., $\mathbb{E}_{\chi}[\mathrm{dln}(1+\tilde{x}_{i,k})] \sim \sum_k [\chi_{i,k}\mathrm{d}\ln(1+\tilde{x}_{i,k})] = 0$. To put it verbally, the tax reform raises export taxes on some industries and lowers it on others, while preserving the sales-weighted average export tax rate. Under this presupposition, the last term in the above equation amounts to the covariance between σ_k and $\mathrm{dln}(1+\tilde{x}_{i,k})$. In particular,

$$\sum_k [\chi_{i,k}\cdot\sigma_k \mathrm{d}\ln(1+\tilde{x}_{i,k})] \sim \mathbb{E}_{\chi}[\sigma_k \mathrm{d}\ln(1+\tilde{x}_{i,k})]$$
$$= \mathbb{E}_{\chi}[\sigma_k \mathrm{d}\ln(1+\tilde{x}_{i,k})] - \mathbb{E}_{\chi}[\sigma_k]\cdot\underbrace{\mathbb{E}_{\chi}[\mathrm{d}\ln(1+\tilde{x}_{i,k})]}_{=0} \sim Cov_{\chi}(\sigma_k, \mathrm{d}\ln(1+\tilde{x}_{i,k})).$$

As a matter of accounting, $\sum_k \rho_{i,k}\mathrm{dln}\,\rho_{i,k} = 0$, so the same logic implies that the second term on the right-hand side of Equation 112 is also the covariance between the change in industry-level employment share, $\mathrm{d}\ln\rho_{i,k}$, and the industry-level markup (relative to the mean):

$$\sum_k \left[\rho_{i,k}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}\right)\mathrm{d}\ln\rho_{i,k}\right] \sim Cov_{\rho}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}, \mathrm{d}\ln\rho_{i,k}\right).$$

Substituting for the above expressions in Equation 112 and plugging the simplified expression for $\mathrm{d}\ln Y_i$ back into our original welfare formula (Equation 111) delivers,

$$\mathrm{d}\ln W_i = \underbrace{\left(1-\pi_i^{\mathscr{X}}\right)Cov_{\rho}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}, \mathrm{d}\ln\rho_{i,k}\right)}_{\text{Allocative Efficiency}} + \overbrace{\left[-\pi_i^{\mathscr{X}}Cov_{\chi}(\sigma_k, \mathrm{d}\ln(1+\tilde{x}_{i,k})) + (1-\lambda_{ii})\,\mathrm{d}\ln w_i\right]}^{\text{Terms of Trade}}, \tag{113}$$

echoing the welfare decomposition provided by Baqaee and Farhi (2019) and Atkin and Donaldson (2021). To offer intuition for this choice of decomposition, the term labeled *Allocative Efficiency* is analogous to the deviation form Hulten's (1978) formula in an inefficient closed economy. More specifically suppose country i was a closed economy hit with a labor productivity shock, $\mathrm{d}\ln A_{i,k}$. Following the same steps as above, the welfare impact of this shock can be decomposed as

$$\mathrm{d}\ln W_i^{\text{closed}} = \overbrace{\sum[\rho_{i,k}\mathrm{dln}\,A_{i,k}]}^{\text{Hulten}} + Cov_{\rho}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}, d\ln\rho_{i,k}\right),$$

where the deviation from Hulten (1978) reflects changes to allocative efficiency. Likewise, *Terms of Trade* effects in Equation 113 are analogous to deviations from Hulten (1978) if country i were open to trade but efficient. In accordance with this logic, the *Terms of Trade* effects in Equation 113 disappear if Country i is closed, in which case $\pi_i^{\mathscr{X}} = 1-\lambda_{ii} = 0$. Relatedly, *Allocative Efficiency* effects disappear if the economy is efficient, in which case $(1+\mu_k)/(1+\bar{\mu}_i) = 1$.

Tension between Allocative Efficiency & Terms of Trade—An export policy shock, $\{\mathrm{d}\ln(1+\tilde{x}_{i,k})\}_k$, that seeks to improve allocative efficiency must reallocate workers from low- to high-μ industries so that $Cov_{\rho}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}, \mathrm{d}\ln\rho_{i,k}\right) > 0$. If demand is elastic and well-behaved, this type of reallocation requires that industry-level export tax reductions to be positively correlated with markups, i.e., $Cov_{\chi}\left(\frac{1+\mu_k}{1+\bar{\mu}_i}, \mathrm{d}\ln(1+\tilde{x}_{i,k})\right) < 0$. Accordingly, if $Cov(\sigma_k, \mu_k) < 0$, the export tax changes will be positively correlated with the trade elasticity, $Cov_{\chi}(\sigma_k, \mathrm{d}\ln(1+\tilde{x}_{i,k})) > 0$. As such, an export tax reform that improves *Allocative Efficiency* (relative to the status quo) worsens the *Terms of Trade* through the term $Cov_\chi\left(\sigma_k, \mathrm{d}\ln\left(1+\tilde{x}_{i,k}\right)\right)$. Now, consider the remaining *Terms of Trade* term that accounts for general equilibrium wage effects. Considering that $Cov_\chi\left(\sigma_k, \mathrm{d}\ln\left(1+\tilde{x}_{i,k}\right)\right) > 0$, the desired export tax alteration consists of raising taxes in high-trade elasticity industries (where export sales are more-sensitive to tax hikes) paired with an proportional tax reduction in low-trade elasticity industries (where export sales are less-responsive to tax cuts). This alternation will, by design, lower overall export sales by country i—resonating with the conventional Ramsey rule. The reduction in export sales will in turn deflate demand for country i's labor and its wage rate relative to rest of the world (i.e., $\mathrm{d}\ln w_i < 0$). That is, the second *Terms of Trade* term is also negative when the export tax reform attempts to improve *Allocative Efficiency* relative to the status quo. To take stock: Suppose $Cov\left(\sigma_k, \mu_k\right) < 0$ and country i is initially in a equilibrium involving uniform (or zero) export/import taxes. In that case, improving *Allocative Efficiency* via piecemeal trade policy adjustments, $\{\mathrm{d}\ln\left(1+\tilde{x}_{i,k}\right)\}$, coincides with a worsening of the terms of trade.

Second-Best Trade Policies are Industry-Blind in Krugman (1980)—When preferences are Cobb-Douglas across industries, country i's second-best trade policy is given by (see Section 3):

$$\text{[2nd-best import tariff]} \qquad 1+t^{**}_{ji,k} = \frac{1+\left(\sigma_k-1\right)\lambda_{ii,k}}{1+\frac{1+\bar{\mu}_i}{1+\mu_k}\left(\sigma_k-1\right)\lambda_{ii,k}}\left(1+\omega_{ji,k}\right)\left(1+\bar{t}_i\right)$$

$$\text{[2nd-best export subsidy]} \qquad 1+x^{**}_{ij,k} = \frac{1+\mu_k}{1+\bar{\mu}_i}\left(\frac{\left(\sigma_k-1\right)\left(1-\lambda_{ij,k}\right)}{1+\left(\sigma_k-1\right)\left(1-\lambda_{ij,k}\right)}\right)\left(1+\bar{t}_i\right),$$

where $\bar{t}_i$ us a uniform trade tax shifter that accounts for the multiplicity of optimal policy schedules (see Lemma 1). Following Alvarez and Lucas (2007), if country i is a small open economy, then $\lambda_{ij,k} = \lambda_{ii,k} = \omega_{ji,k} \to 0$. Moreover, if we assume that the firm- and country-level degrees of market power are identical à la Krugman (1980), we have $1+\mu_k = \frac{\sigma_k}{\sigma_k-1}$. Consolidating these two points, we get the following formula for the 2nd-best trade policy of a small open economy in the multi-industry Krugman (1980) model:

$$1+t^{**}_{ji,k} = 1+\bar{t}_i; \qquad\qquad 1+x^{**}_{ij,k} = \frac{\sigma_k}{\sigma_k-1}\left(\frac{\sigma_k-1}{\sigma_k}\right)\frac{1+\bar{t}_i}{1+\bar{\mu}_i} = \left(1+\bar{t}_i\right)\left(1-\frac{1}{\bar{\sigma}_i}\right),$$

where $\frac{1}{\bar{\sigma}_i} = \sum_k \rho_{i,k}\frac{1}{\sigma_k}$ is the sales-weighted average (inverse) trade elasticity. Evidently, the optimal 2nd-best trade policy consists of a uniform import tariff or export subsidy, which is blind to inter-industry misallocation and industry-level export market power. The logic is that any attempt at exploiting industry-level export market power exacerbates inter-industry misallocation and *vice versa*—leaving the government with no choice but to abandon these targeted policy aspirations. This consideration leads to *industry-blind* optimal trade taxes that solely manipulate the relative aggregate wage $\left(w_i/w_{-i}\right)$ in country i's favor, with minimal reshuffling of resources across industries.

Proposition 2. Suppose country i is initially operating under Laissez-Faire. The government has, moreover, agreed (under a shallow treaty) to limit itself to the efficient or cooperative policy choice specified in Section 3. Our goal is to show that a unilateral implementation of efficient industrial subsidies by country i (without reciprocity by partners) causes a deterioration of country i's terms of trade (ToT) and even immiserizing growth. To this end, we build on the welfare accounting formulas derived earlier. In particular, the welfare impacts of unilateral markup-correcting subsidies by country

i (i.e., $\mathbf{s}_i = \boldsymbol{\mu}$) can be written as

$$\Delta W_i = \int_{\mathbf{s}_i=\mathbf{0}}^{\boldsymbol{\mu}} \mathrm{d} \ln W_i,$$

where the change in welfare in response to modest industrial policy adjustments can, as earlier, be decomposed as

$$\mathrm{d} \ln W_i = \underbrace{\left(1 - \pi_i^{\mathscr{S}}\right) Cov_\rho \left(\frac{1+\mu_k}{1+\overline{\mu}_i}, \mathrm{d} \ln \rho_{i,k}\right)}_{\text{Allocative Efficiency}} + \overbrace{\left[-\pi_i^{\mathscr{S}} Cov_\chi \left(\sigma_k, \mathrm{d} \ln \left(1+s_{i,k}\right)\right) \; + \; \left(1-\lambda_{ii}\right) \mathrm{d} \ln w_i\right]}^{\text{Terms of Trade}}.$$

The above equation can be derived analogously to Equation 113, with $\pi_i^{\mathscr{S}}$ denoting the share of production tax (or subsidy) revenues that are collected from foreign consumers. Intuitively, the fraction of tax revenues collected from domestic consumers deliver income gains that are exactly offset by the corresponding loss from price increases. Hence, the domestically-borne fraction of the tax revenue does not contribute to welfare changes, $\mathrm{d} \ln W_i$, beyond general equilibrium impacts on inter-industry labor allocation and wages.

Considering this background, we can specify the sign of the *Allocative Efficiency* and *Terms of Trade* effects (in the above equation) in response to markup/scale-correcting subsidies. The efficient policy ($s_{i,k} = 0 \to s'_{i,k} = \mu_k$) subsidizes output in high-μ industries, relocates labor from the rest of the economy to these industries, and thereby improves allocative efficiency, i.e., $Cov_\rho \left(\frac{1+\mu_k}{1+\overline{\mu}_i}, \mathrm{d} \ln \rho_{i,k}\right)$. But following the logic presented earlier: If $Cov\left(\sigma_k, \mu_k\right) < 0$, it should be the case that along the path of restoring marginal cost pricing, $Cov_\chi \left(\sigma_k, \mathrm{d} \ln \left(1+s_{i,k}\right)\right) > 0$ and $\mathrm{d} \ln w_i < 0$—both of which contribute to a deterioration of the ToT. In some cases, like the numerical example presented in Section 4.1, the deterioration of the ToT is larger than the corresponding allocative efficiency gains—leading to immiserizing growth in country i.

Adverse Firm-Delocation Effects when Country i is Large—When country i is excessively large, a unilateral adoption of corrective industrial policies worsens its ToT through an additional channel: *firm-delocation* effects. To make this point, let $v_{i,k} \equiv \frac{(1-\lambda_{ii,k})e_{i,k}}{(1-\lambda_{ii})}$ denote the import share pertaining to industry k. Suppose without loss of generality that $\mathbb{E}_v\left[\mu_k\right] = 0$—since following Lemma 1, we can recalibrate the level of markups and wages in the rest of the world without changing welfare. Our welfare decomposition, in that case, takes the following form, internalizing the effect of country i's policy on entry and labor shares in the rest of the world:

$$\begin{aligned}\mathrm{d} \ln W_i = & \left(1 - \pi_i^{\mathscr{S}}\right) Cov_\rho \left(\frac{1+\mu_k}{1+\overline{\mu}_i}, \mathrm{d} \ln \rho_{i,k}\right) + \\ & - \pi_i^{\mathscr{S}} Cov_\chi \left(\sigma_k, \mathrm{d} \ln \left(1+s_{i,k}\right)\right) \; + \; \left(1-\lambda_{ii}\right) \left[\mathrm{d} \ln w_i + Cov_v \left(\mu_k, \mathrm{d}\ln\rho_{-i,k}\right)\right].\end{aligned}$$

Country i's corrective subsidies, by design, relocate labor to high-μ_k industries in the local economy, but have the opposite effect on labor allocation in the rest of the world. Put formally, $Cov_\rho \left(\frac{1+\mu_k}{1+\overline{\mu}_i}, \mathrm{d} \ln \rho_{i,k}\right) > 0$, while $Cov_v \left(\mu_k, \mathrm{d}\ln\rho_{-i,k}\right) < 0$. Improving allocative efficiency with corrective subsidies, therefore, worsens the ToT through an additional term $Cov_v \left(\mu_k, \mathrm{d}\ln\rho_{-i,k}\right)$, which represents *firm-delocation* effects. The logic is that promoting output and entry in high-μ domestic industries, diminishes output and entry in high-μ foreign industries—hence, the term *firm-delocation*. The reduction in foreign firm-level varieties inflates the price of imports $P_{-ii,k} \propto M_{-i,k}^{-\mu_k}$, thereby worsening country i's ToT.

Empirical Appendix

M Cleaning the data on the identity/name of exporting firms

Utilizing the information on the identity of the foreign exporting firm is a critical part of our empirical exercise. Unfortunately, the names of the exporting firms in our dataset are not standardized. As a result, there are instances when the same firm is recorded differently due to using or not using the abbreviations, capital and lower-case letters, spaces, dots, other special characters, etc. To standardize the names of the exporting firms, we used the following procedure.[93]

1. We deleted all observations with the missing exporting names and/or zero trade values.

2. We capitalized firms names and their contact information (which is either email or phone number of the firm).

3. We eliminated abbreviation "LLC," spaces, parentheses, and other special characters (. , ; / @ ' } - & ") from the firms names.

4. We eliminated all characters specified in 3. above and a few others (# : FAX) from the contact information.

5. We dropped observations without contact information (such as, "NOTIENE", "NOREPORTA", "NOREGISTRA," etc.), with non-existent phone numbers (e.g., "0000000000", "1234567890", "1"), and with six phone numbers which are used for multiple firms with different names (3218151311, 3218151297, 6676266, 44443866, 3058712865, 3055935515).

6. Next, we kept only up to first 12 characters in the firm's name and up to first 12 characters in the firm's contact information (which is either email or phone number). In our empirics, we treat all transaction with the same updated name and contact information as coming from the same firm.

7. We also analyzed all observations with the same contact information, but slightly different name spelling. We only focused on the cases in which there are up to three different variants of the firm name. For these cases, we calculated the Levenshtein distance in the names, which is the smallest number of edits required to match one name to another. We treat all export observations as coming from the same firm if the contact phone number (or email) is the same and the Levenshtein distance is four or less.

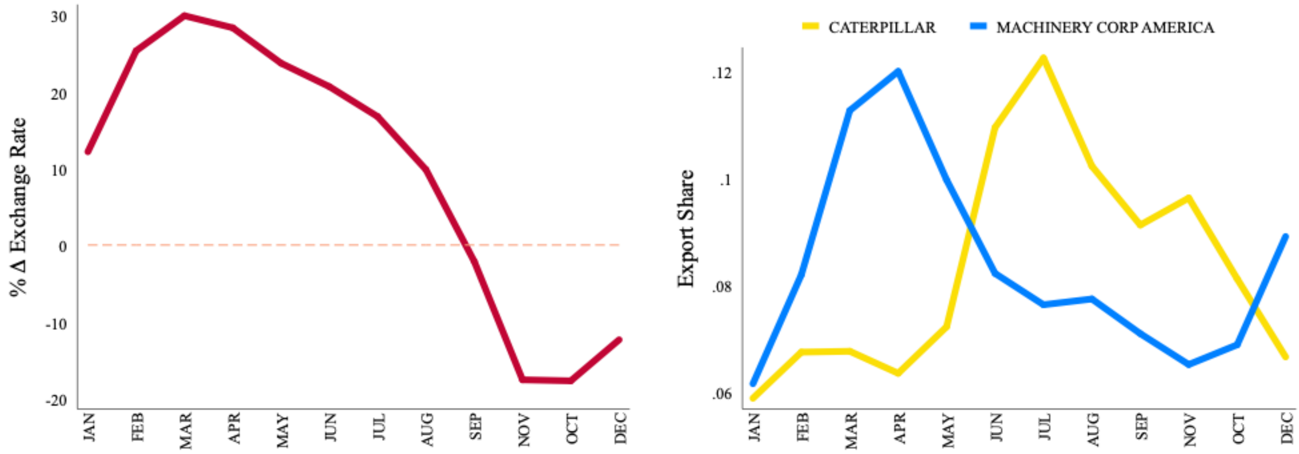
Table 6: *Summary Statistics of the Colombian Import Data.*

	Year						
Statistic	2007	2008	2009	2010	2011	2012	2013
F.O.B. value (billion dollars)	30.77	37.26	31.39	38.41	52.00	55.79	56.92
$\frac{\text{C.I.F. value}}{\text{F.O.B. value}}$	1.08	1.07	1.05	1.06	1.05	1.05	1.05
$\frac{\text{C.I.F. + tax value}}{\text{F.O.B. value}}$	1.28	1.21	1.14	1.19	1.15	1.18	1.15
No. of exporting countries	210	219	213	216	213	221	224
No. of imported varieties	483,286	480,363	457,000	509,524	594,918	633,008	649,561

Notes: Tax value includes import tariff and value-added tax (VAT). The number of varieties corresponds to the number of country-firm-product combination imported by Colombia in a given year.

[93] The corresponding Stata code is in the cleanFirmsNames.do.

Figure 8: *Monthly export shares and exposure to aggregate exchange rate shocks*



Notes: The left panel reports the year-over-year change in the Peso-to-Dollar exchange rate for each month in 2009. The right panel reports monthly export sales shares for the two largest US firms serving product code HS8431490000—namely, Caterpillar and Machinery Corp. of America.

N Illustrative Example for our Instrumental Variable

This section presents an example to elucidate the logic behind our shift-share instrument, presented in Section 6. The example compares two major U.S. firms that dominate exports to Colombia in product code HS8431490000 (PARTS AND ATTACHMENTS OTHER FOR DERRICKS). We have chosen this product code because it features two of the biggest exporters to Colombia: "CATERPILLAR" and "MACHINERY CORPORATION OF AMERICA."

The left panel of Figure 8 shows the year-over-year change in the Peso-to-Dollar exchange rate for each month in 2008. The right panel plots monthly export shares for "CATERPILLAR" and "MACHINERY CORPORATION OF AMERICA" of HS8431490000. Notice that export patterns from "CATERPILLAR" and "MACHINERY CORPORATION OF AMERICA" are markedly different. The former exports primarily in the first half of the fiscal year, while the latter exports primarily in the second half. The prices charged by these two firms are, thus, differentially affected by aggregate exchange rate shocks.

O Robustness Checks: *Import Demand Estimation*

This appendix reports three robustness checks that we described in Section 6. The first check addresses the possibility that firms set prices in forward-looking manner. To restate the issue, when there are lags in inventory clearances, firms' optimal pricing decisions may be forward-looking. If true, such price-setting behaviors can violate assumption (a1). To address this concern, we reconstruct our shift-share instrument using 4 lags instead of 1. If inventories clear in at most 4 years, we can deduce that pricing decisions do not internalize expected demand shocks beyond the 4 year mark. As a result, $\mathbb{E}\left[\tilde{p}_{jkt-4}(\omega, m)\,\Delta \ln \varphi_{\omega jkt}\right] = 0$, and this more-stringent instrument will satisfy the exclusion restriction. The *top* panel in Figure 9 compares the estimated σ_k and μ_k under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

The second check addresses the possibility that, in the presence of cross-inventory effects, $\Delta \ln \varphi_{\omega jkt}$ may encompass omitted variables that concern firms' dynamic inventory management decisions. These decisions internalize exchange rate movements, which may violate the identifying assumption (a2), i.e., $\mathbb{E}\left[\Delta \ln \mathcal{E}_{jt}(m)\, \Delta \ln \varphi_{\omega jkt}\right] \neq 0$. To address this concern, we reestimate the firm-level import demand function while directly controlling for changes on the annual exchange rate. In that case, $\mathbb{E}\left[z_{jk,t}(\omega)\, \Delta \ln \varphi_{\omega jkt} \mid \Delta \ln \mathcal{E}_{jt}\right]$, and the exclusion restriction will be satisfied insofar as dynamic demand optimization is a concern. The *middle* panel in Figure 9 compares the estimated σ_k and $\mu_k$under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

The third check addresses large multi-product firms that export multiple product varieties to Colombia in a given year. Suppose a multi-product firm ω exports many products including products k and g to Colombia in year t. If demand shock are correlated across the varieties supplied by this firm (i.e., $\mathbb{E}\left[\Delta \ln \varphi_{\omega jkt}\, \Delta \ln \varphi_{\omega jgt}\right] \neq 0$), Assumption (a2) may be violated despite each variety's market share being infinitesimally small. To address this issue, we reestimate the firm-level import demand function on a restricted sample that drops excessively large firms with a within-national market share that exceeds 0.1%. The *bottom* panel in Figure 9 compares the estimated σ_k and $\mu_k$under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

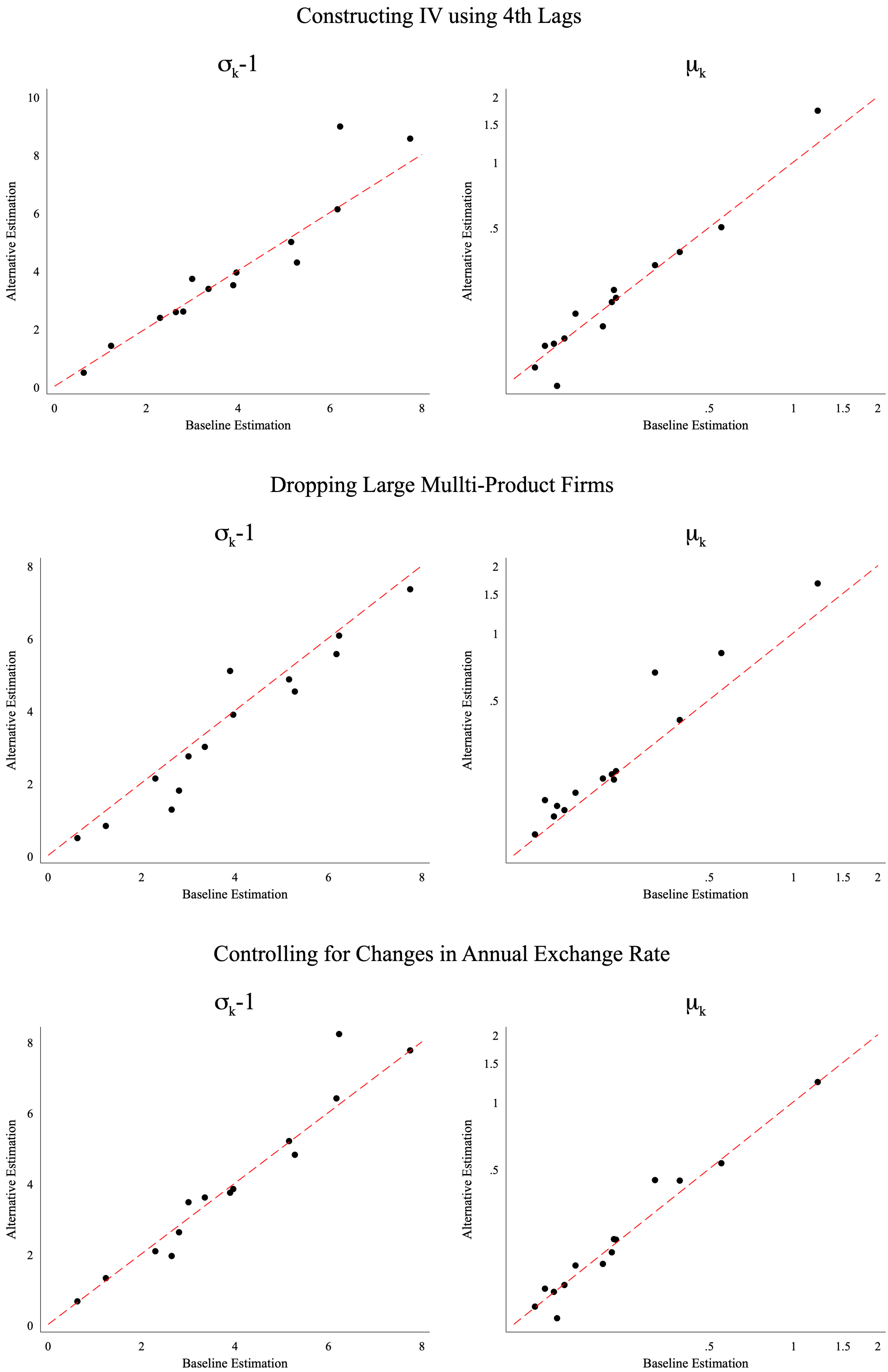
P Estimating the Import Demand Function in Levels

Our preferred estimates for μ_k and σ_k are obtained by estimating a firm-level import demand function in first-differences—see Section 6. The first-difference approach for estimating elasticities in this context can be traced back to the seminal work of Feenstra (1994) and Broda and Weinstein (2006)—although both studies rely on *country-level* rather than *firm-level* data. Another body of literature estimates the trade elasticity by fitting a country-level import demand function in log-levels, while controlling for appropriate fixed effects (e.g., Hummels, Lugovskyy, and Skiba (2009); Caliendo and Parro; Shapiro (2016)).

Recently, Boehm et al. (2020) have outlined the advantages and disadvantages of each approach: On the one hand, the first-difference approach performs better at handling the identification challenge poised by endogenous policy choices and omitted variable bias. On the other hand, the first difference estimator—at least when applied to country-level data—may not necessarily identify the long-run elasticity, which is the desired target for static trade models.

These issues pose a lesser problem for our firm-level estimation. We articulate this claim in two steps. First, we detail the *long-* versus *short-run* dilemma identified by Boehm et al. (2020), and explain why the same dilemma does not necessarily plague our firm-level estimation. Second, we establish our claim empirically by re-estimating our firm-level import demand function in levels. This exercise encouragingly confirms that our estimation in levels yields very similar results to our baseline estimation in differences.

Figure 9: *Robustness checks to address challenges to the identification of σ_k and μ_k*



The dilemma facing country-level estimations. Country level trade flows—which are traditionally used to estimate the trade elasticity—can be decomposed as follows:

$$\tilde{X}_{ji,k} = N_{ji,k}\tilde{p}_{ji,k}q_{ji,k}$$

where $\tilde{X}_{ji,k}$ denotes gross sales corresponding to *origin j–destination i–industry k*; $\tilde{p}_{ji,k}q_{ji,k}$ denotes average sales per firm (i.e., the intensive margin) and $N_{ji,k}$ denotes the total mass of firms associated with transaction ji, k (i.e., the extensive margin). Accordingly, the *long-run* trade elasticity is composed of an extensive and an intensive margin component:

$$\text{trade elasticity} \sim \frac{\partial \ln \tilde{X}_{ji,k}}{\partial \ln(1+t_{ji,k})} = \underbrace{\frac{\partial \ln N_{ji,k}}{\partial \ln(1+t_{ji,k})}}_{\varepsilon_n} + \underbrace{\frac{\partial \ln \tilde{p}_{ji,k}q_{ji}}{\partial \ln(1+t_{ji,k})}}_{\varepsilon_x}.$$

The issue raised by Boehm et al. (2020) concerns the fact that researchers do not separately observe $N_{ji,k}$ and $\tilde{p}_{ji,k}q_{ji,k}$ in country-level datasets. A standard solution to this limitation is to assume away firm-selection (i.e., set $N_{ji,k} = N_{j,k}$). Under this assumption, one can recover the trade elasticity by estimating an import demand function that controls for $N_{j,k}$ with *origin-industry* fixed effects. Crudely speaking, this solution is analogous to omitting the extensive margin component, i.e., setting $\varepsilon_n = 0$.

In practice, however, $N_{ji,k}$ may feature a bilateral element that accounts for firm-selection and which varies with the bilateral tariff rate—even after we control for a full set of origin and destination fixed effects. As noted above, traditional techniques that estimate the import demand function in *levels* with origin/destination fixed effects, are unable to account for the bilateral nature of $N_{ji,k}$. As such, traditional log-level estimators often suffer from an omitted variable bias.

Boehm et al. (2020) argue that we can overcome the omitted variable bias by estimating the country-level import demand function in differences rather than levels. Under this approach, however, one must employ long differences (over a sufficiently long time horizon) to credibly estimate the extensive margin component, ε_n. Nonetheless, the long-difference estimator may still fall short if tariff changes occur unevenly over the time-differencing horizon. In such cases, a correction must be applied to the estimated trade elasticity to account for lumpy longitudinal tariff changes.

Importantly, these limitations do not plague our firm-level estimation. We directly observe firm-level sales and need not to *infer* changes in $N_{ji,k}$ from changes in country level trade flows. Our data explicitly encompasses information on $N_{ji,k}$ and our identification strategy relies on the cross-sectional variation in firm-level variables within *importer–HS10 product–year* cells. With this level of disaggregation, our estimation is closer in spirit to the *Industrial Organization* literature on markup estimation. This literature routinely uses first difference estimators to recover markups (see, for example, equations 17-19 and related discussion in De Loecker and Warzynski (2012)). These markups estimates have been routinely used to discipline steady state models in the Macroeconomics literature (e.g., Baqaee and Farhi (2020b)).

Re-estimating our firm-level import demand function in levels. Above, we presented a conceptual argument that (compared to traditional country-level estimations) firm-level estimations should yield relatively similar results whether the import demand is estimated in *levels* or in *first differences*—provided that appropriate instruments are employed to adequately handle reverse causality. To illustrate the same point empirically, we re-estimate our firm-level import demand function in levels with *two-ways fixed effects*. We then compare the *two-ways-fixed-effects* estimates for $\mu_k = \frac{1}{\gamma_k - 1}$ and σ_k with our baseline estimates. The estimating equation in log-levels can be expressed as follows:

$$\ln \tilde{x}_{j,kt}(\omega) = (1-\sigma_k) \ln \tilde{p}_{j,kt}(\omega) + \left[1 - \frac{\sigma_k - 1}{\gamma_k - 1}\right] \ln \lambda_{j,kt}(\omega) + \underbrace{D_{kt}}_{\text{HS10-year FE}} + \underbrace{\varphi_{jk}(\omega)}_{\text{HS10-firm FE}} + \varphi_{\omega jkt}. \quad (114)$$

Recall that $\tilde{x} \equiv \tilde{p}q$ denotes gross firm-level sales value; $\tilde{p}$ denotes the consumer price which includes taxes and tariffs; $\lambda_{j,kt}(\omega)$ denotes the *within-origin* $j\times$*product* k expenditure share on firm-level variety ω; D_{kt} accounts for product–year fixed effects, while $\varphi_{jk}(\omega)$ accounts for product-firm-origin fixed effects. The above equation differs from our baseline estimating equation in that the firm-product fixed effect, $\varphi_{jk}(\omega)$, is not differenced out. Instead the equation is estimated in levels.

As in the baseline case, we estimate the Equation (114) using a 2SLS estimator. To this end, we modify our original shift-share instrument to make it consistent with the fixed-effects estimation, which is conducted in levels. The new instrument is calculated as follows

$$\acute{z}_{j,kt}(\omega) = \sum_{m=1}^{12} s_{j,kt-1}(\omega, m) \ln \mathcal{E}_{jt}(m)$$

where $s_{j,kt-1}(\omega, m)$ denotes the lagged share of Month m sales in firm ω's total annual export sales. $\mathcal{E}_{jt}(m)$, as before, denotes the exchange rate (between Origin j's currency and the Colombian Peso) in Month m of the current year. The other instrumental variables are adjusted accordingly, to be consistent with our estimation that is conducted in levels rather than in differences.

The estimation results are reported in Table 7. The estimated values for σ_k and $\mu_k = \frac{1}{\gamma_k} - 1$ are encouragingly similar to the baseline (first-differences) estimates. Most importantly, the new estimation quasi-maintains the ranking of industries in terms of the underlying degree of national-level market power (σ_k) and firm-level market power. Later, in Appendix X, we recalculate the gains from optimal policy using the newly-estimated μ_k's and σ_k's. Encouragingly, the implied gains are starkly similar to those implied by our baseline estimates.

Q Comparison of Scale Elasticity Estimation Techniques

This appendix overviews the various approaches to scale elasticity estimation, offering some perspective on the advantages of our demand-based estimation technique. To provide a fair description of the existing techniques, we use an extended theoretical framework that accommodates (i) scale economies due to love-for-variety à la Krugman (1980), (ii) scale economies due to Marshallian externalities, and (iii) diseconomies of scale due to quasi-fixed factors of production. To this end, we begin this appendix by introducing a richer firm-level production function that accommodates Marshallian externalities and quasi-fixed inputs.

General Production Function—Firm ω located in origin i–industry k employs labor (L) and quasi-fixed inputs (F) using the following production function:

$$q_{i,k}(\omega) = \varphi_{i,k}(\omega) \left(L_{i,k}(\omega)^{1-\beta_{i,k}} F_{i,k}(\omega)^{\beta_{i,k}} \right) \times L_{i,k}^{\psi_k}.$$

Quasi-fixed inputs ($F_{i,k}(\omega)$) correspond to land, physical capital, or industry-specific human capital, the supply of which is fixed at the industry-level, i.e., $\sum_{\omega} F_{i,k}(\omega) = \overline{F}_{i,k}$. The last term in the production function accounts for Marshallian externalities, whereby the TFP of firm ω increases with industry-

Table 7: *Two-ways fixed effects estimation results*

Sector	ISIC4 codes	Estimated Parameter $\sigma_k - 1$	$\frac{\sigma_k-1}{\gamma_k-1}$	μ_k	Obs.	Weak Ident. Test
Agriculture & Mining	100-1499	4.563 (1.739)	0.698 (0.132)	0.153 (0.089)	10,762	3.07
Food	1500-1699	2.476 (0.818)	0.927 (0.050)	0.374 (0.284)	17,594	5.01
Textiles, Leather & Footwear	1700-1999	3.256 (0.297)	0.685 (0.023)	0.210 (0.024)	110,925	59.94
Wood	2000-2099	2.093 (1.196)	0.893 (0.191)	0.427 (0.801)	5,282	2.12
Paper	2100-2299	7.858 (3.953)	0.895 (0.154)	0.114 (0.177)	35,058	2.65
Petroleum	2300-2399	0.397 (0.342)	0.698 (0.081)	1.758 (1.584)	3,675	2.53
Chemicals	2400-2499	4.738 (0.496)	0.913 (0.031)	0.193 (0.071)	127,946	29.71
Rubber & Plastic	2500-2599	4.025 (0.791)	0.664 (0.062)	0.165 (0.045)	101,730	9.95
Minerals Basic & Fabricated Metals	2600-2699 2700-2899	3.390 (0.453)	0.681 (0.036)	0.201 (0.035)	173,432	20.03
Machinery	2900-3099	4.402 (1.765)	0.710 (0.080)	0.161 (0.065)	257,788	19.88
Electrical & Optical Equipment	3100-3399	0.756 (0.300)	0.609 (0.015)	0.806 (0.099)	246,597	19.25
Transport Equipment N.E.C. & Recycling	3400-3599 3600-3800	2.156 (0.462)	0.514 (0.032)	0.238 (0.053)	147,772	11.37

Notes. Estimation results of Equation (16). Standard errors in parentheses. The estimation is conducted with HS10 product-year fixed effects. All standard errors are simultaneously clustered by product-year and by origin-product, which is akin to the correction proposed by Adao et al. (2019). The weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist et al. (1996).

wide employment, $L_{i,k}$, at a constant elasticity ψ_k.

Aggregation of Firm-Level Prices into Industry-Level Prices Indexes—Cost-minimization and profit-maximization imply that firm ω sets a price equal to $p_{in,k}(\omega) = \left(\frac{\gamma_k}{\gamma_k-1}\right)\frac{d_{ij,k}}{\varphi_{i,k}(\omega)} w_i^{1-\beta_k} v_{i,k}^{\beta_k} L_{i,k}^{\psi_k}$. Variable $v_{i,k}$ denotes the unit price of the quasi-fixed input in industry k, which per cost minimization satisfies $v_{i,k} = \frac{1-\beta_k}{\beta_k}\frac{w_i L_{i,k}}{\bar{F}_{i,k}}$. Supposing that preferences have a nested-CES parameterization (per Assumption A1), we can use the logic in Section 2 to aggregate firm-level prices into industry-level price indexes subject to free entry ($M_{i,k} = L_{i,k}/\gamma_k f_k^e$). Doing so yields the following producer price index for goods associated with origin i–destination n–industry k,

$$P_{in,k} = \left(\frac{\gamma_k}{\gamma_k - 1}\right) \frac{\tau_{in,k}}{\overline{\varphi}_{i,k}} \, w_i \, L_{i,k}^{-\left(\frac{1}{\gamma_k - 1} + \psi_k\right) + \beta_k}, \tag{115}$$

where $\overline{\varphi}_i$ encompasses constant parameters including (average) firm productivity. Our baseline Krugman model is a special case of this equation, in which $\psi_k = \beta_k = 0$. Though, as we argue shortly, our estimation of the scale elasticity is insensitive to $\beta_k = 0$. Based on Equation 115, the scale of employment, $L_{i,k}$, affects producer prices through increasing-returns to scale (i.e., *Jacobian* (love-for-variety) + *Marshallian* externalities) and decreasing returns to scale due to quasi-fixed inputs. More specifically, $\partial \ln P_{in,k} / \partial \ln L_{i,k} = -\left(\frac{1}{\gamma_k - 1} + \psi_k\right) + \beta_k$, where the sub-elasticity $\left(\frac{1}{\gamma_k - 1} + \psi_k\right)$ accounts for increasing-returns to scale that disrupt allocative efficiency, while β_k accounts for decreasing-returns to scale that do *not* undermine allocative efficiency given equilibrium constraints. Hence, for policy analysis, it is crucial to separately identify the former sub-elasticity from the latter—as the degree of allocative inefficiency depends solely on the sub-elasticity $\left(\frac{1}{\gamma_k - 1} + \psi_k\right)$. The following remark formalizes this point.

Remark 1. *For policy evaluation, it is crucial to separately identify* $\frac{1}{\gamma_k - 1} + \psi_k$ *from* β_k*. The logic is that if* $\frac{1}{\gamma_k - 1} + \psi_k = 0$*, the market equilibrium is constrained-efficient irrespective of* β_k*, and there is no scope for improving allocative efficiency with policy. Correspondingly, the corrective gains from policy depend on the following notion of scale elasticity that differs from the reduced-form elasticity,* $\frac{\partial \ln P_{in,k}}{\partial \ln L_{i,k}}$*:*

$$\mu_k = \underbrace{(\gamma_k - 1)^{-1}}_{\text{Jacobian (love-for-variety)}} + \underbrace{\psi_k}_{\text{Marshallian}} \sim \text{scale elasticity}$$

The above remark is an immediate corollary of the First Welfare Theorem. In particular, letting $\gamma_k \to \infty$ and $\psi_k = 0$, our theoretical model reduces to a simple Arrow-Debreu model to which the fundamental welfare theorems apply. Our emphasis on $\mu_k \neq \partial \ln P_{in,k} / \partial \ln L_{i,k}$, as we elaborate shortly, speaks to one of the possible techniques for scale elasticity estimation, which infers μ_k from the reduced-form elasticity, $\ln P_{in,k} / \partial \ln L_{i,k}$.

Section 4 unveiled another consideration when estimating scale elasticities. We, in particular, argued that the cross-industry covariance between the scale elasticity (μ_k) and the trade elasticity (σ_k) is a crucial determinant of policy outcomes in open economies. Hence, it is advantageous to estimate these elasticities in a manner that ascertains mutual consistency.

Remark 2. *Policy outcomes in open economies depend crucially on the cross-industry covariance between the scale and trade elasticities, i.e.,* $Cov\left(\mu_k, \sigma_k\right)$*. So, for policy evaluation, it is advantageous to jointly estimate* μ_k *and* σ_k *in a way that ascertains mutual consistency.*

Taking these remarks into consideration, we describe three techniques for estimating scale elasticities and identify their advantages and disadvantages. We begin with the demand-based estimation technique developed in Section 6 of this paper.

Technique 1: Firm-Level Demand Estimation

Firm-level demand estimation can identify the scale elasticity, insofar as scale effects are driven by love-for-variety à la Krugman (1980). Demand estimation can also simultaneously identify the trade elasticity, σ_k, which is advantageous considering Remark 2. To unpack these points, let us revert to our generalized Krugman model for a moment while retaining the assumption that production exhibits diseconomies of scale (i.e., $\beta_{i,k} > 0$). In this setting, which widely used for trade policy analysis, the scale and trade elasticities become

$$\mu_k = \frac{1}{\gamma_k - 1} \sim \text{scale elasticity}; \qquad\qquad \sigma_k \sim \text{trade elasticity}.$$

The reason we can infer μ_k from demand parameters is that the scale elasticity in Krugman (1980) reflects the extent of *love-for-variety*—the social benefits of which are not internalized by firms' entry decisions. Recall from Section 2 (Assumption A1) that the nested-CES demand function facing firm ω can be written in terms of sales ($\tilde{x} \equiv \tilde{p} \times q$) as follows

$$\tilde{x}_{ni,k}(\omega) = \xi_{ni,k}(\omega) \left(\frac{\tilde{p}_{ni,k}(\omega)}{\tilde{P}_{ni,k}} \right)^{1-\gamma_k} \left(\frac{\tilde{P}_{ni,k}}{P_{i,k}} \right)^{1-\sigma_k} Y_{i,k}.$$

One immediately notices that estimating the above function simultaneously determines the scale elasticity ($\mu_k = 1/(\gamma_k - 1)$) and the trade elasticity (σ_k). To perform the estimation, we take inspiration from Berry (1994) and rearrange and log-linearize the above demand function to obtain our estimating equation:

$$\ln \tilde{x}_{ni,k}(\omega) = (1 - \sigma_k) \ln \tilde{p}_{i,k}(\omega) + \left(1 - \frac{\sigma_k - 1}{\gamma_k - 1} \right) \ln \lambda_{ni,k}(\omega) + D_{i,k} + \varepsilon_{in,k}(\omega),$$

where $\lambda_{ni,k}(\omega) = \frac{\tilde{x}_{in,k}(\omega)}{\sum_{\omega'} \tilde{x}_{in,k}(\omega')}$ is firm ω's observed conditional market share within nest (in, k); $D_{i,k}$ account for importer-industry fixed effects; and the demand residual $\varepsilon_{in,k}(\omega)$ encompasses good-specific demand-shifters and measurement error. Our identification of the demand function relies on a shift-share instrument that constitutes a firm-specific cost- or supply shifter—see Section 6 for specific details.

Advantages and Disadvantages of Technique 1—The key advantage of our the demand-based estimation technique (relative to supply-based alternatives) is that it simultaneously identifies σ_k and μ_k—which is crucial following Remark 2. Moreover, our demand-based estimation (unlike supply-based techniques) is robust to the presence of diseconomies of scale, as measured by β_k. Our estimation technique is not merely limited to the Krugman (1980) model either, as it also identifies the scale elasticity in the more general Melitz (2003)-Pareto setting (see Appendixes D and Q.1). A clear limitation of our approach, on the other hand, is its inability to identify Marshallian externalities, as measured by ψ_k.

Technique 2: National Labor Content Supply Estimation

The genesis of this technique is the observation that the producer price index, $P_{in,k}$, can be regarded as the of price of country i's labor services in destination n. Under this interpretation, the product of the trade and scale elasticity can be recovered from the labor content supply elasticity, $\frac{\partial \ln P_{in,k}}{\partial \ln L_{i,k}}$, *insofar* as production involves no quasi-fixed inputs ($\beta_k = 0$). As for the actual estimation, the trick is that even though aggregate price indexes ($P_{in,k}$) are unobserved, they can be proxied by aggregate sales. To sketch out the logic, let $\tilde{X}_{in,k} = \tilde{P}_{in,k} Q_{in,k}$ denote gross sales which satisfy the gravity equation in our framework. In particular,

$$\frac{1}{1 - \sigma_k} \ln \tilde{X}_{in,k} = \left(\frac{1}{\gamma_k - 1} + \psi_k - \beta_k \right) \ln L_{i,k} + D_{i,k} + D_{n,k} + \phi_{ij,k},$$

where $D_{i,k}$ and $D_{n,k}$ are labor-size-adjusted exporter and importer fixed effects and $\phi_{in,k}$ is the bilateral resistance term. Suppose $D_{i,k} = \tilde{D}_i + \tilde{D}_k + \varepsilon^A_{i,k}$ and $\phi_{in,k} = \phi_{in} + \phi_{nk} + \varepsilon^B_{in,k}$, where ε^A and ε^B are mean-zero. Appealing to the above equation and noting that $\frac{1}{\gamma_k - 1} + \psi_k = \mu_k$, we can produce the following equation relating gross industry-level sales to employment size,

$$\frac{1}{1-\sigma_k} \ln \sum_n \tilde{X}_{in,k} = (\mu_k - \beta_k) \ln L_{i,k} + D_k + D_i + \varepsilon_{i,k}, \tag{116}$$

where $D_i \equiv \tilde{D}_i + \ln \sum_n (\exp(\phi_{in}))$ and $D_k \equiv \tilde{D}_k + \ln \sum_n [\exp(\phi_{nk})]$ are country and industry fixed effects, while the error term, $\varepsilon_{i,k}$, collects $\varepsilon^A_{i,k}$ (production cost shifters) and $\varepsilon^B_{in,k}$'s (trade cost shifters). Notice that $\varepsilon_{i,k}$ is akin to a supply shock here, but this interpretation rests on the implicit assumption that bilateral resistance terms have no demand-driven component. Importantly, the left-hand side variable in Equation 116 can be regarded a proxy for the price of country i's labor services in industry k. To elucidate this connection, note that $\sum_n \tilde{X}_{in,k} = P^{1-\sigma_k}_{ii,k} \sum_n \left[\tau_{ni,k} d_{ni,k} \left(\tilde{P}^{\sigma_k - 1}_{n,k}\right) Y_{n,k}\right]$ where $\tau_{ni,k}$ collects all tax instruments associated with triplet in, k. It then follows that $\frac{1}{1-\sigma_k} \ln \sum_n \tilde{X}_{in,k} \sim P_{ii,k} + \delta_{n,k}$, where $\delta_{n,k} \equiv \sum_n \left[\tau_{ni,k} d_{ni,k} \left(\tilde{P}^{\sigma_k - 1}_{n,k}\right) Y_{n,k}\right]$ can be broken down into components that are absorbed by D_k, D_i, and $\varepsilon_{i,k}$. Putting the pieces together, Equation 116 can be regarded as a supply function for country i's labor services, with $\varepsilon_{i,k}$ representing idiosyncratic supply shocks.

One can utilize macro-level sales and employment data to estimate the following combination of parameters based on Equation 116:

$$\frac{\partial \ln \sum_n \tilde{X}_{in,k}}{\partial \ln L_{i,k}} \sim (\mu_k - \beta_k)(\sigma_k - 1)$$

Identification in this case relies on plausibly exogenous demand-shifters that are orthogonal to $\varepsilon_{i,k}$—see Bartelme et al. (2019) for one such application. The product of the trade and scale elasticity can be, subsequently, recovered from $(\mu_k - \beta_k)(\sigma_k - 1)$ *insofar* as production involves no quasi-fixed inputs, i.e., $\beta_k = 0$. To isolate the scale elasticity (μ_k) from the trade elasticity ($\sigma_k - 1$), one must additionally rely on externally-estimated values for the trade elasticities.

Advantages and Disadvantages of Technique 2—This technique can, in principle, detect Marshallian externalities, but this advantage comes with limitations. Technique 2 cannot separately identify the scale elasticity, μ_k, from β_{ik}—which is crucial following Remark 1. This limitation can be especially problematic in industries like petroleum, mining, or heavy manufacturing that rely extensively on quasi-fixed inputs (Morrison (2012)). Another drawback of this technique is its inability to isolate the scale elasticity from the trade elasticity. So, to isolate the scale elasticity, one must rely on externally-estimated trade elasticity values or vice versa—which, following Remark 2, is far from ideal insofar as one seeks to use these elasticities for policy evaluation in open economies.

Technique 3: Production Function Estimation

This technique is an augmentation of the standard production function estimation technique. Suppose we posses firm-level data on real output, $q_{i,k}(\omega)$, and input quantities, $\mathbf{X}_{i,k}(\omega) = \{L_{i,k}(\omega), F_{i,k}(\omega), ...\}$. We can, then, estimate the following log-linear production function, which includes industry-level employment as an additional covariate to identify the Marshallian component of the scale elasticity, ψ_k:

$$\ln q_{i,k}(\omega) = \boldsymbol{\beta}_k \cdot \ln \mathbf{X}_{i,k}(\omega) + \psi_k \ln L_{i,k} + \varepsilon_{i,k}(\omega).$$

The residual term $\varepsilon_{i,k}(\omega)$, in this specification, encompasses idiosyncratic firm productivity shifters and measurement error. The above function can be all but impossible to estimate at scale given the scarcity of firm-level data on real input and output quantities. To bypass this challenge, existing applications of the production function technique often estimate an aggregate version of the above equation that regresses industry-wide output, $Q_{i,k} = \sum_{\omega} q_{i,k}(\omega)$, on input quantities, $\mathbf{X}_{i,k} = \sum_{\omega} \mathbf{X}_{i,k}(\omega)$—see e.g., Basu and Fernald (1997). The scale elasticity is then recovered as $\mu_k = \sum_f (\beta_{f,k}) - 1$, where f indexes production inputs. Under this approach, $Q_{i,k}$ and $\mathbf{X}_{i,k}$ are calculated by deflating nominal sales and cost data using price indexes calculated by statistical agencies. The identification challenges relating to production function estimation of this sort are well-documented in the literature, so we refer readers to Ackerberg, Caves, and Frazer (2015) for a comprehensive synthesis of these issues.

Advantages and Disadvantages of Technique 3—The production function technique can detect Marshallian externalities, similar to Technique 2. It is also robust to the presence of quasi-fixed inputs, like Technique 1. Despite these appealing properties, the production function technique exhibits crucial limitations given its reliance on externally-constructed price indexes. This approach can credibly identify the scale elasticity, μ_k, only if the price indexes constructed by statistical agencies have adequately accounted for product quality and love-for-variety—which is often not the case. Another disadvantage of this approach is that it relies on domestic production data, meaning that the same data cannot be used to identify the trade elasticity $(\sigma_k - 1)$. Instead, one must rely on completely different data to estimate $(\sigma_k - 1)$, which can compromise mutual consistency as emphasized by Remark 2.

Q.1 Demand-Based Estimation: *Krugman vs. Melitz*

As discussed earlier, the demand parameters, γ_k, fully determine the markups and scale elasticities in our baseline Krugman model (i.e., $\mu_k = \frac{1}{\gamma_k - 1}$). Section 2 also noted that the relationship between demand parameters and markups/scale elasticities is amended in richer environments. One such canonical case is the Melitz-Pareto model where firms incur a fixed overhead cost to serve individual markets. Following Appendix D, the markup and scale elasticities in this environment depend on the shape of the Pareto firm-level productivity distribution, ϑ_k, in addition to demand parameter, γ_k. In particular,

$$\text{[Melitz-Pareto Model]} \qquad \mu_k^{\text{RE}} = \frac{\gamma_k \vartheta_k}{(\gamma_k - 1)(\vartheta_k + 1) - \vartheta_k} - 1 \sim \text{markup} \qquad \mu_k^{\text{FE}} = \frac{1}{\vartheta_k} \sim \text{scale elasticity}$$

To provide some intuition, the adjusted markup, $\frac{\gamma_k \vartheta_k}{(\gamma_k - 1)(\vartheta_k + 1) - \vartheta_k}$, corresponds to the gross markup, $\frac{\gamma_k}{\gamma_k - 1}$, net of fixed cost payments. From a policy standpoint, the fraction of the markup paid to cover fixed costs is not a source of misallocation. In fact, if fixed cost payments consume the entire gross markup, the market equilibrium will be *constrained* efficient. We present a quantitative analysis of the Melitz-Pareto model in Appendix X, elaborating more on the model's implications. Below, we discuss other settings in which the markup and scale elasticity values depend on factors other than the demand parameter γ_k.

Q.2 Markups under Alternative Market Conducts

Our analysis thus far assumed that firms compete under monopolistic competition. Beyond this case, markups depend not only on demand parameters but also the conduct parameter. The markup associated with goods from origin i–industry k is, in particular, given by

$$\mu_{i,k} = v_{i,k} \times \frac{\gamma_k}{\gamma_k - 1},$$

where $v_{i,k}$ denotes the conduct parameter. Following Weyl and Fabinger (2013),(1) $v_{i,k} = 1$ under monopolistic or Bertrand competition, (2) $v_{i,k} = 0$ under perfect competition,[94] and (3) $v_{i,k} = 1/N_{i,k}$ under Cournot competition. In the spirit of Berry et al. (1995) and Berry (1994), our main analysis recovered markups from demand parameters by setting $v_{i,k} = 1$. We also discussed, in detail, how our optimal policy results change if we were to assume perfect or Bertrand competition instead.

Below, we discuss how our quantitative results may be be impacted by Cournot competition. The crucial takeaway from our baseline markup estimation was that trade elasticities and markups are negatively correlated across industries. This pattern may weaken or even reverse if the number of firms,$N_{i,k}$, is systematically correlated with γ_k. We investigate this possibility, using the World Bank's Exporter Dynamics Database (EDD) described in Fernandes, Klenow, Meleshchuk, and Pierola (2018). The publicly-available version of the EDD features data on firm-level exports provided by customs agencies from 60 countries for the 1997–2013 period. One of these datapoints is the number of exporters per origin and HS6 product code, from which we can infer $N_{i,k}$. Using this information, we update our baseline markup estimates as, $\mu_{i,k} = \frac{1}{N_{i,k}} \times \frac{\gamma_k}{\gamma_k - 1}$ to make them compatible with Cournot competition. We then regress the our estimated trade elasticity $(\sigma_k - 1)$ on the Cournot-compatible markup values to investigate wether the negative correlation is persevered.

The results reported in Table 8 indicate that the negative relationship between the trade elasticities and firm-level markups are robust to relaxing the monopolistic competition assumption with Cournot competition. The negative relationship becomes slightly weaker but remains significant and strong, nonetheless. Note once more, this structural relationship is the crucial driving force behind our quantitative findings that center around immiserizing growth. As detailed in Section 4, if $\text{Cov}_k\left(\sigma_k, \mu_{i,k}\right) < 0$, non-cooperative second-best trade policies are ineffective at correcting misallocation in domestic industries and cooperative domestic policies trigger immiserizing growth unless they are internationally coordinated.

Table 8: *The tension between ToT and sectoral misallocation under Cournot competition*

	dependent: trade elasticity $(\sigma_k - 1)$		
$\mu_{i,k} = \frac{1}{N_{i,k}} \times \frac{\gamma_k}{\gamma_k - 1}$	-0.138***	-0.138***	-0.279***
	(0.0111)	(0.0111)	(0.0157)
Year fixed effects	No	Yes	Yes
Origin fixed effects	No	No	Yes
Observations	3,221	3,221	3,221

Note: This paper relationship between the firm-level markup under Cournot competition, $\frac{1}{N_{i,k}} \times \frac{\gamma_k}{\gamma_k - 1}$, and the trade elasticity, $\sigma_k - 1$. Data for $N_{i,k}$ are from the World Bank's Exporter Dynamics Database. Parameters σ_k and γ_k are from the demand estimation conducted in Section 6, where k denotes a WIOD industry. *** denotes significant at the 1\% level.

Q.3 Scale Elasticities under Arbitrary Love-for-Variety

In our baseline Krugman model, there is a one-to-one link between the degree of firm-level market power and the love-for-variety in each industry. Baqaee and Farhi (2020a) demonstrate that this link

[94] Likewise, under Bertrand competition with homogeneous sub-products, $v_{i,k} = 0$.

has deep root, beyond CES models. This tight link, however, can be broken by introducing arbitrary love-for-variety into the CES demand aggregator à la Benassy (1996). In particular, suppose the sub-national CES aggregator in industry k is adjusted as follows:

$$Q_{ji,k} = \left(N_{j,k}^{\varsigma_k} \times \int_{\omega \in \Omega_{j,k}} \varphi_{ji,k}(\omega)^{\frac{1}{\gamma_k}} q_{ji,k}(\omega)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega \right)^{\frac{\gamma_k}{\gamma_k - 1}}.$$

Parameter ς_k regulates the love-for-variety as measured by the number of firm-level varieties, $N_{j,k}$. The above CES aggregator coincides with our baseline CES aggregator if $\varsigma_k = 0$. It is straightforward to check that firm-level markups are unaffected by ς_k, as individual firms treat $N_{j,k}$ as given when setting their prices. The scale elasticity, however, should be adjusted as follows:

$$\text{[Krugman+Benassy]} \qquad \mu_k = \frac{1}{\gamma_k - 1} \sim \text{markup} \qquad 1 + \psi_k = \left(1 + \frac{1}{\gamma_k - 1}\right)(1 + \varsigma_k) \sim \text{scale leasticity}$$

The optimal domestic subsidy in this case is $1 + s_k^\star = \left(1 + \frac{1}{\gamma_k - 1}\right)(1 + \varsigma_k)$, and the gains from restoring efficiency are, accordingly, amplified. Notice, markup heterogeneity is no longer necessary to justify policy intervention. The heterogeneity in ς_k is sufficient, which echos Epifani and Gancia's (2011) findings in a single-sector economy.

Estimating the love-for-variety parameter, ς_k, with sales and price data is, however, challenging. In our *firm-level* estimation, $N_{j,k}^{\varsigma_k}$ will appear as an origin-and-industry-specific demand shifter and will be absorbed by our extensive set of fixed effects. Estimating ς_k with *national-level* sales and price data faces the same complications as external economies of scale. In particular, one cannot purge elasticity, ς_k, from the diseconomies of scale elasticity without explicit data on quasi-fixed factors of production. As detailed in Section 6.5, the latter elasticity does not contribute to inefficiency and must be excluded from the optimal Pigouvian subsidy.

R Examining the Plausibility of Estimates

In this Appendix we examine the plausibility of our estimated parameters from a different angle. We show that when our estimated parameters are plugged into a workhorse trade model, they resolve the *income-size* elasticity puzzle. This puzzle, as noted by Ramondo et al. (2016), concerns the fact that a large class of quantitative trade models—including Krugman (1980), Eaton and Kortum (2001), and Melitz (2003)—predict a counterfactually high income-size elasticity (i.e., the elasticity at which real per capita income increases with population size). One straightforward remedy for this counterfactual prediction is introducing domestic trade frictions into the aforementioned models. This treatment, however, is only a partial remedy. As shown by Ramondo et al. (2016), even after controlling for direct measures of internal trade frictions, the predicted income-size elasticity remains counterfactually strong.

To test macro-level predictions, we first produce economically-representative estimates for σ_k and μ_k. We do so by pooling data across all manufacturing and non-manufacturing industries and estimating Equation 16 on theses two pooled samples.The estimation results are reported in Table 9, and imply that $\sigma \approx 3.8$ and $\frac{\sigma - 1}{\gamma - 1} \approx 0.66$ across manufacturing industries. For the sake of comparison, the same table also reports estimates produced using the standard OLS estimator.

To understand the income-size elasticity puzzle, consider a single-industry version of the model

Table 9: Pooled estimation results

	Manufacturing		Non-Manufacturing	
Variable (log)	2SLS	OLS	2SLS	OLS
Price, $1-\sigma$	-2.766***	0.202***	-5.540***	0.102***
	(0.186)	(0.003)	(0.706)	(0.006)
Within-national share, $1-\mu(\sigma-1)$	0.340***	0.816***	0.167***	0.804***
	(0.010)	(0.002)	(0.033)	(0.009)
Weak Identification Test	259.90	...	28.83	...
Under-Identification P-value	0.00	...	0.00	...
Within-R^2	...	0.78	...	0.73
N of Product-Year Groups	21,416		8,903	
Observations	1,130,742		204,828	

Notes: *** denotes significant at the 1% level. The Estimating Equation is (16). Standard errors in brackets are robust to clustering within product-year. The estimation is conducted with HS10 product-year fixed effects. The reported R^2 in the OLS specifications correspond to within-group goodness of fit. Weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The p-value of the under-identification test of instrumented variables is based on the Kleibergen-Paap LM test. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist et al. (1996).

presented in Section 2. Such a model implies the following expression relating country i's real income per worker or TFP ($W_i = w_i/P_i$) to its structural efficiency, A_i, population size, L_i, trade-to-GDP ratio, λ_{ii}, and a measure of internal trade frictions, τ_{ii}:

$$W_i = \gamma\, A_i\, L_i^{\mu}\, \lambda_{ii}^{-\frac{1}{\sigma-1}}\, \tau_{ii}^{-1}. \tag{117}$$

The standard Krugman model assumes extreme love-of-variety (or extreme scale economies), which implies $\mu = 1/(\sigma-1)$ and precludes internal trade frictions, which results in $\tau_{ii} = 1$. Given these two assumptions, we can compute the real income per worker predicted by the standard Krugman model and contrast it to actual data for a cross-section of countries.

For this exercise, we use data on the trade-to-GDP ratio, real GDP per worker, and population size for 116 countries from the PENN WORLD TABLES in the year 2011. Given our micro-estimated trade elasticity, $\sigma - 1$, and plugging $\tau_{ii} = 1$ as well as $\mu = 1/(\sigma-1)$ into Equation 117, we can compute the real income per worker predicted by the Krugman model. Figure 10 (top panel) reports these predicted values and contrasts them to factual values. Clearly, there is a sizable discrepancy between the income-size elasticity predicted by the standard Krugman model (0.36, standard error 0.03) and the factual elasticity (-0.04, standard error 0.06). To gain intuition, note that small countries import a higher share of their GDP (i.e., posses a lower λ_{ii}), which partially mitigates their size disadvantage. However, even after accounting for observable levels of trade openness, the scale economies underlying the Krugman model are so strong that they lead to a counterfactually high income-size elasticity.

One solution to the income-size elasticity puzzle is introducing internal trade frictions into the Krugman model (i.e., relaxing the $\tau_{ii} = 1$ assumption). Ramondo et al. (2016) perform this task using direct measures of domestic trade frictions. Their calibration is suggestive of $\tau_{ii} \propto L_i^{0.17}$. Plugging this implicit relationship into Equation 117 and using data on population size and trade openness, we compute the model-predicted real income per worker and contrast it with actual data in Figure 10 (middle panel). Expectedly, accounting for internal frictions shrinks the income-size elasticity. However, as pointed out by Ramondo et al. (2016), the income-size elasticity remains puzzlingly large.

We ask if our micro-estimated scale elasticity can help resolve the remaining income-size elasticity puzzle. To this end, in Equation 117, we set the scale elasticity to $\mu = \alpha/(\sigma-1)$ where α is set to 0.65 as implied by our micro-level estimation. Then, using data on population size and trade-to-GDP ratios, we compute the real income per capita predicted by a model that features both domestic trade frictions *and* adjusted scale economies. Figure 10 plots these predicted values, indicating that this adjustment indeed resolves the income-size elasticity puzzle. In particular, the income-size elasticity predicted by the Krugman model with adjusted scale economies is statistically insignificant (0.02, standard error 0.03), aligning very closely with the factual elasticity.

S Mapping Second-Best Tax Formulas to Data

In this appendix, we present an analog to Proposition 3, but for second-best trade taxes under restricted entry (as specified by Theorem 2). As in Section 7, we assume that preferences have a CES-Cobb-Douglas parametrization. We use the "$**$" superscript indicates that a variable is being evaluated in the counterfactual *second-best* optimal policy equilibrium. We assume hereafter that countries do not apply domestic subsidies in the factual equilibrium, i.e., $s_{n,k} = 0$ for all $n \in \mathbb{C}$. Using the hat-algebra notation and the expression of the good-specific supply elasticity, $\omega_{ji,k}$ (Equation 10), we can write the second-best tax formulas in changes as follows:

$$\begin{aligned}
&\text{[optimal import tax]} \quad 1+t^{**}_{ji,k} = \frac{1+(\sigma_k-1)\hat{\lambda}_{ii,k}\lambda_{ii,k}}{1+\frac{1+\overline{\mu}^{*}_{i}}{1+\mu_k}(\sigma_k-1)\hat{\lambda}_{ii,k}\lambda_{ii,k}}\left(1+\omega^{**}_{ji,k}\right)\\
&\text{[optimal export subsidy]} \quad 1+x^{**}_{ij,k} = \frac{(\sigma_k-1)\sum_{n\neq i}\left[(1+\omega^{**}_{ni,g})\hat{\lambda}_{nj,k}\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\hat{\lambda}_{ij,k}\lambda_{ij,k})}\left(\frac{1+\mu_k}{1+\overline{\mu}^{**}_{i}}\right),\\
&\text{[change in taxes]} \quad \widehat{1+s_{i,k}} = 1; \qquad \widehat{1+t_{ji,k}} = \frac{1+t^{**}_{ji,k}}{1+t_{ji,k}}; \qquad \widehat{1+x_{ij,k}} = \frac{1+x^{**}_{ij,k}}{1+x_{ij,k}}.
\end{aligned} \tag{118}$$

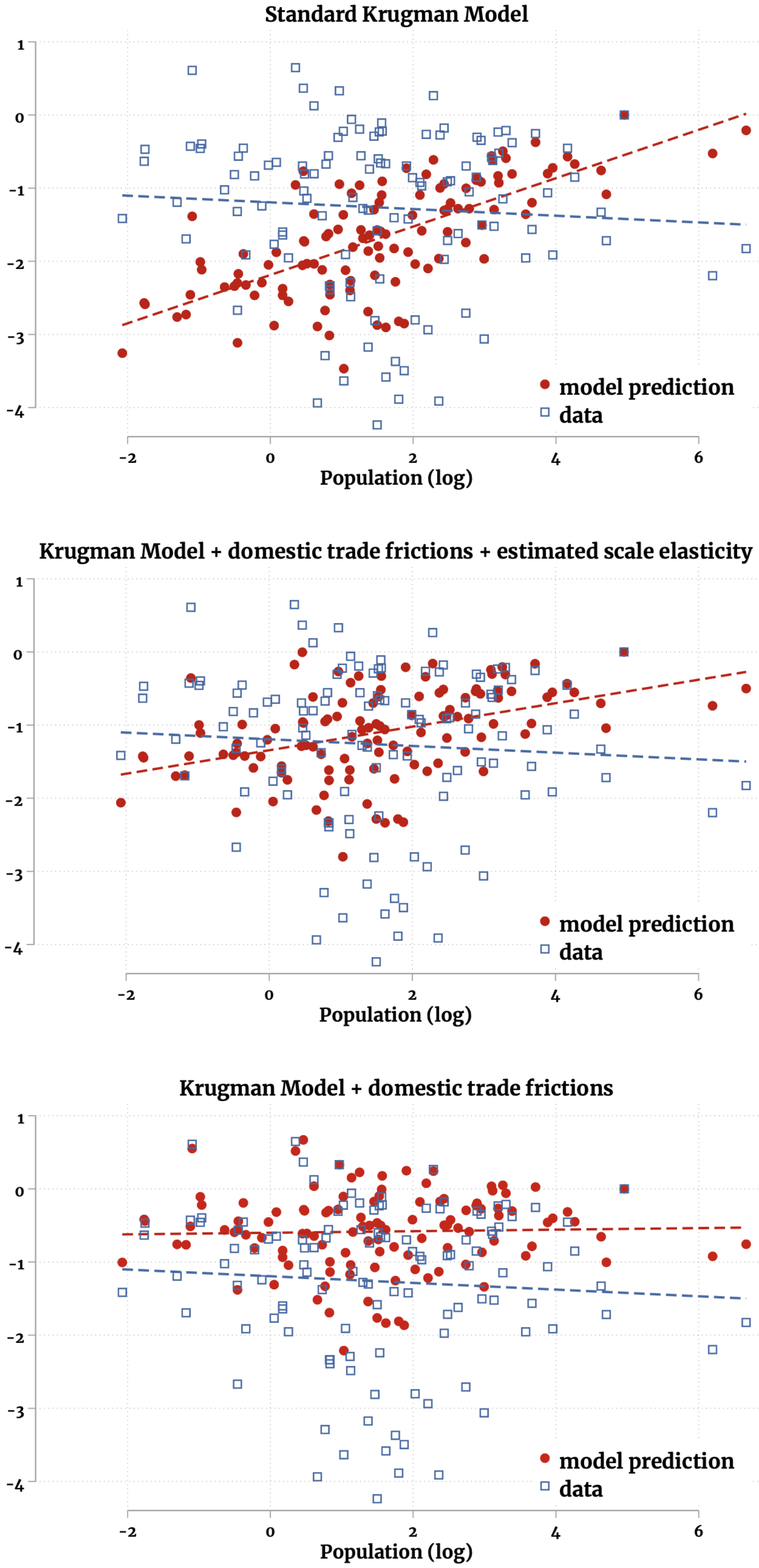
Since the rest of the world is passive in their use of taxes, $\widehat{1+s_{n,k}} = \widehat{1+t_{jn,k}} = \widehat{1+x_{nj,k}} = 1$ for all $n \neq i$. To determine the change in expenditure shares, $\hat{\lambda}_{ji,k}$, we need to determine the change in consumer price indexes. Invoking the CES structure of within-industry demand, we can express the change in *market i–industry k*'s consumer price index as

$$\text{[price indexes]} \quad \hat{\tilde{P}}_{i,k} = \sum_{n=1}^{N}\left(\lambda_{ni,k}\left[\frac{\widehat{1+t_{ni,k}}}{\widehat{1+x_{ni,k}}}\widehat{w}_n\right]^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}. \tag{119}$$

Given $\hat{\tilde{P}}_{i,k}$, we can calculate the change in expenditure shares $\hat{\lambda}_{ji,k}$ and revenue shares $\hat{r}_{ji,k}$ as

$$\begin{aligned}
&\text{[expenditure shares]} \quad \hat{\lambda}_{ji,k} = \left[\frac{\widehat{1+t_{ji,k}}}{\widehat{1+x_{ji,k}}}\widehat{w}_j\right]^{1-\sigma_k}\hat{\tilde{P}}^{\sigma_k-1}_{i,k}\\
&\text{[revenue shares]} \quad \hat{r}_{ji,k} = \left(\frac{\widehat{1+x_{ji,k}}}{\widehat{1+t_{ji,k}}}\hat{\lambda}_{ji,k}\hat{Y}_i\right)\left(\sum_{n=1}^{N}\frac{\widehat{1+x_{jn,k}}}{\widehat{1+t_{jn,k}}}\hat{\lambda}_{jn,k}\hat{Y}_n\right)^{-1}.
\end{aligned} \tag{120}$$

Figure 10: Resolving the income-size elasticity puzzle



The change in wage rates, $\hat{w}_i$, and labor shares, $\hat{\rho}_{i,k}$, are dictated by the labor market clearing (LMC) condition, which ensures that industry-level sales match wage payments:

$$[\text{LMC}] \qquad (1+\overline{\mu}_i^{**})\hat{w}_i w_i L_i = \sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1+x_{ji,k}^{**}}{1+t_{ji,k}^{**}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]. \tag{121}$$

where the output-weighted average markup in the counterfactual equilibrium is given by

$$1+\overline{\mu}_i^{**} = \frac{\sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1+x_{ji,k}^{**}}{1+t_{ji,k}^{**}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]}{\sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1+x_{ji,k}^{**}}{(1+\mu_k)(1+t_{ji,k}^{**})}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]}. \tag{122}$$

The change in national expenditure, $\hat{Y}_i$, is governed by the balanced budget (BB) condition, which ensures that total expenditure matches total income from wage payments and tax revenues:

$$[\text{BB}] \quad \hat{Y}_i Y_i = +(1+\overline{\mu}_i^{**})\hat{w}_i w_i L_i + \sum_{j\neq i}\sum_{k}\left(\frac{t_{ji,k}^{**}}{1+t_{ji,k}^{**}}\lambda_{ji,k}\hat{\lambda}_{ji,k}e_{i,k}\hat{Y}_i Y_i + \frac{1-(1+x_{ij,k}^{**})}{1+t_{ij,k}^{**}}\lambda_{ij,k}\hat{\lambda}_{ij,k}e_{j,k}\hat{Y}_j Y_j\right). \tag{123}$$

Equations 118-123 represent a system of $2N+NK+2(N-1)K$ independent equations and unknowns. The independent unknowns are, namely, $\hat{w}_i$ (N unknowns), $\hat{Y}_i$ (N unknowns), $\hat{\rho}_{i,k}$ (NK unknowns), $\widehat{1+t_{ji,k}}$ ($(N-1)K$ unknowns), and $\widehat{1+x_{ij,k}}$ ($(N-1)K$ unknowns). Solving the aforementioned system is possible with information on observable data points, $\mathbb{D}$, and estimable parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. Once we solve this system, the welfare consequences of country i's optimal policy are also fully determined. The following proposition outlines this result.

Proposition 4. *Suppose we have data on observable shares, national accounts, and applied taxes,* $\mathbb{D}$=$\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, Y_i, w_iL_i$ *and information on structural parameters,* $\Theta \equiv \{\mu_k, \sigma_k\}$. *We can determine the economic consequences of country i's second-best optimal policy by calculating* $\mathbb{X} = \left\{\hat{Y}_i, \hat{w}_i, \hat{\rho}_{i,k}, \widehat{1+t_{ji,k}}, \widehat{1+x_{ij,k}}\right\}$ *as the solution to the system of Equations 118-123. After solving for* $\mathbb{X}$, *we can fully determine the welfare consequence of country i's optimal policy as*

$$\hat{W}_n = \hat{Y}_n / \prod_{k\in\mathbb{K}} \hat{\tilde{P}}_{n,k}^{e_{n,k}}, \qquad (\forall n \in \mathbb{C})$$

where $\hat{\tilde{P}}_{n,k}$ *can be computed as function of* $\mathbb{X}$ *and data,* $\mathbb{D}$, *using Equation 119.*

T Additional Details about the World Input-Output Database

This appendix presents additional details about the World Input-Output Database analyzed in Section 7. Table 10 describes our aggregation of WIOD industries into 16 industries. To summarize the information in this table, we aggregate the 'Agriculture' and Mining' industries into one non-manufacturing industry. We also follow Costinot and Rodríguez-Clare (2014) in two details: First, we aggregate the 'Textile' and 'Leather' industries into one industry. Second, we lump all service-related industries together treating them as one semi-non-tradable sector.

Following Proposition 3 in Section 7, we need data on observable shares, national accounts, and applied taxes ($\mathbb{D} = \{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_iL_i, x_{ij,k}, t_{ji,k}, s_{i,k}\}_{j,i,k}$) to compute the gains from policy. The WIOD reports data on trade values, $X_{ji,k} \equiv P_{ji,k}Q_{ji,k}$, for each origin j–destination i–industry k. The aggregated version of the data covers $N = 33$ countries (including the rest of the world) and $K = 16$ industries. Below, we describe how each element in $\mathbb{D}$ is computed based on $X_{ji,k}$ and our estimated values for μ_k. Assuming that countries impose no taxes under the status-quo, we can compute national income and the wage bill in each country i as follows:

$$Y_i = \sum_{k=1}^{K}\sum_{n=1}^{N} X_{ni,k}; \qquad w_i L_i = \begin{cases} \sum_{k=1}^{K}\sum_{n=1}^{N} X_{in,k} & \text{if entry is free} \\ \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{1}{1+\mu_k} X_{in,k} & \text{if entry is restricted} \end{cases}$$

Next, we can compute the within-industry and industry-level expenditure shares for each market i based on the following calculations:

$$\lambda_{ji,k} = \frac{X_{ji,k}}{\sum_{n=1}^{N} X_{ni,k}}; \qquad e_{i,k} = \frac{\sum_{n=1}^{N} X_{ni,k}}{\sum_g \sum_{n=1}^{N} X_{ni,g}} = \frac{\sum_{n=1}^{N} X_{ni,k}}{Y_i}.$$

Lastly, we can compute the within-industry revenue share and the industry-level labor share in each country using the following equations:

$$r_{in,k} = \frac{X_{in,k}}{\sum_{n=1}^{N} X_{in,g}}; \qquad \rho_{i,k} = \frac{\sum_{n=1}^{N} X_{in,k}}{\sum_{g=1}^{K}\sum_{n=1}^{N} X_{in,g}}.$$

U Quantitative Analysis with Exact Formulas

Our optimization-free quantitative approach in Section 7 relied on approximate formulas for the export supply elasticity. The same analysis, however, can also be conducted with exact formulas. In this appendix, we demonstrate this point and show that both approaches deliver virtually identical output. Though, our suggested approximation saves computation time to a notable degree.

As a starting point, we appeal to our exact formula for the (general equilibrium) export supply elasticity,[95]

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \left[\frac{w_i L_i}{w_j L_j} \rho_{i,k} \left(\frac{\partial \ln P_{ii,k}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} + \sum_{n \neq i} \frac{w_n L_n}{w_j L_j} r_{ni,}\rho_{n,k} \left(\frac{\partial \ln P_{ni,k}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} \right].$$

As detailed in Appendix, $\left(\frac{\partial \ln P_{ii,k}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i}$ is a partial derivate holding constant the vector of wages, income, and "consumer" prices associated with economy i. The matrix consisting of these partial derivatives can be evaluated by inverting a system of equations as specified by Equation 59. Namely,

$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} & \cdots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i} \end{bmatrix} = - \underbrace{\begin{bmatrix} \frac{\partial F_{1i,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{1i,k}(.)}{\partial \ln P_{NN,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{Ni,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{Ni,k}(.)}{\partial \ln P_{NN,k}} \end{bmatrix}^{-1}}_{\mathbf{A}_i} \begin{bmatrix} \frac{\partial F_{1i,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{1i,k}(.)}{\partial \ln Q_{Ni,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{Ni,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{Ni,k}(.)}{\partial \ln Q_{Ni,k}} \end{bmatrix}.$$

where $F_{ni,k}(\mathbf{Q}_{i,k}, \mathbf{P}_k) \equiv P_{nn,k} - \bar{\varrho}_{nn,k} w_n \left[\tau_{ni,k} Q_{ni,k} + \sum_{\ell \neq i} \tau_{n\ell,k} Q_{n\ell,k}\right]^{-\frac{\mu_k}{1+\mu_k}} = 0$, with the corresponding derivatives specified in Appendix E.

With the above background, we now explain how the quantitative procedure explained in Section 7.1 can be re-done without appealing to approximation or numerical optimization. In summary, one must now solve the exact optimal tax formulas in conjunction with the equilibrium condition in

[95] Notice, the above expression for $\omega_{ji,k}$ precludes cross-industry effects, given our Cobb-Douglas utility parameterization across industries.

Table 10: List of industries in the World Input-Output Database

WIOD Sector	Sector's Description	Trade Ealsticity	Scale Ealsticity
1	Agriculture, Hunting, Forestry and Fishing	6.227	0.143
2	Mining and Quarrying	6.227	0.143
3	Food, Beverages and Tobacco	2.303	0.393
4	Textiles and Textile Products Leather and Footwear	3.359	0.224
5	Wood and Products of Wood and Cork	3.896	0.229
6	Pulp, Paper, Paper , Printing and Publishing	2.646	0.320
7	Coke, Refined Petroleum and Nuclear Fuel	0.397	1.758
8	Chemicals and Chemical Products	3.966	0.232
9	Rubber and Plastics	5.157	0.140
10	Other Non-Metallic Mineral	5.283	0.167
11	Basic Metals and Fabricated Metal	3.004	0.209
12	Machinery, Nec	7.750	0.120
13	Electrical and Optical Equipment	1.235	0.552
14	Transport Equipment	2.805	0.129
15	Manufacturing, Nec; Recycling	6.169	0.152
16	Electricity, Gas and Water Supply Construction Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods Hotels and Restaurants Inland Transport Water Transport Air Transport Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies Post and Telecommunications Financial Intermediation Real Estate Activities Renting of M&Eq and Other Business Activities Education Health and Social Work Public Admin and Defence; Compulsory Social Security Other Community, Social and Personal Services Private Households with Employed Persons	11	0

changes. The exact optimal tax/subsidy formulas can be expressed as

$$
\begin{aligned}
&\text{[optimal import tax]} \quad 1+t^*_{ji,k} = 1 + \frac{1}{\hat{r}_{ji,k}\hat{\rho}_{j,k} r_{ji,k}\rho_{j,k}} \left[\frac{\hat{w}_i w_i L_i}{\hat{w}_j w_j L_j} \hat{\rho}_{i,k}\rho_{i,k}\varkappa^*_{ij,k} + \sum_{n\neq i} \frac{\hat{w}_n w_n L_n}{\hat{w}_j w_j L_j} \hat{r}_{ni,k}\hat{\rho}_{n,k} r_{ni,k}\rho_{n,k}\varkappa^*_{nj,k} \right] \\
&\text{[optimal export subsidy]} \quad 1+x^*_{ij,k} = \frac{(\sigma_k - 1)\sum_{n\neq i}\left[(1+t^*_{ni,k})\hat{\lambda}_{nj,k}\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\hat{\lambda}_{ij,k}\lambda_{ij,k})}, \\
&\widehat{1+s_{i,k}} = \frac{1+\mu_k}{1+s_{i,k}}; \qquad \widehat{1+t_{ji,k}} = \frac{1+t^*_{ji,k}}{1+t_{ji,k}}; \qquad \widehat{1+x_{ij,k}} = \frac{1+x^*_{ij,k}}{1+x_{ij,k}}. \qquad \text{(A)}
\end{aligned}
$$

The variable $\varkappa^*_{nj,k} \equiv \left(\frac{\partial \ln P_{nn,k}}{\partial \ln Q_{ji,k}}\right)^*_{\mathbf{w},\mathbf{Y},\tilde{\mathbb{P}}_i}$ refers to the partial price derivatives evaluated in the counterfactual optimal policy equilibrium. The entire matrix of $\varkappa^*_{nj,k}$'s can be recovered with information on structural parameters and the change to observable share variables. Namely,

$$
\begin{bmatrix} \varkappa^*_{11,k} & \cdots & \varkappa^*_{1N,k} \\ \vdots & \ddots & \vdots \\ \varkappa^*_{N1,k} & \cdots & \varkappa^*_{NN,k} \end{bmatrix} = -\left(\mathbf{I}_N - \begin{bmatrix} a_{11,k} & \cdots & a_{1N,k} \\ \vdots & \ddots & \vdots \\ a_{N1,k} & \cdots & a_{NN,k} \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{\mu_k}{1+\mu_k} r_{1i,k}\hat{r}_{1i,k} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\mu_k}{1+\mu_k} r_{Ni,k}\hat{r}_{Ni,k} \end{bmatrix}, \qquad \text{(B)}
$$

where the elements of the first matrix on the right-hand side are

$$
a_{nj,k} = \mathbb{1}_{j\neq i}\frac{\mu_k}{1+\mu_k}\sum_{\ell\neq i}\left[\left(\mathbb{1}_{n=j}\sigma_k - (\sigma_k-1)\,\lambda_{j\ell,k}\hat{\lambda}_{j\ell,k}\right) r_{n\ell,k}\hat{r}_{n\ell,k}\right].
$$

Solving Equations (A) and (B) alongside the equilibrium conditions specified by Equations 18-21 in the main text determines the entire vector of counterfactual outcomes after the imposition of optimal taxes/subsidies. The following proposition summarizes this point.

Proposition 5. *Suppose we have data on observables,* $\mathbb{D} = \left\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\right\}_{j,i,k}$*, and information on structural parameters,* $\Theta \equiv \{\mu_k, \sigma_k\}$*. We can determine the economic consequences of country i's optimal policy by calculating* $\mathbb{X} = \left\{\hat{Y}_i, \hat{w}_i, \hat{\rho}_{i,k}, \widehat{1+s_{i,k}}, \widehat{1+t_{ji,k}}, \widehat{1+x_{ij,k}}\right\}$ *as the solution to the system of Equations consisting of (A) and (B) plus equilibrium conditions 18-21. After solving for* $\mathbb{X}$*, we can fully determine the welfare consequence of country i's optimal policy as*

$$
\hat{W}_i = \hat{Y}_i / \prod_{k\in\mathbb{K}} \hat{\tilde{P}}^{e_{i,k}}_{i,k}, \qquad (\forall n \in \mathbb{C})
$$

where $\hat{\tilde{P}}_{i,k}$ *is determined by Equation 18 as a function of* $\mathbb{X}$ *and data,* $\mathbb{D}$*.*

Using Proposition 5, we recalculate the exact gains from optimal policy and compare them with the baseline gains implied by our approximate formulas. The results are displayed in Table 11 for select countries. These are relatively large countries for which our approximation is more suspect. One immediately notices that our approximate formulas deliver identical numbers to the exact formulas. The intuition, as explained in Appendix E, is that the matrix $\mathbf{A}_k = \left[a_{nj,k}\right]$ is sufficiently sparse. To put these results in perspective, Table 11 also reports the gains implied by the small open economy optimal policy formulas. These formulas are presented in Section 3. The small open economy assumption is markedly more error-prone, as it attributes "zero" import market power to each country irrespective of market size and import composition.

Table 11: *Gains from policy: exact vs. approximate optimal tax formulas*

Country	Exact Formula ΔW	Approximated Formula ΔW	Approximated Formula Error	Small Open Economy Formula ΔW	Small Open Economy Formula Error
BEL	1.3088%	1.3088%	0.00%	1.3007%	0.62%
DEU	1.7117%	1.7113%	0.02%	1.6885%	1.37%
NLD	1.3547%	1.3547%	0.00%	1.3450%	0.72%
NOR	1.1889%	1.1889%	0.00%	1.1757%	1.12%
USA	1.5283%	1.5278%	0.03%	1.5178%	0.69%

Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). Policy outcomes in the small open economy case are calculated using the optimal policy specification under 12.

V Elucidating the Tension between ToT and Misallocation

This appendix shows that the inefficacy of 2nd-best *non-cooperative* trade taxes stems from the tension between terms-of-trade (ToT) and misallocation. Our quantitative analysis, recall, indicated that 2nd-best trade taxes can replicate less than 40% of the gains from the 1st-best policy choice, which combines trade taxes with Pigouvian subsidies. In what follows, we argue that this apparent lack of efficacy is not a universal feature that merely reflects the targeting principle. Instead, it is an empirical result based on our estimated trade and scale elasticity values.

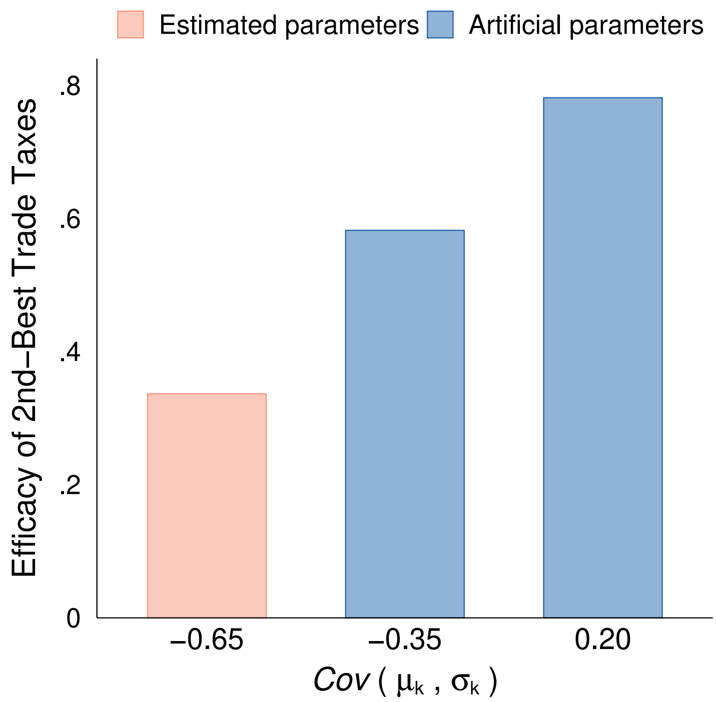
To establish this point, we artificially raise $Cov(\sigma_k, \mu_k)$ and recompute the gains from 2nd-best trade taxes. We then calculate the efficacy of 2nd-best trade taxes as the ratio of the corresponding gains relative to the 1st-best policy choice. Each iteration maintains the estimated vector of trade elasticities and adjusts the scale elasticities (or firm-level markups) to artificially inflate $Cov(\sigma_k, \mu_k)$ relative to its estimated value. Throughout this appendix, we report results for the case of *restricted entry*, noting that similar results hold under free entry.

The results reported in Figure 11 confirm that 2nd-best trade taxes become increasingly more effective as $Cov(\sigma_k, \mu_k)$ is artificially inflated. Under our estimated parameters, $Cov(\sigma_k, \mu_k) \approx -0.60$ and 2nd-best trade taxes can replicate less than 40% of the 1st-best gains from policy. When $Cov(\sigma_k, \mu_k)$ is artificially raised to -0.35, 2nd-best trade taxes can replicate close to 60% of the gains from 1st-best gains from policy. When $Cov(\sigma_k, \mu_k)$ is raised further to 0.30, the efficacy of 2nd-best trade taxes improves to 80%. These results indicate that the inefficacy of 2nd-best trade taxes is not an exclusive reflection of the targeting principle. While one expects a less-than-100% efficacy based on the targeting principle, 2nd-best trade taxes become a remarkably weaker substitute for Pigouvian subsidies under lower values of $Cov(\sigma_k, \mu_k)$.

We repeat the same exercise to elucidate the immiserizing growth effects of unilateral policy markup correction. In particular, we artificially raise $Cov(\sigma_k, \mu_k)$ and recompute the consequences of unilateral markup correction. Each iteration maintains the estimated vector of firm-level markups (or scale elasticities) and adjusts the trade elasticities to artificially inflate $Cov(\sigma_k, \mu_k)$ relative to its estimated value. This choice ensures that the degree of inter-industry misallocation remains approximately the same despite the change in $Cov(\sigma_k, \mu_k)$.

The results reported in Figure 12 indicate that immiserizing growth effects fade and even reverse as $Cov(\sigma_k, \mu_k)$ is artificially inflated. Under our estimated parameters, where $Cov(\sigma_k, \mu_k) \approx -0.60$, unilateral markup correction prompts immiserizing growth and lowers welfare. When $Cov(\sigma_k, \mu_k)$

Figure 11: *2nd-best trade taxes become more effective as $Cov\,(\mu_k, \sigma_k)$ is artificially inflated*



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). Each bar reports the average welfare gains when countries implement their 1st-best policy without retaliation by partners. The artificial parameters are constructed by fixing σ to its estimated values and adjusting $\mu$to artificially inflate $Cov\,(\mu_k, \sigma_k)$.

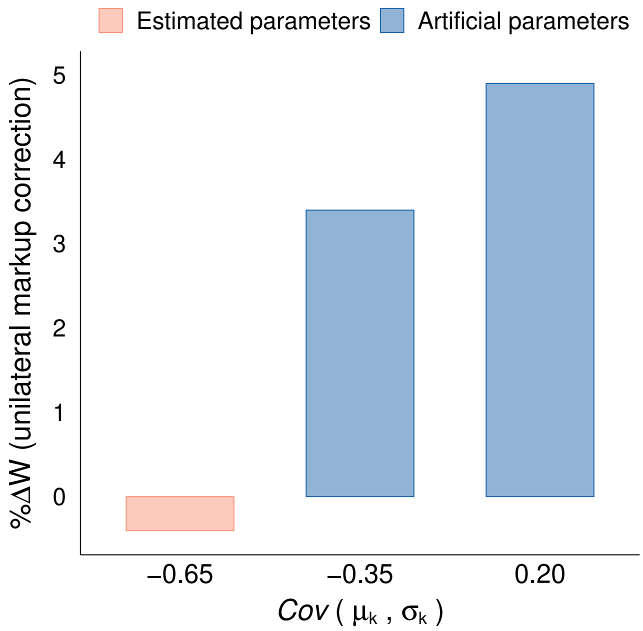
is artificially raised to -0.35, unilateral markup correction no longer triggers immiserizing growth. When $Cov\,(\sigma_k, \mu_k)$ is raised further to 0.30, unilateral markup correction becomes a promising policy choice as it restores allocative efficiency and improves the ToT at the same time.

W Country-Level Exposure to Immiserizing Growth

This appendix digs deeper into the immiserizing growth effects of unilateral industrial policy. Recall from section sec: Tension that immiserizing growth presents a grave challenge to industrial implementation in open economies. In Section 4, we reported the extent of immiserizing growth for the average country. Here we unpack these numbers. First, by reporting immiserizing growth effects on a country-by-country basis. Second, by highlighting that trade-to-GDP is a crucial determinant of the extent to which countries experience immiserizing growth.

Figure 13 displays welfare consequences when countries implement corrective policies without reciprocity by trading partners. The results in Figure 13 highlight two rudimentary points: First, while most countries experience a deterioration of welfare, a few do not. But even for those few countries dampened gains from correcting misallocation than if they were operating as closed economies. Second, trade-to-GDP (measured as the value of imports divided by GDP) is strongly associated with the intensity at which countries experience immiserizing growth. Figure 13, moreover, reveals that countries not experiencing immiserizing growth tend to trade relatively more with each other. Hence, even if these countries adopt corrective industrial policies, it does not spare others from immiserizing growth—hence the importance of multilateral coordination of corrective policies via deep agreements.

Figure 12: *Immiserizing growth effects diminish as $Cov\left(\mu_k, \sigma_k\right)$ is artificially inflated*



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). Each bar reports the average welfare change when countries undertake unilateral markup correction without reciprocity by partners. Thea artificial parameters are constructed by fixing $\boldsymbol{\mu}$ to its estimated values and adjusting $\boldsymbol{\sigma}$to artificially inflate $Cov\left(\mu_k, \sigma_k\right)$.

X Gains from Policy Under Alternative Assumptions

In this appendix we quantify the gains from optimal policy under three alternative scenarios, comparing them to the baseline gains reported in Section 7. In each case, we contrast the new policy gains with the baseline gains along the two dimensions: First, in terms of the gains from first-best trade and industrial policies. Second, in terms of the effectiveness of second-best trade taxes at replicating the first-best outcome.

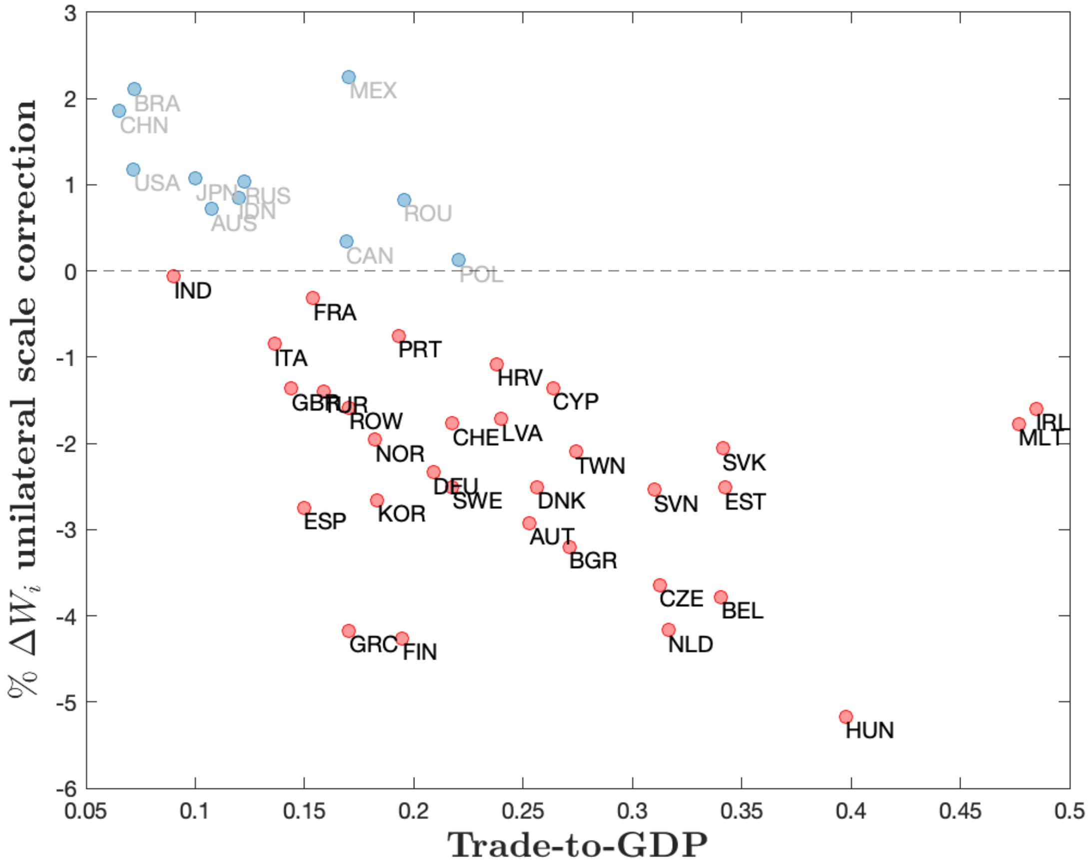
X.1 Gains Implied by the Melitz-Pareto Model

Suppose the data generating process is consistent with a Melitz-Pareto model that accommodates firm-selection effects. In that case, Theorem 1 characterizes the optimal policy under the following reinterpretation of parameters—see Appendix D:

$$1+\mu_k^{\text{Melitz}} = \begin{cases} 1+\frac{1}{\vartheta_k} & \text{if entry is free} \\ \frac{\gamma_k \vartheta_k}{(\gamma_k-1)(\vartheta_k+1)-\vartheta_k} & \text{if entry is restricted} \end{cases}; \qquad \sigma_k^{\text{Melitz}} = 1 + \frac{\vartheta_k}{1+\vartheta_k\left(\frac{1}{\sigma_k-1}-\frac{1}{\gamma_k-1}\right)}.$$

To compute the gains from policy we, therefore, need estimates for σ_k, γ_k, and ϑ_k. We have already produced estimates for the former two parameters. To estimate ϑ_k, we can first recover σ_k^{Melitz} using a standard gravity estimation à la Caliendo and Parro (2015). To explain the estimation procedure, suppose tariffs are applied before markups and industrial and export subsidies are zero ($x_{ji,k} = s_{j,k} = 0$ for all i, j, k). In that case, the national-level import demand function transforms into the following

Figure 13: Higher $\frac{Trade}{GDP}$ is associated with stronger immiserizing growth from unilateral corrective policies



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). *The y-axis* corresponds to welfare gains when a country undertakes unilateral markup scale without reciprocity by partners.

industry-level gravity equation:[96]

$$\tilde{X}_{ji,k} \equiv \tilde{P}_{ji,k}Q_{ji,k} = \Phi_{j,k}\Omega_{i,k}\tau_{ji,k}^{1-\sigma_k^{\text{Melitz}}}(1+t_{ji,k})^{1-\sigma_k^{\text{Melitz}}},$$

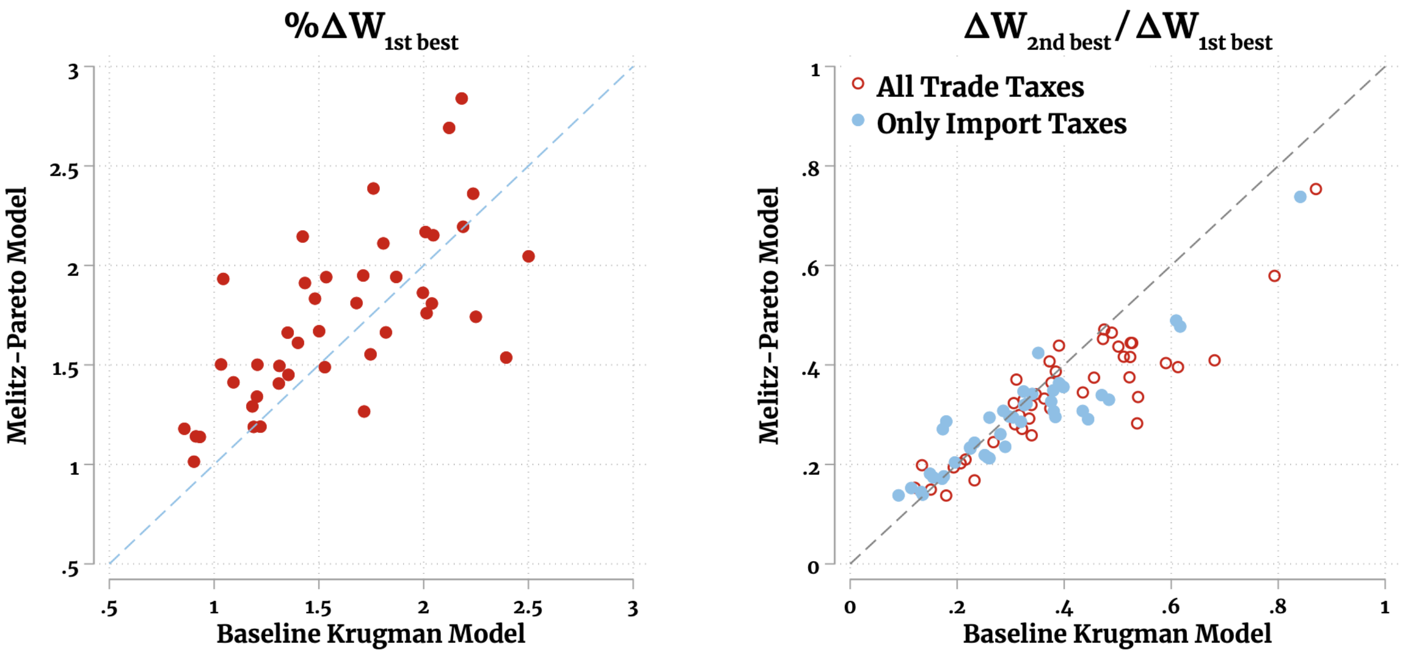
where $\Phi_{j,k} \equiv L_{j,k}^{\mu_k^{\text{Melitz}}}\bar{a}_{j,k}^{1-\sigma_k^{\text{Melitz}}}w_{j,k}^{1-\sigma_k^{\text{Melitz}}}$ and $\Omega_{i,k} \equiv \sum_n \left[\bar{a}_{n,k}w_{n,k}^{1-\sigma_k^{\text{Melitz}}}\tau_{ni,k}^{1-\sigma_k^{\text{Melitz}}}(1+t_{ni,k})^{1-\sigma_k^{\text{Melitz}}}\right] e_{i,k}Y_{i,k}$ can be viewed as the exporter and importer fixed effects in the standard gravity estimation sense. To produce our final estimating equation, we assume that iceberg trade costs are given by $\ln \tau_{ji,k} = \ln d_{ji,k} + \varepsilon_{ji,k}$, where (i) $d_{ji,k} = d_{ij,k}$ is a systematic and symmetric cost component that accounts for the effect of distance, common language, and common border, while (ii) $\varepsilon_{ji,k}$ is a random disturbance term that represents any deviation from symmetry. Invoking this decomposition, we can produce the following estimating equation for any triplet (j,i,n):

$$\ln \frac{\tilde{X}_{ji,k}\tilde{X}_{in,k}\tilde{X}_{nj,k}}{\tilde{X}_{ij,k}\tilde{X}_{ni,k}\tilde{X}_{jn,k}} = -\left(\sigma_k^{\text{Melitz}} - 1\right)\ln \frac{(1+t_{ji,k})(1+t_{in,k})(1+t_{nj,k})}{(1+t_{ij,k})(1+t_{ni,k})(1+t_{jn,k})} + \varepsilon_{jin,k}.$$

The left-hand side variable, in the above equation, is composed of observable national-level trade values in industry k. The right-hand side variable is composed of observable industry-level tariff rates. The error term $\varepsilon_{jin,k} \equiv \theta_k(\varepsilon_{ij,k} - \varepsilon_{ji,k} + \varepsilon_{in,k} - \varepsilon_{ni,k} + \varepsilon_{nj,k} - \varepsilon_{jn,k})$ encompasses any idiosyncratic variation in non-tariff barriers. Under the identifying assumption that applied tariff rates are orthogonal to

[96] The assumption that tariffs are applied before markups, amounts to saying that tariffs act as a cost-shifter. Alternatively, if tariffs are applied after markups, they act as a demand shifter. In the latter case, the elasticity of trade with respect to tariffs diverges from the trade elasticity in its standard definition—see Costinot and Rodríguez-Clare (2014) for more details.

Figure 14: The gains from policy under the Melitz-Pareto model



$\varepsilon_{jin,k}$, *i.e.*, $\mathbb{E}\left[t_{ji,k}\,\varepsilon_{ji,k}\right]=0$, we can estimate σ_k^{Melitz} by estimation the above equation with data on trade values, $\tilde{X}_{ji,k}$, and applied tariffs, $t_{ji,k}$, from the WIOD and TRAINS-UNCTAD datasets. After estimating σ_k^{Melitz}, we can recover ϑ_k for our previously-estimated values for σ_k and μ_k (which are reported in Table 3):

$$\vartheta_k=\frac{\hat{\sigma}_k^{\text{Melitz}}-1}{1+\left(\hat{\sigma}_k^{\text{Melitz}}-1\right)\left(\frac{1}{\gamma_k-1}-\frac{1}{\sigma_k-1}\right)}.$$

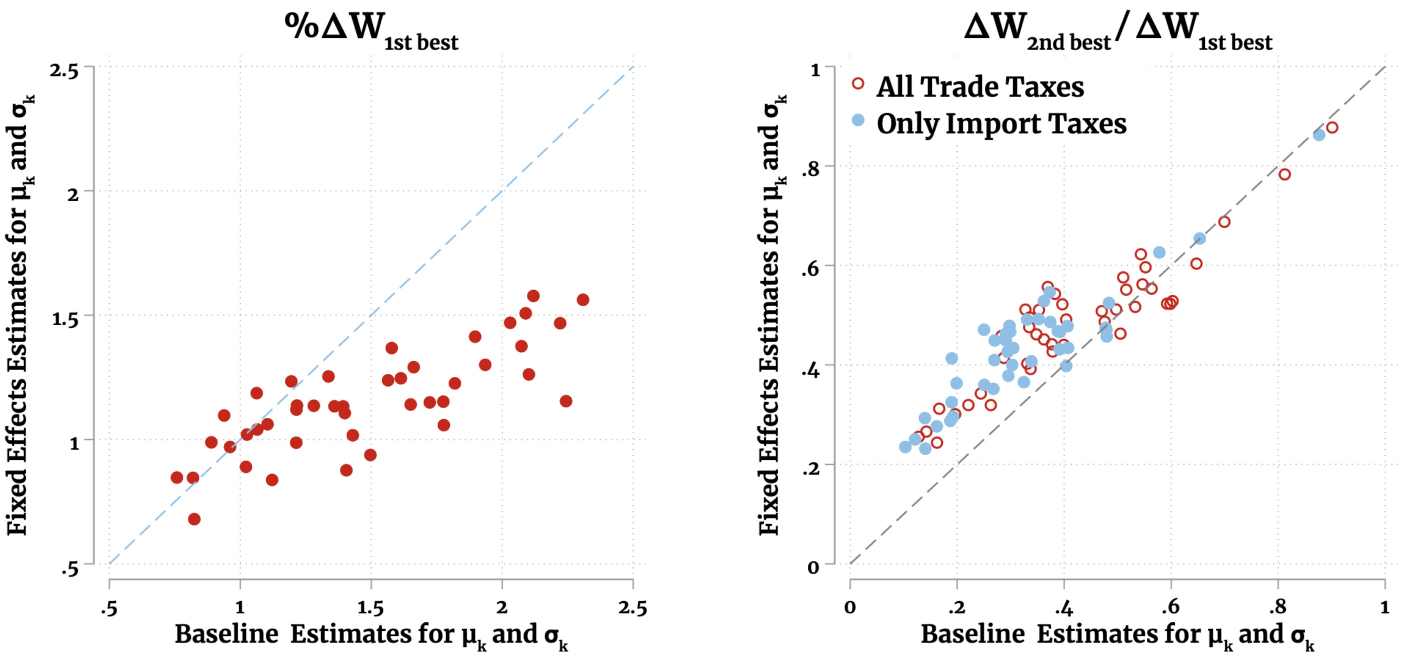
For the analysis that follows, we borrow the estimated values for σ_k^{Melitz} from Lashkaripour (2020a), which is based on the 2014 WIOD and TRAINS-UNCTAD datasets. After pinning down all the necessary parameters, we simply evaluate and plug σ_k^{Melitz} and μ_k^{Melitz} into our optimal tax formulas to compute the gains from optimal policy. The process is akin to that outlined in Section 7. Importantly, one should note that without our micro-level estimates for σ_k and μ_k, it is impossible to recover both σ_k^{Melitz} and μ_k^{Melitz} from macro-level trade and tariff data.

The optimal policy gains implied by the Melitz-Pareto model are reported under Figure 14. The results indicate that accounting for firm-selection (à la Melitz-Pareto) magnifies the gains from the first-best trade and industrial policy schedule. Moreover, accounting for firm-selection dampens the efficacy of second-best trade taxes at replicating the first-best policy gains. If anything, these results indicate that our baseline claim that trade taxes are an ineffective second-best substitute for industrial subsidies is strengthened once we account for firm-selection effects.

X.2 Gains Implied by the Fixed-Effect Estimates for μ_k and σ_k

Our baseline estimation of the gains from policy in Section 7 utilized the first-difference estimates for μ_k and σ_k—these estimates were reported under Table 3. In Appendix P (under Table 7), we reported alternative estimates for μ_k and σ_k based on a two-ways fixed-effects estimation of the firm-level import demand function. In this appendix, we recompute the gains from policy using these

Figure 15: The gains from policy under alternative estimates for σ_k and μ_k



alternative estimates for μ_k and σ_k.

The implied gains from optimal policy are reported under Figure 15. The fixed-effects estimates for σ_k and μ_k imply (on average) smaller gains from first-best trade and industrial policies. This outcome drives from two main factors: First, the fixed-effects estimates for μ_k exhibit smaller heterogeneity across industries. As such, they imply a small degree of misallocation in the economy compared to the baseline estimates. Second, the fixed-effects estimates for σ_k are generally smaller and imply larger unilateral gains from terms-of-trade manipulation.

Another takeaway from Figure 15 is that second-best trade taxes exhibit a greater degree of efficacy compared to the baseline case. This outcome reflects two issues: First, the corrective gains from policy are a smaller fraction of the overall first-best policy gains, once we plug the fixed-effects-estimated values for σ_k and μ_k. Second, the fixed-effects-estimated values for σ_k and μ_k exhibit a smaller negative correlation relative to the baseline estimates. As explained in Section 5, the less negative $\text{Cov}_k(\sigma_k, \mu_k)$, the smaller the implicit tensions between the terms-of-trade-improving and corrective gains from trade taxation—hence, the greater efficacy of second-best trade taxes.

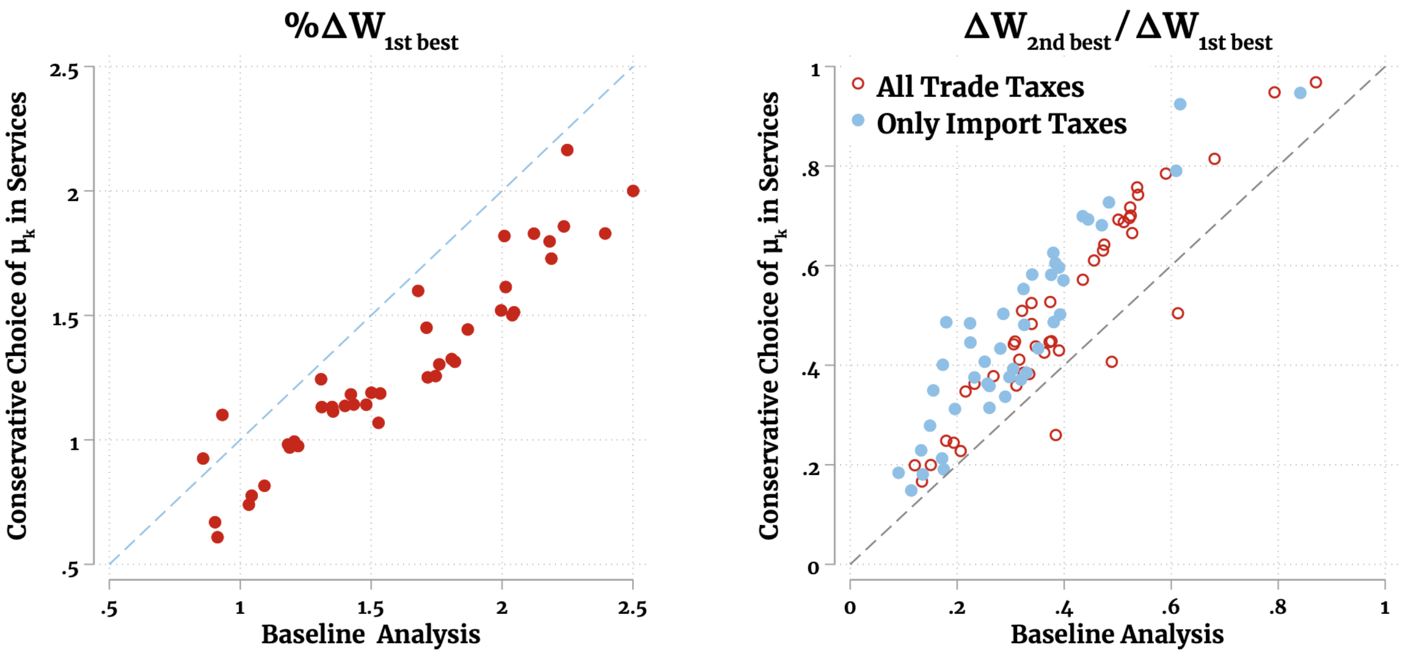
X.3 Assigning Alternative Values to μ_k and σ_k for the Service Sector

Our estimation of σ_k and μ_k in Section 6 relied on transaction-level trade data, which is scarce for (semi-non-traded) service industries. To address this issue, our baseline estimation of the gains from policy normalized the aforementioned parameters in service-related industries as follows:

$$\sigma_k = 11; \qquad \mu_k = 0 \qquad \text{if } k \in \text{Service}$$

The value assigned to σ_k for service-related industries is less consequential for our estimated welfare gains. The reason is that σ_k governs the gains from terms-of-trade manipulation. However, under the status quo, there is little-to-no trade occurring in service industries. With little-to-no service trade under the status quo equilibrium, the scope for terms-of-trade manipulation is limited in service

Figure 16: The gains from policy when the service sector is modeled more conservatively



industries—all irrespective of the value assigned to σ_k.[97]

The value assigned to μ_k, however, can have a profound effect on the estimated gains from optimal policy. To elaborate on this point, recall that one function of optimal policy (in our framework) is to correct misallocation due to markup heterogeneity. The degree of misallocation can be crudely measured as the cross-industry variance in markups, i.e., $\text{Var}_k(\mu_k)$. Data indicate that the service sector constitutes a non-trivial fraction of total output in each country. So, the value assigned to the service sector's μ_k is a non-trivial determinant of misallocation, as measured by $\text{Var}_k(\mu_k)$.

As indicated above, our baseline analysis assumed that the service sector is perfectly competitive. This assumption, which is rather standard in the quantitative trade literature, amounts to setting $\mu_k = 0$ for any service-related industry, k. In this appendix, we contrast our baseline results with those obtained under the alternative but extremely conservative assumption that μ_k in services equals the average μ_k in traded (non-service) industries. This assumption is conservative because it artificially deflates $\text{Var}_k(\mu_k)$ and, accordingly, dampens the corrective gains from optimal policy.

The gains computed under our conservative treatment of the service sector are reported under Figure 16. As expected, the gains from first-best policies are relatively lower under the conservative treatment—simply because the conservative value assigned to the service sector markup artificially lowers the degree of misallocation and the scope for policy intervention. Relatedly, second-best trade

[97] This outcome is an artifact of the CES parametrization of import demand. Specifically, in response to a change, $\hat{\boldsymbol{\tau}}$, in trade taxes, the post-tax-change expenditure shares remain zero if start as zero in the initial equilibrium—all irrespective of the trade elasticity values. Stated in mathematical terms,

$$\lim_{\lambda_{ji,k}\to 0} \hat{\lambda}_{ji,k} = \frac{\lambda_{ji,k}\left(\hat{\tau}_{ji,k}\hat{w}_j\right)^{1-\sigma_k}}{\sum_n \lambda_{ni,k}\left(\hat{\tau}_{ni,k}\hat{w}_n\right)^{1-\sigma_k}} = 0 \qquad \forall \sigma_k \geq 1.$$

Since $\lambda_{ji,k} \approx 0$ in services, trade taxes have little-to-no ability at improving the terms-of-trade, as doing so requires policy to shrink exports/imports in the service sector away from their factual level.

taxes are also more successful at replicating the gains obtainable under the first-best policy schedule. The intuition is that the *corrective* gains from policy constitute a smaller fraction of the first-best policy gains under the conservative model. Hence, the inability of trade taxes to replicate corrective gains becomes less consequential.

Y The Gains from Policy Under Artificial Parameter Values

Under what parameter values will our framework predict larger gains from policy? To answer this question, we simulate an artificial economy (with artificial values assigned to σ_k and μ_k) to examine the degree to which the gains from policy inflate under more extreme parameter values. Our theory indicates that the gains from optimal policy are regulate by two key statistics:

i. The variance in the industry-level scale elasticities, $\text{Var}\left[\log \mu_k\right]$.

ii. The average level of the (inverse) industry-level trade elasticities, $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$.

The first statistic governs the extent to which countries can gain from restoring allocative efficiency. To explain this statistic, we can appeal to the Hsieh and Klenow (2009) exact formula for distance from the efficient frontier. Considering that preferences are Cobb-Douglas across industries, the distance from the efficient frontier in each country (net of trade effects) can be approximated to a first-order as

$$\text{Distance from efficient frontier} \approx \frac{1}{2}\text{Var}\left[\log \mu_k\right].$$

The average level of μ_k is, however, inconsequential. To convey this point, suppose we multiply all the markups by some number $a \in \mathbb{R}_+$. Since this change is akin to offering a uniform industrial subsidy a to all industries, then it preserves real welfare based on Lemma 1.

The second statistic determines the degree of national-level market power and, thus, governs the degree to which countries can gain from ToT manipulation. To explain this statistic succinctly, consider a country that is sufficiently small in relation to the rest of the world. Following Theorem 1, the average optimal trade tax for this country is given by

$$\text{Avg. optimal trade tax} \ \approx \ \mathbb{E}\left[\frac{1}{\sigma_k - 1}\right].$$

If $\sigma_k \to \infty$ for all k, the average optimal trade tax approaches zero, leaving no room for unilateral ToT improvements. Conversely, as σ_k approaches 1 the average optimal trade tax increases and so do the implicit gains from unilateral trade restrictions.[98]

Noting the above background, we recompute the gains from policy by artificially increasing $\text{Var}\left[\log \mu_k\right]$ and decreasing $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$, starting from our estimated vectors of $\{\sigma_k\}$ and $\{\mu_k\}$. The results are reported in 17 for a select set of countries—namely, the United States, China, Indonesia, and Korea. The graph indicates that the gains from policy nearly double if we artificially raise $\text{Var}\left[\log \mu_k\right]$ by a factor of two. A similar effect is borne out if we artificially raise $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$ by a factor of about two. An apparent pattern, here, is that the gains from policy exhibit similar sensitivity levels to $\text{Var}\left[\log \mu_k\right]$ across all countries, but different sensitivity levels to $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$. This pattern is expected, because $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$ governs the gains from ToT-improvement which are smaller (by design) for larger economies like the United States or China. The gains for restoring allocative efficiency, however, depend less on size and

[98] Since there is no choke price in our setup, the optimal export tax can approach infinity in the limit where $\sigma_k \to 1$. Introduce a choke price, then the optimal export tax will exhibit a limit-pricing formulation—see Costinot et al. (2015).

more on a country's industrial pattern of specialization under the status quo—see Kucheryavyy et al. (2016) for the role of specialization patterns.

These findings provide a platform to compare our estimated gains with alternatives in the literature. Our finding that the gains from restoring allocative efficiency are large sits well with the findings in Baqaee and Farhi (2017) that eliminating sectoral markup-heterogeneity in the U.S. economy can raise real GDP by 2.3%.[99] Bartelme et al. (2019), however, estimate smaller gains from similar policies. To understand these differences, note the formula for distance from the efficient frontier. Also note that *true* value for the scale elasticity, $\mu_k^{\text{True}} = \mu_k + \psi_k$, where ψ_k denotes the elasticity of Marshallian externalities. Accordingly, the *true* distance from the frontier can be approximated as follows:

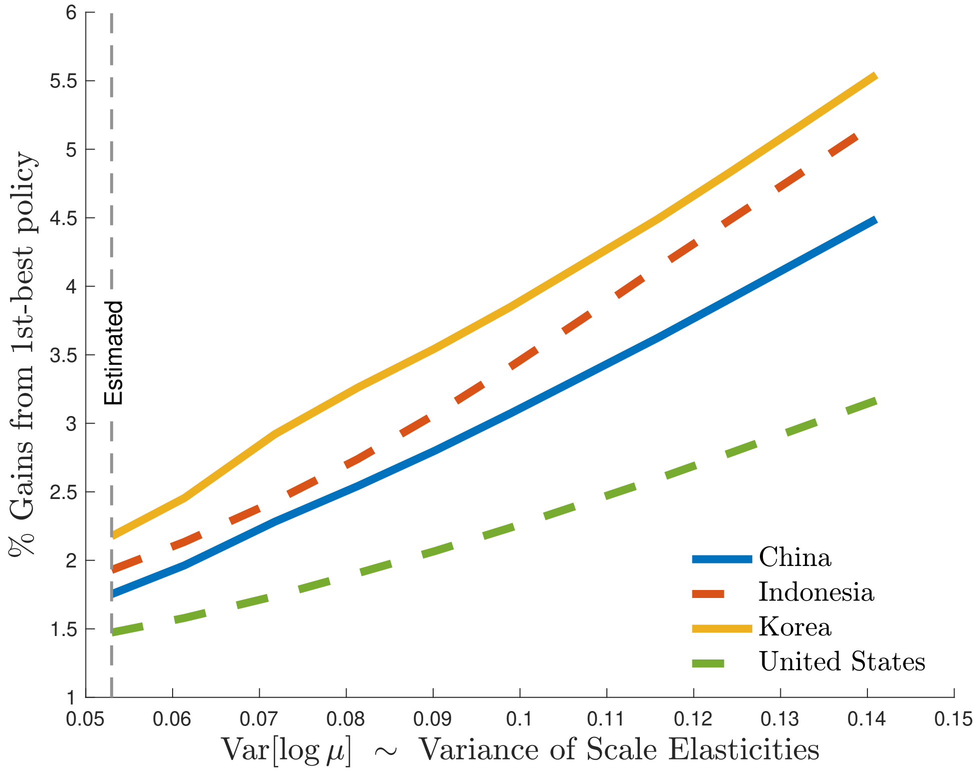
$$\mathcal{L}_{\text{True}} \approx \frac{1}{2}\text{Var}\left[\log\left(\psi_k + \mu_k\right)\right]$$

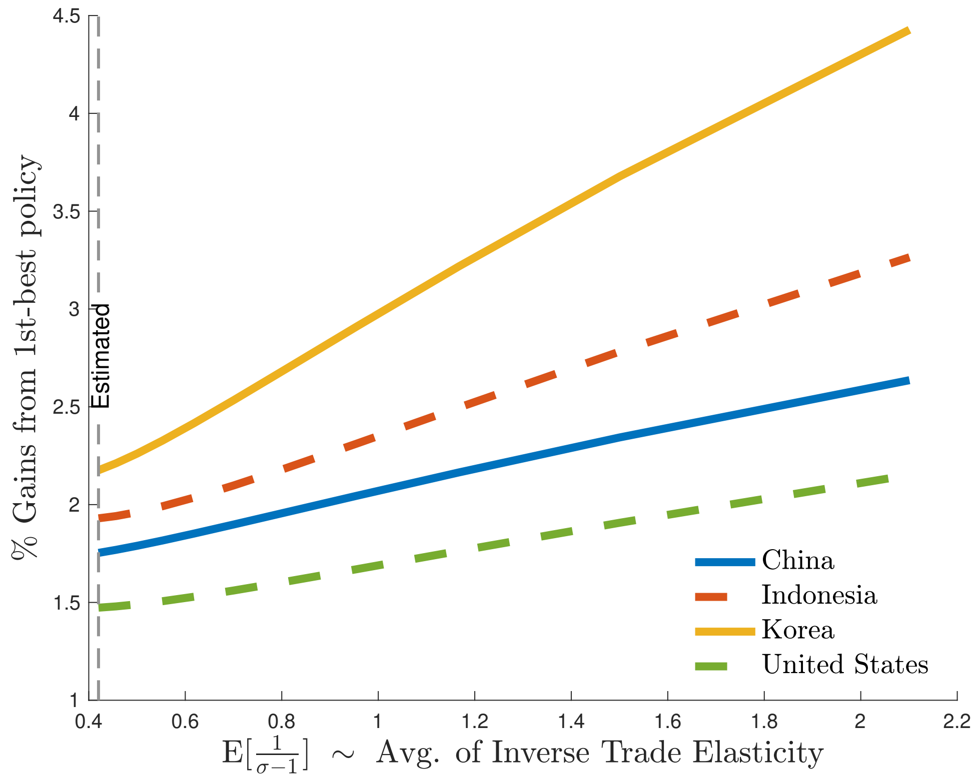
Our analysis like Baqaee and Farhi (2017) sets $\psi_k = 0$, and measures the degree of allocative inefficiency as $\mathcal{L}_{LL} \approx \frac{1}{2}\text{Var}\left[\log\left(\psi_k + \mu_k\right)\right]$. This approach can lead to an overstatement of $\mathcal{L}$ if ψ_k is negatively correlated with firm-level market power, μ_k.[100] In comparison and as noted in Section 6.4, the degree of misallocation in BDCR's analysis is measured as $\mathcal{L}_{BCDR} \approx \frac{1}{2}\text{Var}\left[\log\left(\mu_k + \psi_k - \frac{\beta_k}{\sigma - 1}\right)\right]$, where β_k is the share of industry-specific factors in production. This approach can understate $\mathcal{L}$ when there are significant diseconomies of scale due to a high β_k.

[99] This number corresponds to the average of the numbers reported in the last column of Table 2 in Baqaee and Farhi (2017).

[100] Another issue is that we are assuming away selection effects in our quantitative analysis. In the presence of selection effects, we can still use our estimates for σ_k and μ_k to identify the scale elasticity up-to an externally chosen trade elasticity. Doing so, however, may lead to a lower or higher $\mathcal{L}$.

Figure 17: The gains from policy under artificially higher Var $[\log \mu_k]$ *and* $\mathbb{E}\left[\frac{1}{\sigma_k - 1}\right]$





Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The *1st best* policy is characterized by Theorem 1 for the case of restricted entry.