Profits, Scale Economies, and the Gains from Trade and Industrial Policy*

Ahmad Lashkaripour
Indiana University

Volodymyr Lugovskyy
Indiana University

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Abstract

Trade restrictions are often used as (a) a first-best policy to manipulate the terms-of-trade, or (b) a second-best policy to correct misallocation in domestic industries. We analyze the (in)effectiveness of trade restrictions at achieving these goals. To this end, we derive sufficient statistics formulas for *first-best* and *second-best* trade taxes in an important class of multi-industry, multi-country trade models where misallocation occurs due to scale economies or profits. Guided by these formulas, we estimate the key parameters that govern the gains from policy in these frameworks. Our estimates reveal that (i) the gains from terms-of-trade manipulation are relatively small; (ii) trade policy is remarkably ineffective at correcting misallocation in domestic industries; (iii) a unilateral adoption of domestic industrial policies is also ineffective as it causes *immiserizing growth*; but (iv) industrial policies that are coordinated via deep trade agreements, deliver welfare gains that exceed those of any non-cooperative policy alternative.

1 Introduction

The United States is likely to adopt an explicit industrial policy in the coming decade. Similar developments are well underway in other countries (Aiginger and Rodrik (2020)). With industrial policy back on the scene, we are witnessing a revival of old-but-questionable trade policy practices: governments are turning to protectionist trade policy measures to pursue their industrial policy objectives. This approach is distinctly manifested in the United States' *National Trade Council*'s mission or in the Chinese, *Made in China 2025*, initiative.[1]

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[1]See Bhagwati (1988) and Irwin (2017) for a historical account of trade restrictions being used by governments to promote their preferred industries. One prominent example dates back to 1791, when Alexander Hamilton approached Congress with *"the Report on the Subject of Manufactures,"* which encouraged the implementation of protective tariffs and industrial subsidies. These policies were intended to help the US economy catch up with Britain's economy.

These developments have resurfaced some old but unresolved policy questions: (i) Is trade policy an effective tool for correcting misallocation in domestic industries? (ii) If not, should governments correct misallocation, *unilaterally*, with industrial subsidies, or (iii) should they coordinate their industrial policies via deep trade agreements?

Despite recent advances in quantitative trade theory, the existing literature offers little guidance on these imminent questions. Traditional theories that speak to these questions are typically confined to partial equilibrium, two-good×two-country models that overlook key policy considerations. The quantitative route has proven equally-elusive, as we lack credible estimates for the parameters that govern the gains from trade and industrial policy.[2]

To fill this gap, we present a full analytical characterization of optimal trade/industrial policy in an important class of multi-industry, multi-country quantitative trade models where misallocation occurs due to scale economies or profits. Guided by our theory, we estimate the key parameters that govern the gains from trade and industrial policy in these frameworks. We then plug these estimated parameters into our optimal policy formulas to quantify the gains from trade and industrial policy across 43 major economies.

Our estimation delivers several stark predictions: (i) trade taxes/subsidies are an ineffective second-best policy at correcting misallocation in the domestic economy; (ii) unilateral industrial policy can be equally futile, as it triggers *immiserizing growth* in most countries; but (iii) industrial policies that are coordinated via deep trade agreements, deliver welfare gains that exceed those of any non-cooperative policy alternative.

Section 2 presents our theoretical framework. Our baseline model is a generalized multi-industry Krugman (1980) model that features a non-parametric utility aggregator across industries and a nested CES utility aggregator within industries. This structure has an appealing property wherein the degrees of firm-level and country-level market power are allowed to diverge. We analyze both the restricted and free entry cases of the model to distinguish between the ***short-run*** and ***long-run*** consequences of policy. With a basic reinterpretation of parameters, our framework also nests (a) a multi-industry Melitz (2003) model with a Pareto productivity distribution, and (b) a multi-industry Eaton and Kortum (2002) model with industry-level Marshallian externalities.

Section 3 derives sufficient statistics formulas for *first-best* and *second-best* trade and industrial taxes/subsidies. To summarize these formulas, note that a non-cooperative government can use policy to correct two margins of inefficiency in our framework:

i. *Unexploited terms-of-trade (ToT) gains* vis-à-vis the rest of the world.

ii. *Allocative inefficiency*, which stems from sub-optimal domestic production in high-returns-to-scale (or high-profit) industries.[3]

[2] As we elaborate shortly, the gains from optimal policy are governed by (1) industry-level trade elasticities, and (2) industry-level scale elasticities. A vibrant literature is already devoted to the estimation trade elasticities. Scale elasticities, however, have received less attention. More importantly, we lack a technique to estimate both the trade and scale elasticities in a mutually consistent way. Our theory indicates that mutually consistent estimates are crucial for obtaining credible estimates for the gains from policy.

[3] The link between the degree of scale economies and firm-level market power has deep roots in the literature. Under free entry and monopolistic competition the following relationship holds (Hanoch (1975) and Helpman (1984)): *elasticity of firm-level output w.r.t. input cost = firm-level markup.*

Under the *first-best* optimal policy schedule, trade taxes/subsidies solely target the ToT margin—i.e., they are *misallocation-blind*. A mix of export subsides and import taxes are applied to extract the *national-level* optimal mark-up on exports and the national-level optimal mark-down on imports. First-best industrial subsidies, meanwhile, only correct allocative inefficiency. They are Pigouvian subsidies that equal the inverse of the industry-level scale elasticity (or markup) and restore allocative efficiency in the domestic economy.

Our *second-best* trade policy formulas concern scenarios in which governments are reluctant to use industrial subsidies. This reluctance, which is prevalent in practice, can be driven by either political pressures or institutional barriers.[4] Second-best trade taxes/subsidies are composed of two parts: a standard *ToT-improving* component and a *misallocation-correcting* component. The latter component restricts imports and subsidizes exports in high-returns-to-scale industries in an attempt to mimic first-best industrial subsidies.

Second-best trade policies are subject to a crucial but previously-overlooked trade-off: Restoring allocative efficiency with trade policy often worsens one's ToT. This trade-off limits the effectiveness of trade policy as a second-best misallocation-correcting measure—beyond what is already implied by the *targeting principle*. Similar arguments apply to *third-best* import taxes, which are relevant when the use of both export and domestic subsidies is restricted.[5]

The flip side of this trade-off is that a *unilateral* adoption of Pigouvian industrial subsidies can worsen welfare via *immiserizing growth*; because unilateral Pigouvian subsidies (that are not paired with trade taxes) can severely worsen a country's ToT. To avoid these adverse ToT effects, countries should coordinate their industrial subsidies via a deep trade agreement. In this process, they abolish trade taxes and forgo ToT gains from policy, but benefit from corrective industrial subsidies in the rest of the world. If the extent of misallocation is sufficiently large, such an agreement will deliver welfare gains that dominate those of any non-cooperative policy—even if non-cooperation does not trigger retaliation by trading partners.

Section 6 estimates the structural parameters that govern the gains from policy in our framework. Our optimal policy formulas indicate that the gains from policy depend on (i) industry-level *trade elasticities* that govern the scope for ToT manipulation, and (ii) industry-level *scale elasticities* that govern the extent of misallocation. We develop a new methodology that *simultaneously* estimates these parameters using transaction-level trade data.

Our estimation involves fitting a structural *firm-level* import demand function to the universe of Colombian import transactions from 2007 to 2013. Our data covers 226,288 exporting firms from 251 different countries. The main advantage of our approach is its unique ability to *separately* identify the degree of firm-level market power (that determines the scale elasticity) from the degree of national-level market power (that determines the trade elasticity).

The firm-level nature of our empirical strategy exposes us to an uncharted identification challenge. Standard estimations of import demand are often conducted with country-level data and use tariffs as an exogenous instrument to recover the underlying demand parame-

[4] Trade policy has been regularly used—in place of industrial policy—to promote critical industries (Bhagwati (1988); Harrison and Rodríguez-Clare (2010); Irwin (2017)). Relatedly, see Lane (2020) for a historical account of various industrial policy practices around the world.

[5] Second-best import tariff have served as a focal point in the trade policy literature. We show that second-best import tariffs feature an additional uniform component that mimics a uniform export tax. Based on the Lerner symmetry, this uniform term is redundant when export tax-cum-subsidies are applicable.

ters. This identification strategy is not fully-applicable to our firm-level estimation. To attain identification, we instead compile a comprehensive database on monthly exchange rates. We interact aggregate movements in monthly bilateral exchange rates with (lagged) monthly firm-level export sales to construct a *shift-share* instrument that measures exposure to exchange rate shocks at the *firm-product-year* level.

Section 7 combines our micro-level estimates, our optimal policy formulas, and macro-level data from the 2014 World Input-Output Database to quantify the gains from policy across 43 major economies. Our main findings can be summarized as follows:

i. The pure gains from ToT manipulation are relatively small. Suppose a country implements its first-best (non-cooperative) trade and industrial policy schedule and the rest of the world does not retaliate. In that case, the average country can raise its real GDP by 1.5% under restricted entry and by 2.6% under free entry. However, only one-third of these gains are driven by ToT improvements. The remaining is driven by restoring allocative efficiency in the domestic economy.

ii. Trade policy is remarkably ineffective at correcting misallocation in domestic industries. Under restricted entry, second-best export subsidies and import taxes can raise the real GDP by only 0.59% for the average country. This amounts to only 39% of the gains attainable under the fist-best policy schedule. Third-best import taxes are even less effective as a standalone policy, raising the real GDP by a mere 0.46%. These numbers conform to our previous assertion that restoring allocative efficiency with trade policy worsens the ToT—making trade policy a futile misallocation-correcting policy choice.

iii. Unilateral industrial subsides are equally ineffective at tackling misallocation, as they triggers immiserizing growth in most countries. To provide number, unilateral (Pigouvian) industrial subsides reduce the average country's real GDP by 0.25% under restricted entry and by 0.70% under free entry.[6]

iv. A multilateral adoption of corrective policies via a *deep* trade agreement, delivers welfare gains that are larger than the first-best non-cooperative policy for most countries. This is true even if non-cooperation does not trigger retaliation by trading partners. To provide numbers, the gains from deep cooperation are on average 0.2% percentage points higher than the gains from the first-best non-cooperative policy. In other words, governments are better off forgoing the ToT gains from policy in return for importing more-efficiently-produced goods from the rest of the world.

Related Literature

The literature on *optimal* policy in multi-industry, multi-country quantitative trade models with scale economies or markup distortions is surprisingly thin. In a concurrent paper, Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2019) characterize optimal policy for a small open economy in a multi-sector Ricardian model with Marshallian externalities.[7] By

[6]These losses result from the ToT-worsening effects of Pigouvian subsides. Finding (*iii*) highlights a benefit of international coordination that is often overlooked in standard critiques of global governance (e.g., Rodrik (2019)).

[7]Relatedly, Haaland and Venables (2016) characterize optimal policy for a small open economy in 2-sector×2-

comparison, our optimal policy formulas extend to a large economy, speak to both short- and long-run trade-offs, and accommodate arbitrary cross-demand effects. We also present a novel characterization of optimal policy under political economy pressures and input-output linkages. These results complement Costinot, Rodríguez-Clare, and Werning (2016), who characterize optimal *firm-level* trade policy in a single-sector two-country Melitz model. Our theory is also related to Campolmi, Fadinger, and Forlati (2018) who study trade policy in a two-sector Melitz-Pareto model with two symmetric countries. The aforementioned paper does not explicitly derive optimal policy formulas, but presents a novel welfare decomposition that elucidates the trade-offs facing countries when joining shallow and deep trade agreements.[8]

Our *second-best* trade policy formulas speak to an old literature that distinguishes between the ToT and firm-delocation rationales for trade policy (e.g., Venables (1987); Ossa (2011)). Our work enriches the existing theories in two ways: First, our theory applies to an important class of quantitative multi-country, multi-industry trade models.[9] This feature makes it amenable to state of the art quantitative analysis. Second, we look beyond only import tariffs, and characterize *second-best* export subsidies that have received limited attention in the past.[10]

Our theory builds heavily on Kucheryavyy, Lyn, and Rodríguez-Clare (2016) to establish isomorphism between our baseline model and other workhorse models in the literature. Kucheryavyy et al. (2016) identify the trade and scale elasticities as key determinants of the gains from trade. We estimate these two elasticities and demonstrate that the same set of elasticities govern the optimal design of trade and industrial policies.

Our estimation of the scale and trade elasticities exhibits key differences from the prior literature. Above all, our approach separately identifies the degree of firm-level market power (the scale elasticity) from country-level market power (the trade elasticity). The prior literature, in comparison, has devoted most of its attention to estimating the trade elasticity, with the scale elasticity often normalized to either *zero* or the inverse of the trade elasticity.[11] A notable exception is Bartelme et al. (2019), who concurrent with us have developed a strategy to estimate the *product* of scale and trade elasticities. Their approach has the advantage of detecting industry-level Marshallian externalities but relies on the assumption that there are

country Krugman and Melitz-Pareto models. A novel feature of Haaland and Venables (2016) is that labor can exhibit imperfect sectoral mobility—a feature that mimics diseconomies of scale at the sector level.

[8]Outside multi-industry models, Demidova and Rodriguez-Clare (2009) and Felbermayr, Jung, and Larch (2013) characterize optimal tariffs in a *single industry* Melitz-Pareto model. The single industry assumption ensures that the market equilibrium is efficient (Dhingra and Morrow (2019) and that import and export taxes are equivalent by the Lerner symmetry. As such, the first-best allocation can be reached with only import taxes.

[9]As indicated earlier, we extend our first-best optimal policy formulas to a setup with arbitrary input-output networks. Recently, Lashkaripour (2020b) and Caliendo, Feenstra, Romalis, and Taylor (2021) have extended the *small open economy* case of our second-best optimal tariff formulas to accommodate input-output linkages. These extensions suggest that input-output linkages dampen the optimal second-best tariffs.

[10]Our paper also relates to a growing literature that analyzes trade policy from the lens of quantitative or new trade theories—e.g., Costinot and Rodríguez-Clare (2014); Campolmi, Fadinger, and Forlati (2014); Costinot, Donaldson, Vogel, and Werning (2015); Bagwell and Lee (2018); Caliendo, Feenstra, Romalis, and Taylor (2015); Demidova (2017); Beshkar and Lashkaripour (2019, 2020). These papers focus on a wide range of policy issues that are strictly different from those studied in this paper. As such, they *either* (a) abstract from scale economies (or markup distortions) and do not speak to second-best scenarios, (b) are partial equilibrium in nature, *or* (c) characterize the marginal welfare effects of piecemeal tariff reforms rather than the optimal tariff rate.

[11]See Broda and Weinstein (2006), Simonovska and Waugh (2014), Caliendo and Parro (2015), Soderbery (2015), and Feenstra, Luck, Obstfeld, and Russ (2018) for different approaches to estimating the trade elasticity. Benassy (1996) was one of the first scholars to criticize the normalization of the scale elasticity, noting that such a normalization creates an arbitrary link between firm-level and country-level market power.

no diseconomies of scale due to industry-specific factors of production. Our approach cannot detect Marshallian externalities but has the advantage of separately identifying the trade elasticity from the scale elasticity and is robust to the presence of industry-specific factors of production.

Several studies have used the exact hat-algebra methodology to study the consequences of counterfactual tariff reductions (Costinot and Rodríguez-Clare (2014); Caliendo and Parro (2015); Ossa (2014, 2016); Spearot (2016)). We contribute to these studies by combining the exact hat-algebra technique with sufficient statistics tax formulas. Doing so simplifies the analysis of *optimal* policy, allowing us to bypass some of the most important computational challenges facing the past literature. Our quantitative analysis of deep trade agreements, meanwhile, has less precedent in the literature. Our closest counterpart is Ossa (2014), who quantifies the gains from an agreement whereby countries cooperate in setting second-best corrective import tariffs. We contribute to Ossa (2014) by analyzing cooperation in first-best corrective policies. Doing so enables us to highlight a previously-neglected tension between the corrective and ToT gains from taxation.

Finally, our paper is related to a vibrant literature that studies the effects of trade openness on misallocation (e.g., De Blas and Russ (2015); Edmond, Midrigan, and Xu (2015); Baqaee and Farhi (2019)). These studies indicate that tariff liberalization can occasionally exacerbate misallocation. We contribute to this literature by highlighting a previously-unknown tension between optimal tariff restrictions and misallocation in domestic industries.

2 Theoretical Framework

Our baseline model is a generalized multi-industry, multi-country Krugman model with non-parametric preferences. In Section 5 we show that our theory readily applies to alternative models featuring firm-selection à la Melitz–Chaney and external economies of scale à la Kucheryavyy et al. (2016). We also extend our theory later to accommodate arbitrary input-output networks and political economy pressures.

We consider a world economy consisting of multiple countries and industries. Countries are indexed by of $i, j, n \in \mathbb{C}$. Industries are indexed by $g, k \in \mathbb{K}$. Industries can differ in fundamentals such as the degree of scale economies or trade elasticity. Each country $i \in \mathbb{C}$ is populated by L_i individuals who supply one unit of labor inelasticity. Labor is the sole primary factor of production in each economy. Workers cannot relocate between countries but are perfectly mobile across industries within a country, and are paid a country-wide wage, w_i.

2.1 Preferences

Each good in our model is indexed by a triplet, which signifies its location of production (origin), it location of final consumption (destination), and the industry under which the good is classified. To give an example: Good "ji, k" denotes a good corresponding to *origin country j–destination country i–industry k*.

Cross-Industry Demand. The representative consumer in country $i \in \mathbb{C}$ faces a vector of industry-level consumer price indexes $\tilde{\mathbf{P}}_i = \{\tilde{P}_{i,k}\}$, where index $\tilde{P}_{i,k} \equiv \tilde{P}_{i,k}(\tilde{P}_{1i,k}, ..., \tilde{P}_{Ni,k})$ aggregates over industry k goods sourced from various origins. The consumer choses their demand for industry-level bundles $\mathbf{Q}_i \equiv \{Q_{i,k}\}$ to maximize a non-parametric utility function subject to a budget constraint. This choice yields an indirect utility, which is a function of the consumer's income, Y_i, and the vector of industry-level *"consumer"* price indexes in market i, $\tilde{\mathbf{P}}_i$:

$$V_i\left(Y_i, \tilde{\mathbf{P}}_i\right) = \max_{\mathbf{Q}_i}\ U_i(\mathbf{Q}_i) \qquad s.t. \sum_{k \in \mathbb{K}} \tilde{P}_{i,k} Q_{i,k} = Y_i. \tag{1}$$

Throughout this paper, the *tilde* notation on price is used to distinguish between "consumer" and "producer" prices. The former includes taxes, whereas the latter does not. Problem 1 yields an *industry-level* Marshallian demand function, which we denote by $Q_{i,k} = \mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$. This function tracks how (given prices and total income) consumers allocate their expenditure across industries. A special case of our general cross-industry demand function is the Cobb-Douglas case, wherein $U_i(\mathbf{Q}_i) = \prod_{k \in \mathbb{K}} Q_{i,k}^{e_{i,k}}$ implying that $Q_{i,k} = e_{i,k} Y_i / \tilde{P}_{i,k}$.

Within-Industry Demand. Each industry-level bundle aggregates over various origin-specific composite varieties: $Q_{i,k} \equiv Q_{i,k}(Q_{1i,k}, ..., Q_{Ni,k})$. Each origin-specific composite variety, itself, aggregates over multiple firm-level varieties: $Q_{ji,k} \equiv Q_{ji,k}(\mathbf{q}_{ji,k})$, where $\mathbf{q}_{ji,k} = \left\{q_{ji,k}\left(\omega\right)\right\}_{\omega \in \Omega_{j,k}}$ is a vector with each element $q_{ji,k}\left(\omega\right)$ denoting the quantity consumed of firm ω's output.[12] We assume that the within-industry utility aggregator, has a nested-CES structure, which enables us to abstract from variable markups and direct our attention to the scale-driven and profit-shifting effects of policy.

Assumption (A1). *The within-industry utility aggregator is nested-CES:*

$$Q_{i,k} = \left(\sum_{j \in \mathbb{C}} Q_{ji,k}^{\frac{\sigma_k - 1}{\sigma_k}}\right)^{\frac{\sigma_k}{\sigma_k - 1}},$$

where $Q_{ji,k}$ is a CES-composite of firm-level varieties from origin country j:

$$Q_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} \varphi_{ji,k}(\omega)^{\frac{1}{\gamma_k}} q_{ji,k}\left(\omega\right)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega\right)^{\frac{\gamma_k}{\gamma_k - 1}}.$$

We assume that $\gamma_k \geq \sigma_k > 1$. The constant demand shifter $\varphi_{ji,k}(\omega)$ accounts for variety ω's perceived quality in destination i.

Given (A1), the demand for *national-level* variety ji, k (origin country j–destination country i–industry k) is given by

$$Q_{ji,k} = \left(\tilde{P}_{ji,k} / \tilde{P}_{i,k}\right)^{-\sigma_k} Q_{i,k}, \tag{2}$$

where $\tilde{P}_{ji,k}$ and $\tilde{P}_{i,k}$ respectively denote the origin-specific and industry-level CES price indexes.[13] Recall that $Q_{i,k}$ denotes industry-level demand, which is given by $Q_{i,k} = \mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$.

[12] $\Omega_{j,k}$ denotes the set of all firms serving industry k from origin j. As we elaborate later, if we introduce firm-selection effects into the model, only a sub-set of firms in set $\Omega_{j,k}$ serve each market. In that case, $\Omega_{ji,k} \subset \Omega_{j,k}$ will denote the set of firms serving market i from country j.

[13] Namely, $\tilde{P}_{ji,k} = \left(\sum_{\omega \in \Omega_{ji,k}} \varphi_{ji,k}(\omega) \tilde{p}_{ji,k}\left(\omega\right)^{1-\gamma_k}\right)^{\frac{1}{1-\gamma_k}}$ and $\tilde{P}_{i,k} = \left(\sum_{j \in \mathbb{C}} \tilde{P}_{ji,k}^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}$.

The demand facing individual firms from country j is, accordingly, given by

$$q_{ji,k}(\omega) = \varphi_{ji,k}(\omega)\left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}}\right)^{-\gamma_k}\left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\sigma_k}\mathcal{D}_{i,k}\left(Y_i, \tilde{\mathbf{P}}_i\right). \tag{3}$$

Importantly, the above parmaterization of demand allows for the *firm-level* and *national-level* degrees of market power to diverge, with γ_k governing the degree of firm-level market power and σ_k governing the degree of national-level market power in industry k.

Elasticity of Demand Facing National-Level Varieties. Following Equation 2, the demand for aggregate variety ji,k is a function of total income in market i, Y_i, and the entire vector of *origin×industry*-specific consumer price indexes in that market: Namely, $Q_{ji,k} = \mathcal{D}_{ji,k}\left(Y_i, \tilde{\mathbf{P}}_i\right)$. To keep track of changes in demand, we define the elasticity of demand for national-level variety ji,k w.r.t. to the price of variety ni,g as follows:

$$\varepsilon^{(ni,g)}_{ji,k} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,g}} \sim \text{price elasticity of demand}$$

Under Cobb-Douglas preferences (i.e., zero cross-substitutability between industries), the national-level demand elasticities are fully determined by the upper-tier CES parameter σ_k and national-level expenditure shares. Specifically, $\varepsilon^{ji,g}_{ji,k} = 0$ if $g \neq k$, while

$$\varepsilon_{ji,k} \sim \varepsilon^{(ji,k)}_{ji,k} = -1 - (\sigma_k - 1)(1 - \lambda_{ji,k}); \qquad \varepsilon^{(\jmath i,k)}_{ji,k} = (\sigma_k - 1)\lambda_{\jmath i,k} \quad (\jmath \neq j),$$

where $\lambda_{ji,k} \equiv \tilde{P}_{ji,k}Q_{ji,k} / \sum_{\jmath} \tilde{P}_{\jmath i,k}Q_{\jmath i,k}$ denotes the (within-industry) share of expenditure on ji,k. In the presence of cross-substitutability between industries, the demand elasticity will feature an additional term that accounts for cross-industry demand effects.

In our setup, optimal policy internalizes the entire matrix of own- and cross-demand elasticities. To present our optimal policy formulas concisely, we use the following matrix notation to track the elasticity of demand w.r.t. goods sourced from various origins and industries.

Definition (D1). *Let $K = |\mathbb{K}|$ denote the number of industries. The $K \times K$ matrix $\mathbf{E}^{(ni)}_{ji}$ describes the elasticity of demand for origin $j \in \mathbb{C}$ goods w.r.t. the price of origin $n \in \mathbb{C}$ goods in market i across all industries:*

$$\mathbf{E}^{(ni)}_{ji} \equiv \begin{bmatrix} \varepsilon^{(ni,1)}_{ji,1} & \dots & \varepsilon^{(ni,K)}_{ji,1} \\ \vdots & \ddots & \vdots \\ \varepsilon^{(ni,1)}_{ji,K} & \dots & \varepsilon^{(ni,K)}_{ji,K} \end{bmatrix}.$$

To simplify the notation, we use $\mathbf{E}_{ji} \sim \mathbf{E}^{(ji)}_{ji}$ to denote the elasticity of origin j goods w.r.t. origin j prices, and use the $K \times (N-1)K$ matrix, $\mathbf{E}^{(-ii)}_{ji} = \left[\mathbf{E}^{(ni)}_{ji}\right]_{n\neq i}$, to summarize the elasticity of demand for origin j goods w.r.t. price of all import varieties in market i (i.e., all varieties source from any origin $n \neq i$). Important for our analysis, $\mathbf{E}_{ji}$ is an invertible matrix—the proof of which is provided in Appendix E using the primitive properties of Marshallian demand.

2.2 Production and Firms

Each economy $i \in \mathbb{C}$ is populated with a mass $M_{i,k} = |\Omega_{i,k}|$ of single-product firms in industry $k \in \mathbb{K}$ that compete under monopolistic competition. Labor is the only factor of production. Firm entry into industry k is either free or restricted. Under restricted entry, $M_{i,k} = \overline{M}_{i,k}$ is invariant to policy. Under free entry, a pool of ex-ante identical firms can pay an entry cost $w_i f_k^e$ to serve industry k from origin i. After paying the entry cost, each firm $\omega \in \Omega_{i,k}$ draws a productivity $z(\omega) \geq 1$ from distribution $G_{i,k}(z)$, and faces a marginal cost $\tau_{ij,k} w_i / z(\omega)$ for producing and delivering goods to destination $j \in \mathbb{C}$, where $\tau_{ij,k}$ denotes a flat iceberg transport cost. Collecting these assumptions, the *"producer"* price index of composite good ij, k (which aggregates over firm-level varieties associated with *origin i–destination j–industry k*) is given by

$$P_{ij,k} = \frac{\gamma_k}{\gamma_k - 1} \bar{a}_{ij,k} w_i M_{i,k}^{-\frac{1}{\gamma_k - 1}}. \tag{4}$$

In the above formulation, $\bar{a}_{ij,k} = \tau_{ij,k} \left[\int_1^\infty z^{\gamma_k - 1} dG_{i,k}(z) \right]^{\frac{1}{1-\gamma_k}}$ encompasses all price components (excluding the markup) that are invariant to policy.[14] To streamline the presentation of our theory, we follow Kucheryavyy et al. (2016) and refer to $1/(\gamma_k - 1) = -\partial \ln P_{ij,k} / \partial \ln M_{i,k}$ as the industry-level ***scale elasticity***:

$$\mu_k \equiv \frac{1}{\gamma_k - 1} \sim \text{scale elasticity} \sim \text{markup}$$

Following Equation 4, μ_k represents both (a) the constant markup over marginal cost in industry k (*i.e.*, $1 + \mu_k = \frac{\gamma_k}{\gamma_k - 1}$), and (b) the elasticity at which variety-adjusted TFP increases with industry-level employment $L_{i,k}$ (noting that $L_{i,k} \propto M_{i,k}$).[15] The equivalence between the industry-level markup and scale elasticity is not a universal property, but a specific feature of our baseline Krugman model.[16] While this feature simplifies our notation going forward, it is not consequential to the theoretical results that follow. As shown in Section 5, our analytical formulas for optimal policy extend to alternative models with external economies of scale or firm-selection (à la Melitz (2003)) in which the scale elasticity and markup levels diverge.

Expressing Producer Prices in terms of Profit-Adjusted Wages

In our framework, net profits (if any) are rebated back to the workers. Considering this feature, we can streamline the presentation of our theory by expressing producer prices in terms of profit-adjusted wages, $\grave{w}_i$, which are defined as follows:

$$\grave{w}_i \equiv (1 + \overline{\mu}_i) w_i \sim \text{profit-adjusted wage.}$$

In the above definition, $\overline{\mu}_i$ denotes economy i's average profit margin across all industries:

$$\overline{\mu}_i = \begin{cases} \dfrac{\sum_{g \in \mathbb{K}} \sum_{j \in \mathbb{C}} \frac{\mu_k}{1+\mu_k} P_{ij,k} Q_{ij,k}}{\sum_{g \in \mathbb{K}} \sum_{j \in \mathbb{C}} \frac{1}{1+\mu_k} P_{ij,k} Q_{ij,k}} & \text{if entry is restricted} \\ 0 & \text{if entry is free} \end{cases}. \tag{5}$$

Under free entry, profits are drawn to zero, resulting in $\overline{\mu}_i = 0$. Under restricted entry, the average profit margin is positive and depends on the industrial composition of country i's

[14] The invariance of $\bar{a}_{ij,k}$ follows from the implicit assumption that firms face no *fixed* exporting cost. This assumption rules out firm-selection in our baseline model. But as elaborated in Section 5, our baseline model is isomorphic to a model that admits firm-selection as long as the productivity distribution, $G_{i,k}(z)$, is Pareto.

[15] With free entry and constant markups, it follows immediately that $L_{i,k} = \bar{c}_{i,k} M_{i,k}$ where $\bar{c}_{i,k}$ is a constant.

[16] The equivalence between the scale elasticity and markup holds under more general models that assume free entry and monopolistic competition. More specifically, under these two assumptions: *elasticity of firm-level output w.r.t. input cost = firm-level markup* (Hanoch (1975); Helpman (1984)).

output—more sales in high-markup (high-μ) industries implies a higher $\overline{\mu}_i$.

Appealing to our definitions for $\tilde{w}_i$ and μ_k, we can restructure Equation 4 to express the producer price of good ij,k (origin i–destination j–industry k) as a function of origin i's *profit-adjusted* wage rate:

$$P_{ij,k} = \begin{cases} \varrho_{ij,k} \, \frac{1+\mu_k}{1+\overline{\mu}_i} \, \tilde{w}_i & \text{if entry is restricted} \\ \varrho'_{ij,k} \, \left[\sum_{j\in\mathbb{C}} \bar{a}_{ij,k} Q_{ij,k}\right]^{-\frac{\mu_k}{1+\mu_k}} \tilde{w}_i & \text{if entry is free} \end{cases}. \tag{6}$$

In the above formulation, $\varrho_{ij,k} = \bar{a}_{ij,k}\bar{M}_{i,k}^{-\mu_k}$ and $\varrho'_{ij,k} = \bar{a}_{ij,k}(1+\mu_k)\left(\mu_k/f_k^e\right)^{\frac{-\mu_k}{1+\mu_k}}$ are constant price shifters; and $\sum_{j\in\mathbb{C}}\left[\bar{a}_{ij,k}Q_{ij,k}\right]$ denotes *origin i–industry k*'s effective output.[17] As we explain shortly, the above formulation of producer prices is useful for tracking the gains from policy in an open economy. The gains from *firm-delocation* channel through changes in $\sum_{j\in\mathbb{C}}\left[\bar{a}_{ij,k}Q_{ij,k}\right]$, while the gains from *profit-shifting* channel through changes in $\overline{\mu}_i$.

2.3 The Instruments of Policy

The government in country i has access to a complete set of revenue-raising trade and domestic policy instruments, including

i. ***import tax***, $t_{ji,k}$, applied to all goods imported from origin $j \neq i$ in industry k;

ii. ***export subsidy***, $x_{ij,k}$, applied to all goods sold to market $j \neq i$ in industry k;

iii. ***industrial subsidy***, $s_{i,k}$, applied to industry k's output irrespective of where it is sold;

iv. ***consumption taxes***, which are redundant given the availability of the other tax instruments and normalized to zero hereafter.

Our specification of policy is quite flexible as it accommodates import subsidies or export taxes ($-1 \leq t < 0$ or $-1 \leq x < 0$) as well as production taxes ($-1 \leq s < 0$). A formal proof for the redundancy of consumption taxes is provided in Appendix A. There is a simple intuition behind this result: Country $i \in \mathbb{C}$ has access to $2(N-1)+2$ different tax instruments in each industry (where $N \equiv |\mathbb{C}|$ denotes the number of countries). These $2(N-1)+2$ tax instruments can directly manipulate $2(N-1)+1$ consumer price indexes: $N-1$ export prices, $N-1$ import prices, and one price associated with the domestically-produced and consumed variety (namely, $\tilde{P}_{ii,k}$). So, by construction, one of the $2(N-1)+2$ tax instruments in each industry is redundant. Here, we treat the industry-level consumption tax as a redundant instrument.[18]

[17] Under free entry, the total entry cost paid by the $M_{i,k}$ entrants should equal gross profits. That is,

$$w_i f_k^e M_{i,k} = \sum_{j\in\mathbb{C}}\left[\frac{\mu_k}{1+\mu_k}P_{ij,k}Q_{ij,k}\right] = \mu_k w_i M_{i,k}^{-\mu_k}\sum_j\left[\bar{a}_{ij,k}Q_{ij,k}\right],$$

where the last line follows from replacing for $P_{ij,k}$ based on 4. The above equation implies that $M_{i,k}{=}\left(\mu_k\sum_j\left[\bar{a}_{ij,k}Q_{ij,k}\right]/f_k^e\right)^{\frac{1}{1+\mu_k}}$. Plugging this expression back into 4 yields the formulation under Equation 6.

[18] With more than two countries ($N > 2$), Country i has access to $2(N-1)+2$ instruments per industry. These instruments can manipulate $2(N-1)+1$ price variables, which implies the same redundancy.

The non-redundant tax instruments create a wedge between consumer price indexes, $\{\tilde{P}_{ji,k}\}$ and producer price indexes, $\{P_{ji,k}\}$, as follows:

$$\tilde{P}_{ji,k} = \frac{1+t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji.k}, \quad \forall j,i \in \mathbb{C},\ k \in \mathbb{K}. \tag{7}$$

These tax instruments also generate/exhaust revenue for the tax-imposing country. The combination of all taxes imposed by country $i \in \mathbb{C}$ produce a tax revenue equal to

$$\begin{aligned} \mathcal{R}_i = & \overbrace{\sum_{k\in\mathbb{K}} \left(\left(\frac{1}{1+s_{i,k}} - 1 \right) P_{ii,k} Q_{ii,k} \right)}^{\text{industrial subsidies}} \\ & + \underbrace{\sum_{k\in\mathbb{K}} \sum_{j\neq i} \left(\frac{t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji,k} Q_{ji,k} + \left[\frac{1}{(1+x_{ij,k})(1+s_{i,k})} - 1 \right] P_{ij,k} Q_{ij,k} \right)}_{\text{import taxes + export subsidies}}. \end{aligned} \tag{8}$$

Tax revenues are rebated to the consumers in a lump-sum fashion. After we account for tax revenues, total income in country i equals the sum of profit-adjusted wage payments, $\varpi_i L_i = (1+\overline{\mu}_i) w_i L_i$, and tax revenues. Namely, $Y_i = \varpi_i L_i + \mathcal{R}_i$, where $\mathcal{R}_i$ can be positive or negative depending on whether country i's policy consists of net taxes or subsidies.

2.4 General Equilibrium

For convenience, we refer to *profit-adjusted* wages as just wages going forward, using $\mathbf{w} \equiv \{\varpi_i\}$ to denote the global vector of wages. We also assume throughout the paper that the underlying parameters of the model are such that the necessary and sufficient conditions for the uniqueness of equilibrium are satisfied.[19] To present our theory, we express all equilibrium outcomes—expect for wages—as a function of global taxes ($\mathbf{x}$, $\mathbf{t}$, and $\mathbf{s}$), treating wages $\mathbf{w}$ as given. As detailed in Appendix E, this formulation derives from solving a system that imposes all equilibrium conditions aside from the labor market clearing conditions. For future reference, we outline this formulation of equilibrium variables below.

Notation. *For a given vector of taxes and wages* $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$, *equilibrium outcomes* $Y_i(\mathbf{T})$, $P_{ji,k}(\mathbf{T})$, $\tilde{P}_{ji,k}(\mathbf{T})$, $Q_{ji,k}(\mathbf{T})$ *are determined such that* (*i*) *producer prices are characterized by 6;* (*ii*) *consumer prices are given by 7;* (*iii*) *industry-level consumption choices are a solution to 1 with demand for national-level varieties,* $Q_{ji,k}$*, given by 2; and* (*iv*) *total income (which dictates total expenditure by country i) equals profit-adjusted wage payments plus tax revenues:*

$$Y_i(\mathbf{T}) = \varpi_i L_i + \mathcal{R}_i(\mathbf{T}),$$

where tax revenues $\mathcal{R}_i(\boldsymbol{T})$ *are described by Equation 8.*

Considering the above formulation of equilibrium variables, welfare, too, can be expressed as a function of taxes and wages as follows:

$$W_i(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}) \equiv V_i\left(Y_i(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}), \tilde{\mathbf{P}}_i(\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})\right).$$

Note that $\mathbf{w}$ is itself an equilibrium outcome. So, a vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ is feasible insofar as $\mathbf{w}$

[19] Following Kucheryavyy et al. (2016), this assumption holds automatically in the two country case, given that $\gamma_k \geq \sigma_k$ per (A1). Otherwise, it will hold if trade costs are sufficiently small.

is the equilibrium wage consistent with $\mathbf{t}$, $\mathbf{x}$, and $\mathbf{s}$. Related to this point, our goal in this paper is to study problems where the government in i choses $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ to maximize $W_i(\mathbf{T})$ subject to feasibility. So, to fix ideas, we define the set of feasible *policy–wage* vectors below.

Definition (D2). *The set of feasible policy–wage vectors, $\mathbb{F}$, consists of any vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ where $\mathbf{w}$ satisfies the labor market clearing condition in every country, given $\mathbf{t}$, $\mathbf{x}$, and $\mathbf{s}$:*

$$\mathbb{F} = \left\{ \mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}) \ \mid \ \grave{w}_i L_i = \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} \left[P_{ij,k}(\mathbf{T}) Q_{ij,k}(\mathbf{T}) \right]; \quad \forall i \in \mathbb{C} \right\}.$$

There is a basic reason for why we formulate equilibrium outcomes as a function of $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ instead of just $(\mathbf{t}, \mathbf{x}, \mathbf{s})$. This choice of formulation allows us to articulate an important intermediate result regarding the equivalence of policy equilibria. This result, which is stated below, simplifies our theoretical derivation of optimal policy to a great degree.

Lemma 1. *[**Equivalence of Policy Equilibria**] For any a and $\tilde{a} \in \mathbb{R}_+$ (i) if $\mathbf{T} = (\mathbf{1} + \mathbf{t}_i, \mathbf{t}_{-i}, \mathbf{1} + \mathbf{x}_i, \mathbf{x}_{-i}, \mathbf{1} + \mathbf{s}_i, \mathbf{s}_{-i}; \grave{w}_i, \mathbf{w}_{-i}) \in \mathbb{F}$, then $\mathbf{T}' = (a(\mathbf{1} + \mathbf{t}_i), \mathbf{t}_{-i}, a(\mathbf{1} + \mathbf{x}_i), \mathbf{x}_{-i}, \frac{1}{\tilde{a}}(\mathbf{1} + \mathbf{s}_i), \mathbf{s}_{-i}; \frac{a}{\tilde{a}} \grave{w}_i, \mathbf{w}_{-i}) \in \mathbb{F}$. Moreover, (ii) welfare is preserved under $\mathbf{T}$ and $\mathbf{T}'$: $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ for all $n \in \mathbb{C}$.*

The above lemma is proven in Appendix B. It is a basic extension of the Lerner symmetry to an environment where domestic taxes are also applicable. It implies that there are multiple optimal tax combinations for each country i, which simplifies our forthcoming task of characterizing the optimal policy. To give some detail: The contribution of general equilibrium wage and income effects to the optimal tax schedule is often summarized in uniform tax shifters. The redundancy established by Lemma 1, simplifies the task of handling of these uniform tax shifters to a great degree.

3 Sufficient Statistics Formulas for Optimal Policy

In this section, we derive sufficient statistics formulas for optimal trade and industrial taxes/subsidies. These formulas are later employed to quantify the gains from policy across many different countries. Before proceeding to the derivation, let us briefly discuss the different rationales for policy intervention in our setup. From the perspective of a *non-cooperative* government, taxes can correct two margins of inefficiency:

a) Unexploited *terms-of-trade (ToT)* gains vis-à-vis the rest of the world, and

b) Misallocation in domestic industries due to the cross-industry heterogeneity in markups or scale elasticities.

Improving the ToT will always lead to a Pareto sub-optimal outcome; because such a correction has a beggar-thy-neighbor effect that transfers surplus from the rest of the world to the tax-imposing economy. Correcting allocative inefficiency, however, is Pareto improving. There is sub-optimal global output in high-profit or high-returns-to-scale (high-μ) industries. When this type of distortion is corrected, consumers all around the world reap the benefits.

Table 1: Summary of Key Variables

Variable	Description
$\tilde{P}_{ji,k}$	Consumer price index (origin j–destination i–industry k)
$P_{ji,k}$	Producer price index (origin j–destination i–industry k)
Y_i	Total income in country i
$\mathcal{R}_i$	Total tax revenue in country i (Equation 8)
w_i and $\bar{w}_i$	pure and profit-adjusted wage rates in country i: $\bar{w}_i = (1+\overline{\mu}_i)w_i$
$x_{ji,k}$	Export subsidy applied to good ji,k (if $j \neq i$)
$t_{ji,k}$	Import tax applied on good ji,k (if $j \neq i$)
$s_{i,k}$	Industrial subsidy applied to all goods from *origin i–industry k*
$\lambda_{ji,k}$	Within-industry expenditure share (good ji,k): $\tilde{P}_{ji,k}Q_{ji,k} / \sum_\jmath \tilde{P}_{\jmath i,k}Q_{\jmath i,k}$
$r_{ji,k}$	Within-industry sales share (good ji,k): $P_{ji,k}Q_{ji,k} / \sum_\iota P_{j\iota,k}Q_{j\iota,k}$
$e_{i,k}$	Industry-level expenditure share (destination i–industry k)
$\rho_{i,k}$	Industry-level sales share (origin i–industry k)
μ_k	industry-level markup ~ industry-level scale elasticity
$\overline{\mu}_i$	Average profit margin in origin i (Equation 5)
σ_k	Cross-national CES parameter ~ (1 + trade elasticity)
$\varepsilon_{ji,k}^{(ni,g)}$	Elasticity of demand for good ji,k w.r.t. the price of ni,g
$\omega_{ji,k}$	Inverse of good ji,k's supply elasticity (Equation 28)

3.1 Optimal Cooperative Policy

As a useful benchmark, we first characterize the optimal cooperative policy in our framework. Such a policy maximizes global welfare, as defined by a population-weighted sum of welfare across all countries:

$$\max_{(\mathbf{t},\mathbf{x},\mathbf{s};\mathbf{w})\in\mathbb{F}} \quad \sum_{i\in\mathbb{C}} \delta_i W_i\left(\mathbf{t},\mathbf{x},\mathbf{s};\mathbf{w}\right).$$

It is straightforward to show that the solution to above problem involves *zero* trade taxes and Pigouvian subsidies that restore marginal-cost-pricing in all countries:

$$t^*_{ji,k} = x^*_{ji,k} = 0 \quad \forall ji,k; \qquad 1+s^*_{i,k} = 1+\mu_k \quad \forall i,k \tag{9}$$

The above characterization applies to both the free and restricted entry cases, and is analogous to the cooperative policy structure studied in Bagwell and Staiger (2001).[20] It can be supported as the solution to a multi-country Nash bargaining game with side transfers. The assertion that cooperative policy restores *global* allocative efficiency is an artifact of non-politically-weighted welfare functions. Indeed, the above policy schedule corresponds to what would be implemented under a ***deep trade agreement***. As we discuss in Section 5, industrial subsidies offered by politically-motivated governments can worsen allocative efficiency at a global level, without necessarily improving the tax-imposing country's ToT.

[20]Despite apparent similarities, the gains from cooperative policy are larger under free entry where optimal policy corrects distortions due to sub-optimal entry decisions—we elaborate on this point in Section 7.

3.2 First-Best Unilaterally Optimal Policy

We now characterize the unilaterally optimal policy schedule for a non-cooperative country $i \in \mathbb{C}$. We consider cases where country i choses tax vectors $\mathbf{t}_i \equiv \{t_{ji,k}\}$, $\mathbf{x}_i \equiv \{x_{ij,k}\}$, and $\mathbf{s}_i \equiv \{s_{i,k}\}$, taking policy choices in the rest of the world as given—i.e., $\mathbf{t}_{-i} = \mathbf{x}_{-i} = \mathbf{s}_{-i} = \mathbf{0}$. Accordingly, we hereafter express equilibrium variables as a function of only country i's taxes. Noting this minor switch in notation, country i's first-best policy solves the following problem:

$$\max_{(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w})} \quad W_i\left(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}\right) \qquad s.t. \;\; \left(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}\right) \in \mathbb{F} \qquad \text{(P1)}.$$

We analytically solve Problem (P1) under both the restricted and free entry cases. We perceive the restricted entry case to be a more appropriate benchmark if governments are concerned with ***short-run*** gains from policy. The free entry case, on the other hand, is more relevant if governments are concerned with ***long-run*** gains. These two cases exhibit an important difference: Producer prices respond differently to contractions in export supply under restricted and free entry. Below, we elaborate on this difference.

General Equilibrium Export Supply Elasticity

The terms-of-trade gains from policy, in our framework, channel through changes in the price of imported and exported goods. The government in $i \in \mathbb{C}$ cannot directly dictate the *producer* price of say good, ji,k, that is imported from origin $j \neq i$. Instead, it can deflate ji,k's producer price ($P_{ji,k}$) *indirectly* by contracting or expanding its export supply ($Q_{ji,k}$). The contraction in $Q_{ji,k}$ also affects the producer price of goods supplied by other locations through general equilibrium linkages. To track these effects, we define the general equilibrium ***export supply elasticity of good*** ji,k as follows:

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_g \left[\frac{\tilde{w}_i L_i}{\tilde{w}_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i} + \sum_{n \neq i} \frac{\tilde{w}_n L_n}{\tilde{w}_j L_j} r_{ni,g} \rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i} \right].$$

In the above definition, $\mathbb{T}_i \equiv (\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i)$ denotes country i's vector of taxes and subsidies, while $r_{ni,g} \equiv P_{ni,g} Q_{ni,g} / \sum_\iota P_{n\iota,g} Q_{n\iota,g}$ and $\rho_{n,g} \equiv \sum_\iota P_{n\iota,g} Q_{\iota,g} / \tilde{w}_n L_n$ respectively denote the *good-specific* and *industry-wide* sales shares associated with origin n. As defined above, $\omega_{ji,k}$ describes how the producer prices linked to economy i respond to a change in $Q_{ji,k}$. Correspondingly, it embodies different economic forces under the free and restricted entry cases, as we detail below.

Under restricted entry, producers prices from origin $j \in \mathbb{C}$ are fully determined by the (profit-adjusted) wage rate, $\tilde{w}_j$, and the aggregate profit margin, $\overline{\mu}_j$ (see Equation 6). Policy, thus, has two distinct effects on producer prices under restricted entry: One effect that channels through wages, $\mathbf{w}$; and another that channels through aggregate profit margins. To explain the latter, hold $\mathbf{w}$ constant: contracting the export supply of good ji,k with taxes will alter all producer prices associated with origin j through a change in origin j's aggregate profit margin, $\overline{\mu}_j$. The change in $\overline{\mu}_j$ derives from the fact that industries have differential markup margins, and that taxing good ji,k alters the industrial composition of output in origin $j \in \mathbb{C}$.

Under free entry, producer prices from origin $j \in \mathbb{C}$ are determined by the wage rate, $\tilde{w}_j$, and the *origin j–industry k*-specific scale of production. So, aside from wage-related effects, policy has a second effect on producer prices that channels through industry-level scale economies. To elaborate, consider an import tax on good ji, k (origin j–destination i–industry k). Such a tax contracts the supply of ji, k and the scale of production in *origin j–industry k*. Given Equation 6, this contraction in scale increases the entire vector of producer price indexes associated with *origin j–industry k*—all through additional firm entry.

In both cases, $\omega_{ji,k}$ describes how expanding or contracting good ji, k's export supply impacts country i's terms-of-trade via either profit-shifting or industry-level scale economies. Importantly, $\omega_{ji,k}$ can be characterized (to a first-order approximation) as a simple function of sales shares, scale elasticities, and Marshallian demand elasticities (see Appendix E):[21]

$$\omega_{ji,k} \approx \begin{cases} \dfrac{\left(1-\frac{1+\bar{\mu}_j}{1+\mu_k}\right)\sum_g r_{ji,g}\rho_{j,g}}{1+\sum_g \sum_{\iota\neq i}\left[1+\left(1-\frac{1+\bar{\mu}_j}{1+\mu_g}\right)r_{j\iota,g}\rho_{j,g}\varepsilon_{j\iota,g}\right]} & \text{if entry is restricted} \\ \dfrac{-\frac{\mu_k}{1+\mu_k}r_{ji,k}}{1-\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}}\left[1-\frac{\mu_k}{1+\mu_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k}r_{in,k}}{\rho_{j,k}r_{jn,k}}\varepsilon^{(jn,k)}_{in,k}\right] & \text{if entry is free} \end{cases}. \tag{10}$$

The above formulation for $\omega_{ji,k}$ is quite intuitive: Under restricted entry, $\omega_{ji,k}$ governs the relationship between export supply and the average markup paid on imports. Accordingly, $\omega_{ji,k}$ is non-zero only when industries exhibit differential markup levels. Otherwise, $\omega_{ji,k}$ collapses to zero as the average markup (or profit margin) paid on imports is constant and invariant to changes in export supply, *i.e.*, $\bar{\mu}_j = \mu_k = \mu \Longrightarrow \omega_{ji,k} = 0$. Under free entry, $\omega_{ji,k}$ regulates the terms-of-trade gains from policy that channel through scale economies. Accordingly, in the limit where industries operate based on constant-returns to scale, $\omega_{ji,k}$ once again collapses to zero—namely, $\lim_{\mu_k\to 0}\omega_{ji,k} = 0$.[22]

Our Dual Approach to Characterizing Optimal Policy

Our characterization of optimal policy employs the dual approach and is presented in Appendix E. The derivation is relatively involved, occupying around 20 pages and utilizing several intermediate lemmas. Below, we provide a bullet point summary of our approach:

a) We first simplify Problem (P1) by reformulating it into a problem where country i's government chooses the vector of prices $\mathbb{P}_i = \left\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\right\}$ associated with its own economy. Country i's optimal tax/subsidy schedule $\mathbb{T}_i^* \equiv (\mathbf{t}_i^*, \mathbf{x}_i^*, \mathbf{s}_i^*)$ is then recovered from the optimal price vector $\mathbb{P}_i^*$.

b) The second step characterizes the first-order conditions (F.O.C.) associated with country i's reformulated optimal policy problem. This step uses two tricks to overcome the complications associated with general equilibrium analysis: First, we use the envelope conditions associated with optimal demand choices to net out redundant behavioral responses. Second, we identify additional neutrality conditions that are specific to Problem

[21]The above approximation derives from Wu, Yin, Vosoughi, Studer, Cavallaro, and Dick's (2013) first-order approximated inverse of a diagonally-dominant matrix. Figure (2) in Appendix E illustrates the precision of this approximation. The same appendix also presents an exact (approximation-free) formulation for $\omega_{ji,k}$.

[22]Note that $\omega_{ji,k}$ can be negative under free entry if σ_k is sufficiently small. Intuitively, each *origin j–industry k* exhibits a "backward-falling" supply curve, which implies that expanding the supply of ji, k deflates the producer price index associated with *origin j–industry k* via increased firm entry.

(P1). Most importantly, we observe that the terms in the F.O.C.s that account for general equilibrium wage and income effects are redundant at the optimum.

c) The last step of our derivation combines the F.O.C.s and solves them simultaneously as part of one system. In this process, we first appeal to the multiplicity of optimal tax/subsidy schedules (Lemma 1) to eliminate uniform tax shifters, which are difficult to characterize but redundant from a welfare standpoint. We then appeal to well-known properties of the Marshallian demand function (e.g., *Cournot aggregation* and *homogeneity of degree zero*) to prove that our system of F.O.C.s is invertible and has a unique solution.

The above steps lead us to simple sufficient statistics formulas for unilaterally optimal policy. The following theorem presents these formulas.

Theorem 1. *Country i's optimal policy is unique up to two uniform tax shifters $1+\bar{s}_i$ and $1+\bar{t}_i \in \mathbb{R}_+$, and is implicitly given by*

$$\begin{aligned} &[\text{domestic subsidy}] && 1+s^*_{i,k} = (1+\mu_k)(1+\bar{s}_i) \\ &[\text{import tax}] && 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i) \\ &[\text{export subsidy}] && \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right); \end{aligned}$$

where $\omega_{ji,k}$ denotes the inverse of good ji,k's supply elasticity as given by 10, while $\mathbf{E}_{ij} \sim \mathbf{E}_{ij}^{(ij)}$ and $\mathbf{E}_{ij}^{(-ij)}$ denote matrixes of Marshallian demand elasticities as defined under (D1).[23]

The uniform tax shifters, $\bar{s}_i$, and $\bar{t}_i$ account for the multiplicity of optimal policy equilibria (as indicated by Lemma 1). These shifters can be assigned any arbitrary value, provided that $1+\bar{s}_i$ and $1+\bar{t}_i \in \mathbb{R}_+$. For instance, if we assign a sufficiently high value to $\bar{t}_i$ and $\bar{s}_i$, the optimal policy will involve import tariffs, export subsidies, and industrial subsidies. Conversely, if we assign a sufficiently low value to $\bar{t}_i$ and $\bar{s}_i$, the optimal policy will involve import subsidies, export taxes, and industrial production taxes.

Intuition Behind Optimal Tax Formulas. Theorem 1 states that country i's unilaterally optimal policy consists of (1) Pigouvian subsidies that restore marginal cost pricing in economy i; (2) import taxes/subsidies that exploit country i's collective import market power, delivering an optimal mark-down on the producer price $P_{ji,k}$ of imported goods; and (3) export taxes/subsidies that exploit country i's collective export market power, charging an optimal mark-up on the consumer price $\tilde{P}_{ij,k}$ of exported goods.

When evaluating Theorem 1, note that Marshallian demand elasticities, $\varepsilon^{(ni,g)}_{ji,k}$, are fully-determined by expenditure shares, $\lambda_{ji,k}$, and σ_k. Likewise, the export supply elasticity, $\omega_{ji,k}$, is fully-determined by sales shares, $r_{ji,k}$, scale elasticities, μ_k, and Marshallian demand elasticities. As such, Theorem 1 characterizes optimal policy in terms of three sets of *sufficient statistics*: (i) observable shares, $r_{ij,k}$, and $\lambda_{ij,k}$, (ii) industry-level trade elasticities, σ_k-1, and

[23]To be clear, $\mathbf{E}_{ij}^{(-ij)} = \left[\mathbf{E}_{ij}^{(nj)}\right]_{n\neq i}$ is a $K\times(N-1)K$ matrix and $\mathbf{1}\equiv\mathbf{1}_{(N-1)K\times 1}$ is a column vector of *ones*. Also, in the general case with asymmetric income elasticities of demand, $\mathbf{E}_{ij}$ should be replace with $\tilde{\mathbf{E}}_{ij} \equiv [\frac{e_{ij,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{ij,g}]_{g,k}$. Otherwise , the symmetry of the Slutsky matrix implies that $\frac{e_{ij,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{ij,g} = \varepsilon^{(ij,g)}_{ij,k}$, which implies that $\mathbf{E}_{ij}=\tilde{\mathbf{E}}_{ij}$.

(iii) industry-level scale elasticities, μ_k. This particular feature of Theorem 1 greatly simplifies our quantitative analysis of optimal policy in Section 7.

A canonical special case of our model is the multi-industry Armington model, in which $\mu_k = 0$ for all $k \in \mathbb{K}$. Under this special case, $\omega_{ji,k} = 0$ for all ji, k and optimal import tariffs are uniform, i.e., $t^*_{ji,k} = \bar{t}_i$ for all ji, k. This result can be understood as follows: Absent scale economies or profits, import tariffs cannot impact the producer price of imported goods on a good-by-good basis. At best, import taxes can induce a uniform reduction in import prices (per origin j) by deflating $\mathbf{w}_{-i}$ relative to w_i. This uniform reduction, though, can be perfectly mimicked with a uniform increase in export taxes per destination j. As such, optimal import taxes are either uniform or redundant by choice of $\bar{t}_i = 0$.

A notable feature of the optimal policy schedule is that $x^*_{ij,k}$ depends on the entire matrix of own- and cross-price demand elasticities associated with good ij, k. The explanation is that $x^*_{ij,k}$ (in Theorem 1) corresponds to the optimal markup of a multi-product monopolist. To better understand this point, assign $\bar{t}_i = 0$, in which case $x^*_{ij,k}$ acts as a tax on good ij, k (rather than a subsidy). The optimal tax rate on ij, k is equal to the optimal mark-up on that good if country i's government was pricing its exports as a multi-product monopolist rather than an individual single-product firm. The difference is that the government's optimal pricing decision internalizes the effect of raising $\tilde{P}_{ij,k}$ on its sales of other products in destination j.

Finally, note the resemblance between $t^*_{ji,k}$ (as implied by Theorem 1) and the traditional optimal tariff formula. In both cases, the optimal tariff rate is related to the inverse of foreign's export supply elasticity. Our formula, however, is based on a general equilibrium multi-country model rather than a traditional partial equilibrium two-country model. This distinction aside, $\omega_{jn,k}$ in our model describes a *backward falling* supply curve that stems from increasing returns to scale at the industry level. In traditional models, $\omega_{jn,k}$ describes an upward-sloping supply curve resulting from diseconomies of scale. As such, our model is consistent with a possibly negative $\omega_{jn,k}$, which conforms to recent evidence in Farrokhi and Soderbery (2020).

Special Case with Cobb-Douglas Preferences. To gain deeper intuition about Theorem 1, consider a special case where preferences are Cobb-Douglas across industries. In that case, the formulas specified by Theorem 1 reduce to[24]

$$\begin{aligned}
&[\textit{domestic subsidy}] && 1 + s^*_{i,k} = (1 + \mu_k)(1 + \bar{s}_i) \\
&[\textit{import tax}] && 1 + t^*_{ji,k} = (1 + \omega_{ji,k})(1 + \bar{t}_i) \\
&[\textit{export subsidy}] && 1 + x^*_{ij,k} = \frac{(\sigma_k - 1)\sum_{n \neq i}\left[(1 + \omega_{ni,k})\lambda_{nj,k}\right]}{1 + (\sigma_k - 1)(1 - \lambda_{ij,k})}(1 + \bar{t}_i),
\end{aligned} \tag{11}$$

A well-known special case of the above formula is the *single-industry×two-country* formula in Gros (1987). To demonstrate this, drop the industry subscript k and reduce the global economy

[24]In the Cobb-Douglas case: (a) $\varepsilon^{(ij,k)}_{nj,k} = -\sigma_k \mathbb{1}_{n=j} + (\sigma_k - 1)\lambda_{ij,k}$ and (b) $\varepsilon^{(ij,k)}_{nj,g} = 0$ if $g \neq k$. Plugging the expression for $\varepsilon_{ji,k}$ into Equation 6, the inverse of the export supply under restricted entry is given by

$$\omega_{ji,k} = \frac{\left(1 - \frac{\bar{\mu}_j}{\mu_k}\right)\sum_g r_{ji,g}\rho_{j,g}}{1 + \sum_{\iota \neq i}\left[1 - \left(1 - \frac{\bar{\mu}_j}{\mu_k}\right) r_{j\iota,k}\left(1 + (\sigma_k - 1)(1 - \lambda_{j\iota,k})\right)\right]}.$$

The parmaterization of $\omega_{ji,k}$ under free entry can be derived in a similar fashion.

into two countries, i.e., $\mathbb{C} = \{i, j\}$. Noting that $1 - \lambda_{ij} = \lambda_{jj}$ in the two-country case, we can deduce from the above formulas that

$$\frac{1 + t^*_{ji}}{1 + x^*_{ij}} = 1 + \frac{1}{(\sigma - 1)\lambda_{jj}}.$$

By the Lerner symmetry, export and import taxes are equivalent in the single-industry model.[25] Hence, without loss of generality, we can set $x^*_{ij} = 0$ and arrive at the familiar-looking optimal tariff formula in Gros (1987), i.e., $t^*_{ji} = 1/(\sigma - 1)\lambda_{jj}$.

The Cobb-Douglas case of Theorem 1 is also a strict generalization of the formula derived concurrently by Bartelme et al. (2019) for a small open economy with multiple sectors. Specifically, enforcing the ***small open economy*** assumption—i.e., setting $\omega_{ji,k} \approx \lambda_{ij,k} \approx 0$; $\lambda_{jj,k} \approx 1$—our optimal policy formulas in the Cobb-Douglas case reduce to:

$$1 + s^*_{i,k} = 1 + \mu_k; \qquad t^*_{ji,k} = 0; \qquad 1 + x^*_{ij,k} = \frac{\sigma_k - 1}{\sigma_k}. \tag{12}$$

3.3 Second-Best Unilaterally Optimal Trade Taxes

Suppose the government in $i \in \mathbb{C}$ cannot use domestic subsidies due to say institutional barriers or political pressures. It is optimal, in that case, to use trade taxes as a second-best policy to restore allocative efficiency in the domestic economy. In this section, we derive analytic formulas for *second-best* optimal trade taxes in such circumstances. Country i's optimal policy problem, in this case, includes an added constraint that $\mathbf{s}_i = \mathbf{0}$:

$$\max_{(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w})} \quad W_i\left(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}\right) \quad s.t. \begin{cases} (\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbf{w}) \in \mathbb{F} \\ \mathbf{s}_i = \mathbf{0} \end{cases} \qquad \text{(P2)}.$$

Using the dual approach discussed earlier, we analytically solve Problem (P2) and derive sufficient statistics formulas for second-best optimal trade taxes. The following proposition presents these formulas, with a formal proof provided in Appendix F.

Theorem 2. *Suppose industrial subsidies $\mathbf{s}_i$ are unavailable to the government: Second-best optimal trade taxes are unique up to a uniform tax shifter $\bar{t} \in \mathbb{R}_+$ and are implicitly given by:*

$$\text{[import tariff]} \qquad \mathbf{1} + \mathbf{t}^\star_{ji} = (1 + \bar{t}_i)\left(\mathbf{1} + \mathbf{\Omega}_{ji}\right) \oslash \left(\mathbf{1} + \mathbf{E}^{-1}_{-ii}\,\mathbf{E}^{(ii)}_{-ii}\left[1 - \frac{1 + \mu_k}{1 + \bar{\mu}_i}\right]_k\right)$$

$$\text{[export subsidy]} \qquad \mathbf{1} + \mathbf{x}^\star_{ij} = -(1 + \bar{t}_i)\left(\mathbf{E}^{-1}_{ij}\mathbf{E}^{(-ij)}_{ij}\left(\mathbf{1} + \mathbf{\Omega}_{-ii}\right)\right) \odot \left[\frac{1 + \mu_k}{1 + \bar{\mu}_i}\right]_k,$$

where $\mathbf{\Omega}_{ji} = \left[\omega_{ji,k}\right]_k$ is a vector of (inverse) export supply elasticities as given by 10; $\bar{\mu}_i$ denotes the output-weighted average markup in economy i as described by 5; and $\mathbf{E}_{-ii}$, $\mathbf{E}^{(ii)}_{-ii}$, $\mathbf{E}_{ij}$, and $\mathbf{E}^{(-ij)}_{ij}$ are matrixes of Marshallian demand elasticities as defined under Definition (D1).[26]

Theorem 2 asserts that, when governments cannot use industrial subsidies, (i) the optimal export subsidy is adjusted to promote exports in high-returns-to-scale (high-μ) industries,

[25] The Lerner symmetry is a special case of the equivalence result presented under Lemma 1. Also, note that the market equilibrium is efficient in the single industry Krugman model studied by Gros (1987). As such, the optimal industrial subsidy can be also normalized to zero, i.e., $s^*_i = 0$.

[26] Letting N and K denote the number of countries and industries: $\mathbf{E}_{-ii} \sim \mathbf{E}^{(-ii)}_{-ii} = \left[\mathbf{E}^{(ji)}_{ni}\right]_{n \neq i, j \neq i}$ is a square $(N -$

and (ii) the optimal import tax is adjusted to restrict import competition in high-returns-to-scale (high-μ) industries. Intuitively, the government's objective when solving (P2) is to mimic Pigouvian industrial subsidies with trade taxes/subsidies. To reach this objective, import taxes and export subsidies should increase in high-returns-to-scale industries relative to the *first-best* benchmark. While these adjustments promote domestic production in high-μ industries, they cannot perfectly replicate the first-best outcome due to a lack of sufficient policy instruments.[27]

Special Case with Cobb-Douglas Preferences. We can invoke the Cobb-Douglas assumption to further elucidate the second-best tax formulas under Theorem 2. Under this assumption, there are zero cross-demand effects between industries and the optimal policy formulas specified by Theorem 2 can be simplified as follows:

$$\text{[import tariff]} \qquad 1+t^{\star}_{ji,k} = \frac{1+(\sigma_k-1)\lambda_{ii,k}}{1+\frac{1+\overline{\mu}_i}{1+\mu_k}(\sigma_k-1)\lambda_{ii,k}} \overbrace{(1+t^{*}_{ji,k})}^{\text{1st-best}}$$

$$\text{[export subsidy]} \qquad 1+x^{\star}_{ij,k} = \frac{1+\mu_k}{1+\overline{\mu}_i} \underbrace{(1+x^{*}_{ij,k})}_{\text{1st-best}},$$

where $1+t^{*}_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i)$ and $1+x^{*}_{ji,k} = \frac{(\sigma_k-1)\sum_{n\neq i}\left[(1+\omega_{ni,k})\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\lambda_{ij,k})}(1+\bar{t}_i)$ denote the first-best optimal rate (Equation 11). For a ***small open economy***, the formulas further reduce to

$$1+t^{\star}_{ji,k} = \frac{1+(\sigma_k-1)\lambda_{ii,k}}{1+\frac{1+\overline{\mu}_i}{1+\mu_k}(\sigma_k-1)\lambda_{ii,k}}(1+\bar{t}_i); \qquad\qquad 1+x^{\star}_{ij,k} = \frac{1+\mu_k}{1+\overline{\mu}_i}\left(\frac{\sigma_k-1}{\sigma_k}\right)(1+\bar{t}_i).$$

In summary, the above formulas indicate that second-best *import* taxes are higher in (1) industries with a greater-than-average markup, and (2) industries in which country i has a comparative advantage (i.e., high-$(\sigma_k-1)\lambda_{ii,k}$ industries). These two properties allow second-best import taxes to mimic Pigouvian subsidies to the best extent possible. Likewise, second-best *export* subsidies feature a *misallocation-correcting* component that favors industries with a higher-than-average scale elasticity or markup.

Importantly, if the markup or scale elasticity is uniform across industries (i.e., $\mu_k = \mu = \overline{\mu}_i$), the above formulas yield the *first-best* or purely ToT-improving tax rate—i.e., $t^{\star}_{ji,k} = t^{*}_{ji,k}$ and $x^{\star}_{ij,k} = x^{*}_{ij,k}$. The intuition is that the Krugman model *without* cross-industry markup heterogeneity is efficient; leaving no room for policy interventions to restore allocative efficiency.

Third-Best Import Taxes. Now suppose that in addition to industrial subsidies, the use of export subsidies is also restricted. The government's optimal policy problem in this case features two additional constraints, $\mathbf{s}_i = \mathbf{x}_i = \mathbf{0}$:

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1)$K \times (N-1)K$ matrix, where $\mathbf{E}^{(ji)}_{ni} \equiv \left[\varepsilon^{(ji,g)}_{ni,k}\right]_{k,g}$ as defined under Definition (D1). Likewise, $\mathbf{E}^{(-ij)}_{ij} = \left[\mathbf{E}^{(nj)}_{ij}\right]_{n\neq i}$ and $\boldsymbol{E}^{(ii)}_{-ii} = \left[\boldsymbol{E}^{(ii)}_{ni}\right]_{n\neq i}$ are respectively $K \times (N-1)K$ and $(N-1)K \times K$ matrixes. In all the equations, $\mathbf{1} \equiv \mathbf{1}_{(N-1)K\times 1}$ is a columns vector of ones. Meanwhile, $\boldsymbol{\Omega}_{-ii} = \left[\omega_{ni,k}\right]_{n\neq i,k}$ is a $(N-1)K \times 1$ vector; and the operators $\odot$ and $\oslash$ denote element-wise multiplication and division.

[27] As noted earlier, the government needs at least $2(N-1)K+1$ tax instruments (per industry) to achieve the first-best outcome. In *homogeneous* good models, though, trade taxes can fully mimic industrial subsidies, because fewer instruments suffice to achieve the first-best.

$$\max_{(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w})} \quad W_i\left(\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}\right) \quad s.t. \begin{cases} (\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i;\mathbf{w}) \in \mathbb{F} \\ \mathbf{s}_i = \mathbf{x}_i = \mathbf{0} \end{cases} \qquad \text{(P3)}.$$

Some variation of the above problem has been studied by an expansive literature on optimal tariffs. Though, nearly all existing studies are limited to partial equilibrium two-by-two models. Here, we use the same dual approach described earlier to analytically solve Problem (P3) within our multi-country, multi-industry general equilibrium framework. Our derivation, as before, yields simple sufficient statistics formulas for optimal third-best import taxes. The following theorem presents these formulas, with a formal proof provided in Appendix G.

Theorem 3. *Suppose both industrial and export subsidies are unavailable to the government: Third-best optimal import taxes are uniquely given by:*

$$\mathbf{1} + \mathbf{t}^{\star}_{ji} = (1+\bar{\tau}^{\star}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right) \oslash \left(\mathbf{1} + \mathbf{E}^{-1}_{-ii}\,\mathbf{E}^{(ii)}_{-ii}\left[1 - \frac{1+\mu_k}{1+\overline{\mu}_i}\right]_k\right)$$

where $\bar{\tau}^{\star}_i = \left[-\sum_{g,s}\sum_{j\neq i}\chi_{ij,g}\left(1+\varepsilon^{(ij,s)}_{ij,g}\right)\right]^{-1}$ *is a uniform tariff shifter that represents the elasticity of international demand for country i's labor (with* $\chi_{ij,g} \equiv P_{ij,g}Q_{ij,g}/\sum_{n\neq i}\mathbf{P}_{in}\cdot\mathbf{Q}_{in}$ *denoting export shares).* $\overline{\mu}_i$ *denotes the output-weighted average markup in economy i as described by 5; and* $\mathbf{E}_{-ii}$ *and* $\mathbf{E}^{(ii)}_{-ii}$ *are matrixes of Marshallian demand elasticities as defined under Definition (D1).*

Unlike Theorems 1 and 2, the third-best optimal tariff schedule identified by Theorem 3 is unique. That is because the multiplicity implied by Lemma 1 no longer applies when both export and industrial subsidies are restricted to zero. Nevertheless, the third-best tariff specified by Theorem 3 differs from the second-best tariffs (in Theorem 2) by only a uniform tariff shifter, $1+\bar{\tau}^{\star}_i$. So, barring the uniform component, $1+\bar{\tau}^{\star}_i$, we can understand the above formula based on the same intuition provided under Theorem 2.

The uniform tariff component, $1+\bar{\tau}^{\star}_i$, compensates for the unavailability of export tax-cum-subsidies to the government. By the Lerner symmetry, which is implicit in Lemma 1, import taxes can perfectly mimic a uniform export tax. This ability was previously redundant (under Theorems 1 and 2) because export taxes/subsidies were directly applicable, and there was no point in using other instruments to mimic them. But since export taxes are restricted under Problem (P3), it is optimal to uniformly raise all tariffs by a factor $1+\bar{\tau}_i$, using them as a second-best substitute for optimal export taxes/subsidies.

4 Tension between ToT and Misallocation-Correcting Objectives

Following Theorems 2 and 3, second-best trade policies pursue two main objectives: (a) contract exports in nationally-differentiated (low-σ) industries, and (b) promote domestic production in high-returns-to-scale (high-μ) industries. The former improves the ToT and the latter corrects allocative inefficiency. Second-best trade policies are, therefore, plagued by a possible tension: If $\text{Cov}_k(\sigma_k,\mu_k) < 0$, improving allocative efficiency with trade taxes worsens the ToT. That is, reaching objective (b) undermines objective (a). The opposite holds if $\text{Cov}_k(\sigma_k,\mu_k) > 0$, but we hereafter focus on the case where $\text{Cov}_k(\sigma_k,\mu_k) < 0$, because it is supported by our forthcoming estimation. Importantly, the tension described above, diminishes the efficacy of trade taxes as a second-best misallocation-correcting measure—beyond what is implied by the *targeting principle* (Bhagwati and Ramaswami (1963)).[28]

Proposition 1. *If the industry-level trade and scale elasticities are negatively correlated, i.e., $Cov_k(\sigma_k,\mu_k)<0$, then restoring allocative efficiency with "trade policy" worsens a country's ToT. This tension diminishes the efficacy of trade taxes as a second-best misallocation-correcting policy measure, beyond what is already implied by the targeting principle.*

The above proposition has a notable flip side: If $\text{Cov}_k(\sigma_k, \mu_k) < 0$, a unilateral adoption of Pigouvian subsidies will worsen the ToT, resulting in possibly adverse welfare consequences. This outcome is a basic manifestation of ***immiserizing growth*** (Bhagwati, 1958). If $\text{Cov}_k(\sigma_k, \mu_k) < 0$, Pigouvian subsidies (that restore marginal-cost-pricing) expand domestic output in high-σ industries. These are nationally-differentiated industries in which countries enjoy significant export market power. Raising output and, correspondingly, exports in these industries can worsen the ToT to the point of triggering immiserizing growth. This tension reveals that unilateralism in industrial policies is unlikely to succeed. We unpack this issue in the following subsection, shedding new light on the gains from deep trade agreements.

Avoiding Immiserizing Growth via Deep Trade Agreements

Proposition 1 indicated that, when $\text{Cov}_k(\sigma_k, \mu_k) < 0$, trade taxes are a poor second-best measure for correcting misallocation in domestic industries. A unilateral adoption of corrective industrial policies can be equally futile, as it may trigger immiserizing growth. The remedy is that countries either ***(a)*** pair industrial subsidies with trade taxes that are globally inefficient, or ***(b)*** coordinate their industrial policies via a deep trade agreement.

To elaborate on choice (b), recall that corrective industrial subsidies expand overall production in high-μ industries. The ToT rationale for policy, meanwhile, requires that countries contract export sales in low-σ industries. So, if $\text{Cov}_k(\sigma_k, \mu_k) < 0$, corrective industrial subsidies worsen the ToT as they inadvertently expand export sales in the wrong (low-σ) industries from a ToT standpoint. This tension can be resolved if other countries, too, adopt corrective subsidies and concurrently expand their export sales in the same low-σ industries.

To put it differently, corrective (Pigouvian) industrial subsidies in the rest of the world have a positive externality on economy i. These externalities imply that joining deep trade agreements is beneficial beyond concerns about retaliation by trading partners. To articulate this point, we can compare the gains from the first-best *non-cooperative* policy (choice a) to the gains from multilateral cooperation (choice b), using the following decomposition:[29]

$$\frac{W_i(\mathbf{s}_i^*, \mathbf{0}, \mathbf{0} \mid \mathbf{s}_{-i} = \mathbf{s}_{-i}^*)}{W_i(\mathbf{s}_i^*, \mathbf{t}_i^*, \mathbf{x}_i^* \mid \mathbf{s}_{-i} = \mathbf{0})} = \underbrace{\frac{W_i(\mathbf{s}_i^*, \mathbf{0}, \mathbf{0} \mid \mathbf{s}_{-i} = \mathbf{s}_{-i}^*)}{W_i(\mathbf{s}_i^*, \mathbf{t}_i^*, \mathbf{x}_i^* \mid \mathbf{s}_{-i} = \mathbf{s}_{-i}^*)}}_{\text{forgone ToT gains}} \times \underbrace{\frac{W_i(\mathbf{s}_i^*, \mathbf{t}_i^*, \mathbf{x}_i^* \mid \mathbf{s}_{-i} = \mathbf{s}_{-i}^*)}{W_i(\mathbf{s}_i^*, \mathbf{t}_i^*, \mathbf{x}_i^* \mid \mathbf{s}_{-i} = \mathbf{0})}}_{\text{spilled-over corrective gains}}.$$

[28] Relatedly, when trade taxes are restricted, governments will use industrial subsidies as a second-best instrument to improve their ToT. Moving from the *first-best* to the *second-best* production tax-cum-subsidy schedule, in that case, improves the ToT inefficiency at the expense of worsening allocative efficiency.

[29] To clarify the notation, $W_i(\mathbf{s}_i^*, \mathbf{t}_i^*, \mathbf{x}_i^* \mid \mathbf{s}_{-i} = \mathbf{0})$ denotes country i's welfare under the first-best non-cooperative policy as characterized by Theorems 1. $W_i(\mathbf{s}_i^*, \mathbf{0}, \mathbf{0} \mid \mathbf{s}_{-i} = \mathbf{s}_{-i}^*)$ denotes i's welfare under a deep trade agreement that prohibits trade taxes but implements Pigouvian subsidies all over the world. The above decomposition is theoretically grounded in the fact that $\mathbf{t}^*$ and $\mathbf{x}^*$ only target the ToT inefficiency, whereas $\mathbf{s}^*$ only targets allocative inefficiency under the first-best polict schedule.

The term labeled "forgone ToT gains" concerns the loss in welfare when trade taxes are abolished as part one's commitments to the deep agreement. The "spilled-over corrective gains," account for the gains that accrue to economy i when the rest of the world restore efficiency in their domestic economies. Extrapolating from Theorem 1, we can verify that:

a) The foregone ToT gains are determined primarily by $\text{Avg}_k(1/(\sigma_k - 1))$;

b) The spilled-over corrective gains are determined primarily by $\text{Var}_k(\mu_k)$.[30]

Therefore, if $\text{Var}_k(\mu_k)$ is sufficiently large relative to $\text{Avg}_k(1/(\sigma_k - 1))$, it is beneficial for countries to forgo ToT gains and, in return, reap the benefits of corrective subsidies in the rest of world. These arguments are summarized in the following proposition.

Proposition 2. *(i) If $Cov_k(\sigma_k, \mu_k) < 0$, a unilateral adoption of corrective industrial policies (without trade taxes) worsens the ToT and may trigger immiserizing growth. However, if $Var_k(\mu_k)$ is sufficiently large relative to $Avg_k(1/(\sigma_k - 1))$, (ii) a multilateral adoption of corrective industrial policies via deep trade agreements delivers welfare gains that dominate those of any non-cooperative policy alternative. This latter assertion holds even if non-cooperation does not prompt retaliation by trading partners.*

In light of Propositions 1 and 2, the optimal implementation of industrial policy and the resulting gains depend crucially on σ_k and μ_k. We later develop a methodology to estimate these crucial policy parameters. But before moving on to the estimation, we briefly discuss how our theory readily applies to other canonical trade models or extends to richer environments with political pressures and input-output linkages.

5 Extensions and Application to other Canonical Models

In this section, we first show that our theoretical results readily apply to two other canonical trade models. We then extend our baseline theoretical results to richer environments featuring input-output linkages and political economy pressures.

5.1 Application to Other Canonical Trade Models

The optimal policy formulas specified by Theorems 1-3 apply to two other canonical trade models. Though, parameters σ_k and μ_k in these formulas adopt different interpretations, which reflects the different micro-foundation underlying these frameworks.

The Eaton-Kortum model with Marshallian externalities. As in Kucheryavyy et al. (2016), consider a multi-industry Eaton and Kortum (2002) model where production in each industry is subject to agglomeration economies. Let ψ_k denote the constant agglomeration elasticity in industry k, and let θ_k denotes the industry-level Eaton-Kortum Fréchet shape parameter. Theorem 1 characterizes the optimal policy in this model under the following reinterpretation of parameters: $\mu_k^{\text{EK}} = \psi_k$ and $\sigma_k^{\text{EK}} = 1 + \theta_k$. The tension between the *ToT* and *misallocation-correcting* rationales for policy—as outlined by Proposition 1—extends to this model if $\text{Cov}_k(\psi_k, \theta_k) < 0$.

[30] To establish these points, consider two extreme cases: First, suppose $\text{Var}_k(\mu_k) \approx 0$, in which case Theorem 1 indicate that $\mathbf{s}^*_{-i} \approx 0$ by choice of shifter $\bar{s}_i$, implying zero "spilled-over corrective gains." Second, consider the case where $\sigma_k \to \infty$, in which case $\mathbf{t}_i^* \approx \mathbf{x}_i^* \approx \mathbf{0}$ by choice of shifter $\bar{t}_i$, implying zero "foregone ToT gains."

The fact that our theory readily extends to the Eaton-Kortum model is an artifact of the isomorphism established in Kucheryavyy et al. (2016). We extend this isomorphism result in Appendix C, demonstrating that the nested-CES import demand function implied by (A1) may analogously arise from within-industry specialization á la Eaton-Kortum.

The Melitz-Pareto model. Consider a multi-industry Melitz (2003) model that features the same nested-CES demand function specified by (A1). Suppose the firm-level productivity distribution is Pareto in each industry with a shape parameter, ϑ_k. Appendix D establishes that the Melitz-Pareto model is isomorphic to our baseline Krugman model insofar as macro-level representation is concerned. Hence, Theorem 1 characterizes the optimal policy in the Melitz-Pareto model under the following reinterpretation of parameters: (1) $\mu_k^{\text{Melitz}} = (\vartheta_k + 1)/(\mu_k\vartheta_k - 1)$ if entry is restricted and $\mu_k^{\text{Melitz}} = \vartheta_k$ is entry is free; and (2) $\sigma_k^{\text{Melitz}} = 1 + \vartheta_k / \left[1 + \vartheta_k\mu_k\left(\frac{1}{\mu_k(\sigma_k-1)} - 1\right)\right]$. This mapping indicates that we need to estimate parameter ϑ_k, in addition to σ_k and μ_k, to quantify the gains from policy under firm-selection effects—a procedure we formally undertake and elaborate on in Section 7.

5.2 Extension #1: Accounting for Input-Output Networks

Suppose production employs both labor and intermediate inputs, which are distinguished from final goods by superscript $\mathcal{I}$. Cost minimization entails that the producer price of good ij,k (origin i–destination j–industry k) depends on (i) the wage rate in origin i and (ii) the price of all intermediate inputs, $\tilde{\mathbf{P}}_i^{\mathcal{I}} \equiv \{\tilde{P}_{nj,k}^{\mathcal{I}}\}$, available to firms in origin i. Namely,

$$P_{ij,k} = \bar{\rho}_{ij,k} C_{i,k}(w_i, \tilde{\mathbf{P}}_i^{\mathcal{I}}) \mathcal{Q}_{i,k}^{-\frac{\mu_k}{1+\mu_k}}, \tag{13}$$

where $C_{i,k}(.)$ is a homogeneous of degree one cost function w.r.t. w_i and $\tilde{\mathbf{P}}_i^{\mathcal{I}}$.[31] The dependence of $P_{ij,k}$ on *origin i–industry k*'s effective output, $\mathcal{Q}_{i,k} \equiv \sum_{j\in\mathbb{C}}\left[\bar{a}_{ij,k}Q_{ij,k}\right]$, accounts for scale economies under free entry. The formal definition of general equilibrium in the presence of input-output (IO) linkages is presented in Appendix H. The same appendix characterizes optimal policy using our previously-described dual approach, while appealing to additional supply-side envelope conditions.[32] Our characterization indicates that optimal industrial subsidies and import taxes are IO-blind—i.e., they are described by the same formulas as in Theorem 1. The intuition is that after fixing the price of exported goods with export subsidies, import tariffs and industrial subsidies have no impact on prices in the rest of the world. For these policies to affect prices in the rest of the world, they need to propagate through re-exportation. But any possible gains the channel through re-exportation, will be already internalized by the optimal choice w.r.t. export subsidies. Consistent with this intuition, optimal export subsidies depend on the fraction of export value that is reimported via the global IO network. Putting

[31] Without loss of generality, we assume that good ji,k can be used as either an intermediate input or a final consumption good, with taxes being applied on a good irrespective of the intended final use, i.e., $\tilde{P}_{ij,k}^{\mathcal{I}} = \tilde{P}_{ij,k}$. This assumption is innocuous, because we can fragment every industry k into a final good version k' and an intermediate good version k''. Since we do not impose any restrictions on the number of industries, our theory extends to the case where differential taxes are imposed on fragments k' and k''.

[32] Beshkar and Lashkaripour (2020) borrow the dual approach developed in this paper to derive analogous formulas for optimal trade taxes in a perfectly competitive model with diseconomies of scale and input-output linkages. These formulas are then used to compute the cost of a global trade war.

the pieces together, country i's first-best optimal policy under IO linkages is given by:

$$\begin{aligned}
\text{[domestic subsidy]} \qquad & 1+s^*_{i,k} = (1+\mu_k)(1+\bar{s}_i) \\
\text{[import tax]} \qquad & 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i) \\
\text{[export subsidy]} \qquad & \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}^{-1}_{ij}\left[\mathbf{E}^{(-ij)}_{ij}\left(\mathbf{1}+\mathbf{t}^*_i\right)+\mathbf{\Lambda}_{ij}(1+\bar{t}_i)\right],
\end{aligned}$$

where $\mathbf{\Lambda}_{ij} \equiv \left[\Lambda_{ij,k}\right]_k$, with $\Lambda_{ij,k}$ denoting the fraction of good ij,k's value that is reimported via the IO network.[33] The above formula indicates that export subsidies are relatively higher on upstream industries for which the subsidy is partially passed back to domestic consumers. This detail aside, the *ToT-improving* motive for policy still requires a contraction of exports in low-σ industries, whereas the *misallocation-correcting* motive asks for an expansion of output in high-μ industries. So, unless downstream industries exhibit a systemically lower σ, Proposition 1 maintains its validity. That is, if $\mathrm{Cov}_k(\sigma_k, \mu_k) < 0$, improving allocative efficiency with trade policy worsens the ToT.

5.3 Extension #2: Accounting for Political Economy Pressures

A possible challenge to Proposition 1 is that optimal policy choices are often influenced by political pressures. To address this concern, we follow Ossa's (2014) adaptation of Grossman and Helpman (1994). In particular, our baseline analysis assumed that the government in country i maximizes $W_i \equiv V_i(w_iL_i + \mathcal{R}_i + \Pi_i, \tilde{\mathbf{P}}_i)$, where $\Pi_i \equiv \overline{\mu}_i w_i L_i$ denotes total profits in economy i. Now, we assume instead that the government maximizes a politically-weighted welfare function, $W_i \equiv V_i(w_iL_i + \mathcal{R}_i + \sum_k \pi_{i,k}\Pi_{i,k}, \tilde{\mathbf{P}}_i)$; where $\pi_{i,k}$ is the political economy weight assigned to industry k's profits (with $\sum_k \pi_{i,k}/K = 1$). It follows trivially from Theorem 1 that the first-best optimal policy in the political setup is given by

$$\begin{aligned}
\text{[domestic subsidy]} \qquad & 1+s^*_{i,k} = (1+\mu^{\mathcal{P}}_{i,k})(1+\bar{s}_i) \\
\text{[import tax]} \qquad & 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i) \\
\text{[export subsidy]} \qquad & \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}^{-1}_{ij}\mathbf{E}^{(-ij)}_{ij}\left(\mathbf{1}+\mathbf{t}^*_i\right);
\end{aligned}$$

where $\mu^{\mathcal{P}}_{i,k} = \frac{\mu_k}{\pi_{i,k}-(1-\pi_{i,k})\mu_k}$ is the *political economy-adjusted* markup of industry k. Considering the above formulas: if $\mathrm{Cov}_k(\pi_{i,k}, \mu_k)<0$, the optimal policy may tax high-μ industries to the detriment of social welfare. In that case, even if $\mathrm{Cov}_k(\sigma_k, \mu_k) > 0$, the misallocation-correcting and ToT motives for trade taxation will clash. However, if $\mathrm{Cov}_k(\pi_{i,k}, \mu_k)\geq 0$ Proposition 1 remains valid despite political economy pressures.[34]

6 Estimating the Key Policy Parameters

It should be clear by now that computing the gains from policy requires credible estimates for (i) the industry-level trade elasticities, $\sigma_k - 1$, which reflect the degree of *national-level* mar-

[33] If country i is a small open economy, $\Lambda_{ij,k} \approx \mathbf{0}$. Correspondingly, optimal policy formulas for a small open economy under IO linkages perfectly overlap with the baseline formulas specified under Equation 12.

[34] The above formula has another basic implication: When governments are politically-motivated, their policy objectives may no longer align insofar as domestic policies are concerned. In particular, some governments may assign a greater weight to low-profit industries such as agriculture. These political economy considerations can prompt governments to subsidize and promote the wrong domestic industries from the perspective of the rest of the world. This concern is at the root of some existing opposition to industrial policy.

ket power in industry k; and (ii) the industry-level scale elasticities, $\mu_k = 1/(\gamma_k - 1)$, which reflect the degree of *firm-level* market power in industry k. The trade literature has paid considerable attention to estimating σ_k, but less attention has been devoted to disentangling μ_k (or γ_k) from σ_k. Instead, μ_k is often disciplined according to following two normalizations: (i) $\mu_k = 1/(\sigma_k - 1)$ in monopolistically competitive models, and (ii) $\mu_k = 0$ in perfectly competitive models.[35] The latter normalization eliminates market imperfections. The former allows researchers to pin down μ_k based on existing estimates for the trade elasticity. But as noted by Benassy (1996), such a choice creates an arbitrary link between the market power of individual firms and the national-level market power in each industry.[36]

To credibly analyze the *ToT-improving* and *misallocation-reducing* gains from policy, we need to (a) estimate σ_k and μ_k, and (b) make sure that these estimates are mutually-consistent. To this end, we propose a new methodology that simultaneously estimates σ_k and μ_k from the same data.[37] Our approach involves fitting the structural *firm-level* import demand function implied by A1 to the universe of Colombian import transactions from 2007–2013. We outline this approach below, starting with a description of the data used in our estimation.

Data Description. Our estimation uses data on import transactions from the Colombian Customs Office for the 2007–2013 period.[38] The data include detailed information about each transaction, such as the Harmonized System 10-digit product category (HS10), importing and exporting firms, f.o.b. (free on board) and c.i.f. (customs, insurance, and freight) values of shipments in US dollars, quantity, unit of measurement (of quantity), freight in US dollars, insurance in US dollars, value-added tax in US dollars, country of origin, and weight. A unique feature of this data set is that it reports the identities of all foreign firms exporting to Colombia, allowing us to define import varieties as firm-product combinations—in comparison, most papers focusing on international exports to a given location typically treat varieties as more aggregate country-product combinations. Table 5 (in the appendix) reports a summary of basic trade statistics in our data.[39]

When working with the above data set, we face two challenges: First, exporters are not identified by a unique standardized ID. Instead, they are identified by a name, a number, and an address.[40] We deal with this problem by standardizing the spelling and the name lengths along with utilizing information on firms' phone numbers, with details are provided

[35] See Ossa (2016) and Costinot and Rodríguez-Clare (2014) for a synthesis of the previous literature.

[36] Beyond the trade policy literature, De Loecker and Warzynski (2012), Edmond et al. (2015), and De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) use firm-level production data to estimate the degree of firm-level market power within industries. The production-side approach implicitly pins down μ_k without determining σ_k. The challenge for trade policy analysis is to jointly estimate σ_k and μ_k with the same data.

[37] In the presence of firm-selection effects, our estimated parameters are necessary but not sufficient to pin down the trade and scale elasticities. In addition to our estimated parameters, we need information on the shape of the Pareto productivity distribution—see Appendixes D and P for details.

[38] The data is obtained from DATAMYNE, a company that specializes in documenting import and export transactions in Americas. For more detail, please see `www.datamyne.com`.

[39] Our estimation also employs data on monthly average exchange rates, which are taken from the Bank of Canada: `http://www.bankofcanada.ca/rates/exchange/monthly-average-lookup/`.

[40] The identification of the Colombian importing firms is standardized by the national tax ID number. For the foreign exporting firms, the data provide the name of the firm, phone number, and address. The names of the firms are not standardized, and thus there are instances in which the name of a firm and its address are recorded differently (e.g., using abbreviations, capital and lower-case letters, dashes, etc.).

in Appendix I Second, for some products, Colombia has been changing the HS10 classification between 2007 and 2013. Fortunately, the Colombian Statistical Agency, DANE, has kept track of these changes,[41] and we utilized this information to concord the Colombian HS10 codes over time. In the process, we follow the guidelines provided by Pierce and Schott (2012) for the concordance of the U.S. HS10 codes over time.[42] Overall, changes in HS10 codes between 2007 and 2013 affect a very small portion (less than 0.1%) of our dataset.

6.1 Estimating Equation

Since we are focusing on one importer, we hereafter drop the importer's subscript i and add a year subscript t to account for the time dimension of our data. With this switch in notation, the demand facing firm ω located in country j and supplying product k in year t is given by:

$$q_{j,kt}(\omega) = \varphi_{j,kt}(\omega)\left(\frac{\tilde{p}_{j,kt}(\omega)}{\tilde{P}_{j,kt}}\right)^{-\gamma_k}\left(\frac{\tilde{P}_{j,kt}}{\tilde{P}_{kt}}\right)^{-\sigma_k} Q_{kt}, \tag{14}$$

Note that subscript k has been used thus far to reference industries. But in our empirical analysis, k will designate the most disaggregated industry/product category in our dataset, which is an HS10 product category. The quadruplet "ωjkt" therefore denotes a unique imported variety corresponding to *firm ω–country of origin j–HS10 product k–year t*. Rearranging Equation 14, we can produce the following equation

$$q_{j,kt}(\omega) = \varphi_{jkt}(\omega)\ \tilde{p}_{j,kt}(\omega)^{-\sigma_k}\ \lambda_{j,kt}(\omega)^{1-\frac{\sigma_k-1}{\gamma_k-1}}\ \tilde{P}_{kt}^{\sigma_k} Q_{kt}, \tag{15}$$

where $\lambda_{j,kt}(\omega)$ denotes the share of expenditure on firm ω *conditional* on buying good k from country of origin j,

$$\lambda_{j,kt}(\omega) \equiv \varphi_{j,kt}(\omega)\left(\frac{\tilde{p}_{j,kt}(\omega)}{\tilde{P}_{j,kt}}\right)^{1-\gamma_k} = \frac{\tilde{p}_{j,kt}(\omega)q_{j,kt}(\omega)}{\sum_{\omega'\in\Omega_{j,kt}}\tilde{p}_{j,kt}(\omega')q_{j,kt}(\omega')}.$$

Taking logs from Equation 15, noting that $\mu_k \equiv 1/(\gamma_k - 1)$, and letting $\tilde{x}(\omega) \equiv \tilde{p}(\omega)q(\omega)$ denote sales, yields the following firm-level import demand function:

$$\ln\tilde{x}_{j,kt}(\omega) = (1-\sigma_k)\ln\tilde{p}_{j,kt}(\omega) + [1-\mu_k(\sigma_k-1)]\ln\lambda_{j,kt}(\omega) + \delta_{kt} + \varphi_{j,kt}(\omega), \tag{16}$$

where $\delta_{kt} \equiv \ln P_{kt}^{\sigma_k} Q_{kt}$ can be treated as a product-year fixed effect. We assume that $\varphi_{jkt}(\omega) = \bar{\varphi}_{j,k}(\omega) + \varphi_{\omega jkt}$ can be decomposed into a time-invariant *firm*×*product*-specific quality component, $\bar{\varphi}_{j,k}(\omega)$, and a time-varying component $\varphi_{\omega jkt}$, that encompasses (i) idiosyncratic variations in consumer taste, (ii) measurement errors, and (iii) omitted variables that account for dynamic demand optimization. To eliminate $\bar{\varphi}_{j,k}(\omega)$ from the estimating equation, we employ a first-difference estimator, which also drops observations pertaining to one-time exporters. We deem the first-difference estimator appropriate given the possibility that $\varphi_{\omega jkt}$'s are sequentially correlated. As a robustness check, we also report estimation results based on a two-ways fixed effects estimator in Appendix L.[43] Stated in terms of first-differences, our

[41] We thank Nicolas de Roux and Santiago Tabares for providing us with this information.

[42] To preserve the industry identifier of the product codes, and in contrast to Pierce and Schott (2012), we try to minimize the number of the synthetic codes. The concordance data and do files are provided in the data appendix.

[43] Following Boehm, Levchenko, and Pandalai-Nayar (2020), the first-difference estimation offers a partial rem-

estimating equation takes the following form

$$\Delta \ln \tilde{x}_{j,kt}(\omega) = (1-\sigma_k)\Delta \ln \tilde{p}_{j,kt}(\omega) + [1-\mu_k(\sigma_k-1)]\,\Delta \ln \lambda_{j,kt}(\omega) + \Delta\delta_{kt} + \Delta\varphi_{\omega jkt}, \qquad (17)$$

where $\Delta\varphi_{\omega jkt}$, crudely speaking, represents a variety-specific demand shock; and $\Delta\delta_{kt}$ is a product-year fixed effect.[44] Of the remaining variables, $\Delta \ln \tilde{p}_{j,kt}(\omega)$ and $\Delta \ln \tilde{x}_{j,kt}(\omega)$ are directly observable for each import variety, while changes in the *within-national* market share, $\Delta \ln \lambda_{j,kt}(\omega)$, can be calculated using the universe of firm-level sales to Colombia.[45]

As noted earlier, k indexes an HS10 product category in Equation 17. To conduct our forthcoming quantitative analysis, however, we need to estimate σ_k and μ_k for 14 broadly-defined industries based on the World Input-Output Database (WIOD) classification. Considering this, we pool all HS10 products belonging to the same WIOD industry κ together, and estimate Equation 17 on this pooled sample assuming that σ_k and μ_k are uniform across products within the same industry (i.e., $\mu_k = \mu_\kappa$ and $\sigma_k = \sigma_\kappa$ for all $k \in \mathbb{K}_\kappa$). In principle, we can also estimate the import demand function separately for each HS10 product category to attain HS10-level elasticities. However, such elasticities will be of little use for our quantitative policy analysis, as multi-country data on trade, production, and expenditure shares are scarce at such levels of disaggregation.

Before moving forward, a discussion regarding the role of $\mu_k(\sigma_k-1)$ in Equation 17 is in order. $\mu_k(\sigma_k-1) = \frac{\sigma_k-1}{\gamma_k-1}$ corresponds to the spread between the national-level and firm-level degrees of market power. Broadly speaking, variety ωjkt is (i) either imported from a thick market like China in which case it competes with many other Chinese varieties, hence a low $\lambda_{j,kt}(\omega)$, or (ii) it is imported from a thin market like Taiwan where it competes with a few other Taiwanese varieties, hence a high $\lambda_{j,kt}(\omega)$. If—after controlling for prices—varieties originating from thick markets generate lower sales, it should be the case that goods from the same origin country are relatively more substitutable. Accordingly, our import demand function identifies this case as one where $1 > \frac{\sigma_k-1}{\gamma_k-1} = \mu_k(\sigma_k-1) > 0$.[46]

6.2 Identification Strategy

The identification challenge we face is that $\Delta \ln \tilde{p}_{j,kt}(\omega)$ and $\Delta \ln \lambda_{j,kt}(\omega)$ are endogenous variables that can covary with the demand shock, $\Delta\varphi_{\omega jkt}$.[47] Traditional *country-level* import demand estimations overcome a similar challenge by instrumenting for prices with plausibly

edy for omitted variable bias and reverse causality due to endogenous policy choices. Both of these issues pose a serious challenge to traditional log-level estimations of import demand. Depending on the application, though, the first-difference estimator may not necessarily identify the desired long-run elasticity. As detailed in Appendix L, this limitation is less severe in our *firm-level* estimation—as we explicitly control for the extensive margin of trade and utilize the cross-sectional variation in firm-level price and demand changes within *product-year* cells. We illustrate this point formally in Appendix L by re-estimating Equation 16 in levels and comparing the estimation results to the baseline values. This point notwithstanding, our theoretical model accommodates both the free- and restricted-entry cases and, therefore, speaks to various policy horizons.

[44] In the previous version of the paper, we included all observations that reported a non-missing $\Delta \ln \tilde{p}_{j,kt}$. Some of our industry-level estimates, however, display sensitivity to outlier observations. Considering this, our current estimation trims the sample to exclude observations that report a price change, $\Delta \ln \tilde{p}_{j,kt}$, above the 99th percentile or below the 1st percentile of HS10 product code k in year t.

[45] Some readers may notice parallels between our estimating equation and the nested demand function analyzed in Berry (1994); but given the structure of our data, we adopt a distinct identification strategy.

[46] Note that only $\mu_k(\sigma_k-1) \leq 1$ is consistent with our earlier assumption on the uniqueness of the equilibrium. As we will see shortly, this condition is satisfied by our estimated parameter values.

[47] Another challenge is that unit price data may be contaminated with measurement errors, as they are averaged

exogenous tariff rates.[48] This strategy, however, does not suit our *firm-level* estimation, because tariffs discriminate by country-of-origin but not across firms from the same country.

To achieve identification, we need to construct a *firm×origin×product×year*-specific cost shifter that is uncorrelated with $\Delta\varphi_{\omega jkt}$. To this end, we capitalize on the monthly frequency of import transactions in our data. We compile an external database on aggregate monthly exchange rates and interact the monthly variation in aggregate exchange rates with the lagged monthly composition of firm-level exports to construct the following *shift-share* instrument:

$$z_{j,kt}(\omega) = \sum_{m=1}^{12} \frac{\tilde{x}_{j,kt-1}(\omega;m)}{\tilde{x}_{j,kt-1}(\omega)} \Delta\mathcal{E}_{jt}(m).$$

In the above expression, $\Delta\mathcal{E}_{jt}(m) \equiv \mathcal{E}_{jt}(m) - \mathcal{E}_{jt}(m-1)$ denotes the change in country j's exchange rate with Colombia in month m of year t; $\tilde{x}_{j,kt-1}(\omega;m)$ denotes the month m sales of firm ω (from origin country j in product category k) in the prior year, $t-1$; and $\tilde{x}_{j,kt-1}(\omega) = \sum_{m=1}^{12} \tilde{x}_{j,kt-1}(\omega;m)$ denotes firm ω's total annual export sales in $t-1$. Crudely speaking, $z_{j,kt}(\omega)$ measures the *firm×origin×product×year*-specific exposure to exchange rate shocks. The idea being that aggregate exchange rate movements have differential effects on different firms depending on the monthly composition of their prior export activity to Colombia.[49] Encouragingly, this idea is backed by the fact that z and $\Delta \ln \tilde{p}$ exhibit a strong and statistically significant correlation in our data. Appendix J illustrates this point using the example of two U.S.-based exporting firms.

Our instrument utilizes lagged monthly sales, $\tilde{x}_{jkt-1}(\omega, m) = \tilde{x}(\tilde{p}_{jkt-1}(\omega, m); ...)$, which depend on lagged prices and other market-level indexes. Therefore, the exclusion restriction, $\text{Corr}\,[z, \Delta\varphi] = 0$, hinges on two identifying assumptions:

(a1) Prior price-setting decisions are orthogonal to concurrent demand shocks: $\text{Corr}\left[\tilde{p}_{jkt-1}(\omega), \Delta\varphi_{\omega jkt}\right] = 0$.

(a2) National-level exchange rate movements are orthogonal to variety-level demand shocks: $\text{Corr}\left[\Delta\mathcal{E}_{jt}(m), \Delta\varphi_{\omega jkt}\right] = 0$.

These assumptions can be challenged if there are cross-inventory linkages or if individual export varieties account for a significant fraction of a country's total exports to Colombia. We will discuss and address these identification challenges in Section 6.4.

Instruments for $\Delta \ln \lambda_{j,kt}(\omega)$. Following Khandelwal (2010), we construct two standard instruments for the annual variation in the within-national market shares: (i) annual changes in the total number of origin j firms serving the Colombian market in product category k, and (ii) changes in the total number of HS10 product categories actively served by firm ω in year t. These count measures will be correlated with $\Delta \ln \lambda_{j,kt}(\omega)$ but uncorrelated with $\Delta\varphi_{\omega jkt}$ if

across many transactions. Following Berry (1994), this type of measurement error is fairly innocuous when dealing with log-linear demand functions. Furthermore, our instrumental variable approach will handle measurement errors, provided that lagged monthly sales patterns are uncorrelated with concurrent measurement errors.

[48] A prominent example is Caliendo and Parro (2015) who use tariff data to identify the trade elasticity.

[49] One can draw parallels between our instrument and the widely-used *Bartik* instrument. The latter builds on the idea that different regions are affected differentially by national labor market shocks depending on their initial industrial composition of the local labor market.

variety-level entry and exit occurs prior to, or independent of, the demand shock realization of competing varieties. As noted by Khandelwal (2010), this assumption is widely-invoked when estimating discrete choice demands curves—see also Berry, Levinsohn, and Pakes (1995).[50]

6.3 Estimation Results

The industry-level estimation results are reported in Table 2. We also report estimation results corresponding to a pooled sample of all industries in Table 7 of the appendix. This table also compares the 2SLS and OLS estimates to ensure that our IV strategy is operating in the expected direction. Our estimates point to an average trade elasticity of $\sigma - 1 = 3.6$ and an average scale elasticity of $\mu \approx 0.19$. Our pooled estimation yields a heteroskedasticity-robust Kleibergen-Paap Wald rk F-statistic of 95. Hence, we can reject the null of weak instruments given the Stock-Yogo critical values. A similar, albeit weaker, argument applies to the industry-level estimation.

The industry-level elasticities reported in Table 2 display considerable variation across industries. The estimated scale elasticity or markup margin is highest in the 'Electrical & Optical Equipment' ($\mu = 0.45$) and 'Petroleum' ($\mu = 1.7$) industries; both of which are associated with high R&D or fixed costs. The estimated scale elasticity is lowest in 'Agricultural & Mining' ($\mu = 0.14$) and 'Machinery' ($\mu = 0.10$) industries. Furthermore, with the exception of 'Agriculture & Mining,' we cannot reject the prevalence of scale economies. The finding that returns-to-scale are negligible in the agricultural sector aligns with a large body of evidence on the inverse farm-size productivity (IFSP) relationship—see Sen (1962) and subsequent references to IFSP.

Our industry-level trade elasticity estimates, $\sigma_k - 1$, also display some novel properties. To our knowledge, our estimation is the first to identify the industry-level trade elasticities using (i) firm-level data, and (ii) while controlling for intra-national cross-demand effects. Qualitatively speaking, our estimated trade elasticities are similar in magnitude to those estimated by Simonovska and Waugh (2014) but slightly lower than those estimated by Caliendo and Parro (2015). Aside from the firm-level nature of our estimation, these differences may be driven by the fact that instead of controlling for f.o.b. prices with exporter fixed effects, we directly use data on f.o.b. price levels.

Importantly, our estimates indicate that $\mu_k(\sigma_k - 1) = \frac{\sigma_k - 1}{\gamma_k - 1} < 1$ in nearly all industries. This finding rejects the arbitrary link often assumed between the firm-level and national-level degrees of market power in the literature.[51] Our estimates also indicate that $\mathrm{Cov}_k(\sigma_k, \mu_k) < 0$,

[50] Although trade taxes count as a weak instrument in our firm-level estimation, we nonetheless include them as an additional instrument to comply with the literature. These taxes include applied ad-valorem tariffs and the Columbian value-added tax (VAT). We exclude the VAT in the 'Transportation' and 'Petroleum' industries since the VAT in these industries discriminates by the method of delivery and level of luxury, both of which may be correlated with $\Delta\varphi_{\omega jkt}$.

[51] In other words, our estimation rejects the *independence of irrelevant alternatives* (IIA) because product varieties or technologies are less differentiated intra-nationally than inter-nationally. While the IIA assumption has garnered considerable attention in the industrial organization literature, the trade literature has only recently tested this assumption against data. Redding and Weinstein (2016) estimate an international demand system that relaxes the IIA assumption by accommodating heterogeneous taste across consumers. Adao, Costinot, and Donaldson (2017) estimate a trade model that (unlike standard CES models) permits varieties from certain countries to be closer substitutes. Our results contribute to this emerging literature by highlighting another aspect of the trade data that is at odds with the IIA assumption.

which empirically confirms the innate tension between the *ToT-improving* and *misallocation-correcting* motives for trade taxation, as highlighted by Proposition 1.

Table 2: *Industry-level estimation results*

Sector	ISIC4 codes	Estimated Parameter $\sigma_k - 1$	$\frac{\sigma_k - 1}{\gamma_k - 1}$	μ_k	Obs.	Weak Ident. Test
Agriculture & Mining	100-1499	6.212	0.875	0.141	11,962	2.51
		(2.112)	(0.142)	(0.167)		
Food	1500-1699	3.333	0.883	0.265	20.042	6.00
		(0.815)	(0.050)	(0.131)		
Textiles, Leather & Footwear	1700-1999	3.413	0.703	0.207	126,483	63.63
		(0.276)	(0.020)	(0.022)		
Wood	2000-2099	3.329	0.899	0.270	5,962	1.76
		(1.331)	(0.181)	(0.497)		
Paper	2100-2299	2.046	0.813	0.397	37,815	2.65
		(0.960)	(0.216)	(0.215)		
Petroleum	2300-2399	0.397	0.698	1.758	4,035	2.03
		(0.342)	(0.081)	(1.584)		
Chemicals	2400-2499	4.320	0.915	0.212	134,413	42.11
		(0.376)	(0.027)	(0.069)		
Rubber & Plastic	2500-2599	3.599	0.582	0.162	107,713	7.22
		(0.802)	(0.041)	(0.039)		
Minerals	2600-2699	4.561	0.847	0.186	28,197	3.19
		(1.347)	(0.096)	(0.129)		
Basic & Fabricated Metals	2700-2899	2.959	0.559	0.189	155,032	16.35
		(0.468)	(0.024)	(0.032)		
Machinery	2900-3099	8.682	0.870	0.100	266,628	8.54
		(1.765)	(0.080)	(0.065)		
Electrical & Optical Equipment	3100-3399	1.392	0.631	0.453	260,207	17.98
		(0.300)	(0.015)	(0.099)		
Transport Equipment	3400-3599	2.173	0.289	0.133	86,853	5.09
		(0.589)	(0.028)	(0.036)		
N.E.C. & Recycling	3600-3800	6.704	0.951	0.142	70,974	8.51
		(1.133)	(0.100)	(0.289)		

Notes. Estimation results of Equation (17). Standard errors in parentheses. The estimation is conducted with HS10 product-year fixed effects. All standard errors are simultaneously clustered by product-year and by origin-product, which is akin to the correction proposed by Adao, Kolesár, and Morales (2019). The weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist, Imbens, and Rubin (1996).

6.4 Challenges to identification

Our man identifying assumptions, (a1), and (a2), can be challenged under certain circumstances. Below, we discuss these challenges and present additional evidence to address them.

Within-Cluster Correlation in Error Terms. Adao et al. (2019) show that identification based on shift-share instruments exhibits an over-rejection problem if regression errors are cross-correlated. In the context of our estimation, this problem will arise if demand shocks are correlated across *firm-origin-product-year* varieties with a similar monthly export composition. To handle this issue, we adopt a conservative two-way clustering of standard errors by *product-year* and *origin-product*. Clustering standard errors this way is akin to the correction proposed by Adao et al. (2019).

Cross-Inventory Effects. Lags in inventory clearances can challenge our identifying assumptions on two fronts. First, firms' optimal pricing decisions may be forward-looking, which violates assumption (a1). To address this concern, we reconstruct our shift-share instrument using 4 lags instead of 1. If inventories clear in at most 4 years, we can deduce that pricing decisions do not internalize expected demand shocks beyond the 4-year mark. Hence, $\text{Corr}\left[\tilde{p}_{jkt-4}(\omega), \Delta\varphi_{\omega jkt}\right] = 0$, and the new instrument will satisfy the exclusion restriction. The trade-off is that using such an instrument amounts to losing more observations, as the instrument can be constructed for firms that continuously export in the 4 different years. The *top* panel of Figure 4 (in Appendix K) compares estimation results under this alternative instrument to the baseline results. The ordering and magnitude of the estimated elasticities are rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption underlying Proposition 1.

Second, with cross-inventory effects, $\Delta\varphi_{\omega jkt}$ may encompass omitted variables that govern firms' dynamic inventory management decisions. One of these omitted variables is presumably the exchange rate. If so, $\text{Corr}\left[\Delta\mathcal{E}_{jt}(m), \Delta\varphi_{\omega jkt}\right] \neq 0$, and our identifying assumption (a2) will be violated. To address this concern, we reestimate Equation 17 while directly controlling for the annual change in the exchange rate, $\Delta\mathcal{E}_{jt}$. Even if changes in inventory-related demand depend on the changes in exchange rate, we can still assert that $\text{E}\left[z_{j,kt}(\omega)\ \Delta\varphi_{\omega jkt} \mid \Delta\mathcal{E}_{jt}\right] = 0$—i.e., the exclusions restriction is satisfied with the added control, $\Delta\mathcal{E}_{jt}$. The *middle* panel of Figure 4 (in Appendix K) compares estimation results from this alternative specification to the baseline results. Reassuringly, the ordering and magnitude of the estimated elasticities are preserved under the new specification; and so is the negative correlation between σ_k and μ_k.

Export Varieties with Significant Market Share. Our identification can come under challenge if individual varieties account for a significant fraction of a country's sales to Colombia. In such a case, variety-specific demand shocks can influence the bilateral exchange between the Colombian Peso and the origin country's currency, thereby violating identifying assumption (a2). This concern, however, does not apply to our sample of exporters. The variety with the highest 99th percentile within-national market share accounts for only 0.1% of the origin country's total exports to Colombia. The variety with the highest 90th percentile within-national market share accounts for only 0.0008% of the origin country's total exports to Colombia.

One may remain concerned about large multi-product firms that export multiple product varieties to Colombia in a given year. Consider, for instance, a multi-product firm ω that exports goods k and g to Colombia in year t. If demand shocks are correlated across varieties supplied by this firm (i.e., $\mathrm{E}\left[\Delta\varphi_{\omega jkt}\,\Delta\varphi_{\omega jgt}\right] \neq 0$), Assumption (a2) may be violated despite each variety's market share being infinitesimally small. To address this issue, we reestimate Equation 17 on a restricted sample that drops excessively large firms with a total within-national market share that exceeds 0.1%. The *bottom* panel of Figure 4 (in Appendix K) compares estimation results from the trimmed sample to the baseline results. Encouragingly, the ordering and magnitude of the estimated elasticities are preserved across industries. The new estimation also retains the negative correlation between σ_k and μ_k.

6.5 Plausibility of Estimates

Before moving on to the quantitative analysis, let us briefly discuss the plausibility of our estimates. We do so by exploring their macro-level implications and by benchmarking them against alternative estimates in the literature.

Plausibility from the Lens of Macro-Level Predictions. Our estimated elasticities deliver sharp predictions about the cross-national income-size elasticity. As pointed out by Ramondo, Rodríguez-Clare, and Saborío-Rodríguez (2016), the factual relationship between real per capita income and population size (i.e., the income-size elasticity) is negative and statistically insignificant. Quantitative trade models featuring the normalization $\mu(\sigma-1) = \frac{\sigma-1}{\gamma-1} = 1$, however, predict a strong and positive income-size elasticity that remains significant even after the introduction of domestic trade frictions. Ramondo et al. (2016) call this observation the income-size elasticity puzzle. Considering this puzzle, in Appendix M we compute the income-size elasticity implied by our estimated value of $\frac{\sigma-1}{\gamma-1} \approx 0.67$. Encouragingly, we find that our estimated value for $\frac{\sigma-1}{\gamma-1}$ completely resolves the aforementioned puzzle. In other words, our micro-estimated elasticities are consistent with the macro-level cross-national relationship between population size and real per capita income.

Comparison to Industry-Specific Estimates in the Literature. Reassuringly, our estimates align closely with well-known industry-level case studies. Take, for example, our elasticity estimates for the 'Petroleum' industry, which appear somewhat extreme. First, our estimate for σ_k aligns with the consensus in the Energy Economics literature that national-level demand for petroleum products is price-inelastic.[52] Second, our estimated μ_k for the 'Petroleum' industry closely resembles existing estimates in the Industrial Organization literature. Considine (2001), for instance, estimates $\mu \approx 2$ using detailed data on the U.S. petroleum industry. Moreover, our finding that the 'Petroleum' industry is the most scale-intensive industry is consistent with the finding in Antweiler and Trefler (2002), which is based on more aggregated data. Likewise, consider the 'Transportation' or auto industry, where our estimated $\mu_k = 0.13$ implies an optimal markup of 13%. This estimate aligns with existing estimates from various industry-level studies. Recently, Coşar, Grieco, Li, and Tintelnot (2018) have estimated markups for the auto industry that range between roughly 6% to 13%. Previously, Berry et al. (1995) have estimated markups of around 20% in the U.S. auto industry using data from 1971-1990.

[52] See Pesaran, Smith, and Akiyama (1998) for specific estimates and Fattouh (2007) for a survey of this literature.

Comparison to Bartelme et al. (2019). Concurrent with us, Bartelme et al. (2019, BCDR) have developed an alternative methodology to estimate the scale elasticity. In particular, they estimate the elasticity of export sales with respect to industry-level employment, namely α_k^{BCDR}. From the lens of gravity trade models, this elasticity assumes the following interpretation:

$$\alpha_k^{BCDR} = (\psi_k + \mu_k)(\sigma_k - 1) - \beta_k,$$

where ψ_k denotes the industry-level agglomeration elasticity and β_k denotes the share of industry-specific factors in total production.

BCDR's estimation implicitly assumes that there are *no* are industry-specific factors of production, which amounts to $\beta_k = 0$. They borrow estimates for the trade elasticity, $\sigma_k - 1$, from several sources in the literature and recover the scale elasticity as $\mu_k + \psi_k = \alpha_k^{BCDR}/(\sigma_k - 1)$. The advantage of their approach is that it detects Marshallian (or agglomeration) externalities. The advantages of our approach are two-fold: (a) we separately identify σ_k from μ_k;[53] and (b) our estimates are robust to the presence of industry-specific factors of production, i.e., our identification does not rely on $\beta_k = 0$.[54] These differences can also explain why we estimate a larger scale elasticity than BCDR in some industries. Consider, for instance, the 'Petroleum' industry where we estimate a considerably larger scale elasticity than BCDR. This difference can be driven by the 'Petroleum' industry employing a large amount of industry-specific factors of production, like natural resources or offshore platforms. This situation corresponds to a high β_k that can attenuate the estimates in BCDR relative to ours.

7 Quantifying the Gains from Policy

As a final step, we use our estimated values for μ_k and σ_k to quantify the gains from trade and industrial policy for a wide range of countries. Before outlining our quantitive approach, we describe the macro-level data used to discipline our quantitative model.

Trade, Production, and Tariff Data. We take macro-level data on domestic and international production and expenditure from the 2014 World Input-Output Database (WIOD, Timmer, Erumban, Gouma, Los, Temurshoev, de Vries, Arto, Genty, Neuwahl, Francois, et al. (2012)). This database spans 56 industries and 43 countries plus an aggregate of the rest of the world. The list of countries in the sample includes all 27 members of the European Union plus 16 other major economies—all of which are listed in Table 3. Following Costinot and Rodríguez-Clare (2014), we aggregate the 56 WIOD industries into 15 traded industries (for which we have estimated μ_k and σ_k) plus a service sector. Details for our industry aggregation are reported in Table 8 of the appendix. Our baseline analysis normalizes $\mu_k = 0$ and $\sigma_k = 11$ for all service-related industries. In Appendix P, however, we test the sensitivity of our results to alternative normalization choices. Finally, we complement the WIOD data with matching data on applied tariffs from the UNCTAD-TRAINS database. In this process, we closely follow the cleaning

[53] As noted in Appendix D, in the presence of selection effect, our approach can identify the scale elasticity only up-to an externally estimated trade elasticity. Second, our approach can identify the scale elasticity even when there are diseconomies of scale at the industry-level.

[54] Our estimation of μ_k and σ_k relies solely on Assumption (A1). Our estimates are, thus, compatible with an arbitrary production function that admits multiple (and possibly industry-specific) factors of production.

and matching procedure described in Kucheryavyy et al. (2016).[55]

7.1 Mapping Optimal Policy Formulas to Data

The sufficient statistics formulas provided by Theorems 1-3, let us compute the gains from *optimal* policy free of numerical optimization. This feature is particularly advantageous, because numerical optimization routines (like MATLAB's FMINCON) have well-know limitations when applied to non-linear models with many free-moving variables.[56] Below, we present the procedure by which the gains from policy are computed using our optimal policy formulas. We present our procedure for the case of first-best policies under free entry—a similar procedure can be applied to the other cases with small modifications.

To map our theory to data, we need to take a stance on the cross-industry utility aggregator. As is common in literature, we assume a Cobb-Douglas parmaterization, $U_i(\mathbf{Q}_i) = \prod_k Q_{i,k}^{e_{i,k}}$. As explained earlier, we posses data on observable shares, national accounts, and applied taxes. We use $\mathbb{D} = \left\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\right\}_{j,i,k}$ to denote such data.[57] We have estimated the trade and scale elasticity across many industries, and use $\Theta = \{\sigma_k, \mu_k\}$ to denote our set of estimated parameters.

The main idea behind our procedure is to express optimal tax/subsidy formulas in changes and solve them alongside equilibrium conditions. To this end, we use the exact hat-algebra notation, whereby $\hat{z} = z^*/z$ denotes the change in a generic variable when moving from the factual value, z, to the counterfactual value under optimal policy, z^*.

As discussed under Theorem 1, country i's first-best policy schedule in the Cobb-Douglas case is described by the following formulas:

$$1 + s_{i,k}^* = 1 + \mu_k; \qquad 1 + t_{ji,k}^* = 1 + \omega_{ji,k}^*; \qquad 1 + x_{ij,k}^* = \frac{(\sigma_k - 1)\sum_{n \neq i}\left[(1 + \omega_{ni,g}^*)\lambda_{nj,k}^*\right]}{1 + (\sigma_k - 1)(1 - \lambda_{ij,k}^*)};$$

where the "$*$" superscript indicates that a variable is being evaluated in the counterfactual optimal policy equilibrium. Using the hat-algebra notation and our expression for the good-specific supply elasticity, $\omega_{ji,k}$ (Equation 10), we can write the above formulas in changes as

[55]To make the final data consistent with our theoretical model, we need to purge it from trade imbalances. We, therefore, rebalance our raw data using the procedure described in Costinot and Rodríguez-Clare (2014).

[56]Costinot and Rodríguez-Clare (2014) note that computing optimal policy via numerical optimization can become increasingly burdensome when dealing with many free-moving tax instruments. Their optimal tariff analysis is, therefore, limited to a uniform tariff applied to all industries (see P. 227 and the discussion following Figure 4.1)

[57]As explained in Section 2, under free entry, the number of firms operating in *origin n–industry k* can be expressed as $M_{i,k} = \bar{m}_{i,k}\rho_{i,k}$, where $\bar{m}_{i,k}$ is composed of parameters and variables that are invariant to policy. We can, therefore, use $\rho_{i,k}$ to track scale economies that channel through entry—as detailed under Equation 6.

follows:[58]

$$\begin{aligned}
&[\text{optimal import tax}] \quad 1+t^*_{ji,k} = \frac{-\frac{\mu_k}{1+\mu_k}\hat{r}_{ni,k}r_{ni,k}\Phi'_{ni,k}}{1-\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i}\left(\hat{r}_{n\iota,k}r_{n\iota,k}\left[1+(\sigma_k-1)(1-\hat{\lambda}_{n\iota,k}\lambda_{n\iota,k})\right]\right)} \\
&[\text{optimal export subsidy}] \quad 1+x^*_{ij,k} = \frac{(\sigma_k-1)\sum_{n\neq i}\left[(1+t^*_{ni,g})\hat{\lambda}_{nj,k}\lambda_{nj,k}\right]}{1+(\sigma_k-1)(1-\hat{\lambda}_{ij,k}\lambda_{ij,k})}, \\
&[\text{change in taxes}] \quad \widehat{1+s_{i,k}} = \frac{1+\mu_k}{1+s_{i,k}}; \qquad \widehat{1+t_{ji,k}} = \frac{1+t^*_{ji,k}}{1+t_{ji,k}}; \qquad \widehat{1+x_{ij,k}} = \frac{1+x^*_{ij,k}}{1+x_{ij,k}}.
\end{aligned} \tag{18}$$

Since the rest of world is passive, then $\widehat{1+s_{n,k}} = \widehat{1+t_{jn,k}} = \widehat{1+x_{nj,k}} = 1$ for all $n \neq i$. To determine the change in expenditure shares, $\hat{\lambda}_{ji,k}$, we need to determine the change in consumer price indexes. Invoking the CES structure of within-industry demand, we can express the change in *market i–industry k*'s consumer price index as

$$[\text{price indexes}] \quad \hat{\tilde{P}}_{i,k} = \sum_{n\in\mathbb{C}}\left(\lambda_{ni,k}\left[\frac{\widehat{1+t_{ni,k}}}{(\widehat{1+x_{ni,k}})(\widehat{1+s_{n,k}})}\hat{w}_n\hat{\rho}_{n,k}^{-\mu_k}\right]^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}. \tag{19}$$

Recall that $\rho_{n,k} = L_{n,k}/L_n$ denotes industry k's sales share in origin n, which—under free entry—is equal to the share of origin n's workers employed in that industry. The above formulation uses the fact that, by free entry, $\hat{M}_{i,k} = \hat{\rho}_{i,k}$. Given $\hat{\tilde{P}}_{i,k}$, we can calculate the change in expenditure and revenue shares as follows:

$$\begin{aligned}
&[\text{expenditure shares}] \quad \hat{\lambda}_{ji,k} = \left[\frac{\widehat{1+t_{ji,k}}}{(\widehat{1+x_{ji,k}})(\widehat{1+s_{j,k}})}\hat{w}_j\hat{\rho}_{j,k}^{-\mu_k}\right]^{1-\sigma_k}\hat{\tilde{P}}_{i,k}^{\sigma_k-1} \\
&[\text{revenue shares}] \quad \hat{r}_{ji,k} = \left(\frac{\widehat{1+t_{ji,k}}}{\widehat{1+x_{ji,k}}}\hat{\lambda}_{ji,k}\hat{Y}_i\right)\left(\sum_{n\in\mathbb{C}}\frac{\widehat{1+t_{jn,k}}}{\widehat{1+x_{jn,k}}}\hat{\lambda}_{jn,k}\hat{Y}_n\right)^{-1}.
\end{aligned} \tag{20}$$

The change in the wage rate, $\hat{w}_i$, and industry-level sales shares, $\hat{\rho}_{i,k}$, are dictated by the labor market clearing (LMC) condition, which ensures that industry-level sales match wage payments, industry by industry:

$$[\text{LMC}] \quad \hat{\rho}_{i,k}\rho_{i,k}\hat{w}_iw_iL_i = \sum_{j\in\mathbb{C}}\left[\frac{(1+x^*_{ij,k})(1+s^*_{i,k})}{1+t^*_{ij,k}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_jY_j\right]; \qquad \sum_{k\in\mathbb{K}}\hat{\rho}_{i,k}\rho_{i,k} = 1. \tag{21}$$

The change in national expenditure, $\hat{Y}_i$, is governed by the balanced budget (BB) condition, which ensures that total expenditure matches total income from wage payments and tax revenues:

$$\begin{aligned}
[\text{BB}] \quad \hat{Y}_iY_i =& \hat{w}_iw_iL_i - \sum_{k\in\mathbb{K}}\left[s^*_{i,k}\hat{\lambda}_{ii,k}\lambda_{ii,k}e_{i,k}\hat{Y}_iY_i\right] \\
&+\sum_{j\neq i}\sum_{k\in\mathbb{K}}\left(\frac{t^*_{ji,k}}{1+t^*_{ji,k}}\lambda_{ji,k}\hat{\lambda}_{ji,k}e_{i,k}\hat{Y}_iY_i + \frac{1-(1+x^*_{ij,k})(1+s^*_{i,k})}{1+t^*_{ij,k}}\lambda_{ij,k}\hat{\lambda}_{ij,k}e_{j,k}\hat{Y}_jY_j\right).
\end{aligned} \tag{22}$$

[58]The multiplier $\Phi'_{ni,k} = 1-\left(1-\frac{1}{\mu_k}\right)(\sigma_k-1)\sum_{\iota\neq i}\left[\frac{\hat{r}_{ii,k}r_{ii,k}}{\hat{r}_{ji,k}r_{ji,k}}\hat{\lambda}_{j\iota,k}\lambda_{j\iota,k}\right]\frac{\hat{\rho}_{i,k}\rho_{i,k}\hat{w}_iw_iL_i}{\hat{\rho}_{j,k}\rho_{j,k}\hat{w}_jw_jL_j}$ accounts for cross-demand effects in foreign markets—see Equation 10 from Section 3.

Equations 18-22 represent a system of $2N + NK + (2(N-1)+1)K$ independent equations and unknowns. The independent unknown variables are $\hat{w}_i$ (N unknowns), $\hat{Y}_i$ (N unknowns), $\hat{\rho}_{i,k}$ (NK unknowns), $\widehat{1+s_{i,k}}$ (K unknowns), $\widehat{1+t_{ji,k}}$ ($(N-1)K$ unknowns), and $\widehat{1+x_{ij,k}}$ ($(N-1)K$ unknowns). Solving the aforementioned system is possible with information on observable data points, $\mathbb{D}$, and estimated parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. Once we solve this system, the welfare consequences of country i's optimal policy are automatically determined. The following proposition outlines this result.

Proposition 3. *Suppose we have data on observable shares, national accounts, and applied taxes, $\mathbb{D} = \left\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\right\}_{j,i,k}$, and information on structural parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. We can determine the economic consequences of country i's optimal policy by calculating $\mathbb{X} = \left\{\hat{Y}_i, \hat{w}_i, \hat{\rho}_{i,k}, \widehat{1+s_{i,k}}, \widehat{1+t_{ji,k}}, \widehat{1+x_{ij,k}}\right\}$ as the solution to the system of Equations 18-22. After solving for $\mathbb{X}$, we can fully determine the welfare consequence of country i's optimal policy as*

$$\hat{W}_n = \hat{Y}_n / \prod_{k \in \mathbb{K}} \hat{\tilde{P}}_{n,k}^{e_{n,k}}, \qquad (\forall n \in \mathbb{C})$$

where $\hat{\tilde{P}}_{n,k}$ is determined by Equation 19 as a function of $\mathbb{X}$ and data, $\mathbb{D}$.

To take stock, the optimization-free procedure described by Proposition 3 simplifies the task of computing the gains from *first-best* trade and industrial policies. A similar procedure can be used (in combination with Theorems 2 and 3) to compute the gains from *second-best* trade policies—see Appendix N. Without Proposition 3, we would have to rely on numerical optimization to recover country i's optimal policy.[59] As noted earlier, numerical optimization can become increasingly difficult-to-implement when dealing with many free-moving policy variables. Furthermore, in many instances, obtaining credible results from numerical optimization requires specialized commercial solvers like SNOPT or KNITRO. Propositions 3's optimization-free procedure allows us to bypass any such complications, delivering notable gains in both computational speed and accuracy.

7.2 The Gains from *First-* and *Second-Best* Non-Cooperative Policies

Table 3 reports the gains from optimal non-cooperative policies under free and restricted entry. In all cases, the welfare gains are computed assuming the rest of the world does not retaliate. The first column of each panel reports the gains from the first-best non-cooperative tax schedule (Theorems 1). The second column reports the gains from second-best optimal import and export taxes/subsidies (Theorem 2). The third column reports the gains from third-best optimal import taxes (Theorem 3).

We can draw two general conclusions from Table 3. First, the gains from ToT manipulation are *relatively* small. This is partly evident from the fact that when governments are restricted to only trade taxes, the resulting gains are significantly smaller than the first-best case. We can also directly verify that *pure* ToT gains account for less than 1/3 of the total gains from first-best policy, which average around 1.5% under restricted entry and 2.6% under free entry.[60]

[59] Such a problem is typically formulated as a Mathematical Programming with Equilibrium Constraints (MPEC) problem–see Ossa (2014) for further details.

[60] Note that the gains from policy are larger under free entry than restricted entry, despite similar optimal tax rates. This nuanced observation suggests that *firm-delocation* gains from policy dominate the *profit-shifting* gains.

Table 3: The Gains from Optimal Non-Cooperative Policies

Country	Restricted Entry			Free Entry		
	1st best	2nd best trade taxes	3rd best import taxes	1st best	2nd best trade taxes	3rd best import taxes
AUS	0.82%	0.20%	0.16%	2.08%	0.54%	0.31%
AUT	1.22%	0.65%	0.48%	1.38%	1.18%	0.58%
BEL	1.21%	0.67%	0.49%	1.60%	0.94%	0.53%
BGR	1.94%	0.65%	0.52%	3.46%	1.90%	0.81%
BRA	1.78%	0.25%	0.22%	4.03%	0.63%	0.34%
CAN	1.50%	0.43%	0.30%	3.16%	1.22%	0.44%
CHE	0.94%	0.56%	0.45%	1.24%	0.75%	0.49%
CHN	1.61%	0.26%	0.23%	3.18%	0.39%	0.27%
CYP	1.56%	0.60%	0.57%	5.10%	1.55%	1.34%
CZE	1.66%	0.94%	0.59%	2.11%	1.79%	0.76%
DEU	1.58%	0.75%	0.53%	2.62%	1.45%	0.65%
DNK	1.06%	0.54%	0.43%	1.35%	0.88%	0.46%
ESP	1.40%	0.53%	0.41%	2.29%	1.05%	0.46%
EST	1.06%	0.58%	0.40%	2.49%	1.42%	0.53%
FIN	1.28%	0.51%	0.42%	1.77%	0.84%	0.44%
FRA	1.12%	0.45%	0.33%	1.93%	1.15%	0.50%
GBR	1.02%	0.48%	0.40%	1.97%	1.04%	0.56%
GRC	1.65%	0.55%	0.50%	2.51%	1.03%	0.61%
HRV	0.89%	0.53%	0.43%	1.22%	0.74%	0.49%
HUN	2.22%	1.13%	0.66%	4.01%	2.56%	1.00%
IDN	1.78%	0.30%	0.25%	4.24%	1.46%	0.46%
IND	1.72%	0.38%	0.33%	3.13%	1.12%	0.36%
IRL	0.82%	0.67%	0.47%	1.12%	0.93%	0.39%
ITA	1.39%	0.46%	0.37%	2.50%	0.98%	0.47%
JPN	1.43%	0.38%	0.27%	2.76%	0.79%	0.42%
KOR	2.12%	0.77%	0.57%	2.66%	1.69%	0.70%
LTU	2.31%	0.85%	0.69%	3.06%	1.38%	0.77%
LUX	0.96%	0.87%	0.84%	1.43%	1.19%	1.05%
LVA	0.76%	0.46%	0.36%	1.24%	0.78%	0.45%
MEX	2.10%	0.59%	0.40%	4.80%	1.41%	0.74%
MLT	1.19%	0.84%	0.78%	1.92%	1.36%	1.01%
NLD	1.21%	0.63%	0.48%	1.68%	0.93%	0.52%
NOR	1.03%	0.35%	0.26%	1.58%	0.77%	0.36%
POL	2.07%	0.73%	0.60%	3.86%	1.67%	0.80%
PRT	1.82%	0.63%	0.53%	3.62%	1.73%	0.76%
ROU	1.90%	0.75%	0.63%	3.72%	1.73%	0.95%
RUS	2.24%	0.29%	0.23%	4.55%	1.36%	0.31%
SVK	2.03%	1.10%	0.76%	3.21%	2.29%	1.10%
SVN	1.34%	0.87%	0.65%	2.69%	1.30%	0.88%
SWE	1.10%	0.55%	0.45%	1.49%	0.80%	0.47%
TUR	1.36%	0.51%	0.41%	2.89%	1.39%	0.60%
TWN	2.09%	0.68%	0.52%	2.90%	1.89%	0.75%
USA	1.40%	0.28%	0.23%	2.90%	0.81%	0.29%
Average	1.48%	0.59%	0.46%	2.64%	1.23%	0.61%

Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The *1st best* policy is characterized by Theorem 1; *2nd best trade taxes* are characterized by Theorem 2; and *3rd best import taxes* are characterized by Theorem 3.

Second, trade taxes are a poor *second-best* substitute for industrial subsidies. This conclusion can be drawn from the fact that trade taxes alone can replicate only a third of the welfare gains attainable under the first-best policy choice.[61] Under restricted entry, for instance, the fist-best policy increases welfare by 1.5% on average, whereas second-best trade taxes/subsidies increase welfare by only 0.6%. Third-best import taxes (that are not paired with export subsidies) are even less effective. Similar results apply to the free entry case. This outcome derives from fact that, based on our estimated parameter values, $\text{Cov}_k(\sigma_k, \mu_k) < 0$. Hence, following Proposition 1, there is an innate tension between improving the ToT and restoring allocate efficiency with trade policy. This tension renders second-best trade taxes as an ineffective tool for both correcting misallocation and improving the ToT.

7.3 The Immiserizing Growth Effects of Industrial Policy

As detailed in Section 4, a unilateral adoption of corrective industrial subsidies can worsen welfare, if (i) these subsidies are not paired with appropriate trade tax measures, or (ii) or similar corrective subsides are not implemented in the rest of the world. The intuition is that a unilateral application of corrective (i.e. Pigouvian) industrial subsidies will improve allocative efficiency but worsen the ToT, thereby triggering *immiserizing growth*. Table 4 reports the extent of this problem. The unilateral case corresponds to a scenario where a country unilaterally implements (Pigouvian) industrial subsidies without raising any trade taxes. The multilateral case corresponds to a coordinated adoption of industrial subsidies via a deep trade agreement.

The results in Table 4 suggest that, for the average country, the ToT losses from *unilaterally* applying corrective industrial subsidies outweigh the misallocation-improving gains. That is, industrial policy triggers immezerising growth when carried out unilaterally. This result highlights the importance of international coordination in industrial policies. The failure of international coordination can deter most countries from undertaking corrective subsidies that are ultimately beneficial—a point that has been overlooked by existing critiques of global governance (e.g., Rodrik (2019)).

Table 4: *Industrial Policy and Immiserizing Growth*

	Restricted Entry		Free Entry	
	Unilateral	Coordinated	Unilateral	Coordinated
Gains from *Corrective* Industrial Subsidies	-0.25%	1.20%	-0.70%	3.22%

Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The columns titled *unilateral* reports welfare gains when a country unilaterally adopts industrial subsidies that restore marginal cost pricing in the domestic economy. The columns titled *multilateral* reports welfare gains when all countries simultaneously adopt industrial subsidies that restore marginal cost pricing globally. The average gains are calculated as the simple average across all 43 countries in the WIOD sample.

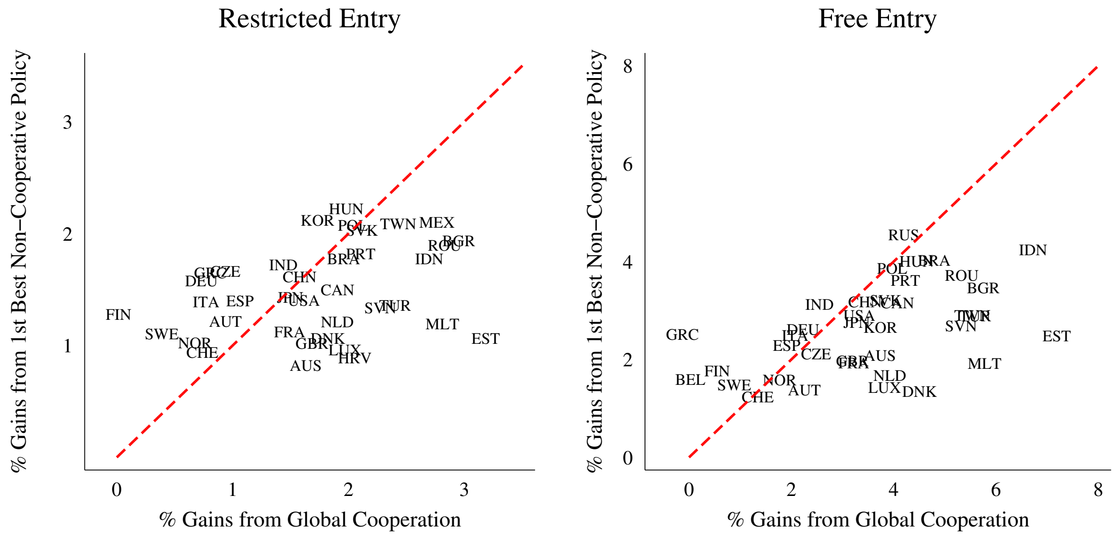
[61] This finding is distinct from Balistreri and Markusen (2009), who argue that optimal tariffs yield smaller gains in the presence of positive firm-level markups. The above finding is a manifestation of Proposition 1, whereas Balistreri and Markusen's assertion is a special case of our Theorem 2. To see this, note that with one traded sector, export and import taxes are equivalent. Invoking this equivalence, Theorem 2 posits that optimal tariffs (and their implied gains) are strictly lower if the traded sector admits a higher markup than the non-traded sector.

7.4 The Gains from *Deep* Trade Agreements

As detailed in Section (3), a deep trade agreement corresponds to a scenario where all countries agree to (*i*) adopt corrective industrial subsidies, and (*ii*) abolish their beggar-thy-neighbor trade taxes. By agreeing to join such an agreement, countries forgo the unilateral *ToT gains* from policy but in return benefit from corrective subsidies in the rest of the word. Our goal is to quantify how this trade-off is borne out in practice.[62]

Our estimation results are displayed in Figure (1). The *x-axis* in the aforementioned figure corresponds to the gains from deep cooperation. The *y-axis* corresponds to the gains from the first-best non-cooperative policy before retaliation. For most economies, the gains from deep cooperation dominate those of the optimal non-cooperative policy. What is perhaps surprising is that this outcome emerges even when non-cooperation does not trigger retaliation by trading partners. It reinstates our earlier finding that the ToT gains from policy are limited in scope. In comparison, the extent of allocative inefficiency in the global economy is sizable, making it beneficial for countries to forgo the ToT gains in exchange for importing more efficient varieties. Also interestingly, the gains from deep cooperation favor small countries that have a comparative disadvantage in high-returns-to-scale (or high-profit) industries, *e.g.*, Estonia, Malta, and Slovenia. The intuition is that these countries depend relatively more on imported varieties in high-returns-to-scale industries and under deep cooperation, these industries are subsidized across the globe.

Figure 1: *Deep cooperation vs. first-best non-cooperative policy*



Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The gains from *1st best non-cooperative policy* are the gains when each country implements the policy characterized by Theorem 1 and the rest of the world is passive. The gains from *global cooperation* correspond to a scenario where all countries forgo trade taxation and apply industrial subsidies that restore marginal cost pricing.

[62] The gains from deep cooperation can be computed with the aid of the optimal policy formulas specified under Equation 9 and the logic presented earlier under Section (7.1).

7.5 Sensitivity Analysis

In Appendix P we recalculate the gains from policy under several alternative specifications. First, we recompute the gains assuming that the data-generating process is a *Melitz-Pareto* model. Second, we recompute the gains based on alternative values for σ_k and μ_k, which are estimated via a two-ways fixed effects estimation (as reported in Appendix L). Lastly, we recompute the gains from policy under a more conservative set of values assigned to μ_k and σ_k in services. In all cases, trade policy turns out to be a poor second-best instrument for resorting allocative efficiency. Another noteworthy observation is that accounting for firm-selection effects à la Melitz (2003), magnifies the gains from (first-best) optimal policies. However, these greater gains are primarily driven by the larger misallocation-correcting gains. If anything, second-best trade taxes/subsidies are even less effective at replicating the firs-best policy gains in the presence of firm-selection into export markets.

What parameter values would produce larger gains from policy? We analyze this question in Appendix Q. To this end, we build on the discussion in Section 4 that the gains from optimal policy are increasing in two statistics: (i) the cross-industry variance of the scale elasticities, $\text{Var}_k(\mu_k)$, and (ii) the average of the (inverse) trade elasticities, $\text{Avg}_k\,(1/(\sigma_k - 1))$. In Appendix Q, we adjust our estimated parameter values to artificially increase both of theses statistics. We then recompute the gains from policy under the artificially-higher values for $\text{Var}_k(\mu_k)$ and $\text{Avg}_k\,(1/(\sigma_k - 1))$. The results are reported in Figure 9 of the same appendix. They reveal that the gains from optimal policy nearly double for all countries if we artificially increase $\text{Var}_k(\mu_k)$ by a factor of about three. The policy gains for different countries, however, exhibit different degrees of sensitivity to an artificial increase in $\text{Avg}_k\,(1/(\sigma_k - 1))$. For larger countries like the U.S. or China, the gains from policy are significantly less sensitive to $\text{Avg}_k\,(1/(\sigma_k - 1))$. The intuition is that $\text{Var}_k(\mu_k)$ governs the gains from correcting misallocation, whereas $\text{Avg}_k\,(1/(\sigma_k - 1))$ regulates the extent to which countries can improve their ToT. For large countries, where trade accounts for a smaller fraction of the GDP, there is less scope for raising real GDP via ToT improvements. Hence, artificially increasing $\text{Avg}_k\,(1/(\sigma_k - 1))$ has a relatively modest effect on the overall gains from policy in these countries.

8 Concluding Remarks

For centuries, scale economies have served as a justification for controversial trade and industrial policy practices. Yet we know surprisingly little about the actual *empirics* of trade and industrial policy in increasing returns to scale industries. Against this backdrop, we took a preliminary step toward identifying the force of industry-level scale economies using micro-level trade data. Our estimates indicated that, trade restrictions are a poor second-best policy for correcting misallocation that is rooted in industry-level scale economies. Unilateral industrial policy can be equally ineffective, as it triggers immiserizing growth in most countries. However, coordinated industrial policies deliver welfare gains that exceed that of any non-cooperative policy alternative.

We used our micro-estimated scale elasticities to uncover a range of macro-level policy implications, but our estimates have an even broader reach. Two implications, which we left out in the interest of space, merit closer attention. First, our scale elasticity estimates can help disentangle the relative contribution of scale economies and Ricardian comparative advantage to patterns of international specialization. This is an old question for which our empirical understanding is surprisingly limited.

Second, our estimates can shed fresh light on the puzzlingly large income gap between rich and poor countries. Economists have always hypothesized that a fraction of this income gap is driven by rich countries specializing in high returns-to-scale industries. An empirical assessment of such hypotheses has been previously impeded by a lack of comprehensive estimates for industry-level scale elasticities. Our micro-level estimates pave the way for an empirical exploration in this direction.

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Theoretical Appendix

A The Redundancy of Consumption Taxes

Without loss of generality suppose country $i \in \mathbb{C}$ imposes a full set of tax instruments, while the rest of the world is passive. Now, consider any arbitrary combination of taxes (indexed by A) that includes (i) industrial (or domestic production) subsidies, $s^A_{i,k}$, (ii) domestic consumption taxes, $\tau^A_{i,k}$, (iii) import taxes, $t^A_{ji,k}$, and (iv) export subsidies, $x^A_{ij,k}$. This set of tax instruments –which includes consumption taxes– produces the following wedges between producer and consumer price indexes for various product varieties:

$$\tilde{P}^A_{ii,k} = \frac{1+\tau^A_{i,k}}{1+s^A_{i,k}} P_{ii,k}; \qquad \tilde{P}^A_{ji,k} = (1+t^A_{ji,k})(1+\tau^A_{i,k})P_{ji,k}; \qquad \tilde{P}^A_{ij,k} = \frac{1}{(1+x^A_{ij,k})(1+s^A_{i,k})} P_{ij,k}; \quad (j \neq i)$$

Our claim here is that the same wedges can be replicated without appealing to consumption taxes, $\tau_{i,k}$. This claim can be established by considering an alternative tax schedule, B, which excludes consumption taxes (i.e., $1+\tau^B_{i,k} = 0$), but includes the following set of production subsidies, export subsidies, and import taxes:

$$1+s^B_{i,k} = \frac{1+s^A_{i,k}}{1+\bar{\tau}^A_{i,k}}; \qquad 1+t^B_{ji,k} = \left(1+t^A_{ji,k}\right)\left(1+\tau^A_{i,k}\right); \qquad 1+x^B_{ij,k} = (1+x^A_{ij,k})(1+\tau^A_{i,k})$$

It is straightforward to see that schedule B can reproduce the same wedge between producer and consumer prices as the original schedule A (i.e., $\tilde{\mathbf{P}}^A = \tilde{\mathbf{P}}^B$). In particular,

$$\begin{aligned}
\tilde{P}^B_{ii,k} &= \frac{1}{1+s^B_{i,k}} P_{ii,k} = \frac{1+\tau^A_{i,k}}{1+s^A_{i,k}} P_{ii,k} = \tilde{P}^A_{ii} \\
\tilde{P}^B_{ji,k} &= (1+t^B_{ji,k})P_{ji,k} = (1+t^A_{ji,k})(1+\tau^A_{i,k})P_{ji,k} = \tilde{P}^A_{ji,k} \\
\tilde{P}^B_{ij,k} &= \frac{1}{(1+x^B_{ij,k})(1+s^B_{i,k})} P_{ij,k} = \frac{1}{(1+x^A_{ij,k})(1+s^A_{i,k})} P_{ij,k} = \tilde{P}^A_{ij,k}.
\end{aligned}$$

It also follows trivially that $\tilde{P}^B_{nj,k} = P_{nj,k} = \tilde{P}^A_{nj,k}$ if $n, j \neq i$, because the rest of the world does not impose taxes.[63] This equivalence indicates that consumption taxes are redundant if the government has access to the other three sets of instruments. Note that the same can be said about production subsidies. More specifically, the effect of industry-level production subsidies can be perfectly replicated with a combination of consumption taxes, import taxes, and export subsidies. However, due to *product differentiation*, if two (of the $2(N-1)+2$) tax instruments are restricted, the replication argument fails. That is, if both production subsidies and consumption taxes are restricted, export subsidies and import taxes cannot fully replicate their effect.

[63]Note, though, that the rest of world imposing or not imposing taxes, does not matter for the redundancy of consumption taxes. The above argument can be easily extrapolated to the case where all countries impose arbitrary taxes.

B Proof of Lemma 1

Consider two policy-wage combinations, $\mathbf{T} = (\mathbf{s}, \mathbf{t}, \mathbf{x}; \mathbf{w})$, and $\mathbf{T}' = (\mathbf{s}', \mathbf{t}', \mathbf{x}'; \mathbf{w}')$, that differ in uniform shifters a and $\tilde{a} \in \mathbb{R}_+$:

$$\begin{cases} \mathbf{1} + \mathbf{x}_i' = a\left(\mathbf{1} + \mathbf{x}_i\right) & \mathbf{1} + \mathbf{x}_{-i}' = \mathbf{1} + \mathbf{x}_{-i} \\ \mathbf{1} + \mathbf{t}_i' = a\left(\mathbf{1} + \mathbf{t}_i\right) & \mathbf{1} + \mathbf{t}_{-i}' = \mathbf{1} + \mathbf{t}_{-i} \\ \mathbf{1} + \mathbf{s}_i' = \left(\mathbf{1} + \mathbf{s}_i\right)/\tilde{a} & \mathbf{1} + \mathbf{s}_{-i}' = \mathbf{1} + \mathbf{s}_{-i} \\ w_i' = \left(a/\tilde{a}\right) w_i & \mathbf{w}_{-i}' = \mathbf{w}_{-i} \end{cases}.$$

Our goal is to prove that (i) if $\mathbf{T} \in \mathbb{F}$ then $\mathbf{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ for all $n \in \mathbb{C}$. To prove these claims, we appeal to two intermediate lemmas. The first lemma establishes the following: Suppose equilibrium quantities are identical under policy-wage vectors $\mathbf{T}$ and $\mathbf{T}'$ (*i.e.*, $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k). Then, the implied nominal income and price levels under $\mathbf{T}$ and $\mathbf{T}'$ are the same up to a scale. The second lemma is a standard result from consumer theory: It indicates the nominal income and price levels implied by the first lemma confirm the original assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Below, we state and prove the first of these lemmas for any $a \in \mathbb{R}_+$.

Lemma 2. *$Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k* $\Longrightarrow \begin{cases} \tilde{\mathbf{P}}_i\left(\mathbf{T}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbf{T}\right); & \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}\right) \\ Y_i\left(\mathbf{T}'\right) = aY_i\left(\mathbf{T}\right); & \mathbf{Y}_{-i}\left(\mathbf{T}'\right) = \mathbf{Y}_{-i}\left(\mathbf{T}\right) \end{cases}$

Proof. Our goal is to compute nominal income and consumer prices under $\mathbf{T}$ and $\mathbf{T}'$ starting from the assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. We start our proof with nominal prices: To simplify the notation, define $\delta_{jn,k}(\mathbf{T}) \equiv \bar{\rho}_{jn,k}\mathcal{Q}_{j,k}(\mathbf{T})^{-\frac{\mu_k}{1+\mu_k}}$. Note that –by assumption– $\delta_{jn,k}(\mathbf{T}) = \delta_{jn,k}(\mathbf{T}') = \bar{\delta}_{jn,k}$. First, consider the price of a typical good ji, k imported by i from origin $j \neq i$. Using Equations 6 and 7, the consumer price of ji, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows:

$$\tilde{P}_{ji,k}(\mathbf{T}') = \bar{\delta}_{ji,k}\frac{1 + t_{ji,k}'}{(1 + x_{ji,k}')(1 + s_{j,k}')}w_j' = \bar{\delta}_{ji,k}\frac{a(1 + t_{ji,k})}{(1 + x_{ji,k})(1 + s_{j,k})}\, w_j = a\tilde{P}_{ji,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + t_{ji,k}' = a(1 + t_{ji,k})$, while $w_j' = w_j$, $x_{ji,k}' = x_{ji,k}$, and $s_{j,k}' = s_{j,k}$ (since $w_j \in \mathbf{w}_{-i}$, $x_{ji,k} \in \mathbf{x}_{-i}$, and $s_{j,k} \in \mathbf{s}_{-i}$). Second, consider a typical good ii, k that is produced and consumed locally in country i. The consumer price of ii, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows

$$\tilde{P}_{ii,k}(\mathbf{T}') = \bar{\delta}_{ii,k}\frac{1}{1 + s_{i,k}'}w_i' = \bar{\delta}_{ii,k}\frac{1}{\frac{1}{\tilde{a}}(1 + s_{i,k})} \times \frac{a}{\tilde{a}}w_i = a\tilde{P}_{ii,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + s_{i,k}' = (1 + s_{i,k})/\tilde{a}$ and $w_i' = aw_i/\tilde{a}$. Third, consider the price of a typical good ij, k export by i to destination market $j \neq i$. The consumer price of ij, k under combination $\mathbf{T}'$ can be related to its price under $\mathbf{T}$ as follows:

$$\tilde{P}_{ij,k}(\mathbf{T}') = \bar{\delta}_{ij,k}\frac{1 + t_{ij,k}'}{(1 + x_{ij,k}')(1 + s_{i,k}')}w_i' = \bar{\delta}_{ij,k}\frac{1 + t_{ij,k}}{a(1 + x_{ij,k}') \times \frac{1}{\tilde{a}}(1 + s_{i,k}')} \times \frac{a}{\tilde{a}}w_i = \tilde{P}_{ij,k}(\mathbf{T}),$$

where the third equality follows from the fact that $1 + x_{ij,k}' = a(1 + x_{ij,k})$, $1 + s_{i,k} = (1 + s_{i,k}')/\tilde{a}$, and $w_i' = aw_i/\tilde{a}$; while $t_{ij,k}' = t_{ji,k}$ since $t_{ji,k} \in \mathbf{t}_{-i}$. Lastly, is follows trivially that $\tilde{P}_{jn,k}(\mathbf{T}') = \tilde{P}_{jn,k}(\mathbf{T})$ if j and

$n \neq i$. Considering that $\tilde{\mathbf{P}}_i = \left\{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\right\}$, the above equations establish that

$$\tilde{\mathbf{P}}_i\left(\mathbf{T}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbf{T}\right), \qquad \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbf{T}\right).$$

Next, we turn to our claim about nominal income levels. To simplify the presentation, we hereafter use $X \equiv X(\mathbf{T})$ and $X' \equiv X(\mathbf{T}')$ to denote the value of a generic variable X under policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$. Keeping in mind this choice of notation, country i's nominal income under $\mathbf{T}'$, i.e., $Y_i' \equiv Y_i(\mathbf{T}')$ is given by:

$$\begin{aligned} Y_i' &= w_i'L_i + \sum_k \left[\left(\frac{1}{1+s_{i,k}'} - 1\right) P_{ii,k}'Q_{ii,k}'\right] + \sum_k\sum_{j\neq i}\left(\frac{t_{ji,k}'}{(1+x_{ji,k}')(1+s_{j,k}')}P_{ji,k}'Q_{ji,k}' + \left[\frac{1}{(1+x_{ij,k}')(1+s_{i,k}')} - 1\right]P_{ij,k}'Q_{ij,k}'\right) \\ &= w_i'L_i + \sum_k\left[\left(1 - [1+s_{i,k}']\right)\tilde{P}_{ii,k}'Q_{ii,k}'\right] + \sum_k\sum_{j\neq i}\left(\left(1 - \frac{1}{1+t_{ji,k}'}\right)\tilde{P}_{ji,k}'Q_{ji,k}' + \left[\frac{1}{1+t_{ij,k}'} - \frac{(1+x_{ij,k}')(1+s_{i,k}')}{1+t_{ij,k}'}\right]\tilde{P}_{ij,k}'Q_{ij,k}'\right). \end{aligned}$$

Note that, by assumption, policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$ exhibit the same output schedule, i.e., $Q_{ii,k}' = Q_{ii,k}$, $Q_{ji,k}' = Q_{ji,k}$, and $Q_{ij,k}' = Q_{ij,k}$. Also, recall that ($\mathbf{T}$ and $\mathbf{T}'$ are constructed such that) $1+t_{ji,k}' = a(1+t_{ji,k})$, $1+x_{ij,k}' = a(1+x_{ij,k})$, $1+s_{i,k} = (1+s_{i,k}')/\tilde{a}$, and $w_i' = aw_i/\tilde{a}$, $t_{ij,k}' = t_{ji,k}$. Considering these relationships and plugging our earlier result that (i) $\tilde{P}_{ii,k} = aP_{ii,k}$, (ii) $P_{ji,k}' = a\tilde{P}_{ji,k}$, and (iii) $\tilde{P}_{ij,k}' = \tilde{P}_{ij,k}$ into the above equation, yields the following expression for Y_i' :

$$\begin{aligned} Y_i' = &\frac{a}{\tilde{a}}w_iL_i + \sum_k\left[\left(1 - \frac{1}{\tilde{a}}(1+s_{i,k})\right)a\tilde{P}_{ii,k}Q_{ii,k}\right] \\ &+ \sum_{j,k}\left[\left(1 - \frac{1}{a(1+t_{ji,k})}\right)a\tilde{P}_{ji,k}Q_{ji,k} + \left[\frac{1}{1+t_{ij,k}} - \frac{a(1+x_{ij,k})\times\frac{1}{\tilde{a}}(1+s_{i,k})}{1+t_{ij,k}}\right]\tilde{P}_{ij,k}Q_{ij,k}\right]. \end{aligned}$$

Appealing to the balanced trade condition, $\sum_k\sum_{j\neq i}\left(\frac{1}{1+t_{ji,k}}\tilde{P}_{ji,k}Q_{ji,k} - \frac{1}{1+t_{ij,k}}\tilde{P}_{ij,k}Q_{ij,k}\right) = 0$, and observing that $(1+s_{i,k})\tilde{P}_{ii,k} = P_{ii,k}$ and $\frac{(1+x_{ij,k})(1+s_{i,k})}{1+t_{ij,k}}\tilde{P}_{ij,k} = P_{ij,k}$, the above equation reduces to

$$Y_i' = \frac{a}{\tilde{a}}w_iL_i + a\sum_k\left[\tilde{P}_{ii,k}Q_{ii,k} + \sum_{j\neq i}\tilde{P}_{ji,k}Q_{ji,k}\right] - \frac{a}{\tilde{a}}\sum_k\left[P_{ii,k}Q_{ii,k} + \sum_{j\neq i}P_{ij,k}Q_{ij,k}\right].$$

Invoking the labor market clearing condition, $w_iL_i - \sum_k\sum_n P_{ijn,k}Q_{in,k} = 0$, the above equation further simplifies as follows

$$Y_i' = a\sum_k\left[\tilde{P}_{ii,k}Q_{ii,k} + \sum_{j\neq i}\tilde{P}_{ji,k}Q_{ji,k}\right] = a\left[w_iL_i + \mathcal{R}_i\right] = aY_i,$$

where $\mathcal{R}_i \equiv \mathcal{R}_i(\mathbf{T})$ denotes country i's tax revenues under $\mathbf{T}$. To bel clear, the third line, in the above equation, follows from country i's balanced budget condition (i.e., total expenditure = total income). Turning to the rest of the world: The fact that $Y_n(\mathbf{T}') = Y_n(\mathbf{T})$ for all $n \neq i$ follows trivially from a similar line of arguments–hence, establishing our claim about nominal income levels:

$$Y_i\left(\mathbf{T}'\right) = aY_i\left(\mathbf{T}\right); \qquad \mathbf{Y}_{-i}\left(\mathbf{T}'\right) = \mathbf{Y}_{-i}\left(\mathbf{T}\right)$$

$\square$

Lemma 2 (proved above) starts from the assumption that $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Our next lemma indicates that this assumption is validated by the nominal income and price levels implied by $\mathbf{T}$ and $\mathbf{T}'$. Below, we state this lemma noting that it follows trivially from the Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, being homogeneous of degree zero.

Lemma 3. $\forall a \in \mathbb{R}_+$: $\begin{cases} \tilde{\mathbf{P}}_i(\mathbf{T}') = a\tilde{\mathbf{P}}_i(\mathbf{T}) \\ Y_i(\mathbf{T}') = aY_i(\mathbf{T}) \end{cases} \implies Q_{ji,k}(\boldsymbol{T}') = Q_{ji,k}(\boldsymbol{T})$ *for all* ji, k

Together, Lemmas 3 and 2 establish that equilibrium quantities should be indeed identical under policy-wage combinations $\mathbf{T}$ and $\mathbf{T}'$—i.e., $Q_{jn,k}(\mathbf{T}') = Q_{jn,k}(\mathbf{T})$ for all jn, k. Hence, if $\mathbf{T} \in \mathbb{F}$ it follows immediately that (i) $\mathbf{T}' \in \mathbb{F}$, and (ii) $W_n(\mathbf{T}) = W_n(\mathbf{T}')$ for all $n \in \mathbb{C}$, which is the claim of Lemma 1.

C Nested-Eaton and Kortum (2002) Framework

Here we show that the nested CES import demand function specified by Assumption (A1), can also arise from within-product specialization à la Eaton and Kortum (2002). To this end, suppose that each industry k is comprised of a continuum of homogenous goods indexed by ν. The sub-utility of the representative consumer in country i with respect to industry k is a log-linear aggregator across the continuum of goods in that industry:

$$Q_{i,k} = \int_0^1 \ln \tilde{q}_{i,k}(\nu) d\nu$$

As in our main model, country j hosts $\overline{M}_{j,k}$ firms indexed by ω, with $\Omega_{j,k}$ denotes the set of all firms serving industry k from country j.[64] Each firm ω supplies good ν to market i at the following *quality-adjusted* price:

$$\tilde{p}_{ji,k}(\nu;\omega) = \tilde{p}_{ji,k}(\omega) / \varphi(\nu;\omega),$$

where $\tilde{p}_{ji,k}(\omega)$ is a nominal price (driven by production costs) that applies to all goods supplied by firm ω in industry k, while the quality component, $\varphi(\nu;\omega)$, is good$\times$firm-specific. Suppose for any given good ν, firm-specific qualities are drawn independently from the following nested Fréchet joint distribution:

$$F_k(\boldsymbol{\varphi}(\boldsymbol{\nu})) = \exp\left[-\sum_{i=1}^{N} \left(\sum_{\omega \in \Omega_{i,k}} \varphi(\nu;\omega)^{-\vartheta_k} \right)^{\frac{\theta_k}{\vartheta_k}} \right],$$

The above distribution generalizes the basic Fréchet distribution in Eaton and Kortum (2002). In particular, it relaxes the restriction that productivities are perfectly correlated across firms within the same country. Instead, it allows for sub-national productivity differentiation and also for the degrees of cross- and sub-national productivity differentiations (ϑ_k and θ_k, respectively) to diverge. A special case of the distribution where $\vartheta_k \longrightarrow \infty$ corresponds to the standard Eaton and Kortum (2002) specification.

The above distribution also has deep theoretical roots. The Fisher–Tippett–Gnedenko theorem states that if ideas are drawn from a (normalized) distribution, in the limit the distribution of the best draw takes the form of a general extreme value (GEV) distribution, which includes the above Fréchet distribution as a special case. A special application of this result can be found in Kortum (1997) who develops an idea-based growth model where the limit distribution of productivities is Fréchet, with $\varphi_{\omega,k}$ reflecting the stock of technological knowledge accumulated by firms ω in category k.

Given the vector of effective prices, the representative consumer in county i (who is endowed

[64] The implicit assumption here is that entry is restricted, so that $\overline{M}_{j,k}$ is exogenous.

with income Y_i) maximizes their real consumption of each good, $\tilde{q}_{i,k}(\nu) = e_{i,k}Y_i/\tilde{p}_{i,k}(\nu)$, by choosing $\tilde{p}_{i,k}(\nu) = \min_\omega\{\tilde{p}_{ji,k}(\omega)\}$. That being the case, the consumer's discrete choice problem for each good ν can be expressed as:

$$\min_{\omega} \tilde{p}_{ji,k}(\omega)/z(\nu;\omega) \sim \max_{\omega} \ln z(\nu;\omega) - \ln \tilde{p}_{ji,k}(\omega).$$

To determine the share of goods for which firm ω is the most competitive supplier, we can invoke the theorem of "General Extreme Value." Specifically, define $G(\tilde{\boldsymbol{p}}_i)$ as follows

$$G_k(\tilde{\boldsymbol{p}}_i) = \sum_{j=1}^{N}\left(\sum_{\omega\in\Omega_{j,k}} \exp(-\vartheta_k \ln \tilde{p}_{ji,k}(\omega))\right)^{\frac{\theta_k}{\vartheta_k}} = \sum_{j=1}^{N}\left(\sum_{\omega\in\Omega_{j,k}} \tilde{p}_{ji,k}(\omega)^{-\vartheta_k}\right)^{\frac{\theta_k}{\vartheta_k}}.$$

Note that $G_k(.)$ is a continuous and differentiable function of vector $\tilde{\boldsymbol{p}}_i \equiv \{\tilde{p}_{ji,k}(\omega)\}$ and has the following properties:

a) $G_k(.) \geq 0$;

b) $G_k(.)$ is a homogeneous function of rank θ_k: $G_k(\rho\tilde{\mathbf{p}}_i) = \rho^{\theta_k}G_k(\tilde{\mathbf{p}}_i)$ for any $\rho \geq 0$;

c) $\lim_{\tilde{p}_{ji,k}(\omega)\to\infty} G_k(\tilde{\mathbf{p}}_i) = \infty$, $\forall\omega$;

d) the m'th partial derivative of $G_k(.)$ with respect to a generic combination of m variables $\tilde{p}_{ji,k}(\omega)$, is non-negative if m is odd and non-positive if m is even.

Manski and McFadden (1981) show that if $G_k(.)$ satisfies the above conditions, and $\varphi(\nu;\omega)$'s are drawn from

$$F_k(\boldsymbol{\varphi}(\nu)) = \exp\left(-G_k(e^{-\ln\boldsymbol{\varphi}})\right) = \exp\left(-\sum_{j=1}^{N}\left(\sum_{\omega\in\Omega_{j,k}} \varphi(\nu;\omega)^{-\vartheta_k}\right)^{\frac{\theta_k}{\vartheta_k}}\right),$$

which is the exact same distribution specified above, then the probability of choosing variety ω is equal to

$$\pi_{ji,k}(\omega) = \frac{\left(\frac{\tilde{p}_{ji,k}(\omega)}{\theta_k}\right)\frac{\partial G_k(\tilde{\mathbf{p}}_i)}{\partial p_{ji,k}(\omega)}}{G_k(\tilde{\mathbf{p}}_i)} = \frac{\tilde{p}_{ji,k}(\omega)\,\tilde{p}_{ji,k}(\omega)^{\vartheta_k-1}\left(\sum_{\omega'\in\Omega_{j,k}}\tilde{p}_{ji,k}(\omega')^{-\vartheta_k}\right)^{\frac{\theta_k}{\vartheta_k}-1}}{\sum_{n=1}^{N}\left(\sum_{\omega'\in\Omega_{j,k}}\tilde{p}_{ji,k}(\omega')^{-\vartheta_k}\right)^{\frac{\theta_k}{\vartheta_k}}}$$

Rearranging the above equation yields the following expression:

$$\pi_{ji,k}(\omega) = \left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}}\right)^{-\vartheta_k}\left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\theta_k},$$

where $\tilde{P}_{ji,k} \equiv \left[\sum_{\omega'\in\Omega_{j,k}}\tilde{p}_{ji,k}(\omega')^{-\vartheta_k}\right]^{-1/\vartheta_k}$ and $\tilde{P}_{i,k} \equiv \left[\sum \tilde{P}_{ji,k}^{-\theta_k}\right]^{-\frac{1}{\theta_k}}$. Given the share of goods sourced from firm ω, total sales of firm ω to market i, in industry k can be calculated as:

$$\tilde{p}_{ji,k}(\omega)q_{ji,k}(\omega) = \tilde{p}_{ji,k}(\omega)\,\frac{\pi_{ji,k}(\omega)e_{i,k}Y_i}{\tilde{p}_{ji,k}(\omega)} = \left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}}\right)^{-\vartheta_k}\left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\theta_k} e_{i,k}Y_i$$

which is identical to the nested-CES function specified by Assumption (A1), with $\gamma_k - 1 = \vartheta_k$ and $\sigma_k - 1 = \theta_k$.

D Firm-Selection under Pareto

In this appendix, we outline the isomorphism between our baseline model and one that admits selection effects. In doing so, we borrow heavily from Kucheryavyy et al. (2016) (KLR, hereafter). We rely on three key assumptions, hereafter:

a) Within-industry demand is governed by the same nested-CES utility function presented under Assumption (A1). As in the baseline mode, σ_k and γ_k respectively denote the upper- and lower-tier elasticities of substitution.

b) The firm-level productivity distribution, $G_{i,k}(z)$, is Pareto with shape parameter, ϑ_k.

c) The fixed "marketing" cost is paid in terms of labor in the destination market.

d) Taxes are applied before the markup, and operate as a cost-shifter.

Following KLR, we also assume that cross-industry utility aggregator is Cobb-Douglas, with $e_{i,k}$ denoting the constant share of country i's expenditure on industry k. Following the derivation in KLR, we can define the effective supply of production labor in country i as

$$\widetilde{L}_i = \left[1 - \sum_k e_{i,k}\left(\frac{\vartheta_k - \gamma_k + 1}{\vartheta_k \gamma_k}\right)\right] L_i.$$

The labor market clearing condition is, accordingly, given by $\sum w_i L_{i,k} = w_i \widetilde{L}_i$. With regards to aggregate markup levels, we can appeal to the well-known result that the profit margin in each industry is constant and given by the following expression:

$$\text{mark-up} \sim \frac{\sum_n P_{in,k} Q_{in,k}}{w_i L_{i,k}} = \frac{\vartheta_k - \gamma_k + 1}{(\vartheta_k + 1)(\gamma_k - 1)}.$$

With regards to aggregate demand functions, we can follow the derivation in Appendix B.2 of KLR to express demand for national-level variety ji, k as

$$Q_{ji,k} = \left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\sigma_k^{\text{Melitz}}} Q_{i,k},$$

where $\sigma_k^{\text{Melitz}} \equiv 1 + \vartheta_k / \left[1 + \vartheta_k\left(\frac{1}{\sigma_k - 1} - \frac{1}{\gamma_k - 1}\right)\right]$ denotes the trade elasticity under firm-selection. Moreover, we can show that national-level producer price indexes are given by the following formulation:

$$P_{ij,k}^{\text{Melitz}} = \begin{cases} \bar{\rho}_{ij,k} w_i & \text{if entry is restricted} \\ \bar{\rho}'_{ij,k} w_i Q_{i,k}^{-\frac{\vartheta_k}{1+\vartheta_k}} & \text{if entry is free} \end{cases},$$

where $\bar{\rho}_{ij,k}$ and $\bar{\rho}'_{ij,k}$ are composed of structural parameters that are invariant to policy–this includes ϑ_k that regulates firm selection.[65] Abstracting from taxes, $\tilde{P}_{i,k} = \left(\sum P_{ji,k}^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}$ is the CES industry-level consumer price index that shows up in indirect utility $V_i(.)$. Referring to our earlier result about

[65]Unlike $\tilde{P}_{i,k}$, the national-level indexes, $\tilde{P}_{ji,k}$, are not the same as the CES price indexes defined in the main text, but this is not problematic from the point of the isomorphism result we are seeking.

constant markup margins, aggregate profits in country i given by

$$\Pi_i^{\text{Melitz}} = \begin{cases} \sum_k \sum_j \left(\frac{\frac{\vartheta_k - \gamma_k + 1}{(\vartheta_k+1)(\gamma_k-1)}}{1+\frac{\vartheta_k-\gamma_k+1}{(\vartheta_k+1)(\gamma_k-1)}} P_{ij,k} Q_{ij,k} \right) & \text{if entry is restricted} \\ 0 & \text{if entry is free} \end{cases}.$$

To fixe ideas, recall that we used μ_k to denote both (1) the scale elasticity under free entry, and (2) the profit margin under restricted entry in the baseline model. This overlapping choice of notation was motivated by the observation that in the generalized Krugman model, the scale elasticity (under free entry) and the profit margin (under restricted entry) are identical: $\mu_k^{\text{FE}} = \mu_k^{\text{RE}} = \mu_k = 1/(\gamma_k - 1)$. This equivalence, though, was not used to derive any of our theorems. Instead, it was only invoked to simplify the presentation of our theorems. Evidently, under the Melitz-Pareto model the equivalence between the scale elasticity and the profit margin crumbles. Taking note of this nuance, the Melitz-Pareto model is isomorphic to our baseline model with the following reinterpretation of parameters:

$$\mu_k^{\text{Melitz}} = \begin{cases} 1/\vartheta_k & \text{if entry is free} \\ \frac{\vartheta_k \mu_k - 1}{1+\vartheta_k} & \text{if entry is restricted} \end{cases}; \qquad \sigma_k^{\text{Melitz}} = 1 + \frac{\vartheta_k}{1 + \vartheta_k \mu_k \left(\frac{1}{\mu_k(\sigma_k - 1)} - 1 \right)}.$$

The Marshallian demand elasticities in the Melitz-Pareto model are accordingly given by the following equations as a function σ_k^{Melitz} and expenditure shares:

$$\varepsilon_{ji,k}^{(ji,k)} = -1 - (\sigma_k^{\text{Melitz}} - 1) \left(1 - \lambda_{ji,k}\right); \qquad \varepsilon_{ji,k}^{(ji,k)} = \sigma_k^{\text{Melitz}} \lambda_{ji,g}$$

In the above expressions, μ_k and σ_k can be taken directly from our firm-level demand estimation. Doing so, identifies the Melitz-Pareto model's key parameters up to a Pareto shape parameter, ϑ_k. To obtain an estimate for ϑ_k, we can estimate the trade elasticity, $\sigma_k^{\text{Melitz}} - 1$, using macro-level trade data and standard techniques from the literature. Given the estimated trade elasticities, we can simply recover ϑ_k by plugging our micro-level estimates for μ_k and σ_k into the expression for σ_k^{Melitz}.

D.1 The Case where Taxes are Applied After Markups

Our derivation, above, assumed that taxes are applied before the markup, and act as a cost shifter. Below, we discuss how relaxing this assumption may affect the arguments listed above. To this end, we focus on the spacial case where preferences are non-nested. Namely,

$$\text{non-nested preferences} \sim \sigma_k = \gamma_k, \qquad \forall k \in \mathbb{K}.$$

Following the Online Appendix 5 in Costinot and Rodríguez-Clare (2014), the trade elasticity in the Melitz-Pareto model with non-nested preferences is described by the following formulation:

$$\sigma_k^{\text{Melitz}} = \begin{cases} 1 + \vartheta_k & \text{tax applied before markup} \\ \frac{\sigma}{\sigma-1} \vartheta & \text{tax applied after markup} \end{cases}.$$

Appealing to the above formulation, we can show that *Theorem 1* nests, as a special case, the optimal tariff formula derived by Demidova and Rodriguez-Clare for a small open economy in a *single-industry×two-country* Melitz-Pareto model. To demonstrate this, drop the industry subscript k and reduce the global economy into two countries, i.e., $\mathbb{C} = \{i, j\}$. Noting that $1 - \lambda_{ij} = \lambda_{jj}$ in the two-country case, we can deduce from the above formulation and Theorem 1 that

$$\frac{1+t_{ji}^{*}}{1+x_{ij}^{*}}=1+\frac{1}{(\sigma^{\text{Melitz}}-1)\lambda_{jj}}=\frac{1}{(\frac{\sigma}{\sigma-1}\vartheta-1)\lambda_{jj}}.$$

By the Lerner symmetry, export and import taxes are equivalent in the single-industry model.[66] Hence, without loss of generality, we can set $x_{ij}^{*}=0$. Moreover, if country i is a small open economy, then $\lambda_{jj}\approx 1$. Combining these two observations, we can arrive at the familiar-looking optimal tariff formula in Demidova and Rodriguez-Clare:

$$t_{ji}^{*}=\frac{\frac{\sigma-1}{\sigma}}{\vartheta-\frac{\sigma-1}{\sigma}}\quad\sim\text{ small open economy w/ one traded sector.}$$

E Proof of Theorem 1

Our proof proceeds in five steps. The first four steps characterize the optimal tax/subsidy schedule for country $i\in\mathbb{C}$ under *free entry*. The last step demonstrates that this characterization can be extrapolated to the case with *restricted entry*.

Step #1: Express Equilibrium Variables as function of $\mathbb{P}_i$ and w

Our goal is to characterize optimal policy for country $i\in\mathbb{C}$ assuming the rest of the world is passive: $\mathbf{t}_{-i}=\mathbf{x}_{-i}=\mathbf{s}_{-i}=\mathbf{0}$. To simplify the proof, we reformulate country i's optimal policy problem as one where the government chooses the optimal consumer prices (rather than the actual taxes) associated with its economy. By construction, country i's optimal tax schedule can be recovered from its optimal consumer-to-producer price ratios. The first step in reformulating the optimal policy problem is to express equilibrium variables (e.g., $Q_{ji,k}$, Y_i, etc.) as a function of (1) the vector of consumer prices associated with economy i, $\mathbb{P}_i\equiv\left\{\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji},\tilde{\mathbf{P}}_{ij}\right\}$, where

$$\tilde{\mathbf{P}}_{ii}\equiv\{P_{ii,k}\}_k;\qquad\tilde{\mathbf{P}}_{ji}\equiv\left\{P_{ji,k}\right\}_{j\neq i,k};\qquad\tilde{\mathbf{P}}_{ij}\equiv\left\{P_{ij,k}\right\}_{j\neq i,k}\tag{23}$$

and (2) the vector of national-level wage rates across the world,

$$\mathbf{w}=\{w_1,...,w_N\}.$$

The following lemma shows that our desired formulation of equilibrium variables follows from (a) treating $\mathbb{P}_i$ and $\mathbf{w}$ as given, and (b) solving a system that satisfies all equilibrium conditions excluding the labor market clearing condition.

Lemma 4. *All equilibrium outcomes (excluding $\mathbb{P}_i$ and $\mathbf{w}$) can be uniquely determined as a function of $\mathbb{P}_i\equiv\left\{\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji},\tilde{\mathbf{P}}_{ij}\right\}$, and $\mathbf{w}$.*

Proof. As noted above, the proof follows from solving all equilibrium conditions excluding the equilibrium expression for consumer prices, $\tilde{P}_{ji,k}$ (which are encompassed by $\mathbb{P}_i$), and the country-specific balanced trade conditions (which pin down $\mathbf{w}$).[67] Stated formally, we need to solve the following

[66]The Lerner symmetry is a special case of the equivalence result presented under Lemma 1. Also, note that the market equilibrium is efficient in the single industry Krugman model studied by Gros (1987). As such, the optimal industrial subsidy can be normalized to zero, i.e., $s_i^{*}=0$.

[67]Note that by Walras' law, the balanced trade condition is equivalent to the labor market clearing condition in each country.

system treating $\mathbb{P}_i$, and $\mathbf{w}$ as given:

$$\begin{aligned}
&\text{[optimal pricing]} && P_{jn,k} = \bar{\rho}_{ji,k} w_j \left[\sum_i \bar{a}_{ji,k} Q_{ji,k}\right]^{-\frac{\mu_k}{1+\mu_k}} \\
&\text{[optimal consumption]} && Q_{jn,k} = \mathcal{D}_{jn,k}(Y_n, \tilde{\mathbf{P}}_{1n}, ...\tilde{\mathbf{P}}_{Nn}) \\
&\text{[RoW imposes zero taxes]} && \tilde{P}_{jn,k} = P_{jn,k} \quad (\tilde{P}_{jn,k} \notin \mathbb{P}_i); \qquad Y_n = w_n L_n \quad (n \neq i) \\
&\text{[Balanced Budget in } i\text{]} && Y_i = w_i L_i + \left(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}\right) \cdot \mathbf{Q}_{ii} + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right) \cdot \mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right) \cdot \mathbf{Q}_{ji},
\end{aligned}$$

where "$\cdot$" denotes the inner product operator for equal-sized vectors (i.e., $\mathbf{a} \cdot \mathbf{b} = \sum_n a_n b_n$). Since there is a unique equilibrium, the above system is exactly identified in that it uniquely determines $P_{jn,k}$, $Q_{jn,k}$, and Y_n as a function of $\mathbb{P}_i$ and $\mathbf{w}$. $\square$

Following Lemma 4, we can express total income in country i, Y_i, as well as the entire demand schedule in that country as follows:

$$Y_i \equiv Y_i(\mathbb{P}_i; \mathbf{w}); \qquad Q_{ji,k} \equiv Q_{ji,k}(\mathbb{P}_i; \mathbf{w}) = \mathcal{D}_{ji,k}\left(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\right).$$

Recall that $\mathcal{D}_{ji,k}(.)$ denotes the Marshallian demand function facing variety ji,k. Taking note of the above representation, our main objective is to reformulate country i's policy problem as one where the government chooses $\mathbb{P}_i$ (as opposed to directly choosing tax rates). This reformulation, though, needs to take into account that $\mathbf{w}$ is an equilibrium outcome that implicitly depends on the choice of $\mathbb{P}_i$. To track this constraint, we define the $(\mathbb{P}_i; \mathbf{w})$ combinations that are feasible as follows.

Definition 1. *A policy-wage combination $(\mathbb{P}_i; \boldsymbol{w})$ is **feasible** iff given $\mathbb{P}_i$, the vector of wages, $\boldsymbol{w}$, satisfies the balanced trade condition in every country $n \in \mathbb{C}$. In particular,*

$$(\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P \quad \Longleftrightarrow \quad \begin{cases} \sum_{j \neq n} \sum_{k \in \mathbb{K}} \left[P_{jn,k}(\mathbb{P}_i; \mathbf{w}) Q_{jn,k}(\mathbb{P}_i; \mathbf{w}) - P_{nj,k}(\mathbb{P}_i; \mathbf{w}) Q_{nj,k}(\mathbb{P}_i; \mathbf{w})\right] = 0 & \text{if } n \neq i \\ \sum_{j \neq n} \sum_{k=1}^{K} \left[P_{ji,k}(\mathbb{P}_i; \mathbf{w}) Q_{jn,k}(\mathbb{P}_i; \mathbf{w}) - \tilde{P}_{ij,k} Q_{nj,k}(\mathbb{P}_i; \mathbf{w})\right] = 0 & \text{if } n = i \end{cases}.$$

To elaborate on the above definition: The balanced trade condition for a generic country $n \in \mathbb{C}$ can be expresses as $\sum_{j \neq n,k} \left[\frac{1}{1+t_{jn,k}} \tilde{P}_{nj,k} Q_{jn,k} - \frac{1}{1+t_{nj,k}} \tilde{P}_{nj,k} Q_{nj,k}\right]$. The expression for the balanced trade condition, above, follows from the assumption that only country i imposes taxes and the rest of the world is passive. We should emphasize one more time that by Walras' law the satisfaction of the balanced trade condition is analogous to the satisfaction of the labor market clearing condition in each country. Relatedly, take note of the equivalence between $\mathbb{F}_P$ and $\mathbb{F}$–with the latter being defined in the main text under Definition (D2). Taking note of these implicit details, we now proceed to reformulate the optimal policy problem (P1).

Step #2: Reformulate the Optimal Tariff Problem

Before proceeding with the second step of the proof, we formally present our notation for partial derivatives. We will rely heavily on this choice of notation, especially in the subsequent steps of the proof where we derive the first-order conditions.

Notation [*Partial Derivative*] *Let $f(x_1, x_2)$ be a function of two variables, where $x_2 = g(x_1)$ is possibly an implicit function of x_1. We henceforth use*

$$\left(\frac{\partial f(x_1, x_2)}{\partial x_1}\right)_{x_2} = \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1}$$

to denote the derivative of $f(.)$ w.r.t. x_1, treating $x_2 = \bar{x}_2$ as a constant.[68]

Moving on with Step 2, recall the original formulation of the optimal policy problem (P1) from Section 2:

$$\max_{\mathbb{T}_i} \; W_i(\mathbb{T}_i; \mathrm{w}) \qquad s.t. \;\; (\mathbb{T}_i; \mathrm{w}) \in \mathbb{F} \qquad \text{(P1)}$$

In the above formulation, $\mathbb{T}_i \equiv (\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i)$ denotes country i's vector of taxes and $\mathbb{F}$ is defined according to Definition (D2, Section 2) and analogously to $\mathbb{F}_P$. Our next intermediate result shows that Problem (P1) can be alternatively cast as one where the government chooses the optimal vector of consumer prices $\mathbb{P}_i$ associated with its economy. After determining $\mathbb{P}_i^*$, the optimal tax vectors, $\mathbf{t}_i^*$, $\mathbf{x}_i^*$, and $\mathbf{s}_i^*$ can be automatically recovered from the optimal consumer-to-producer price ratios.

Lemma 5. *Country i's vector of optimal taxes, $\{\mathbf{t}_i^*, \mathbf{x}_i^*, \mathbf{s}_i^*\}$, can be determined by solving the following problem instead of (P1):*

$$\max_{\mathbb{P}_i} \; W_i(\mathbb{P}_i; \mathrm{w}) \equiv V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \quad s.t. \; \begin{cases} (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P \\ \mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}, \end{cases} \qquad (\widetilde{\text{P1}})$$

Proof. The proof consists of two parts. First, we can trivially verify that there is a one-to-one correspondence between the optimal choice w.r.t. $\mathbb{P}_i^* \equiv \left\{\tilde{\mathbf{P}}_{ii}^*, \tilde{\mathbf{P}}_{ji}^*, \tilde{\mathbf{P}}_{ij}^*\right\}$ and $\mathbb{T}_i^* \equiv \{\mathbf{t}_i^*, \mathbf{x}_i^*, \mathbf{s}_i^*\}$. More specifically, given information on $\mathbb{P}_i^*$ (and the accompanying wage vector $\mathbf{w}^*$), we can uniquely recover the optimal tax/subsidy rates using the following set of equations:

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}(\mathbb{P}_i^*, \mathbf{w}^*)}; \qquad 1 + x_{ij,k}^* = \frac{P_{ji,k}(\mathbb{P}_i^*, \mathbf{w}^*)/\tilde{P}_{ji,k}^*}{P_{ii,k}(\mathbb{P}_i^*, \mathbf{w}^*)/\tilde{P}_{ii,k}^*}; \qquad 1 + s_{i,k}^* = \frac{P_{ii,k}(\mathbb{P}_i^*, \mathbf{w}^*)}{\tilde{P}_{ii,k}^*}.$$

The correspondence presented above, indicates an equivalence between choosing $\mathbb{P}_i$ versus choosing $\mathbb{T}_i$ directly. That is,

$$\max_{\mathbb{P}_i} \; W_i(\mathbb{P}_i; \mathrm{w}) \;\; s.t. \; (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P \; \sim \; \max_{\mathbb{T}_i} \; W_i(\mathbb{T}_i; \mathrm{w}) \;\; s.t. \; (\mathbb{T}_i; \mathbf{w}) \in \mathbb{F}.$$

The next step is to show that the constraint$\mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}$ is non-binding *at the optimum*. With two countries, the constraint $\mathbf{w}_{-i} = \overline{\mathbf{w}}_{-i}$ is non-binding by Walras' law, as wages in the rest of the world can be normalized to zero by choice of numeraire. More generally, this constraint is non-binding based on the targeting principle (Bhagwati and Ramaswami (1963)). In particular, fix the policy vector to its optimal level, $\mathbb{P}_i^*$ and assign $w_n \in \mathbf{w}_{-i}$ as the numeraire: Since $W_i(\mathbb{P}_i; \mathbf{w}) = V_i(Y_i(\mathbb{P}_i; \mathbf{w}); \tilde{\mathbf{P}}_i)$ and $\tilde{\mathbf{P}}_i \in \mathbb{P}_i$, the Envelope Theorem indicates that

$$\left(\frac{\partial W_i(\mathbb{P}_i^*; \mathbf{w})}{\partial \mathbf{w}_{-i}}\right)_{w_n} = \frac{\partial V_i(.)}{\partial Y_i} \left(\frac{\partial Y_i(\mathbb{P}_i^*; \mathbf{w})}{\partial \mathbf{w}_{-i}}\right)_{w_n}.$$

[68] Based on the above notation and the chain rule, the full derivative of $f(.)$ w.r.t. x_1 is given by

$$\frac{\mathrm{d}f(x_1, x_2)}{\mathrm{d}x_1} = \left(\frac{\partial f(x_1, x_2)}{\partial x_1}\right)_{x_2} + \left(\frac{\partial f(x_1, x_2)}{\partial x_2}\right)_{x_2} \frac{\mathrm{d}g(x_1)}{\mathrm{d}x_1}$$

But it is easy check from the lens of the Targeting Principe that $\left(\frac{\partial Y_i(\mathbb{P}_i^*;\mathbf{w})}{\partial \mathbf{w}_{-i}}\right)_{w_n}$. Specifically, assigning $w_n \in \mathbf{w}_{-i}$ as the numeriare, a change in w_j/w_n can affect W_i indirectly through its effect on country i's tax revenues, $\mathcal{R}_i$. But it is easy to check in the spirit of Lemma 1 that any such effect can be perfectly mimicked with an adjustment in the vector of export/import prices included in $\mathbb{P}_i$. Hence, by the *targeting principle*, any indirect gains from raising w_j/w_i will be already internalized by the optimal choice, $\mathbb{P}_i^*$ implying that $\left(\frac{\partial Y_i(\mathbb{P}_i^*;\mathbf{w})}{\partial \mathbf{w}_{-i}}\right)_{w_n} = 0.$ $\square$

The above lemma is significant for two reasons. First, it allows us to characterize the F.O.C. w.r.t. consumer prices, allowing us to present them succinctly in terms of Marshallian demand elasticities. Second, Lemma 5 allows us to derive the F.O.C. while treating $\mathbf{w}_{-i}$ as given, knowing that the change in both of these vectors are welfare-neutral at the optimum, $\mathbb{P}_i^*$.

Step #3. Deriving and Simplifying the System of First-Order Conditions

This step derives and solves the system of first-order necessary conditions (F.O.C.s) associated with Problem $\widetilde{\text{P1}}$. This system of F.O.C.s can be formally expressed as follows:

$$\nabla_{\tilde{P}} W_i(\mathbb{P}_i;\mathbf{w}) + \nabla_{\mathbf{w}} W_i \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\tilde{P}}\right)_{(\mathbb{P}_i;\mathbf{w})\in\mathbb{F}_P} = 0, \qquad \forall \tilde{P} \in \mathbb{P}_i.$$

where recall that $\mathbb{P}_i = \left\{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{ji}\right\}$ includes all consumer price variables associated with economy i. To elaborate the right-hand side of the above equation consists of two terms, as implied by the chain rule: The first term accounts for the change in welfare holding $\mathbf{w}$ fixed. The second term account for the change in $\mathbf{w}$ w.r.t. $\tilde{P} \in \mathbb{P}_i$ in order to satisfy feasibility.

Our characterization of optimal policy employs the dual approach, the presentation of which relies heavily on Marshallian demand elasticities. So, for future reference, we formally define these elasticities below.

Notation [*Marshallian Demand Elasticities*] *Let $Q_{ji,k} \equiv \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$ denote the Marshallian demand function facing variety ji,k. This demand function is characterized by the following set of demand elasticities:*

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}_i)}{\partial \ln \tilde{P}_{ni,g}} \sim \textit{price elasticity}$$

$$\eta_{ji,k} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\boldsymbol{P}}_i)}{\partial \ln Y_i} \sim \textit{income elasticity,}$$

where $\tilde{\mathbf{P}}_i = \left\{\tilde{\mathbf{P}}_{1i}, \tilde{\mathbf{P}}_{2i}, ..., \tilde{\mathbf{P}}_{Ni}\right\}$ corresponds to the entire of vector of consumer prices in market i. Also, recall from the main text that $V(Y_i, \tilde{\mathbf{P}}_i)$ denotes the indirect utility associated with the Marshallian demand function, $\mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$.

In what follows, we appeal the above definition to characterize the first-order condition w.r.t. each element of $\mathbb{P}_i$. We start with country i's import prices, $\tilde{\mathbf{P}}_{ji}$, and then proceed to domestic and export price instruments, $\tilde{\mathbf{P}}_{ii}$, and $\tilde{\mathbf{P}}_{ij}$.

Step 3.A: Deriving the F.O.C. w.r.t. $P_{ji,k}^* \in \mathbb{P}_i$.

Consider the price of import variety ji,k, supplied by *origin j–industry k* (where $j \neq i$). To present the first-order necessary condition (F.O.C.) w.r.t. the price of ji,k, we use $\mathbb{P}_{-ji,k}$ to denote all elements of $\mathbb{P}_i$ excluding $\tilde{P}_{ji,k}$:

$$\mathbb{P}_{-ji,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ji,k}\} \sim \text{entire policy vector excluding } \tilde{P}_{ji,k}$$

Next, recall that $W_i(\mathbb{P}_i; \mathbf{w}) \equiv V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji})$ where income, $Y_i(\mathbb{P}_i; \mathbf{w}) = \tilde{w}_i L_i + \mathcal{R}_i(\mathbb{P}_i; \mathbf{w})$, is dictated by the balanced budget condition. Applying the chain rule to $W_i(\mathbb{P}_i; \mathbf{w})$, the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ (holding the remaining elements of $\mathbb{P}_i$ constant) can be stated as follows:[69]

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i; \mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \overbrace{\frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}}^{\left(\frac{\partial W_i(\mathbb{P}_i; \mathbf{w})}{\partial \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}} + \left(\frac{\partial W_i(\mathbb{P}_i; \mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0 \tag{24}$$

The first term on the right-hand side of the above equation accounts for the direct welfare effects of a change in the price of good ji, k (holding Y_i and $\mathbb{P}_i - \{\tilde{P}_{ji,k}\}$ constant). The second term accounts for welfare effects that channel through revenue-generation (holding $\mathbf{w}$ and $\mathbb{P}_i - \{\tilde{P}_{ji,k}\}$ constant). The last term accounts for general equilibrium wage effects. Below, we characterize each of these elements one-by-one.

The term accounting for direct price effects can be simplified by appealing to Roy's identity, $\frac{\partial V_i / \tilde{P}_{ji,k}}{\partial V_i / \partial Y_i} = -Q_{ji,k}$, which indicates that

$$[\text{Roy's identity}] \qquad \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right). \tag{25}$$

To characterize $\left(\partial Y_i(\mathbb{P}_i; \mathbf{w}) / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}}$, note that total income in country i (which dictates total expenditure) is the sum of wage payments plus tax revenues:[70]

$$Y_i(\mathbb{P}_i; \mathbf{w}) = w_i L_i + \underbrace{\sum_{n \neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}\right) \cdot \mathbf{Q}_{ni}\right]}_{\text{import tax revenues}} + \underbrace{\left(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}\right) \cdot \mathbf{Q}_{ii}}_{\text{production tax revenues}} + \underbrace{\sum_{n \neq i}\left[\left(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}\right) \cdot \mathbf{Q}_{in}\right]}_{\text{export tax revenues}},$$

Holding w and $\mathbb{P}_{-ji,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ji,k}\}$ fixed, $\tilde{P}_{ji,k}$ has no effect on wage payments: $\left(\partial w_i L_i / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} =$

[69] We can alternatively formulate the above optimization problem using the method of Lagrange multipliers, and by appealing to Lagrange sufficiency theorem. In that case the objective function can be formulated as follows:

$$\max_{\mathbb{P}_i, Y_i} \mathcal{L}_i(\mathbb{P}_i; \mathbf{w}) = V_i(Y_i, \tilde{\mathbf{P}}_i) + \lambda_y\left(Y_i - \tilde{w}_i L_i - \mathcal{R}_i(\mathbb{P}_i; \mathbf{w})\right).$$

The F.O.C.with respect to Y_i entails that $\lambda_Y = \frac{\partial V_i(.)}{\partial Y_i}$. Hence, the F.O.C. with respect to $\tilde{P}_{ji,k} \in \mathbb{P}_i$ can be expressed as

$$\frac{\mathrm{d}\mathcal{L}_i(\mathbb{P}_i; \mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ji,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \lambda_y\left(\frac{\partial(\tilde{w}_i L_i + \mathcal{R}_i(\mathbb{P}_i; \mathbf{w}))}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} + \left(\frac{\partial \mathcal{L}_i(\mathbb{P}_i; \mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0,$$

which is equivalent to the F.O.C. expressed above.

[70] To be clear with regards to the notation: The operator "$\cdot$" denotes the inner product of two equal-sized vectors. Also, since we are focused on the free entry case, for now, the profit-adjusted wage rate is equal to the actual (unadjusted) wage rate, i.e., $\tilde{w}_i = w_i$.

0. The effect of $\tilde{P}_{ji,k}$ on import tax revenues can be unpacked as follows:

$$\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ji,k}}\right)=\tilde{P}_{ji,k}Q_{ji,k}+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left[\sum_{j\neq i}\frac{P_{ji,g}Q_{ji,g}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial P_{ji,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}+\sum_{\ell\in\mathbb{C}}\frac{P_{i\ell,g}Q_{i\ell,g}}{P_{i\ell,g}Q_{i\ell,g}}\left(\frac{\partial P_{i\ell,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}\right]\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{26}$$

The first term in the above expression accounts for the direct, arithmetic effect of $\tilde{P}_{ji,k}$ on import tax revenues. The second term accounts for the change in revenue due to the change in country i's import demand schedule as a result of changing $\tilde{P}_{ji,k}\in\tilde{\mathbf{P}}_{i}$. The change in demand can itself be decomposed into two components:

$$\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=\underbrace{\frac{\partial \ln \mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_{i},Y_{i})}{\partial \ln \tilde{P}_{ji,k}}}_{\text{price effect}}+\underbrace{\frac{\partial \ln \mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_{i},Y_{i})}{\partial \ln Y_{i}}\left(\frac{\partial \ln Y_{i}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}}_{\text{income effect}}=\varepsilon_{ni,g}^{(ji,k)}+\eta_{ni,g}\left(\frac{\partial \ln Y_{i}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}, \tag{27}$$

where $\varepsilon_{ni,g}^{(ji,k)}$ and $\eta_{ni,g}$ denote the Marshallian price and income elasticities of demand. The presence of $\left(\partial \ln Y_{i}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ in the above expression, manifests the circular nature of our general equilibrium setup. We will not unpack this term for now. Instead, we show later that income effects sum up to zero at the optimum.

The last term in Equation 26, accounts for scale effects: Noting that $P_{ni,g}=\bar{\rho}_{ni,g}w_{n}\left[\sum_{\iota}\bar{a}_{n\iota,k}Q_{n\iota,k}\right]^{-\frac{\mu_k}{1+\mu_k}}$, a change in the export supply of good ni,g (due to a change in $\tilde{P}_{ji,k}$) alters the scale of production in *origin n–industry g* and the producer prices associated with that location. Due to cross demand effects, this change also impacts the producer price of domestic suppliers as well as foreign suppliers outside of origin n. To keep track of the general equilibrium scale effects, we use $\omega_{ni,g}$ to denote (the inverse of) good ni,g's export supply elasticity:

$$\omega_{ni,g}\equiv\sum_{j\neq i}\frac{P_{ji,g}Q_{ji,g}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial P_{ji,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}+\sum_{\ell\in\mathbb{C}}\frac{P_{i\ell,g}Q_{i\ell,g}}{P_{i\ell,g}Q_{i\ell,g}}\left(\frac{\partial P_{i\ell,g}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}$$
$$=\frac{1}{r_{ji,k}\rho_{j,k}}\sum_{g}\left[\frac{\bar{w}_{i}L_{i}}{\bar{w}_{j}L_{j}}\rho_{i,g}\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{i}}+\sum_{n\neq i}\frac{\bar{w}_{n}L_{n}}{\bar{w}_{j}L_{j}}r_{ni,g}\rho_{n,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{T}_{i}}\right]\sim\text{export supply elasticity} \tag{28}$$

The second line in the above definition derives from the fact that $\left(\frac{\partial \ln P_{i\ell,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{i}}=\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{i}}$ for all $\ell\in\mathbb{C}$ (as the price of origin i's good sold to different locations differ in only a constant iceberg cost shifter) and that sales shares for each origin $n\in\mathbb{C}$ are defined as follows:

$$r_{ni,g}\equiv\frac{P_{ni,g}Q_{ni,g}}{\sum_{\iota\in\mathbb{C}}\left(P_{n\iota,g}Q_{n\iota,g}\right)}\sim\text{good-specific sales share;}\qquad \rho_{n,g}=\frac{\sum_{\iota\in\mathbb{C}}\left(P_{n\iota,g}Q_{n\iota,g}\right)}{\bar{w}_{n}L_{n}}\sim\text{industry-wide sales share.}$$

For now, we do not unpack the supply elasticity, $\omega_{ni,g}$. We relegate this task instead to Step #4 of the proof, where we solve our full system of F.O.C.s. Using the above definition for $\omega_{ni,g}$, we can simplify

Equation 26 as follows:

$$\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k}+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{29}$$

Moving on, the effect of a change in $\tilde{P}_{ji,k}$ on country i's production and export tax revenues can be unpacked as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\left(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii}\right)\cdot\mathbf{Q}_{ii,g}+\sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in}\right)\cdot\mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$$

$$=\sum_{g}\left[\left(\tilde{P}_{ii,g}-P_{ii,g}\right)Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]-\sum_{g}\sum_{n}\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{30}$$

The first term in the above equation accounts for revenue effects that channel through a change in the demand for domestic varieties (i.e., ii,g). The second term accounts for scale effects—i.e., a change in $Q_{ii,g}$ alters the scale of production in origin i–industry k, and the producer prices associated with country i in all export markets. To simplify the term accounting for scale effects, we invoke the following observation, which follows from the *Free Entry* condition:[71]

$$\sum_{n}\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}\right]=\sum_{n}\left[\frac{P_{in,g}Q_{in,g}}{P_{ii,g}Q_{ii,g}}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}\right]P_{ii,g}Q_{ii,g}= \tag{32}$$

$$=\sum_{n}\left[\frac{r_{in,g}}{r_{ii,g}}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}\right]P_{ii,g}Q_{ii,g}=-\frac{\mu_g}{1+\mu_g}P_{ii,g}Q_{ii,g},$$

The last line in the above equation follows from the fact that *(a)* $\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_{i}}=-\frac{\mu_g}{1+\mu_g}r_{ii,g}$, and *(b)* $\sum_n r_{in,g}=1$, by construction. We can plug the above equation back into Equation 30 to simplify it as follows:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\sum_{n}\left[\left(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in}\right)\cdot\mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=\sum_{g}\left[\left(\tilde{P}_{ii,g}-\left[1-\frac{\mu_g}{1+\mu_g}\right]P_{ii,g}\right)Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{33}$$

Note that $\left(\partial \ln Q_{ii,g}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses price and income effects as indicated by Equation 27. Combining Equations 29 and 33, we can express the sum of all tax revenue-related effects as

$$\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=\tilde{P}_{ji,k}Q_{ji,k}+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right]$$

$$+\sum_{g}\left[\left(\tilde{P}_{ii,g}-\frac{1}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right]+\Delta_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}. \tag{34}$$

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[71] In particular, note that $P_{in,g}=\tau_{in,g}P_{ii,g}$, where by Free Entry, $P_{ii,g}=\bar{\rho}_{ii,g}w_i\mathcal{Q}_{i,g}^{-\frac{\mu_g}{1+\mu_g}}$, with $\mathcal{Q}_{i,g}=\sum_n\bar{a}_{in,g}Q_{in,g}$ denoting country i's effective output in industry g. Hence, holding $\mathbf{w}$ and $\mathbb{P}_i-\{\tilde{P}_{ji,k}\}$ constant, we can show that

$$\sum_{n}\left[\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\frac{r_{in,g}}{r_{ij,g}}\right]=\sum_{n}\left[\left(\frac{\partial \ln P_{ii,g}}{\partial \ln \mathcal{Q}_{i,g}}\frac{\partial \ln \mathcal{Q}_{i,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\frac{r_{in,g}}{r_{ij,g}}\right]=\frac{\partial \ln P_{ii,g}}{\partial \ln \mathcal{Q}_{i,g}}=-\frac{\mu_g}{1+\mu_g}, \tag{31}$$

where the second line follows from the fact that $\partial \ln \mathcal{Q}_{i,g}/\partial \ln Q_{ij,g}=r_{ij,g}$, by definition.

The uniform term $\Delta_i(\mathbb{P}_i)$ regulates the net force of (circular) general equilibrium income effects. It correspondingly depends on the Marshallian income elasticities of demand:

$$\Delta_i(\mathbb{P}_i;\mathbf{w}) \equiv \sum_g \sum_{n\neq i} \left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right) Q_{ni,g}\eta_{ni,g}\right] + \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1}{1+\mu_g}P_{ii,g}\right) Q_{ii,g}\eta_{ii,g}\right]. \quad (35)$$

To characterize the general equilibrium wage effects in the F.O.C. (i.e., the last term on the right-hand side of Equation 24), we invoke our earlier result under Lemma 5: By the targeting principle $\mathbf{w}_{-i}$ is welfare neutral at the optimum (i.e., $\mathbb{P}_i = \mathbb{P}_i^*$), which implies that

$$\left(\frac{\partial W_i(\mathbb{P}_i^*;\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \left(\frac{\partial W_i(\mathbb{P}_i^*;\mathbf{w})}{\partial w_i}\right)_{\mathbf{w}_{-i},\mathbb{P}_i} \left(\frac{\mathrm{d}w_i}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}.$$

That is, we can characterize the term that encompasses wage effects, treating $\mathbf{w}_{-i}$ as given. Accordingly, the term $\left(\mathrm{d}w_i/\mathrm{d}\ln \tilde{P}_{ji,k}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}$ can be calculated by applying the Implicit Function Theorem to country i's balanced trade condition,[72]

$$\text{[Balanced Trade]} \qquad \mathrm{T}_i\left(\mathbb{P}_i,\mathbf{w}\right) \equiv \sum_{n\neq i}\left[\mathbf{P}_{ni}(\mathbb{P}_i;\mathbf{w})\cdot\mathbf{Q}_{ni,g}(\mathbb{P}_i;\mathbf{w}) - \tilde{\mathbf{P}}_{ni}\cdot\mathbf{Q}_{ni,g}(\mathbb{P}_i;\mathbf{w})\right],$$

while treating $\mathbf{w}_{-i} = \bar{\mathbf{w}}_{-i}$ as if it were given. This step yields the following equation

$$\begin{aligned}
\left(\frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}} &= -\left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \Big/ \left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})}{\partial \ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i} \\
&= \frac{-\sum_{n\neq i}\left[(\mathbf{P}_{ni}\odot\mathbf{Q}_{ni})\cdot\left(\frac{\partial \ln \mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + (\mathbf{P}_{ni}\odot\mathbf{Q}_{ni})\cdot\left(\boldsymbol{\Omega}_{ni}\odot\frac{\partial \ln \mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]}{\left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})}{\partial \ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}}. \quad (36)
\end{aligned}$$

where $\boldsymbol{\Omega}_{ni} \equiv \{\omega_{ni,k}\}_k$ is a vector composed of export supply elasticities (as defined under Equation 28) and $\odot$ denotes the element-wise product of two equal-sized vectors (i.e., $\mathbf{a}\odot\mathbf{b} = [a_n b_n]_n$). The second line in the above equation follows from the fact that $\left(\partial \ln Q_{in,g}(\mathbb{P}_i,\mathbf{w})/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0$ if $n \neq i$. That is, if we fix the vector of wages, $\mathbf{w}$, the choice of $\tilde{P}_{ji,k}$ has no effect on the demand schedule in the rest of the world. In other words, the only way the effect of $\tilde{P}_{ji,k}$ transmits to foreign markets is through its effect on $\mathbf{w}$. Now, define the importer-wide term, $\bar{\tau}_i$, as follows:

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}\left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})/\partial \ln w_i\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}}. \quad (37)$$

Importantly, note that $\bar{\tau}_i$ does not feature an industry-specific subscript. Combining Equation 36 with

[72] To be clear about the notation, we can write country i's balanced trade condition without appealing to the inner product operator as follows:

$$\mathrm{T}_i\left(\mathbb{P}_i,\mathbf{w}\right) \equiv \sum_g \sum_{n\neq i}\left(P_{ni,g}(\mathbb{P}_i,\mathbf{w})Q_{ni,g}(\mathbb{P}_i,\mathbf{w}) - \tilde{P}_{in,g}Q_{in,g}(\mathbb{P}_i,\mathbf{w})\right) = 0.$$

the expression for $\bar{\tau}_i$, we can summarize the wage effects in the F.O.C. (Equation 24) as follows

$$\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = -\sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\tau_i P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right]$$
$$-\sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_i P_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}. \quad (38)$$

Finally, plugging Equations 25, 34, and 38 back into the F.O.C. (Equation 24); yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ji,k}\in\mathbb{P}_i$:

$$[\text{FOC w.r.t. } \tilde{P}_{ji,k}] \qquad \sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right]$$
$$+\sum_g\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}=0. \quad (39)$$

The uniform term $\widetilde{\Delta}_i(.)$ is defined analogously to $\Delta_i(.)$, but adjusts for the interaction of general equilibrium wage and income effects:

$$\widetilde{\Delta}_i(\mathbb{P}_i;\mathbf{w})\equiv\sum_g\sum_{n\neq i}\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\omega_{ni,g})(1+\bar{\tau}_i)\right)P_{ni,g}Q_{ni,g}\eta_{ni,g}\right]+\sum_g\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right]. \quad (40)$$

Before moving forward, a remark on the uniform term $\bar{\tau}_i$ is in order. We do not unpack this term because the multiplicity of country i's optimal tax schedule (per Lemma 1) will render the exact value assigned to $\bar{\tau}_i$ as redundant. We will elaborate more on this point when we combine the F.O.C.s *w.r.t.* all tax instruments in step #4 of the proof.

***Step 3.B*: Deriving the W.O.C. w.r.t. $P_{ii,k}\in\mathbb{P}_i$.**

Next, we derive the F.O.C. w.r.t. to a locally produced and locally consumer variety ii,k. Recall that the objective function can is given by $W_i = V_i(Y_i(\mathbb{P}_i;\mathbf{w}),\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji})$. The F.O.C. w.r.t. $\tilde{P}_{ii,k}$, holding the remaining elements of $\mathbb{P}_i$ (namely, $\mathbb{P}_{-ii,k}\equiv\mathbb{P}_i-\{\tilde{P}_{ii,k}\}$) constant, can be stated as

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ii,k}}\right)_{\mathbb{P}_{-ii,k}}=\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ii,k}}+\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ii,k}}\right)_{\mathbf{w},\mathbb{P}_{-ii,k}}+\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ii,k}}\right)_{\mathbb{P}_{-ii,k}}=0. \quad (41)$$

Each element of the right-hand side can be characterized in a manner identical to *Step 3.A*. Specifically, the first term can be simplified using Roy's identity. The second term, which accounts for revenue-raising effects can be characterized using cross-demand elasticities w.r.t. $\tilde{P}_{ii,k}$ instead of $\tilde{P}_{ji,k}$. The same goes for the last term accounting for general equilibrium wage effects. Repeating the derivations in

Step 3.A, the F.O.C. characterized by Equation 41 can be unpacked as follows:

$$[\text{FOC w.r.t. } \tilde{P}_{ii,k}] \qquad \sum_{n\neq i}\sum_{g}\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ii,k)}\right]$$
$$+\sum_{g}\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ii,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ii,k}}\right)_{\mathbf{w},\mathbb{P}_{-ii,k}}=0, \tag{42}$$

where the uniform terms, $\widetilde{\Delta}_i(.)$, and $\bar{\tau}_i$, have the same definition as that introduced under Equations 40 and 37.

***Step 3.C*: Deriving the W.O.C. w.r.t. $P^*_{ij,k}$.**

Finally, we derive the F.O.C. w.r.t. to export variety ij,k, which is sold to destination $j\neq i$ in industry k. Note again that the objective function is given by $W_i = V_i(Y_i(\mathbb{P}_i;\mathbf{w}),\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji})$. The F.O.C. w.r.t. $\tilde{P}_{ij,k}$, holding the remaining elements of $\mathbb{P}_i$ (namely, $\mathbb{P}_{-ij,k}\equiv\mathbb{P}_i-\{\tilde{P}_{ij,k}\}$) constant, can be stated as

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ij,k}}+\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}+\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=0. \tag{43}$$

The first term as before accounts for the direct effect of a price change on consumer surplus. This term is trivially equal to zero in this case, since $\tilde{P}_{ij,k}\notin\tilde{\mathbf{P}}_i$. That is, since ij,k is not part of the domestic consumption bundle, raising its price has no direct effect on consumer surplus in country i:

$$\tilde{P}_{ij,k}\notin\tilde{\mathbf{P}}_i \implies \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ij,k}}=0. \tag{44}$$

The second term in Equation 43 accounts for the revenue-raising effects of a change in $\tilde{P}_{ij,k}\in\mathbb{P}_i$. To unpack this term note that total income (or expenditure) in country i is dictated by the sum of wage payments and tax revenues:

$$Y_i(\mathbb{P}_i;\mathbf{w})=w_iL_i+\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot\mathbf{Q}_{ni}\right]+(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii})\cdot\mathbf{Q}_{ii}+\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right],$$

Hence, holding wages $\mathbf{w}$ constant, the change in country i's income amounts to the change in import, domestic, and export tax revenues. The effect on import tax revenues can be unpacked as follows:

$$\left(\frac{\partial\sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot\mathbf{Q}_{ni}\right]}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\sum_{g}\sum_{n\neq i}\left[(\tilde{P}_{ni,g}-P_{ni,g})\,Q_{ni,g}\left(\frac{\partial\ln Q_{ni,g}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\omega_{nj,g}\left(\frac{\partial\ln Q_{nj,g}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]. \tag{45}$$

where $\omega_{nj,g}$ is the export supply elasticity as defined by 28. The first term on the right-hand side accounts for general equilibrium income effects: Specifically, a change in $\tilde{P}_{ij,k}$ can raise country i's income Y_i through higher tax revenues, and alter the entire demand schedule, $Q_{ni,g}=\mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_i,Y_i)$, in the local market. The second term accounts for scale effects: To elaborate, a change in $\tilde{P}_{ij,k}$ distorts origin i's export supply schedule in market $j\in\mathbb{C}$. This change alters the scale of production and the producer prices associated with *origin n–industry g* that serves market j (this includes $P_{ni,g}$ which is associated with economy i). It also changes the scale of production and producer prices from foreign suppliers through cross-demand effects. These changes in international producer prices, impacts country i's terms-of-trade by changing its import tax revenues. Also, note that since the rest of the world (including country j) is passive, their income is pinned to their wage rate and vector $\mathbf{w}$. Hence, $\left(\partial Y_j / \partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0$, which implies that $\left(\partial \ln Q_{nj,g} / \partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \partial \ln \mathcal{D}_{nj,g}(\bar{Y}_j, \tilde{\mathbf{P}}_j) / \partial \ln \tilde{P}_{ij,k} = \varepsilon^{(ij,k)}_{nj,g}$. Likewise, since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$, its only effect on the demand schedule in the local market i is through general equilibrium income effects. Putting these results together, we can posit that

$$\left(\frac{\partial \ln Q_{n\iota,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \begin{cases} \varepsilon^{(ij,k)}_{nj,g} & \text{if } \iota = j \\ \eta_{ni,g}\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} & \text{if } \iota = i \end{cases}.$$

Considering the above expressions and noting our earlier definition for $\omega_{ni,g}$ under Equation 28, Equation 45 can be simplified as

$$\begin{aligned}\left(\frac{\partial \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni})\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} &= -\sum_g\sum_{n\neq i}\left[\omega_{nj,g}P_{ni,g}Q_{ni,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} \\ &= -\sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}.\end{aligned} \tag{46}$$

The last line in the above equation follows from (1) the definition of ω, which entails that $\omega_{nj,g}r_{ni,g} = \omega_{ni,g}r_{nj,g}$, and (2) the fact that $r_{ni,g}/r_{nj,g} = P_{ni,g}Q_{ni,g}/P_{nj,g}Q_{nj,g}$, since the markup is uniform across output sold to different destinations in the same industry.

The effect of a change in $\tilde{P}_{ij,k}$ on country i's production and export tax revenues can be unpacked as follows:[73]

$$\begin{aligned}\left(\frac{\partial}{\partial \ln \tilde{P}_{ij,k}}\sum_n\left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right]\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} &= \sum_g\left[\left(\tilde{P}_{ii,g} - P_{ii,g}\right)Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right] \\ + \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - P_{ij,g}\right)Q_{ij,g}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right] &+ \sum_g\sum_n\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbf{w},\mathbb{P}_i}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right].\end{aligned} \tag{47}$$

The first term (in the first line) account for the effect on domestic tax revenues that channel through general equilibrium income effects. The second term on the right-hand side ($\tilde{P}_{ij,k}Q_{ij,k}$) accounts for the direct, arithmetic effect of $\tilde{P}_{ij,k}$ on export tax revenues. The third term account for revenue effects that channel through a change in the demand for all varieties sold to destination j (i.e., ij, g). The last term accounts for scale effects—i.e., a change in $Q_{ij,g}$ alters the scale of production in *origin i–industry g*, and modifies all the producer prices associated with that industry. As noted in *Step 3.A*, the last term in Equation 47 can be simplified using the Free Entry condition, which entails that (See Equation 32):

$$\sum_{n\in\mathbb{C}}\left[P_{in,g}Q_{in,g}\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbf{w},\mathbb{P}_i}\right] = -\frac{\mu_g}{1+\mu_g}P_{ij,g}Q_{ij,g},$$

Also, recall from our earlier discussion that since country $j \neq i$ collects no tax revenues by assumption,

[73] To be clear, $\sum_n\left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right] = (\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii})\cdot \mathbf{Q}_{ii,g} + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right]$ denotes the sum of domestic and export tax revenues.

$\left(\partial Y_j/\partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0$, which implies that $\left(\partial \ln Q_{nj,g}/\partial \ln \tilde{P}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \partial \ln \mathcal{D}_{nj,g}(\bar{Y}_j, \tilde{\mathbf{P}}_j)/\partial \ln \tilde{P}_{ij,k} = \varepsilon^{(ij,k)}_{nj,g}$. Plugging these expressions back into Equation 47 simplifies it as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ij,k}} \sum_n \left[(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}) \cdot \mathbf{Q}_{in}\right]\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \frac{\mu_g}{1+\mu_g}] P_{ij,g}\right) Q_{ij,g} \varepsilon^{(ij,k)}_{ij,g}\right] + \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1}{1+\mu_g} P_{ii,g}\right) Q_{ii,g} \eta_{ii,g}\right] \left(\frac{\partial \ln Y_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{48}$$

Combining Equations 46 and 48, we can express the sum of tax revenue-related effects as

$$\left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \frac{\mu_g}{1+\mu_g}] P_{ij,g}\right) Q_{ij,g} \varepsilon^{(ij,k)}_{ij,g}\right] - \sum_g \sum_{n \neq i} \left[\omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon^{(ij,k)}_{nj,g}\right] + \Delta_i(\mathbb{P}_i; \mathbf{w}) \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}, \tag{49}$$

where $\Delta_i(.)$ encompasses the terms accounting for circular income effects and is given by Equation 35. No we turn to characterizing the general equilibrium wage effects in the F.O.C.—namely, the last term on the right-hand side of Equation 24. To this end, we invoke our observation based on the *targeting principle* (as stated under Lemma 5) that $\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \left(\frac{\partial W_i(.)}{\partial w_i}\right)_{\mathbf{w}_{-i},\mathbb{P}_i} \left(\frac{\mathrm{d} w_i}{\mathrm{d} \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ij,k}}$. The term $\left(\frac{\mathrm{d} w_i}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ji,k}}$ can be calculated by applying the Implicit Function Theorem to country i's balanced trade condition,

$$[\text{Balanced Trade}] \qquad \mathrm{T}_i\left(\mathbb{P}_i, \mathbf{w}\right) \equiv \sum_{n \neq i} \left[\mathbf{P}_{ni}(\mathbb{P}_i; \mathbf{w}) \cdot \mathbf{Q}_{ni,g}(\mathbb{P}_i; \mathbf{w}) - \tilde{\mathbf{P}}_{ni} \cdot \mathbf{Q}_{ni,g}(\mathbb{P}_i; \mathbf{w})\right],$$

while treating $\mathbf{w}_{-i} = \bar{\mathbf{w}}_{-i}$ as given. This application yields the following equation (*Notation*: $\mathbf{\Omega}_{nj} \equiv \{\omega_{nj,k}\}_k$ is a vector composed of export supply elasticities, while $\odot$ and $\cdot$ denotes the element-wise and inner products of two equal-sized vectors):

$$\left(\frac{\mathrm{d} \ln w_i}{\mathrm{d} \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}_{-i},\mathbb{P}_{-ij,k}} = -\left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i, \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} / \left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i, \mathbf{w})}{\partial \ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}$$
$$= \frac{-\tilde{P}_{ij,k} Q_{ij,k} - \left(\tilde{\mathbf{P}}_{ij} \odot \mathbf{Q}_{ij}\right) \cdot \left(\frac{\partial \ln \mathbf{Q}_{ij}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \sum_{n \neq i} \left[\left(\mathbf{P}_{ni} \odot \mathbf{Q}_{ni}\right) \cdot \left(\frac{\partial \ln \mathbf{Q}_{ni}}{\partial \ln \tilde{P}_{ij,k}} + \mathbf{\Omega}_{nj} \odot \frac{\partial \ln \mathbf{Q}_{nj}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]}{\left(\frac{\partial \mathrm{T}_i(\mathbb{P}_i, \mathbf{w})}{\partial \ln w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}} \tag{50}$$

The numerator in the second line of the above equation is composed of three terms: The first term accounts for the arithmetic effect of $\tilde{P}_{ji,k}$ on country i's trade balance. The second term account for own- and cross-price effects that are specific to market j—the market to which good ij,k is being exported. The last term accounts for scale effects: Specifically, a change in $\tilde{P}_{ij,k}$ interacts with the *balanced trade condition* by modifying the producer of a generic good ni,g imported from *origin i–industry g*. As before, define the uniform importer-wide term, $\bar{\tau}_i$, as follows

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial w_i}\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}\left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})/\partial \ln w_i\right)_{\bar{\mathbf{w}}_{-i},\mathbb{P}_i}}. \tag{51}$$

Combining Equation 50 with the expression for $\bar{\tau}_i$, we can summarize the wage effects in the F.O.C. (Equation 24) as follows:

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{d\mathbf{w}}{d\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i\tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] - \sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{52}$$

Finally, plugging Equations 44, 49, and 52 back into the F.O.C. (Equation 43); and dividing all the expressions by $(1+\bar{\tau}_i)$ yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ij,k}\in\mathbb{P}_i$:

$$[\text{FOC w.r.t. } \tilde{P}_{ij,k}]\qquad \tilde{P}_{ij,k}Q_{ij,k}+\sum_g\left[\left(1-\frac{1}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right] + \widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0, \tag{53}$$

where $\widetilde{\Delta}_i(.)$ is defined as in Equation 40. Also, we are not unpacking the term $\bar{\tau}_i$, for the same reasons discussed under *Step 3.A*.

Step #4: Solving the System of F.O.C.s and Establishing Uniqueness

To determine the optimal tax schedule we need to collect the each of first order conditions and simultaneously solve them under one system. For the ease of reference, we will restate the F.O.C. w.r.t. to each element of $\mathbb{P}_i$ below. Following Equations 39 and 42, the F.O.C. w.r.t. $\tilde{P}_{\ell i,k}\in\mathbb{P}_i$ (where $\ell = i$ or $\ell = j\neq i$) is given by the following equation:

$$(1)\quad \sum_{n\neq i}\sum_g\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)e_{ni,g}\varepsilon^{(\ell i,k)}_{ni,g}\right]+$$
$$\sum_g\left[\left(1-\frac{1}{1+\mu_g}\frac{P_{ii,g}}{\tilde{P}_{ii,g}}\right)e_{ii,g}\varepsilon^{(\ell i,k)}_{ii,g}\right]+\widetilde{\Delta}_i(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial\ln Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{\ell i,k}}\right)_{\mathbf{w},\mathbb{P}_{-\ell i,k}} = 0.$$

where $e_{ni,g} = \tilde{P}_{ni,g}Q_{ni,g}/Y_i$ denotes the (unconditional) expenditure share on good ni,g. Likewise, dividing Equation 53 by $\tilde{P}_{ij,k}Q_{ij,k}$, the F.O.C. w.r.t. export price $\tilde{P}_{ij,k}\in\mathbb{P}_i$ is given by the following equation:

$$(2)\quad 1+\sum_g\left[\left(1-\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\frac{e_{ij,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{ij,g}\right]$$
$$-\sum_{n\neq i}\sum_g\left[\omega_{ni,g}\frac{e_{nj,g}}{e_{ij,k}}\varepsilon^{(ij,k)}_{nj,g}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\frac{Y_i}{Y_j}\left(\frac{\partial\ln Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0.$$

Going forward note two points: First, the system of F.O.C.s labeled (1) can be solved independent of (2). Second, if we establish that the unique solution to System (1) is the trivial solution, then it follows immediately that $\widetilde{\Delta}_i(\mathbb{P}_i^*; \mathbf{w}) = 0$–that is, circular income effects are welfare neutral at the optimum under the trivial solution.

To establish uniqueness, note that for any $\tilde{P} \in \mathbb{P}_i$, the prohibitive rate, i.e., $\tilde{P} \to \infty$, satisfies the necessary conditions specified by Equations (1) and (2). But prohibitive prices are trivially suboptimal because they eliminate the good from consumption basket without raising any revenue to compensate the consumers. So, we hereafter restrict attention to non-prohibitive prices whereby $e_{ni,g} > 0$ for all ni, g. Taking note of this observation, we now how that System (1) has a unique trivial solution. To this, end we express System (1) in matrix notation, under the assumption that $\widetilde{\Delta}_i(\mathbb{P}_i^*, \mathbf{w}) = 0$. As noted earlier, this assumption will be automatically justified if it turns our that the resulting system admits a unique trivial solution. This step, yields the following matrix equation:

$$\underbrace{\begin{bmatrix} e_{1i,1}\varepsilon_{1i,1}^{(1i,1)} & \cdots & e_{Ni,}\varepsilon_{Ni,1}^{(1i,1)} & \cdots & e_{1i,K}\varepsilon_{1i,K}^{(1i,1)} & \cdots & e_{Ni,K}\varepsilon_{Ni,K}^{(1i,1)} \\ \vdots & & \ddots & & \ddots & & \vdots \\ e_{1i,1}\varepsilon_{1i,1}^{(Ni,K)} & \cdots & e_{Ni,}\varepsilon_{Ni,1}^{(Ni,K)} & \cdots & e_{1i,K}\varepsilon_{1i,K}^{(Ni,K)} & \cdots & e_{Ni,K}\varepsilon_{Ni,K}^{(Ni,K)} \end{bmatrix}}_{\widetilde{\mathbf{E}}_i} \begin{bmatrix} 1 - (1+\omega_{1i,k})(1+\bar{\tau}_i)\frac{P_{1i,1}}{\tilde{P}^*_{1i,k}} \\ \vdots \\ 1 - \frac{1}{1+\mu_k}\frac{P_{ii,k}}{\tilde{P}^*_{ii,k}} \\ \vdots \\ 1 - (1+\omega_{Ni,k})(1+\bar{\tau}_i)\frac{P_{Ni,k}}{\tilde{P}^*_{Ni,k}} \end{bmatrix}_k = \mathbf{0}.$$

To establish that the above equation exhibits a unique, trivial solution it suffices to show that the demand-adjusted elasticity matrix, $\mathbf{E}_i = \left[e_{ji,k}\varepsilon_{ji,k}^{(ni,g)}\right]_{jk,ng}$ is non-singular. The following intermediate lemma establishes this result using the primitive properties of the Marshallian demand function.

Lemma 6. *The $NK \times NK$ matrix $\widetilde{\mathbf{E}}_i \equiv \left[e_{ji,k}\varepsilon_{ji,k}^{(ni,g)}\right]_{jk,ng}$ is non-singular.*

Proof. We can appeal to Proposition 2.E.2 in Mas-Colell, Whinston, Green, et al. (1995), which indicates that the Marshallian demand function satisfies $e_{ji,k} =\mid e_{ji,k}\varepsilon_{ji,k}^{(ji,k)} \mid - \sum_{n,g\neq j,k} \mid e_{ni,g}\varepsilon_{ni,g}^{(ji,k)} \mid$—a property often referred to as Cournot aggregation. Since $e_{ji,k} > 0$ (as we have ruled out prohibitive prices), Cournot aggregation ensures the matrix $\widetilde{\mathbf{E}}_i$ is strictly diagonally dominant. The Lèvy-Desplanques Theorem (Horn and Johnson (2012)), accordingly, ensures that $\widetilde{\mathbf{E}}_i$ is non-singular. The lower bound on $\det(\widetilde{\mathbf{E}}_i)$ follows trivially from Gerschgorin's circle theorem. Specifically, following Ostrowski (1952),

$$\mid \det\left(\widetilde{\mathbf{E}}_i\right) \mid \geq \prod_{j\in\mathbb{C}}\prod_{k\in\mathbb{K}}\left(\left|e_{ji,k}\varepsilon_{ji,k}^{(ji,k)}\right| - \sum_{(n,g)\neq(j,k)}\left|e_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right|\right) = \prod_{j\in\mathbb{C}}\prod_{k\in\mathbb{K}} e_{ji,k} > 0.$$

□

Appealing to above lemma, it is immediate that the unique solution to the above matrix equation is indeed the trivial solution, given by:

$$\frac{\tilde{P}^*_{ji,k}}{P_{ji,1}} = (1+\omega_{ji,k})(1+\bar{\tau}_i); \qquad \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}} = \frac{1}{1+\mu_g}. \tag{54}$$

It is straightforward to check that the above solution constitutes a global maximum by contradiction. To present the logic: Since $\lim_{\tilde{\mathbf{P}}_i\to\infty} W_i(\mathbb{P}_i, \mathbf{w}) \to 0$, the above solution identifies a vector of consumer prices at home, $\tilde{\mathbf{P}}_i^* \in \mathbb{P}_i^*$, that yields a strictly higher welfare level than prohibitive prices. As such, $\mathbb{P}_i^*$ cannot constitute a global minimum.

With the aid of the above result, we can proceed to solving System (2), knowing that $\widetilde{\Delta}_i(\mathbb{P}_i^*, \mathbf{w}) = 0$. To this end, let us economize on the notation by defining χ as follows:

$$\chi_{ij,k} \equiv \frac{1}{(1+\mu_g)(1+\bar{\tau}_i)} \frac{P_{ij,g}}{\tilde{P}_{ij,g}}.$$

Invoking this minor switch of notation, the F.O.C. specified by System (2) implies the following optimality condition:

$$1 + \sum_g \left[(1-\chi_{ij,g}) \frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}} \right] - \sum_{n\neq i}\sum_g \left[\omega_{ni,g} \frac{e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}} \right] = 0. \tag{55}$$

To simplify the above expression we will appeal to the Cournot aggregation property–a well-known primitive property of Marshallian demand as discussed earlier (see Mas-Colell et al. (1995)):

$$[\text{Cournot aggregation}] \qquad 1 + \sum_g \left[\frac{e_{ij,g}}{e_{ij,k}} \varepsilon_{ij,g}^{(ij,k)} \right] = -\sum_{n\neq i}\sum_g \left[\frac{e_{nj,g}}{e_{ij,k}} \varepsilon_{nj,g}^{(ij,k)} \right].$$

Next, combine the above expression with Equation 55, while noting that by Slutsky's equation $\frac{e_{nj,g}}{e_{ij,k}} \varepsilon_{nj,g}^{(ij,k)} = \varepsilon_{ij,k}^{(nj,g)}$ if $\eta_{ni,g} = 1$ for all ni, g. Performing these steps yields the following:

$$-\sum_g \left[\chi_{ij,g}\varepsilon_{ij,k}^{(ij,g)} \right] - \sum_{n\neq i}\sum_g \left[(1+\omega_{ni,g})\varepsilon_{ij,k}^{(nj,g)} \right] = 0 \qquad \forall (ij,k).$$

We can rewrite the above equation in matrix algebra as follows:

$$-\mathbf{E}_{ij}\mathbf{X}_{ij} - \mathbf{E}_{ij}^{(-ij)} \left(\mathbf{1}_{(N-1)K} + \mathbf{\Omega}_i \right) = 0, \tag{56}$$

where $\mathbf{X}_{ij} \equiv \left[\chi_{ij,k}\right]_k$ is a $K \times 1$ vector. The $K \times K$ matrix $\mathbf{E}_{ij} \sim \mathbf{E}_{ij}^{(ij)} \equiv \left[\varepsilon_{ij,k}^{(ij,g)}\right]$ encompasses the own- and cross-price elasticities between the different varieties sold by origin i to market j—see Definition (D1). Analogously, $\mathbf{E}_{ij}^{(-ij)} \equiv \left[\varepsilon_{ij,k}^{(nj,g)}\right]_{k,n\neq i,g}$ is a $K \times (N-1)K$ matrix summarizing the cross-price elasticity of market j's demand between varieties sold by origin i and all other (non-i) origin countries. $\mathbf{\Omega}_i \equiv \left[\omega_{ni,g}\right]_{n,g}$ is a $(N-1)K \times 1$ vector of all *import good-specific* inverse supply elasticities. To invert the above system we need to establish that $\mathbf{E}_{ij}$ is non-singular, which is done under the following lemma.

Lemma 7. *The $K \times K$ matrix $\mathbf{E}_{ij} \equiv \left[\varepsilon_{ij,k}^{(ij,g)}\right]_{k,g}$ is non-singular.*

Proof. The proof proceeds similar to Lemma 6: The Marshallian demand function's homogeneity of degree zero implies that $\mid \varepsilon_{ij,k}^{(ij,k)} \mid = \eta_{ij,k} + \sum_{n,g\neq i,k} \mid \varepsilon_{ij,k}^{(nj,g)} \mid$. Based on this property, since $\eta_{ij,k} > 0$, the matrix $\mathbf{E}_{ij}$ is strictly diagonally dominant. The Lèvy-Desplanques Theorem (Horn and Johnson (2012)), therefore, ensures that $\mathbf{E}_{ij}$ is non-singular. $\square$

Following the above lemma we can invert the system specified by Equation 56 to obtain the optimal level of $\mathbf{X}_{ij} = \left[\chi_{ij,k}\right]_k$:

$$\mathbf{X}_{ij}^* = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)} \left(\mathbf{1}_{(N-1)K} + \mathbf{\Omega}_i \right). \tag{57}$$

Next, there remains the task of recovering the optimal tax/subsidy rates from the optimal price wedges implies by Equations 54 and 57. Noting the following relationship between taxes/subsidies and price wedges,

$$1+t^*_{ji,k}=\frac{\tilde{P}_{ji,k}}{P_{ji,k}};\qquad 1+s^*_{i,k}=\frac{P_{ii,k}}{\tilde{P}^*_{ii,k}};\qquad 1+x_{ij,k}=\frac{P_{ij,k}/\tilde{P}^*_{ij,k}}{P_{ii,k}/\tilde{P}^*_{ii,k}};$$

country i's unilaterally optimal tax schedule can be expressed as follows:

$$\begin{aligned}[\text{domestic subsidy}]\quad & 1+s^*_{i,k}=1+\mu_k\\ [\text{import tariff}]\quad & 1+t^*_{ji,k}=(1+\omega_{ji,k})(1+\bar{\tau}_i)\\ [\text{export subsidy}]\quad & \mathbf{1}+\mathbf{x}^*_{ij}=-\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right).\end{aligned}\tag{58}$$

The last step is to invoke the multiplicity of optimal tax schedules as indicated by Lemma 1. Doing so indicates that the uniform term $\bar{\tau}_i$ is redundant and need not be unpacked. To elaborate, Lemma 1 indicates that any policy schedule that includes an import tax equal to $(1+\bar{t}_i\in\mathbb{R}_+)$

$$1+t_{ji,k}=(1+\omega_{ji,k})(1+\bar{\tau}_i)(1+\bar{t}_i)$$

is also optimal, since it delivers an identical level of welfare to the original optimal policy schedule specified by 58. As such, the exact value assigned to $\bar{\tau}_i$ is redundant for a welfare standpoint. This is why we did not unpack the term $\bar{\tau}_i$ earlier in Step #3. Lemma 1 indicates that there is another dimension of multiplicity, whereby any uniform shift in domestic production subsidies (paired with a proportional adjustment to w_i) preserves the equilibrium. Considering these points, the optimal policy schedule (after accounting for all dimensions of multiplicity) is given by:

$$\begin{aligned}[\text{domestic subsidy}]\quad & 1+s^*_{i,k}=(1+\mu_k)(1+\bar{s}_i)\\ [\text{import tariff}]\quad & 1+t^*_{ji,k}=(1+\omega_{ji,k})(1+\bar{t}_i)\\ [\text{export subsidy}]\quad & \mathbf{1}+\mathbf{x}^*_{ij}=-\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right),\end{aligned}$$

where $1+\bar{s}_i=1+\bar{t}_i\in\mathbb{R}_+$ are arbitrary tax shifters. What remains is a formal characterization of the good-specific supply elasticity, $\omega_{ji,k}$, which is presented below.

Characterizing the (Inverse) Export Supply Elasticity, $\omega_{ji,k}$. To fix ideas, it is helpful to repeat the definition of the export supply elasticity presented earlier:

$$\omega_{ji,k}\equiv\frac{1}{r_{ji,k}\rho_{j,k}}\sum_g\left[\frac{\check{w}_iL_i}{\check{w}_jL_j}\rho_{i,g}\left(\frac{\partial\ln P_{ii,g}}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i}+\sum_{n\neq i}\frac{\check{w}_nL_n}{\check{w}_jL_j}r_{ni,g}\rho_{n,g}\left(\frac{\partial\ln P_{ni,g}}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{T}_i}\right],\tag{59}$$

where $r_{ni,g}=P_{ni,g}Q_{ni,g}/\sum_{\iota\in\mathbb{C}}\left(P_{n\iota,g}Q_{n\iota,g}\right)$ and $\rho_{n,g}=\sum_{\iota\in\mathbb{C}}\left(P_{n\iota,g}Q_{n\iota,g}\right)/\check{w}_nL_n$ respectively denote the good ni,g-specific and industry-wide sales shares associated with origin $n\in\mathbb{C}$. Also, note that the producer price of good ni,g under free entry is given by $P_{ni,g}=\tau_{ni,g}P_{nn,g}$, where

$$P_{nn,g}=\bar{\varrho}_{nn,g}w_n\sum_{\iota\in\mathbb{C}}\left[\bar{a}_{n\iota,g}Q_{n\iota,g}\right]^{-\frac{\mu_g}{1+\mu_g}}\qquad\forall(n,g)$$

To characterize $\omega_{ji,k}$, we need to characterize $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{T}_i} = \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{T}_i}$ for each origin n–industry g. To this end we can apply the Implicit Function Theorem to the following function:

$$F_{ni,g}(Q_{1i,g},...,Q_{Ni,g},P_{11,g},...,P_{NN,g}) = P_{nn,g} - \bar{\varrho}_{nn,g}w_n \left[\bar{a}_{ni,g}Q_{ni,g} + \sum_{\ell \neq i} \bar{a}_{n\ell,g}Q_{n\ell,g}(\underbrace{\boldsymbol{\tau}_{-i\ell} \odot \mathbf{P}_{-i}}_{\tilde{\mathbf{P}}_{-i\ell}})\right]^{-\frac{\mu_g}{1+\mu_g}} = 0.$$

where $\boldsymbol{\tau}_{-in} \odot \mathbf{P}_{-i} \sim \left\{\tau_{jn,g}P_{jj,g}\right\}_{j\neq i,g}$ denotes the vector of consumer prices in market $n \neq i$ from all origins aside from i. The above function implicitly characterizes the producer prices in each origin j–industry g as a function of export supply levels to market i (i.e., $Q_{1i,g},...,Q_{Ni,g}$). Importantly, the above function treats both $\bar{\mathbb{P}}_i$ and $\mathbf{w}$ as given, as all elements of $\bar{\mathbb{P}}_i$ are chosen directly the by the government in i. Accordingly, the function $Q_{n\iota,g}(.)$ on the right-hand side derives from the Marshallian demand function,

$$Q_{jn,g}(\underbrace{\boldsymbol{\tau}_{-in} \odot \mathbf{P}_{-i}}_{\tilde{\mathbf{P}}_{-in}}) = \mathcal{D}_{n\iota,g}(\tilde{\mathbf{P}}_{-in}, \overline{\tilde{\mathbf{P}}_{in}}, \underbrace{\overline{w_n L_n}}_{Y_n}),$$

treating $\overline{\tilde{\mathbf{P}}_{in}} \in \mathbb{P}_i$ and $w_n \in \mathbf{w}$ as given. This function accounts for the fact that any change in the producer price of varieties associated with *origin n–industry g* will affect the consumer prices and the demand schedule in all market excluding i. The reason is that prices in international markets (excluding i) are not directly pinned down by the choice, $\mathbb{P}_i$. For the sake of presentation, abstract from cross-industry demand effects. Applying the Implicit Function Theorem to the system of equations specified by $F_{ni,g}(.)$, yields the following:

$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\mathbb{P}_i} & \cdots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\mathbb{P}_i} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbf{w},\mathbb{P}_i} & \cdots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbf{w},\mathbb{P}_i} \end{bmatrix} = -\underbrace{\begin{bmatrix} \frac{\partial F_{1,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{1,k}(.)}{\partial \ln P_{NN,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{N,k}(.)}{\partial \ln P_{NN,k}} \end{bmatrix}^{-1}}_{\mathbf{A}_i} \begin{bmatrix} \frac{\partial F_{1,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{1,k}(.)}{\partial \ln Q_{Ni,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{N,k}(.)}{\partial \ln Q_{Ni,k}} \end{bmatrix}. \tag{60}$$

The elements of the matrixes on the right-hand side of the above equation are given by

$$\frac{\partial F_{n,k}(.)}{\partial \ln P_{jj,k}} = \mathbb{1}_{j=n} + \mathbb{1}_{j\neq i}\frac{\mu_g}{1+\mu_g}\sum_{\ell\neq i} r_{n\ell,k}\varepsilon_{n\ell,k}^{(j\ell,k)}; \qquad \frac{\partial F_{n,k}(.)}{\partial \ln Q_{ji,k}} = \mathbb{1}_{j=n}\frac{\mu_g}{1+\mu_g}r_{ji,k}.$$

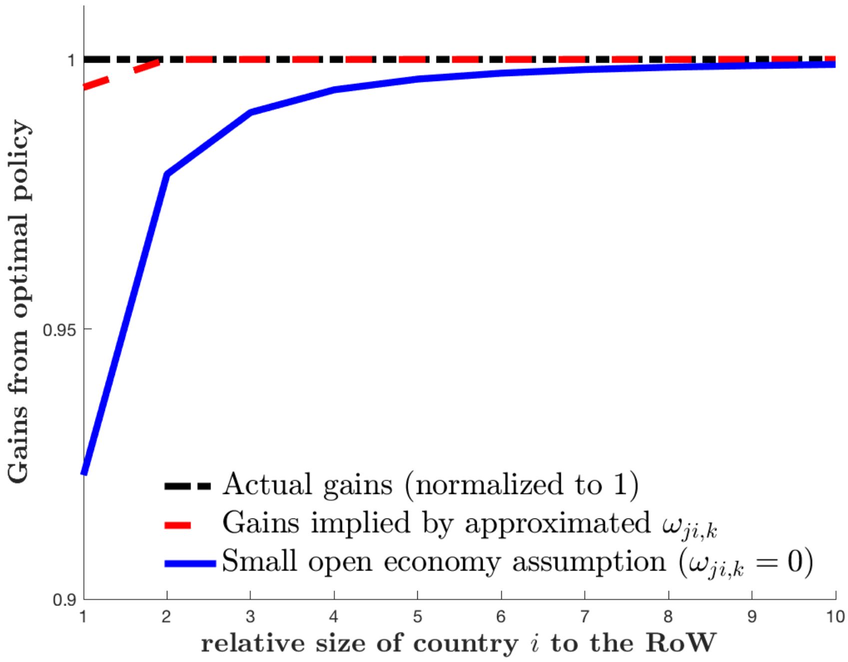
It is straightforward to see that $\mathbf{A}_i$ is diagonally-dominant. Hence, we can apply the method proposed by Wu et al. (2013) to characterize $\mathbf{A}_i^{-1}$ to a first-order approximation around $r_{ji,k} \approx 0$ (for $j \neq i$). Doing so yields the following expression based on the matrix Equation 60:

$$\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{ji,g}}\right)_{\mathbf{w},\mathbb{P}_i} \approx \begin{cases} \frac{-\frac{\mu_g}{1+\mu_g}r_{ni,g}}{1+\frac{\mu_g}{1+\mu_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(n\iota,g)}} & n = j \\ \frac{\frac{\mu_g}{1+\mu_g}r_{ji,g}}{1+\frac{\mu_g}{1+\mu_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(n\iota,g)}}\left(\frac{\mu_g}{1+\mu_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(j\iota,g)}\right) & n \neq j \end{cases}$$

Plugging the above expression back into the definition specified by Equation 59, while noting that $r_{ni,g} \times r_{ji,g} \approx 0$ if $j \neq i$ and $n \neq i$, yields the following approximation for the export supply elasticity:

$$\omega_{ji,k} \approx \frac{-\frac{\mu_k}{1+\mu_k}r_{ji,k}}{1+\frac{\mu_k}{1+\mu_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}}\left[1 - \frac{\mu_k}{1+\mu_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k}r_{in,k}}{\rho_{j,k}r_{ji,k}}\varepsilon_{in,k}^{(jn,k)}\right].$$

Figure 2: *The efficacy of the approximated $\omega_{ji,k}$ at predicting gains from policy*



Note: the above simulation is based on a two country–two industry model with the following specifications: (2) $\sigma_1 = \sigma_2 = 5$, (2) $\mu_1 = 0.25$ and $\mu_2 = 0.5\mu_1$; (3) expenditure shares are assigned the following values $\lambda_{21,1} = 0.6$, $\lambda_{12,1} = 0.25/\delta$, $\lambda_{21,2} = 0.25$; $\lambda_{12,2} = 0.4/\rho$ where ρ is relative size.

For the sake of clarity, note that $w_i = \tilde{w}_i$ under free entry—so, we can replace w_i with $\tilde{w}_i$ everywhere in the above approximation. Figure 2 illustrates the goodness of our approximated $\omega_{ji,k}$ using a rather conservative numerical example. We simulate a two-country$\times$two-industry economy in which trade is relatively open and the tax-imposing country is relatively large compared to the rest of the world. We compute the actual gains from optimal policy for the tax-imposing country i, and compare them to gains implied by (1) our approximated $\omega_{ji,.k}$ as well (2) the small open economy approximation, $\omega_{ji,k} \approx 0$. Evidently, our approximated value for $\omega_{ji,k}$ yields indistinguishable results relative to approximation-free benchmark.[74]

Step #5. Extending the Derivation to the Restrict Entry Case

Equipped with a full characterization of optimal policy under free entry, we now switch attention to the case of restricted entry. The main difference between the two cases is in how producer prices vary with export supply: Under restricted entry, holding $\mathbf{w} = \{\tilde{w}_n\}$ fixed, contacting the export supply of good ni, g affects the producer prices associated with origin n through a uniform reduction in the average markup $\overline{\mu}_n$. Namely,

$$P_{ni,g} = \bar{\varrho}_{ni,g} \frac{1+\mu_k}{1+\overline{\mu}_n} \tilde{w}_n \implies \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w},\mathbb{P}_i} = - \left(\frac{\partial \ln (1+\overline{\mu}_n)}{\partial \ln Q_{ni,g}} \right)_{\mathbf{w},\mathbb{P}_i},$$

[74]To be clear, the above approximation is only intended for the quantitative applications. It should not be viewed as a limitation of our theory. The optimal tax formula derived earlier in combination with Equation 60 deliver an exact theoretical specification for the first-best optimal policy schedule.

where economy n's (endogenously-determined) average profit margin is given by

$$1+\bar{\mu}_n = \frac{\sum_{\iota\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[P_{n\iota,k}Q_{n\iota,k}\right]}{\sum_{\iota\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1}{1+\mu_k}P_{n\iota,k}Q_{n\iota,k}\right]}.$$

Another difference is that non-tax-revenue income in country i is the sum of wage payments plus profits. Stated formally, total income in country i can be specified as follows (*notation*: the operator "$\cdot$" denotes the inner product of two equal-sized vectors):

$$Y_i(\mathbb{P}_i;\mathbf{w}) = \underbrace{(1+\bar{\mu}_i)w_iL_i}_{\tilde{w}_iL_i} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}\right] + \left(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{ii}\right)\cdot\mathbf{Q}_{ii} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in}\right)\cdot\mathbf{Q}_{in}\right], \tag{61}$$

In the above formulation, $\tilde{w}_iL_i = (1+\bar{\mu}_i)w_iL_i$, stands for the sum of wage payments plus profits.

With the background information provided above, we can recycle our earlier derivations from the free entry case to characterize the F.O.C. w.r.t. each price instrument in $\mathbb{P}_i$.

First-Order Condition w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k}\in\mathbb{P}_i$. To fix ideas, recall from Step #3 of the proof that the F.O.C. w.r.t. $\tilde{P}_{ji,k}\in\mathbb{P}_i$ (where possibly $j=i$) is given by

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0. \tag{62}$$

As before, $\mathbb{P}_{-ji,k}\equiv\mathbb{P}_i-\{\tilde{P}_{ji,k}\}$ denotes the vector of country i's price instruments excluding $\tilde{P}_{ji,k}$. Each term on the right-hand can be unpacked as in the free entry case, with one difference: holding $\mathbf{w}$ constant, a change in good ji,k's export supply affects the entire vector of prices from origin j. Specifically, noting that $P_{ji,g} = \bar{\varrho}_{ji,g}\frac{1+\mu_g}{1+\bar{\mu}_j}w_j$, indicates that

$$\left(\frac{\partial\ln P_{ji,g}}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i} = -\left(\frac{\partial\ln(1+\bar{\mu}_j)}{\partial\ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i}\qquad\forall g\in\mathbb{K}.$$

Noting this distinction, we now repeat the steps present earlier to unpack each term on the right-hand side of Equation 62. By Roy's identity, the first term on the right-hand side can be unpacked as follows:

$$\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} = -\tilde{P}_{ji,k}Q_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Recall that the second term on the right-hand side of Equation 62 accounts for the revenue-raising effects of policy. Specifically, taking note of Equation 61, the effect on import tax revenues can be unpacked as follows:

$$\left(\frac{\partial\sum_{n\neq i}\left(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni}\right)\cdot\mathbf{Q}_{ni}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k} + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial\ln Q_{ni,g}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]$$
$$-\sum_{g\in\mathbb{K}}\sum_{n\neq i}\left[P_{ni,s}Q_{ni,s}\sum_{s\in\mathbb{K}}\left[\sum_{j\neq i}\frac{P_{ji,s}Q_{ji,s}}{P_{ni,g}Q_{ni,g}}\left(\frac{\partial P_{ji,s}}{\partial\ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_i} + \sum_{\ell\in\mathbb{C}}\frac{P_{i\ell,s}Q_{i\ell,s}}{P_{i\ell,g}Q_{i\ell,g}}\left(\frac{\partial P_{i\ell,s}}{\partial\ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_i}\right]\left(\frac{\partial\ln Q_{ni,g}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right] \tag{63}$$

As in the free entry case, $\left(\partial\ln Q_{ni,g}/\partial\ln\tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses demand adjustments that channel through both price and income effects—see Equation 27. We can simplify the last term on the right-hand side of above equation, by appealing to our definition of the export supply elasticity:

$$\omega_{ni,g} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_g \left[\frac{\bar{w}_i L_i}{\bar{w}_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_i} + \sum_{n \neq i} \frac{\bar{w}_n L_n}{\bar{w}_j L_j} r_{ni,g}\rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i} \right]. \tag{64}$$

$$= \frac{-1}{r_{ji,k}\rho_{j,k}} \left[\frac{\bar{w}_i L_i}{\bar{w}_j L_j} \left(\frac{\partial \ln(1+\overline{\mu}_i)}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_i} + \sum_{n \neq i} \sum_g \left(\frac{\bar{w}_n L_n}{\bar{w}_j L_j} r_{ni,g}\rho_{n,g} \right) \left(\frac{\partial \ln(1+\overline{\mu}_n)}{\partial \ln Q_{ji,k}} \right)_{\mathbf{w},\mathbb{T}_i} \right] \tag{65}$$

The second line indicates our focus on the restricted entry case, wherein $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i} = \left(\frac{\partial \ln(1+\overline{\mu}_n)}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i}$ for all g. That is, holding $\mathbf{w}$ constant, producer prices from each origin change equal-proportionally across all industries with the aggregate profit margin, $1+\overline{\mu}_i$. Plugging the above expression back into Equation 63 yields the following expression that summarizes the (conditional) effect of policy on import tax revenues:

$$\left(\frac{\partial \sum_{n \neq i} \left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni} \right) \cdot \mathbf{Q}_{ni} \right]}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k} Q_{ji,k} + \sum_g \sum_{n \neq i} \left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}] P_{ni,g} \right) Q_{ni,g} \left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \right]. \tag{66}$$

The effect of policy on export and domestic tax revenues can be unpacked as in Equation 30, which was derived earlier for the free entry case. To simplify this equation under restricted entry, we can use the following observation:

$$\sum_n \left[P_{in,g} Q_{in,g} \left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\mathbb{P}_i} \right] = - \sum_{s \in \mathbb{K}} \sum_{n \in \mathbb{C}} \left[\frac{P_{in,s} Q_{in,s}}{P_{ii,g} Q_{ii,g}} \left(\frac{\partial \ln(1+\overline{\mu}_i)}{\partial \ln Q_{ii,g}} \right)_{\mathbf{w},\mathbb{P}_i} \right] P_{ii,g} Q_{ii,g} =$$

$$= \sum_{s \in \mathbb{K}} \sum_{n \in \mathbb{C}} \left[\frac{r_{in,s}\rho_{i,s}}{r_{ii,g}\rho_{i,g}} \left(\frac{\bar{\mu}_i - \mu_g}{1+\mu_g} r_{ii,g}\rho_{i,g} \right) \right] P_{ii,g} Q_{ii,g} = - \left(1 - \frac{1+\overline{\mu}_i}{1+\mu_g} \right) P_{ii,g} Q_{ii,g},$$

To explain, the second line on the above equation follows from that fact that all prices associated with economy i are included in the set $\mathbb{P}_i$. So, holding $\mathbb{P}_i$ and wages $\mathbf{w}$ constant, the policy-induced change in $Q_{ii,g}$ has only a direct arithmetic effect on country i's aggregate profit margin, i.e., $\left(\frac{\partial \ln(1+\overline{\mu}_i)}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_i} = \frac{\bar{\mu}_i - \mu_g}{1+\mu_g} r_{ii,g}\rho_{i,g}$.[75] Plugging the above equation back into Equation 30 yields the following equation describing the (conditional) effects of policy on export and domestic tax revenues:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}} \left\{ \sum_n \left[\left(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in} \right) \cdot \mathbf{Q}_{in} \right] \right\} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \sum_g \left[\left(\tilde{P}_{ii,g} - \frac{1+\overline{\mu}_i}{1+\mu_g} P_{ii,g} \right) Q_{ii,g} \left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w},\mathbb{P}_{-ji,k}} \right]. \tag{67}$$

Recall that $\left(\partial \ln Q_{ni,g} / \partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$, in the above equations, encompasses price- and income-related demand adjustments—see Equation 27. Taking note of this detail, we can combine Equations 66 and 67 to arrive at the following expression that summarizes the (conditional) effect of raising $\tilde{P}_{ji,k}$ on

[75] Note that this argument does not extend to the aggregate profit margin in other countries. Changing the export supply of say good ji,k with policy has a circular effect on origin j's profit margin, $\overline{\mu}_j$, which occurs because the prices associated with economy $j \neq i$ are not pegged to $\mathbb{P}_i$. Specifically, a change in $Q_{ji,k}$ affects the entire vector of origin j's prices outside of market i. This change in prices affects the industrial composition of origin j's output and $\overline{\mu}_j$ in a circular fashion.

country i's tax revenues:

$$\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k}Q_{ji,k} + \sum_g \sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g} - \frac{1+\bar{\mu}_i}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \Delta_i(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}},$$

where $\Delta_i(.)$, as before, encapsulated the circular income effects. The expression for $\Delta_i(.)$ is specified analogously to Equation 35 with two amendments: (1) $\omega_{ni,g}$ is redefined according to 64; and (2) $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.[76] Next, we unpack the last term on the right-hand side of Equation 62, which accounts for general equilibrium wage effects. Repeating the steps presented for the free entry case, while noting the differences discussed above, yields the following:

$$\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = -\sum_g\sum_{n\neq i}\left[\tau_i(1+\omega_{ni,g})P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right],$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i(1+\omega_{ni,g})P_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$$

where $\bar{\tau}_i$ is given by 37. Note that the above expression differs from the analogous expression derived under free entry in the economic forces that regulate export supply elasticity, $\omega_{ni,g}$. Under restricted entry, the export supply elasticity governs the change in aggregate profit margins in response to distortions to export supply. Combining the various terms on the right-hand side of Equation 62, yields the following simplified representation of the F.O.C. w.r.t. $\tilde{P}_{ji,k} \in \mathbb{P}_i$ under restricted entry:

$$\sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}} - (1+\bar{\tau}_i)\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right]$$
$$+\sum_g\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}} - \frac{1+\overline{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \widetilde{\Delta}_i(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0.$$

The uniform term $\widetilde{\Delta}_i(.)$ is described by Equation 40, but with $\omega_{ni,g}$ redefined according to 64 and $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.

First-Order Condition w.r.t. $\tilde{P}_{ij,k}$ ($j \neq i$). Now consider the F.O.C. w.r.t. the price of a generic export good ij,k (where $j \neq i$). Recall from Step #3 that the F.O.C. w.r.t. $\tilde{P}_{ij,k} \in \mathbb{P}_i$ is given by

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \quad (68)$$

[76] To be more specific, $\Delta_i(.)$ is described by the following equation:

$$\Delta_i(\mathbb{P}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(\tilde{P}_{ii,g} - \frac{1+\overline{\mu}_i}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\eta_{ii,g}\right],$$

where $\overline{\mu}_i > 0$ and $\omega_{ni,g} \equiv \sum_{s\in\mathbb{K}}\left[\frac{r_{ni,s}\rho_{n,s}}{\rho_{ni,g}\rho_{n,g}}\left(\frac{\partial \ln P_{ni,s}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_i}\right]$ as we are focused on the restricted rather than free entry.

where $\mathbb{P}_{-ij,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ij,k}\}$ denotes the vector of country i's price instruments excluding $\tilde{P}_{ij,k}$. Building on our previous discussion, each term on the right-hand side is characterized by the same formulas derived in Step #3, with two qualification: (1) The formulation assigned to $\omega_{ni,g}$ should be revised to account for restricted entry (see Equation 10), (2) all equations should be adjusted to admit a non-zero $\overline{\mu}_j$, as is required by restricted entry (see Equation 5).

Without repeating all the details from Step 3, we can unpack the terms on the right-hand side of Equation 68 as follows: Since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$ is not part of the domestic consumer price index, $\partial V_i(Y_i, \tilde{\mathbf{P}}_i)/\partial \ln \tilde{P}_{ji,k} = 0$. The second-term on the right-hand side of Equation 68 is given by:

$$\left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\left(\tilde{P}_{ij,g} - [1 - \frac{\mu_g}{1+\mu_g}] P_{ij,g}\right) Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)}\right]$$
$$- \sum_g \sum_{n \neq i} \left[\omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)}\right] + \Delta_i(\mathbb{P}_i; \mathbf{w}) \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ij,k}},$$

where $\omega_{ni,g}$ is defined as in Equation 64, while $\Delta_i(.)$ is given by Equation 35, with the necessary adjustments described earlier.The last term on the right-hand side of Equation 68 , which accounts for general equilibrium wage effects, can be unpacked as

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i \tilde{P}_{ij,k} Q_{ij,k} + \sum_g \left[\bar{\tau}_i \tilde{P}_{ij,g} Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)}\right]$$
$$- \sum_g \sum_{n \neq i} \left[\bar{\tau}_i \omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)}\right] - \sum_g \sum_{n \neq i} \left[[1 + \omega_{ni,g}] \bar{\tau}_i P_{ni,g} Q_{ni,g} \eta_{ni,g}\right] \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ij,k}}.$$

where $\bar{\tau}_i$ is given by 37. To be clear, the above formula differs from the one derived under free entry in only how $\omega_{ni,g}$ is defined—see Equation 64. Combining the various terms on the right-hand side of Equation 68, yields the following simplified representation of the F.O.C. w.r.t. $\tilde{P}_{ij,k} \in \mathbb{P}_i$:

$$\tilde{P}_{ij,k} Q_{ij,k} + \sum_{g \in \mathbb{K}} \left[\left(1 - \frac{1 + \overline{\mu}_i}{(1+\bar{\tau}_i)(1+\mu_g)} \frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right) \tilde{P}_{ij,g} Q_{ij,g} \varepsilon_{ij,g}^{(ij,k)}\right]$$
$$\sum_{g \in \mathbb{K}} \sum_{n \neq i} \left[\omega_{ni,g} P_{nj,g} Q_{nj,g} \varepsilon_{nj,g}^{(ij,k)}\right] + \widetilde{\Delta}_i(\mathbb{P}_i, \mathbf{w}) \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w}, \mathbb{P}_{-ij,k}} = 0,$$

The uniform term $\widetilde{\Delta}_i(.)$ is described by Equation 40, but with $\omega_{ni,g}$ redefined according to 64 and $1/(1+\mu_g)$ replaced with $(1+\overline{\mu}_i)/(1+\mu_g)$.

Solving the system of F.O.C. Given the tight correspondence between the F.O.C.s derived under the restricted and free entry cases, we can repeat the arguments as in step #4 to solve the system of F.O.C.s and establish the uniqueness of the resulting solution. Doing so yield the following formula for optimal taxes/subsidies under restricted entry:

$$\begin{aligned}
&[\text{domestic subsidy}] \quad && 1 + s_{i,k}^* = (1+\mu_k)/(1+\bar{\mu}_i) \\
&[\text{import tariff}] \quad && 1 + t_{ji,k}^* = (1+\omega_{ji,k})(1+\bar{\tau}_i) \\
&[\text{export subsidy}] \quad && \mathbf{1} + \mathbf{x}_{ij}^* = -\mathbf{E}_{ij}^{-1} \mathbf{E}_{ij}^{(-ij)} (\mathbf{1} + \mathbf{t}_i^*).
\end{aligned}$$

Recall from Lemma 1 that there are two degrees of multiplicity associated with optimal policy schedule. As a result, we need not to unpack the uniform terms $\bar{\tau}_i$ and $\bar{\mu}_i$. Instead, for any arbitrary choice of tax shifters $1+\bar{s}_i$ and $1+\bar{t}_i \in \mathbb{R}_+$, the following tax/subsidy schedule represents an optimal solution:

$$\begin{aligned} &[\text{domestic subsidy}] \quad 1+s^*_{i,k} = (1+\mu_k)(1+\bar{s}_i) \\ &[\text{import tariff}] \quad 1+t^*_{ji,k} = (1+\omega_{ji,k})(1+\bar{t}_i) \\ &[\text{export subsidy}] \quad \mathbf{1}+\mathbf{x}^*_{ij} = -\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}+\mathbf{t}^*_i\right). \end{aligned}$$

The above formula is identical to that derived under free entry, with one qualification. The (inverse) export supply elasticity $\omega_{ji,k}$ has a different interpretation under restricted entry, and is given by 64. So, to conclude the proof, we characterize $\omega_{ji,k}$ under restricted entry next.

Characterizing the (Inverse) Export Supply Elasticity. Following Equation 64, the inverse of the export supply elasticity under restricted entry is defined as

$$\omega_{ni,g} = \frac{-1}{r_{ji,k}\rho_{j,k}}\left[\frac{\tilde{w}_i L_i}{\tilde{w}_j L_j}\left(\frac{\partial \ln(1+\bar{\mu}_i)}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_i} + \sum_{n\neq i}\sum_g\left(\frac{\tilde{w}_n L_n}{\tilde{w}_j L_j} r_{ni,g}\rho_{n,g}\right)\left(\frac{\partial \ln(1+\bar{\mu}_n)}{\partial \ln Q_{ji,k}}\right)_{\mathbf{w},\mathbb{T}_i}\right], \tag{69}$$

where the second line follows from the fact that $P_{ni,s} = \bar{\varrho}_{ni,s}\frac{1+\mu_s}{1+\bar{\mu}_n}\tilde{w}_n$, which implies that $\left(\frac{\partial \ln P_{ni,s}}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_i} = -\left(\frac{\partial \ln(1+\bar{\mu}_n)}{\partial \ln Q_{ni,g}}\right)_{\mathbf{w},\mathbb{P}_i}$. To unpack the above equation, note that (for a given $\mathbb{P}_i$ and $\mathbf{w}$) the aggregate profit margin implicitly solves the following equation:

$$F_n(\bar{\boldsymbol{\mu}},\mathbf{Q}_{ni}) = (1+\bar{\mu}_n) - \underbrace{\frac{\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\boldsymbol{\mu}}_{-i})}{\frac{1}{\mathbf{1}+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\frac{1}{\mathbf{1}+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\boldsymbol{\mu}}_{-i})}}_{g_n(\bar{\mu}_n,\mathbf{Q}_{ni})} = 0.$$

As before, $\odot$ and $\cdot$ respectively denote the *inner* and *element-wise* products of equal-sized vectors (i.e., $\mathbf{a}\cdot\mathbf{b} = \sum_n a_n b_n$ and $\mathbf{a}\odot\mathbf{b} = [a_n b_n]_n$), while with a slight abuse of notation, $\frac{1}{\mathbf{1}+\boldsymbol{\mu}} \equiv \left[\frac{1}{1+\mu_k}\right]_k$. The vector $\mathbf{Q}_{ni}$ represents the export supply of goods from origin $n\neq i$ to market i (which is fully determined by $\mathbb{P}_i$ and $\mathbf{w}$). Outside of market i, consumer prices are not directly pegged to $\mathbb{P}_i$. So, holding $\tilde{w}_n \in \mathbf{w}$ and $\tilde{\mathbf{P}}_{i\iota} \in \mathbb{P}_i$ constant, a change in $\bar{\mu}_i$ affects the producer and consumer price of goods supplied by origin n to any market $\iota \neq i$. Accordingly, $\mathbf{Q}_{n\iota}(\bar{\boldsymbol{\mu}}_{-i}) \equiv \left\{Q_{n\iota,k}(\bar{\boldsymbol{\mu}}_{-i})\right\}_k$ in Equation XXX is implied by Marshallian demand function (treating $\tilde{w}_n \in \mathbf{w}$ and $\tilde{\mathbf{P}}_{i\iota} \in \mathbb{P}_i$ as given):

$$Q_{n\iota,k}(\bar{\boldsymbol{\mu}}_{-i}) = \mathcal{D}_{n\iota,k}(\overline{\tilde{w}_\iota L_\iota}, \overline{\tilde{\mathbf{P}}}_{i\iota}, \tilde{\mathbf{P}}_{-i\iota}(\bar{\boldsymbol{\mu}}_{-i}))$$

Taking note of this detail, we can compute $\left(\partial\ln(1+\bar{\mu}_n)/\partial\ln Q_{ni,g}\right)_{\mathbf{w},\mathbb{P}_i}$ by applying the Implicit Function Theorem to the system of equations specified by $F_n(\bar{\boldsymbol{\mu}},\mathbf{Q}_{ni})$. Namely,

$$\begin{bmatrix} \frac{\partial\ln(1+\bar{\mu}_1)}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial(1+\bar{\mu}_1)}{\partial\ln\mathbf{Q}_{Ni}} \\ \vdots & \ddots & \vdots \\ \frac{\partial\ln(1+\bar{\mu}_N)}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial(1+\bar{\mu}_N)}{\partial\ln\mathbf{Q}_{Ni}} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1(.)}{\partial\ln(1+\bar{\mu}_1)} & \cdots & \frac{\partial F_1(.)}{\partial\ln(1+\bar{\mu}_N)} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_N(.)}{\partial\ln(1+\bar{\mu}_1)} & \cdots & \frac{\partial F_N(.)}{\partial\ln(1+\bar{\mu}_N)} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial F_1}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial F_1}{\partial\ln\mathbf{Q}_{Ni}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_N}{\partial\ln\mathbf{Q}_{1i}} & \cdots & \frac{\partial F_N}{\partial\ln\mathbf{Q}_{Ni}} \end{bmatrix}. \tag{70}$$

Next, we characterize the elements of the matrixes on the right-hand side of the above equation. Considering that $F_n(\bar{\boldsymbol{\mu}},\mathbf{Q}_{ni}) = (1+\bar{\mu}_n) - g(\bar{\boldsymbol{\mu}},\mathbf{Q}_{ni})$, we can unpack the elements of $\left[\frac{\partial F_i(.)}{\partial\ln(1+\bar{\mu}_j)}\right]_{i,j}$ as follows.

Using vector algebra we can show that if $n \neq i$, then

$$\frac{\partial g_n(\bar{\boldsymbol{\mu}}, \mathbf{Q}_{ni})}{\partial \ln(1+\bar{\mu}_n)} = \underbrace{\frac{-\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} - \sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{-(1+\bar{\mu}_n)} + \underbrace{\frac{-\sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\odot\varepsilon_{n\iota}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{-(1+\bar{\mu}_n)\sum_\iota[\mathbf{r}_{n\iota}\cdot\varepsilon_{n\iota}]}$$

$$-(1+\bar{\mu}_i)\left[\underbrace{\frac{-\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} - \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{-1} - \underbrace{\frac{\sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\odot\varepsilon_{n\iota}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{(1+\bar{\mu}_n)\sum_\iota\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{r}_{n\iota}\cdot\varepsilon_{n\iota}\right]}\right].$$

where $\boldsymbol{\varepsilon}_{ni} \equiv \left[\varepsilon_{ni,g}^{(ni,g)}\right]_g$ is a $K\times 1$ vector of own-price elasticities of demand. $\mathbf{r}_{ni} \equiv \left[r_{ni,g}\rho_{n,g}\right]_g$ is a $K\times 1$ vector of sales shares. The above derivation appeals to the definition of sales shares, whereby $r_{ni,k}\rho_{n,k} = \frac{P_{ni,k}Q_{ni,k}}{\sum_j\sum_g P_{nj,g}Q_{nj,g}}$. Likewise, for any n and $\ell \neq i$, we can

$$\frac{\partial g_n(\bar{\boldsymbol{\mu}}, \mathbf{Q}_{ni})}{\partial \ln(1+\bar{\mu}_\ell)} = \frac{-\sum_{\iota\neq i}\left[\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\odot\varepsilon_{n\iota}^{(\ell\iota)}\right] + (1+\bar{\mu}_i)\sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\odot\varepsilon_{n\iota}^{(\ell\iota)}\right]}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}.$$

Combining the above two equations we can characterizes each element of the matrix $\left[\frac{\partial F_n(.)}{\partial\ln(1+\bar{\mu}_\ell)}\right]_{n,\ell}$ as follows:

$$\frac{\partial F_n(\bar{\boldsymbol{\mu}}, \mathbf{Q}_{ni})}{\partial \ln(1+\bar{\mu}_\ell)} = (1+\bar{\mu}_n)\left[\mathbb{1}_{\ell=n} + \mathbb{1}_{\ell\neq i}\sum_k\sum_{\iota\neq i}\left[\left(1-\frac{1+\bar{\mu}_n}{1+\mu_k}\right)r_{n\iota,k}\rho_{n,k}\varepsilon_{n\iota,k}^{(\ell\iota,k)}\right]\right]$$

The elements of the matrix $\left[\frac{\partial F_n}{\partial\ln\mathbf{Q}_{\ell i}}\right]_{n,\ell}$ can be unpacked with a similar logic. Specifically, if $n \neq \ell$ then $\frac{\partial F_n}{\partial\ln\mathbf{Q}_{\ell i}} = 0$. Otherwise, for any $n \in \mathbb{C}$ we can derive the following expression:

$$\frac{\partial g_n(\bar{\boldsymbol{\mu}}, \mathbf{Q}_{ni})}{\partial \ln Q_{ni,k}} = \underbrace{\frac{P_{ni,k}Q_{ni,k}}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{(1+\bar{\mu}_n)r_{ni,k}\rho_{n,k}} - \underbrace{\frac{(1+\bar{\mu}_n)\frac{1}{1+\mu_k}P_{ni,k}Q_{ni,k}}{\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{ni}(\bar{\mu}_n)\cdot\mathbf{Q}_{ni} + \sum_{\iota\neq i}\left[\frac{1}{1+\boldsymbol{\mu}}\odot\mathbf{P}_{n\iota}(\bar{\mu}_n)\cdot\mathbf{Q}_{n\iota}(\bar{\mu}_n)\right]}}_{(1+\bar{\mu}_n)\frac{1+\bar{\mu}_n}{1+\mu_k}r_{ni,k}\rho_{n,k}},$$

which, in turn, characterizes every element of matrix $\left[\frac{\partial F_n}{\partial\ln\mathbf{Q}_{\ell i}}\right]_{n,\ell}$ as follows:

$$\frac{\partial F_n(\bar{\boldsymbol{\mu}}, \mathbf{Q}_{ni})}{\partial \ln Q_{\ell i,k}} = \mathbb{1}_{\ell=n}(1+\bar{\mu}_n)\left[\left(1-\frac{1+\bar{\mu}_n}{1+\mu_k}\right)r_{ni,k}\rho_{n,k}\right].$$

As in the free entry case, it is immediate that $\tilde{\mathbf{A}}_i \equiv \left[\frac{\partial F_i(.)}{\partial\ln(1+\bar{\mu}_j)}\right]_{i,j}$ is diagonally dominant. So, we can once again invoke we first-order approximation proposed by Wu et al. (2013) to characterize $\tilde{\mathbf{A}}_i^{-1}$. Doing so and plugging the implied values of $\frac{\partial\ln(1+\bar{\mu}_1)}{\partial\ln\mathbf{Q}_{1i}}$ back into Equation 69, implies the following approximation for the export supply elasticity under restricted entry:

$$\omega_{ni,g} \approx \frac{-\left(1-\frac{1+\bar{\mu}_n}{1+\mu_g}\right)\sum_k r_{ni,k}\rho_{n,k}}{1+\sum_k\sum_{\iota\neq i}\left[1+\left(1-\frac{1+\bar{\mu}_n}{1+\mu_k}\right)r_{n\iota,k}\rho_{n,k}\varepsilon_{n\iota,k}\right]}.$$

F Proof of Theorem 2

The proof of Theorem 2 has the same basic foundation as Theorem 1. We reformulate the optimal policy problem, expressing equilibrium variables (e.g., $Q_{ji,k}$, Y_i, etc.) as a function of (1) the vector of consumer prices associated with economy i, excluding $\tilde{\mathbf{P}}_{ii}$, i.e., $\mathbb{P}_i \equiv \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$;[77] and (2) the vector of national-level wage rates all over the world, $\mathbf{w} = \{w_1, ..., w_N\}$. To implement this reformulation of equilibrium variables, we need to solve the following system treating $\mathbb{P}_i$, and $\mathbf{w}$ as given:

$$
\begin{array}{rl}
\text{[optimal pricing]} & P_{jn,k} = \bar{\rho}_{ji,k} w_j \\
\text{[optimal consumption]} & Q_{jn,k} = \mathcal{D}_{jn,k}(Y_n, \tilde{\mathbf{P}}_{1n}, ...\tilde{\mathbf{P}}_{Nn}) \\
\text{[RoW imposes zero taxes]} & \tilde{P}_{jn,k} = P_{jn,k} \quad (\tilde{P}_{jn,k} \notin \mathbb{P}_i); \qquad Y_n = \overbrace{(1+\bar{\mu}_n) w_n L_n}^{w_n L_n + \Pi_n} \quad (n \neq i) \\
\text{[Balanced Budget in } i\text{]} & Y_i = (1+\bar{\mu}_i) w_i L_i + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right) \cdot \mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right) \cdot \mathbf{Q}_{ji} \\
\text{[avg. profit margin in } j\text{]} & 1 + \bar{\mu}_j = \dfrac{\sum_{n \in \mathbb{C}} \left[\mathbf{P}_{jn} \cdot \mathbf{Q}_{jn}\right]}{\sum_{n \in \mathbb{C}} \left[\mathbf{P}_{jn} \cdot \left(\mathbf{Q}_{jn} \oslash (\mathbf{1} + \boldsymbol{\mu})\right)\right]}
\end{array}
$$

where "$\cdot$" denotes the inner product operator for vectors of equal size. "$\oslash$" denotes element-wise division of equal-sized vectors, with $\boldsymbol{\mu} \equiv \{\mu_k\}_k$. Since there is a unique equilibrium, the above system is exactly identified in that it uniquely determines $P_{jn,k}(\mathbb{P}_i; \mathbf{w})$, $Q_{jn,k}(\mathbb{P}_i; \mathbf{w})$, $Y_n(\mathbb{P}_i; \mathbf{w})$, and $\bar{\mu}_i(\mathbb{P}_i; \mathbf{w})$ as a function of $\mathbb{P}_i$ and $\mathbf{w}$. Appealing to the above reformulation of the equilibrium, we can reformulate the original optimal policy problem (P2) as follows.

Lemma 8. *Country i's vector of second-best trade taxes, $\{\boldsymbol{t}_i^\star, \boldsymbol{x}_i^\star\}$, can be determined by solving the following problem:*

$$
\max_{\mathbb{P}_i} \; W_i(\mathbb{P}_i; \mathbf{w}) \equiv V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \quad s.t. \;\; (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P \qquad (\widetilde{\text{P2}}),
$$

where the feasibility constraint is satisfied if, given $\mathbb{P}_i$, the wage vector $\boldsymbol{w}$ satisfies balanced trade in each country:

$$
(\mathbb{P}_i; \boldsymbol{w}) \in \mathbb{F}_P \quad \iff \quad \begin{cases} \sum_{j \neq n} \sum_{k \in \mathbb{K}} \left[P_{jn,k}(\mathbb{P}_i; \boldsymbol{w}) Q_{jn,k}(\mathbb{P}_i; \boldsymbol{w}) - P_{nj,k}(\mathbb{P}_i; \boldsymbol{w}) Q_{nj,k}(\mathbb{P}_i; \boldsymbol{w})\right] = 0 & \text{if } n \neq i \\ \sum_{j \neq n} \sum_{k=1}^{K} \left[P_{ji,k}(\mathbb{P}_i; \boldsymbol{w}) Q_{jn,k}(\mathbb{P}_i; \boldsymbol{w}) - \tilde{P}_{ij,k} Q_{nj,k}(\mathbb{P}_i; \boldsymbol{w})\right] = 0 & \text{if } n = i \end{cases}.
$$

The system of F.O.C.'s underlying Problem $(\widetilde{\text{P2}})$ can be expressed as follows:

$$
\nabla_{\tilde{P}} W_i(\mathbb{P}_i; \mathbf{w}) + \nabla_{\mathbf{w}} W_i \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\tilde{P}}\right)_{(\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P} = \mathbf{0}, \qquad \forall \tilde{P} \in \mathbb{P}_i = \left\{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\right\}.
$$

In what follows we characterize and simplify the system of F.O.C., building heavily on the results presented in Appendix E.

Deriving the First-Order Condition w.r.t. $\tilde{\mathbf{P}}_{ji}$

Consider the consumer price index $\tilde{P}_{ji,k} \in \mathbb{P}_i$ associated with a good imported by i from *origin j–industry k.* The F.O.C. w.r.t. this price instrument can be stated as follows:

[77] Recall that vectors $\tilde{\mathbf{P}}_{ji} \equiv \left\{\tilde{P}_{ji,k}\right\}_{j \neq i,k}$ and $\tilde{\mathbf{P}}_{ij} \equiv \left\{\tilde{P}_{ij,k}\right\}_{j \neq i,k}$ encompass only the export and import prices associated with economy i.

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = 0, \tag{71}$$

where $\mathbb{P}_{-ji,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ji,k}\}$ denotes the vector of price instruments excluding $\tilde{P}_{ji,k}$. The above equation is similar to what we characterized in Appendix E under restricted entry, with two distinctions: First, country i's government does not control the price of domestically produced and domestically consumed varieties, i.e., $\tilde{\mathbf{P}}_{ii} \notin \mathbb{P}_i$. Second, country i's income does not include domestic tax revenues:

$$Y_i = (1+\bar{\mu}_i)w_iL_i + \left(\tilde{\mathbf{P}}_{ij} - \mathbf{P}_{ij}\right)\cdot\mathbf{Q}_{ij} + \left(\tilde{\mathbf{P}}_{ji} - \mathbf{P}_{ji}\right)\cdot\mathbf{Q}_{ji}.$$

Taking note of these two differences, we can build on the derivation in Appendix E to simplify Equation 71. By Roy's identity, the first term on the right-hand side of Equation 71 can be stated as

$$\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ji,k}} = -\tilde{P}_{ji,k}Q_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Without repeating the derivations, the second term on the right-hand side of Equation 71 reduces to

$$\begin{aligned}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} =& \tilde{P}_{ji,k}Q_{ji,k} + \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] \\ & \sum_g\left[\left(1-\frac{1+\bar{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \Delta_i'(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\end{aligned}$$

where $\Delta_i'(\mathbb{P}_i;\mathbf{w})$ is a uniform term (without industry subscripts) and is given by

$$\Delta_i'(\mathbb{P}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(1-\frac{1+\bar{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right]. \tag{72}$$

To be clear, the above expressions can be derived by repeating the steps in Appendix E, while dropping domestic tax revenues from the expression for income Y_i. Likewise, the third term on the right-hand side of Equation 71 can be stated as

$$\left(\frac{\partial W_i(.)}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = -\sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\tau_iP_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] - \sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},}$$

where $\bar{\tau}_i$ is given by 83. Combining the above equations the F.O.C. specified by Equation 71 can be simplified as

$$\begin{aligned}&\sum_{n\neq i}\sum_g\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}} - (1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)}\right] \\ &+\sum_g\left[\left(1-\frac{1+\bar{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon_{ii,g}^{(ji,k)}\right] + \widetilde{\Delta}_i'(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0,\end{aligned} \tag{73}$$

where $\widetilde{\Delta}_i'(\mathbb{P}_i;\mathbf{w})$ is specified analogously to $\Delta_i'(\mathbb{P}_i,\mathbf{w})$, but features an adjustment for general equilibrium wage effects:

$$\widetilde{\Delta}_i'(\mathbb{P}_i;\mathbf{w}) \equiv \sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - (1+\bar{\tau}_i)(1+\omega_{ni,g})P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right] + \sum_g\left[\left(1-\frac{1+\bar{\mu}_i}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\eta_{ii,g}\right]. \tag{74}$$

Deriving the First-Order Condition w.r.t. $\tilde{\mathbf{P}}_{ij}$

Now, consider the consumer price index $\tilde{P}_{ij,k} \in \mathbb{P}_i$ associated with a good exported by i from *destination j–industry k.* The F.O.C. w.r.t. this price instrument can be stated as follows:

$$\left(\frac{\mathrm{d}W_i(\mathbb{P}_i;\mathbf{w})}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial\ln\tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \tag{75}$$

where $\mathbb{P}_{-ij,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ij,k}\}$ denotes the vector of price instruments excluding $\tilde{P}_{ij,k}$. As with the previous subsection, The above equation is similar to what we characterized in Appendix E, with two distinctions: First, country i's government does not control the price of domestically produced and domestically consumed varieties, i.e., $\tilde{\mathbf{P}}_{ii} \notin \mathbb{P}_i$. Second, country i's income does not include domestic tax revenues. Noting these two distinctions, we can borrow from the derivation in Appendix E to simplify Equation 75.

Namely, since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$ is not part of the domestic consumer price index in i, $\partial V_i(Y_i,\tilde{\mathbf{P}}_i)/\partial\ln\tilde{P}_{ji,k} = 0$. So, the first term on the right-hand side of Equation 75 collapses to zero. Without repeating the derivations from Appendix E, the second term on the right-hand side of Equation 75 reduces to

$$\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - \frac{1+\bar{\mu}_i}{1+\mu_g}P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right] - \sum_g\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] + \Delta_i'(\mathbb{P}_i;\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}$$

where $\Delta_i'(\mathbb{P}_i;\mathbf{w})$ is a uniform term without industry subscripts, as defined by Equation 72. To elaborate, the above expression can be derived by repeating the steps in Appendix E, while dropping domestic tax revenues from the expression for income Y_i. Likewise, the third term on the right-hand side of Equation 75 can be stated as

$$\left(\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \bar{\tau}_i\tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_g\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] - \sum_g\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}},$$

where $\bar{\tau}_i$ is given by 83. Combining the above equations the F.O.C. specified by Equation 75 can be simplified as

$$\tilde{P}_{ij,k}Q_{ij,k} + \sum_{g\in\mathbb{K}}\left[\left(1 - \frac{1+\bar{\mu}_i}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$\sum_{g\in\mathbb{K}}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right] + \widetilde{\Delta}_i'(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = 0, \tag{76}$$

where $\widetilde{\Delta}_i'(\mathbb{P}_i;\mathbf{w})$ is given by Equation 74.

Solving the System of First-Order Conditions

First, note that we can solve the system specified by Equation 73 independent of 76. To solve the system of Equations 73, we can rely on the intermediate observation that if

$$\left(\mathbf{1} - \frac{1+\overline{\mu}_i}{\mathbf{1}+\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\odot\mathbf{Q}_{ii}\cdot\boldsymbol{\varepsilon}_{ii}^{(ji,k)} + \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni} - (1+\bar{\tau}_i)(\mathbf{1}+\boldsymbol{\Omega}_{ni})\odot\mathbf{P}_{ni})\odot\mathbf{Q}_{ni}\cdot\boldsymbol{\varepsilon}_{ni}^{(ji,k)}\right] = 0, \tag{77}$$

then, to a first-order approximation around $\mu_k \approx \bar{\mu}_i$, $\widetilde{\Delta}_i'(\boldsymbol{\mu}) \approx 0$. So, the optimal choice of $\tilde{\mathbf{P}}_{ji}^*$ (and the implied tariff vector) can be determined by solving Equation 77 instead of 73.[78] Before moving forward, though, let us clarify the vector notation used to express Equation 77. The vector operators "$\cdot$" and "$\odot$" are respectively the inner product and element-wise product operators. The $K \times 1$ vector $\frac{1+\bar{\mu}_i}{1+\boldsymbol{\mu}} = \left[\frac{1+\bar{\mu}_i}{1+\mu_k}\right]_k$ is composed of industry-level The $K \times 1$ vectors $\tilde{\mathbf{P}}_{ni} = \left\{\tilde{P}_{ni,k}\right\}_k$ and $\mathbf{Q}_{ni} = \left\{Q_{ni,k}\right\}_k$ encompass the consumer price and quantity associated with all of country i's import goods for origin $n \neq i$. Analogously, $\boldsymbol{\varepsilon}_{ni}^{(ji,k)} = \left\{\varepsilon_{ni,g}^{(ji,k)}\right\}_g$ encompasses the elasticity of demand for each the goods imported from n w.r.t. the price of ji,k.

We simplify Equation 77 in three steps: First, by noting that $\tilde{\mathbf{P}}_{ii} = \mathbf{P}_{ii}$ and appealing to Cournot's aggregation, $\sum_{j\in\mathbb{C}}\left[\tilde{\mathbf{P}}_{ji}\odot\mathbf{Q}_{ji}\cdot\boldsymbol{\varepsilon}_{ji}^{(ji,k)}\right] = -\tilde{P}_{ji,k}Q_{ji,k}$, we can rewrite Equation 77 as

$$\frac{1+\bar{\mu}_i}{1+\boldsymbol{\mu}}\odot\tilde{\mathbf{P}}_{ii}\odot\mathbf{Q}_{ii}\cdot\boldsymbol{\varepsilon}_{ii}^{(ji,k)} + (1+\bar{\tau}_i)\sum_{n\neq i}\left[(\mathbf{1}+\boldsymbol{\Omega}_{ni})\odot\mathbf{P}_{ni}\odot\mathbf{Q}_{ni}\cdot\boldsymbol{\varepsilon}_{ni}^{(ji,k)}\right] + \tilde{P}_{ji,k}Q_{ji,k} = 0. \tag{78}$$

Second, we invoke the *Slutsky Equation*,[79] to rewrite the first two term in the above equation. Specifically, taking note that

$$\eta_{ii,g} = \eta_{ji,k} = 1 \quad \xrightarrow{\text{Slutsky Equation}} \quad \tilde{P}_{ni,g}Q_{ni,g}\varepsilon_{ni,g}^{(ji,k)} = \tilde{P}_{ji,k}Q_{ji,k}\varepsilon_{ji,k}^{(ni,g)}.$$

We can reduces the F.O.C. described under Equation 78 to

$$1+\sum_g\left[\frac{1+\mu_g}{1+\bar{\mu}_i}\varepsilon_{ji,k}^{(ii,g)}\right] + (1+\bar{\tau}_i)\sum_g\sum_{n\neq i}\left[(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\varepsilon_{ji,k}^{(ni,g)}\right] = 0. \tag{79}$$

Lastly, we use the Marshallian demand function's *homogeneity of degree zero* property, whereby $\eta_{ji,k} + \sum_{j,g}\varepsilon_{ji,k}^{(ji,g)} = 1 + \sum_{j,g}\varepsilon_{ji,k}^{(ji,g)} = 0$. Invoking this property we rewrite Equation 79 as follows

$$\sum_g\left[\left(1-\frac{1+\mu_g}{1+\bar{\mu}_i}\right)\boldsymbol{\varepsilon}_{ji,k}^{(ii,g)}\right] + \sum_g\sum_{n\neq i}\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)\boldsymbol{\varepsilon}_{ji,k}^{(ni,g)}\right] = 0.$$

The above equation, which should hold for all $ji,k \neq ii,k$ specifies a system of FOCS that can be expressed in matrix no notation as

$$\underbrace{\begin{bmatrix}\varepsilon_{1i,1}^{(ii,1)} & \cdots & \varepsilon_{Ni,K}^{(ii,1)}\\ \vdots & \ddots & \vdots\\ \varepsilon_{1i,1}^{(ii,K)} & \cdots & \varepsilon_{Ni,K}^{(ii,K)}\end{bmatrix}}_{\mathbf{E}_{-ii}^{(ii)}}\begin{bmatrix}1-\frac{\mu_1}{\bar{\mu}_i}\\ \vdots\\ 1-\frac{\mu_K}{\bar{\mu}_i}\end{bmatrix} + \underbrace{\begin{bmatrix}\varepsilon_{1i,1}^{(1i,1)} & \cdots & \varepsilon_{i-1i,k}^{(1i,1)} & \varepsilon_{i+1i,k}^{(1i,1)} & \cdots & \varepsilon_{Ni,K}^{(1i,1)}\\ \vdots & \ddots & & & \ddots & \vdots\\ \varepsilon_{1i,1}^{(Ni,K)} & \cdots & \varepsilon_{i-1i,k}^{(Ni,K)} & \varepsilon_{i+1i,k}^{(Ni,K)} & \cdots & \varepsilon_{Ni,K}^{(Ni,K)}\end{bmatrix}}_{\mathbf{E}_{-ii}}\begin{bmatrix}1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{1i,1}}{\tilde{P}_{1i,1}}\\ \vdots\\ 1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{Ni,K}}{\tilde{P}_{Ni,K}}\end{bmatrix} = \mathbf{0}. \tag{80}$$

Following the proof of Lemma 7 from Appendix E, we can easily show the matrix $\mathbf{E}_{-ii}^{(ii)}$ is invertible. We can, thus, invert the system specified by Equation 80 to produce the following formula for optimal

[78] Note that Equation 77 is essentially 73 with $\widetilde{\Delta}_i'(.)$ set to zero.

[79] Recalling that $e_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}/Y_i$ denotes the share of expenditure on ji,k, the Slutsky equation can be formally stated as

$$\text{[Slutsky equation]} \qquad e_{ii,g}\varepsilon_{ii,g}^{(ji,k)} + e_{ji,k}e_{ii,g}\eta_{ii,g} = e_{ji,k}\varepsilon_{ji,k}^{(ii,g)} + e_{ii,g}e_{ji,k}\eta_{ji,k}.$$

import price wedges:

$$\left[(1+\bar{\tau}_i)(1+\omega_{ji,k})\frac{P_{ji,k}}{\tilde{P}^{\star}_{ji,k}}\right]_{j,k}=\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k, \tag{81}$$

where, to be clear, $\mathbf{E}_{-ii}\equiv\left[\mathbf{E}_{ni}^{(ji)}\right]_{j,n\neq i}$ and $\tilde{\mathbf{E}}_{-ii}^{(ii)}\equiv\left[\mathbf{E}_{ni}^{(ii)}\right]_{n\neq i}$ are respectively $(N-1)K\times(N-1)K$ and $(N-1)K\times K$ matrixes of demand elasticities. Note that the optimal choice w.r.t. $\tilde{\mathbf{P}}_{ji}$, ensures that $\widetilde{\Delta}_i'(.)\approx 0$. Hence, the system of F.O.C. specified by Equation 76, transforms to the exact same system we solved in Appendix E. Without repeating the details of our prior derivation, the optimal export price wedges are given by

$$\left[\frac{P_{ij,k}}{\tilde{P}^{\star}_{ij,k}}(1+\bar{\tau}_i)^{-1}\right]_{j,k}=\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K}+\mathbf{\Omega}_{-ii}\right), \tag{82}$$

where $\mathbf{1}_{(N-1)K}$ is a $N(K-1)\times 1$ column vector of ones; $\mathbf{\Omega}_{-ii}=\left[\Omega_{ni,g}\right]_{n\neq i,g}$ is a $N(K-1)\times 1$ vector of (inverse) export supply elasticities; and $\mathbf{E}_{ij}^{(-ij)}$ and $\mathbf{E}_{ij}$ have the same description as in Appendix E. The "$\star$" notation is used to highlight the fact that we are solving for second-best price wedges. Next, we can recover the optimal (second-best) import tax and export subsidy rates from the optimal (second-best) price wedges implies by Equations 81 and 82. Specifically, noting the following relationships,

$$1+t^{\star}_{ji,k}=\frac{\tilde{P}^{\star}_{ji,k}}{P_{ji,k}};\qquad\qquad 1+x^{\star}_{ij,k}=\frac{P_{ij,k}}{\tilde{P}^{\star}_{ij,k}};$$

country i's unilaterally second-best trade tax schedule can be expressed as follows:

$$\begin{aligned}
&[\text{import tariff}] && \mathbf{1}+\mathbf{t}^{\star}_{ij}=(1+\bar{\tau}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right)\oslash\left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right)\\
&[\text{export subsidy}] && \mathbf{1}+\mathbf{x}^{\star}_{ij}=-(1+\bar{\tau}_i)\left(\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}(\mathbf{1}+\mathbf{\Omega}_{-ii})\right)\odot\left[\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k.
\end{aligned}$$

To conclude the proof we can invoke the multiplicity of the optimal trade tax schedules (Lemma 1). As in Theorem 1, this feature indicates that the value assigned to $\bar{\tau}_i$ is redundant. In particular, following Lemma 1, we can multiply $(1+\bar{\tau}_i)$ in the above equation with any non-negative tax shifter $1+\bar{t}_i\in\mathbb{R}_+$, and maintain optimality. That being the case, the exact value assigned to $\bar{\tau}_i$ is redundant and the following describes all possible optimal tax schedules:aa

$$\begin{aligned}
&[\text{import tariff}] && \mathbf{1}+\mathbf{t}^{\star}_{ij}=(1+\bar{t}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right)\oslash\left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right)\\
&[\text{export subsidy}] && \mathbf{1}+\mathbf{x}^{\star}_{ij}=-(1+\bar{t}_i)\left(\mathbf{E}_{ij}^{-1}\mathbf{E}_{ij}^{(-ij)}(\mathbf{1}+\mathbf{\Omega}_{-ii})\right)\odot\left[\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k.
\end{aligned}$$

G Proof of Theorem 3

Theorem 3 concerns the second-best case where the government in i can choose only $\tilde{\mathbf{P}}_{ji}$, which is the vector of import prices (i.e., $\mathbb{P}_i=\{\tilde{\mathbf{P}}_{ji}\}$). To prove this theorem we capitalize on two results from Appendix F: First, the F.O.C. derived w.r.t. $\tilde{P}_{ji,k}\in\tilde{\mathbf{P}}_{ji}$ does not change with the unavailability of $\tilde{\mathbf{P}}_{ij}$ from the government's policy set $\mathbb{P}_i$. Hence, the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ is described by Equation 73 even if $\tilde{P}_{ij,k}\notin\mathbb{P}_i$. Second, recall from Appendix that we were able to solve the system specified by 73 independent of the F.O.C. w.r.t. $\tilde{\mathbf{P}}_{ij}$. Invoking these two observations, the formula for optimal tariff in the case studied by Theorem 3 is given by 81:

$$1+\mathbf{t}^{\star}_{ji} = (1+\bar{\tau}_i)\left(\mathbf{1}+\mathbf{\Omega}_{ji}\right) \oslash \left(\mathbf{1}+\mathbf{E}_{-ii}^{-1}\,\mathbf{E}_{-ii}^{(ii)}\left[1-\frac{1+\mu_k}{1+\bar{\mu}_i}\right]_k\right).$$

Unlike Theorem 2, through, $\bar{\tau}_i$ is no longer redundant. Since export taxes (or equivalently $\tilde{\mathbf{P}}_{ij}$) are excluded from the government's policy set, we can no longer invoke the multiplicity implied by Lemma 1. Instead, we have to formally characterize, $\bar{\tau}_i$, starting from its definition:

$$\bar{\tau}_i \equiv \frac{\left(\frac{\partial W_i(.)}{\partial \ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}}{\left(\partial \mathrm{T}_i(\mathbb{P}_i,\mathbf{w})/\partial \ln w_i\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}}. \tag{83}$$

Also, recall that $W_i(\mathbb{P}_i;\mathbf{w}) = V_i(Y_i(\mathbb{P}_i;\mathbf{w}),\tilde{\mathbf{P}}_{ii},\tilde{\mathbf{P}}_{ji})$, where $\tilde{\mathbf{P}}_{ji}\sim\tilde{\mathbf{P}}_{-ii}\equiv\left\{\tilde{P}_{ji,k}\right\}_{j\neq i,k}$ while income equals wage payments, plus profits, plus import tax revenues: $Y_i = (1+\bar{\mu}_i)w_iL_i + (\tilde{\mathbf{P}}_{ji}-\mathbf{P}_{ji})\cdot\mathbf{Q}_{ji}$. Borrowing from the results in Appendixes E and F, the numerator in Equation 83 can be unpacked as follows:

$$\begin{aligned}
\left(\frac{\partial W_i(.)}{\partial \ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\left(\frac{\partial V_i}{\partial Y_i}\right)^{-1} &= \left(\frac{\partial Y_i}{\partial \ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} + \left(\frac{\partial V_i}{\partial Y_i}\right)^{-1}\frac{\partial V_i(Y_{i,},\tilde{\mathbf{P}}_i)}{\partial \ln\tilde{\mathbf{P}}_{ii}}\cdot\frac{\partial\ln\tilde{\mathbf{P}}_{ii}}{\partial\ln w_i}\\
&= \bar{\mu}_iw_iL_i + \left(\frac{\partial\bar{\mu}_i}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} w_iL_i \;+\; (\tilde{\mathbf{P}}_{-ii}-\mathbf{P}_{-ii})\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} - \mathbf{P}_{ii}\cdot\mathbf{Q}_{ii}\\
&= \sum_{n\neq i}\left[\mathbf{P}_{in}\cdot\mathbf{Q}_{in}\right] + \left(\mathbf{1}-\frac{\bar{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial\mathbf{Q}_{ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} + (\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{-ii})\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}.
\end{aligned} \tag{84}$$

To be clear about the notation, $\frac{\bar{\mu}_i}{\boldsymbol{\mu}}\equiv\left[\frac{\bar{\mu}_i}{\mu_k}\right]_k$, while $\odot$ and $\cdot$ respectively denote *inner* and *element-wise* products of equal-sized vectors, i.e., $\mathbf{a}\cdot\mathbf{b} = \sum_n a_nb_n$ and $\mathbf{a}\odot\mathbf{b} = [a_nb_n]_n$. Next, we move on to characterizing the denominator of Equation 83. Noting that $\mathrm{T}(\mathbb{P}_i,\mathbf{w}) \equiv \sum_{j\neq i}\left[\mathbf{P}_{ji}\cdot\mathbf{Q}_{ji}-\mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}\right]$, we can borrow from the results in Appendixes E and F to unpack the aforementioned term as follows:

$$\left(\frac{\partial\mathrm{T}_i(.)}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} = \left(\frac{\partial}{\partial\ln w_i}\sum_{j\neq i}\left[\mathbf{P}_{ji}\cdot\mathbf{Q}_{ji}-\mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}\right]\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} = \mathbf{P}_{-ii}\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} - \sum_{j\neq i}\left[\left(\frac{\partial\mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\right]. \tag{85}$$

Plugging Equations 84 and 85 back into the expression for $\bar{\tau}_i$ yields the following:

$$\bar{\tau}_i = \frac{\sum_{n\neq i}\left[\mathbf{P}_{in}\cdot\mathbf{Q}_{in}\right] + \left(\mathbf{1}-\frac{\bar{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial\mathbf{Q}_{ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} + \left(\tilde{\mathbf{P}}_{ii}-\mathbf{P}_{-ii}\right)\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}}{\mathbf{P}_{-ii}\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} - \sum_{j\neq i}\left[\left(\frac{\partial\mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\right]}. \tag{86}$$

We can further simplify the above expression by invoking the F.O.C. described by Equation 78. This equation indicates that the following relationship ought to hold at the optimum $\mathbb{P}_i=\mathbb{P}_i^*$:

$$\sum_{j\neq i}\sum_k\left[\left(\mathbf{1}-\frac{\bar{\mu}_i}{\boldsymbol{\mu}}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial\mathbf{Q}_{ii}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}} + \left(\tilde{\mathbf{P}}_{-ii}-(1+\bar{\tau}_i)\mathbf{P}_{-ii}\right)\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln\tilde{P}_{ji,k}}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\right] = 0.$$

Now, we will rearrange and simplify the above relationship in such a way that will help us simply Equation 86. To this, we invoke the property that the Marshallain demand function is homogeneous of degree zero. Combining this property with the fact that $\frac{\partial\ln Y_i}{\partial\ln w_i}\approx\frac{\partial\ln\tilde{P}_{ii,k}}{\partial\ln w_i}=1$, we can simplify the above as follows:

$$\left(\mathbf{1}-\frac{\bar{\mu}_i}{\mu}\right)\odot\mathbf{P}_{ii}\cdot\left(\frac{\partial\mathbf{Q}_{ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}+\left(\tilde{\mathbf{P}}_{ii}-(1+\bar{\tau}_i)\mathbf{P}_{-ii}\right)\cdot\left(\frac{\partial\mathbf{Q}_{-ii}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}=0.$$

Using the above equation, we can cancel out the mirroring expressions in the numerator and denominator of Equation 86. Doing so reduces and simplifies the expression for $\bar{\tau}_i$ to the following:

$$\bar{\tau}_i=\frac{-\sum_{n\neq i}\left(\mathbf{P}_{in}\cdot\mathbf{Q}_{in}\right)}{\sum_{j\neq i}\left[\left(\frac{\partial\mathbf{P}_{ij}\cdot\mathbf{Q}_{ij}}{\partial\ln w_i}\right)_{\mathbb{P}_i,\mathbf{w}_{-i}}\right]}=\frac{-1}{\sum_{j\neq i}\left[\mathbf{X}_{ij}\cdot\left(\mathbf{I}_K+\mathbf{E}_{ij}\right)\mathbf{1}_K\right]}. \tag{87}$$

The $K\times 1$ vector $\mathbf{X}_{ij}=\left[\chi_{ij,k}\right]_k$ is compose of export shares, which are defined as $\chi_{ij,k}\equiv\frac{P_{ij,k}Q_{ij,k}}{\sum_{n\neq i}\mathbf{P}_{in}\cdot\mathbf{Q}_{in}}$. To provide some intuition, the denominator of the above equation corresponds to the elasticity of international demand for origin i's labor. As such, $\bar{\tau}_i$ can be interpreted as country i's optimal markup on its wage rate in international (non-i) markets.

H Optimal Tax Formulas under IO Linkages

We first present a formal description of equilibrium under input-output (IO) linkages. We use the $\mathcal{C}$ superscript to denote final consumption goods and the $\mathcal{I}$ superscript to denote intermediate inputs. To given an example: $Q^{\mathcal{C}}_{ji,k}$ denotes the quantity of a "final" goods associated with *origin j–destination i–industry k*, while $Q^{\mathcal{I}}_{ji,k}$ denotes the quantity of an "intermediate" goods associated with origin j–destination i–industry k. Without loss of generality, we assume that good ji,k exhibits the same price irrespective of whether it is used as a final good or an intermediate input good: $\tilde{P}_{ji,k}\sim\tilde{P}^{\mathcal{C}}_{ji,k}=\tilde{P}^{\mathcal{I}}_{ji,k}$.

On the production side, we impose no restrictions on how intermediate inputs are aggregated in the production process. We, however, assume that the share of labor in production is constant and equal to $1-\bar{\alpha}_{i,k}$ for each *origin i–industry k*. To track the demand for inputs, we use $\mathcal{Y}_{i,k}$ to denote the gross revenue associated with *origin i–industry k*. Correspondingly, $\bar{\alpha}_{i,k}\mathcal{Y}_{i,k}$ denotes *origin i–industry k*'s total expenditure on intermediate inputs.

Marshallian Demand under IO Linkages

We suppose that overall demand for good ji,k, which is the sum of final good demand based on utility maximization and input demand based on cost minimization, is given by the following demand function

$$Q_{ji,k}=Q^{\mathcal{I}}_{ji,k}+Q^{\mathcal{C}}_{ji,k}=\mathcal{D}_{ji,k}(E_i,\tilde{\mathbf{P}}_i),$$

where $E_i=Y_i+\sum_g\bar{\alpha}_{i,g}\mathcal{Y}_{i,g}$ denotes market i's total expenditure on final and intermediate input goods. To make the notation consistent with our previous derivations, we use $\varepsilon^{(ni,g)}_{ji,k}$ and $\eta_{ji,k}$ to denote the price and income elasticities associated with the IO-augmented Marshallian demand function $\mathcal{D}_{ji,k}(E_i,\tilde{\mathbf{P}}_i)$.

General Equilibrium

As in the baseline model, we express all equilibrium outcomes (expect for wages) as a function of global taxes ($\mathbf{x}$, $\mathbf{t}$, and $\mathbf{s}$), treating wages $\mathbf{w}\equiv\{w_i\}_i$ as given. This formulation derives from solving a system that imposes all equilibrium conditions aside from the labor market clearing conditions. We formally outline this formulation below.

Notation. *For a given vector of taxes and wages* $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$*, equilibrium outcomes* $Y_i(\mathbf{T})$*,* $\mathcal{Y}_{i,k}(T)$*,* $P_{ji,k}(\mathbf{T})$*,* $\tilde{P}_{ji,k}(\mathbf{T})$*,* $Q_{ji,k}(\mathbf{T})$ *are determined such that* (i) *producer prices are characterized by 13;* (ii) *consumer prices are given by 7;* (iii) *Consumption and input demand choices are given by* $\mathcal{D}_{ji,k}(E_i, \tilde{\mathbf{P}}_i)$*, where* $E_i = Y_i + \sum_g \bar{\alpha}_{i,g}\mathcal{Y}_{i,g}$*;* (iv) *net income (which dictates total final good expenditure by country i) equals wage payments plus tax revenues:* $Y_i = w_i L_i + \mathcal{R}_i$*,*[80] *where* $\mathcal{R}_i$ *are described by 8 and* (v) *gross industry-level revenues are given by* $\mathcal{Y}_{i,g} = \sum_n P_{in,k} Q_{in,k}$*.*

As in the baseline model, $\mathbf{w}$ is itself an equilibrium outcome. So, a vector $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ is feasible insofar as $\mathbf{w}$ is the equilibrium wage, consistent with $\mathbf{t}$, $\mathbf{x}$, and $\mathbf{s}$. So, to fix ideas we define the set of feasible *policy–wage* vectors as follows.

Definition (D2-IO). *The set of feasible policy–wage vectors,* $\mathbb{F}$*, consists of any vector* $\mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w})$ *where* $\mathbf{w}$ *satisfies the labor market clearing condition in every country, given* $\mathbf{t}$*,* $\mathbf{x}$*, and* $\mathbf{s}$*:*

$$\mathbb{F} = \left\{ \mathbf{T} = (\mathbf{t}, \mathbf{x}, \mathbf{s}; \mathbf{w}) \;\middle|\; w_i L_i = \sum_j \sum_k Qz_{ij,k}(\mathbf{T}) - \sum_j \sum_k P^{\mathcal{I}}_{ji,k}(\mathbf{T}) Q^{\mathcal{I}}_{ji,k}(\mathbf{T}); \quad \forall i \in \mathbb{C} \right\}.$$

Before moving on to the proof, two important details are in order: First, we can easily verify that the labor market clearing condition specified by Definition D2-IO is equivalent to the balanced trade condition. Second, under IO linkages, the choice w.r.t. taxes (or equivalently $\mathbb{P}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$) may affect the entire vector of producer prices, $\{P_{nj,k}\}$, through its effect on input prices. To track these IO-related effects, let $\alpha^{j,g}_{i,k}$ denotes the (possibly variable) cost share of intermediate inputs from *origin* $j\times$ *industry* g used in the output of *origin* $i\times$ *industry* k. By Shepherd's Lemma, the direct effect of raising input price $\tilde{P}^{\mathcal{I}}_{ji,g}$ on the producer price $P_{ij,k}$ can be expressed as follows:

$$\text{[Shepherd's Lemma]} \qquad \left(\frac{\partial \ln P_{ij,k}(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,g}} \right)_{\mathbb{P}_{-ji,g}, \mathbf{w}} = \alpha^{j,g}_{i,k}. \qquad \forall (\jmath, j, i \in \mathbb{C}); \;\; \forall (g, k \in \mathbb{K}).$$

We use the Shepherd's Lemma in combination with our dual approach (from Appendix E) to characterize the optimal policy schedule for each country i. Recall that the optimal policy problem in our dual approach is reformulated as

$$\max_{\mathbb{P}_i} \;\; W_i(\mathbb{P}_i; \mathrm{w}) \equiv V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}_i) \qquad s.t. \;\; (\mathbb{P}_i; \mathbf{w}) \in \mathbb{F}_P,$$

where $\mathbb{P}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ij}\}$ denotes the vector of consumer prices directly associated with economy i. The feasible set $\mathbb{F}_P$ defined analogously to $\mathbb{F}$. Below, we derive and solve the system of F.O.C. associated with the above problem, building heavily on the results introduced earlier under Appendix E.

Step #1: Deriving the F.O.C. w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k} \in \mathbb{P}_i$

First, we derive the F.O.C. w.r.t. to import variety ji, k, supplied by origin j–industry k. Given that $W_i = V_i(Y_i(\mathbb{P}_i; \mathbf{w}), \tilde{\mathbf{P}}^{\mathcal{C}}_{ii}, \tilde{\mathbf{P}}^{\mathcal{C}}_{ji})$, the F.O.C. w.r.t. $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k} \sim \tilde{P}^{\mathcal{C}}_{ji,k}$, holding $\mathbb{P}_{-ji,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ji,k}\}$ constant, can be stated as

$$\left(\frac{\partial W_i}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbb{P}_{-ji,k}} = \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}^{\mathcal{C}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i} \left(\frac{\partial Y_i(\mathbb{P}_i; \mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right)_{\mathbf{w}, \mathbb{P}_{-ji,k}} + \left(\frac{\partial W_i(\mathbb{P}_i, \mathbf{w})}{\partial \mathbf{w}} \right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right)_{\mathbb{P}_{-ji,k}} = 0. \tag{88}$$

[80] *Note that net profits are equal top zero (i.e.,* $\Pi_i = 0$*) as we are focusing on the case of free entry.*

The right-hand side of the above equation can be characterized similar to Appendix E, with two distinctions: First, total demand for good ji,k is the sum of consumption plus input demand: $Q_{ji,k} = Q^{\mathcal{C}}_{ji,k} + Q^{\mathcal{I}}_{ji,k}$. So, we have to distinguish between welfare effects that channel through consumption and those that channel through input demand. Second, we need to account for the effect of a change in input price $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k}$ on the producer prices associated with economy i. To this end, we can invoke Shepherd's Lemma, which implies that

$$\left(\frac{\partial \ln P_{ij,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,g}}\right)_{\mathbb{P}_{-ji,g},\mathbf{w}} = \alpha^{j,g}_{i,k}. \qquad \forall \jmath, j, i \in \mathbb{C};\;\; g,k \in \mathbb{K}.$$

Considering the above caveats, we can proceed as in Appendix E. By Roy's identity, the first term on the right-hand side of the F.O.C. (Equation 88) can be stated as

$$\frac{\partial V_i(Y_i, \tilde{\mathbf{P}}^{\mathcal{C}}_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q^{\mathcal{C}}_{ji,k}\left(\frac{\partial V_i}{\partial Y_i}\right).$$

Next, consider the second term on the right-hand side of Equation 88, which accounts for income effects. Recall that total income in country i equals the sum of wage payments plus import, production and export tax revenues:

$$Y_i(\mathbb{P}_i;\mathbf{w}) = w_i L_i + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}\right)\cdot \mathbf{Q}_{ni}\right] + \left(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}\right)\cdot \mathbf{Q}_{ii} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}\right)\cdot \mathbf{Q}_{in}\right].$$

The effect of $\tilde{P}_{ji,k}$ on import tax revenues can be derived and express exactly as in Appendix E:

$$\left(\frac{\partial \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{ni} - \mathbf{P}_{ni}\right)\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = \tilde{P}_{ji,k} Q_{ji,k} + \sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g} - [1+\omega_{ni,g}]P_{ni,g}\right) Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right] \tag{89}$$

The logic is that holding the vector of wages $\mathbf{w}$ and country i's export prices $\tilde{\mathbf{P}}_{ij} \in \mathbb{P}_i$ fixed, a change in $\tilde{P}_{ji,k}$ has not effect on the producer price of imports $\mathbf{P}_{ji}$ through the input-output network.

The effect of a change in $\tilde{P}_{ji,k}$ on country i's production and export tax revenues can be formulated as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\left(\tilde{\mathbf{P}}_{ii} - \mathbf{P}_{ii}\right)\cdot \mathbf{Q}_{ii,g} + \sum_{n\neq i}\left[\left(\tilde{\mathbf{P}}_{in} - \mathbf{P}_{in}\right)\cdot \mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$$
$$= \sum_{g}\left[\left(\tilde{P}_{ii,g} - P_{ii,g}\right) Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right] + \sum_{g}\sum_{n}\left[P_{in,g} Q_{in,g}\left[\left(\frac{\partial P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbf{w},\mathbb{P}_i}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} + \left(\frac{\partial \ln P_{in,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]\right], \tag{90}$$

The above expression differs from Equation 91 (in Appendix E) in the last term on the second line. This term accounts for the effect of raising input price $\tilde{P}_{ji,k} \sim \tilde{P}^{\mathcal{I}}_{ji,k}$ on the producer prices associated with economy i. As explained above, we can appeal to Shephard's lemma to simplify this extra term as

$$\sum_{g\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left[P_{in,g} Q_{in,g}\left(\frac{\partial \ln P_{in,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ji,k}}\right)_{\mathbb{P}_{-ji,g},\mathbf{w}}\right] = -\sum_{g\in\mathbb{K}}\sum_{n\in\mathbb{C}}\left(Q_{in,g} P_{in,g}\alpha^{j,k}_{i,g}\right) = -\tilde{P}_{ji,k} Q^{\mathcal{I}}_{ji,k}.$$

Plugging the above expression back into Equation 90 and redoing the derivations covered in Appendix E, yields the following expression for the effect of $\tilde{P}_{ji,k}$ on country i's production and export tax revenues:

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ji,k}}\left\{\sum_{n}\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot \mathbf{Q}_{in}\right]\right\}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = -\tilde{P}_{ji,k}Q^{\mathcal{I}}_{ji,k} + \sum_{g}\left[\left(\tilde{P}_{ii,g}-\left[1-\frac{\mu_g}{1+\mu_g}\right]\right)P_{ii,g}Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right]. \tag{91}$$

where recall that $\left(\partial \ln Q_{ii,g}/\partial \ln \tilde{P}_{ji,k}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}$ encompasses price and income effects à la Equation 27 in Appendix E. Combining Equations 89 and 91, and noting that $\tilde{P}_{ji,k}Q_{ji,k} - \tilde{P}_{ji,k}Q^{\mathcal{I}}_{ji,k} = \tilde{P}_{ji,k}Q^{\mathcal{C}}_{ji,k}$ yields the following expression that summarizes all the revenue-related welfare effects in the F.O.C.:

$$\begin{aligned}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} &= \tilde{P}_{ji,k}Q^{\mathcal{C}}_{ji,k} + \sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+ \sum_{g}\left[\left(\tilde{P}_{ii,g}-\frac{1}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right] + \Delta_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}. \end{aligned} \tag{92}$$

The uniform term $\Delta_i(.)$ accounts for circular income effects and is given by Equation 35 in Appendix E. Finally, the last term on the right-hand side of Equation 88, which accounts for general equilibrium wage effects, can be specified in the same exact way as in Appendix E:

$$\left(\frac{\partial W_i(\mathbb{P}_i,\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right)_{\mathbb{P}_{-ji,k}} = -\tau_i\sum_{g}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}}\right],$$

where $\bar{\tau}_i$ is given by Equation 37 in Appendix E. Combining the above expressions, the F.O.C. specified by Equation 88 reduced to

$$\begin{aligned}[\text{FOC w.r.t. } \tilde{P}_{ji,k}] \qquad & \sum_{n\neq i}\sum_{g}\left[\left(\frac{\tilde{P}_{ni,g}}{P_{ni,g}}-(1+\bar{\tau}_i)(1+\omega_{ni,g})\right)P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] \\ &+ \sum_{g}\left[\left(\frac{\tilde{P}_{ii,g}}{P_{ii,g}}-\frac{1}{1+\mu_g}\right)P_{ii,g}Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right] + \widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbb{P}_{-ji,k}} = 0, \end{aligned} \tag{93}$$

where $\widetilde{\Delta}_i(.)$ is given by Equation 40 in Appendix E. Note that the above equation has an identical representation to the F.O.C. in the baseline model. The intuition is that holding country i's export prices $\tilde{\mathbf{P}}_{ij} \in \mathbb{P}_i$ fixed, the choice w.r.t. $\tilde{P}_{ji,k}$ has no first-order effect on country i's terms-of-trade channels through the input-output network. If good ji,k is used as an input in export good in,g, any possible terms-of-trade gains from taxing $\tilde{P}_{ji,k}$ will be internalized by the optimal choice w.r.t. $\tilde{P}_{in,g}$. Furthermore, it is easy to check that Equation 93 characterizes the F.O.C. w.r.t. $\tilde{P}_{ii,k} \in \mathbb{P}_i$ as long as we replace ji,k with ii,k everywhere in that equation. Finally, as in Appendix E, we do not unpack the uniform term $\bar{\tau}_i$ because the multiplicity of country i's optimal tax schedule will render the exact value assigned to $\bar{\tau}_i$ redundant.

***Step* #2: Deriving the W.O.C. w.r.t. $P^{*}_{ij,k}$**

Consider export variety ij,k, which is sold to destination $j \neq i$ in industry k. Noting that $W_i = V_i(Y_i(\mathbb{P}_i;\mathbf{w}), \tilde{\mathbf{P}}^{\mathcal{C}}_{ii}, \tilde{\mathbf{P}}^{\mathcal{C}}_{ji})$, the F.O.C. w.r.t. $\tilde{P}_{ij,k}$, holding $\mathbb{P}_{-ij,k} \equiv \mathbb{P}_i - \{\tilde{P}_{ij,k}\}$ constant, can be stated as

$$\left(\frac{\partial W_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial Y_i}\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial W_i(\mathbb{P}_i,\mathbf{w})}{\partial \mathbf{w}}\right)_{\mathbb{P}_i} \cdot \left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}} = 0. \tag{94}$$

The first term as before accounts for direct price effects. This term is trivially equal to zero since $\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i$. That is, since ij,k is not part of the domestic consumption bundle, raising its price has no direct effect on consumer surplus in country i:

$$\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i \implies \frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}} = 0. \tag{95}$$

The second term in Equation 94 accounts for welfare effects that channel through tax revenues. Specifically, Holding wages $\mathbf{w}$ fixed, the change in country i's income amounts to the change in import, domestic, and export tax revenues. The effect on import tax revenues can be expressed as follows:

$$\left(\frac{\partial \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = \sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left[\left(\frac{\partial P_{ni,g}}{\partial \ln Q_{nj,g}}\right)_{\mathbf{w},\mathbb{P}_i}\left(\frac{\partial \ln Q_{nj,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} + \left(\frac{\partial \ln P_{ni,g}}{\partial \ln \tilde{P}^{\mathcal{I}}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right]\right]. \tag{96}$$

The above equation differs from Equation 45 in Appendix E in only the last term on the second line. This term accounts for that fact that raising the price of input good ij,k can affect the entire vector of producer prices in the rest of the world through input-output networks. Given Shephard's lemma we can simplify this term by noting that

$$\Lambda_{ij,k} \equiv \sum_{n\neq i}\sum_{g\in\mathbb{K}}\left(P_{ni,g}Q_{ni,g}\left(\frac{\partial P_{ni,g}}{\partial \tilde{P}^{\mathcal{I}}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}\right)/\,\tilde{P}_{ij,k}Q_{ij,k}$$

denotes the share of the export value associated with good ij,k that is reimported back into economy i. Plugging the above expression back into 96 and repeating the derivation performed in Appendix E, yields the following:

$$\left(\frac{\partial \sum_{n\neq i}\left[(\tilde{\mathbf{P}}_{ni}-\mathbf{P}_{ni})\cdot \mathbf{Q}_{ni}\right]}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}} = -\,\Lambda_{ij,k}\tilde{P}_{ij,k}Q_{ij,k} - \sum_{g}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right]$$
$$+\sum_{g}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\omega_{ni,g}]P_{ni,g}\right)Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial \ln E_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{97}$$

Repeating the derivation in Appendix E, the effect of a change in $\tilde{P}_{ij,k}$ on country i's production and export tax revenues can be formulated as

$$\left(\frac{\partial}{\partial \ln \tilde{P}_{ij,k}}\sum_{n}\left[(\tilde{\mathbf{P}}_{in}-\mathbf{P}_{in})\cdot\mathbf{Q}_{in}\right]\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=\tilde{P}_{ij,k}Q_{ij,k}+\sum_{g}\left[\left(\tilde{P}_{ij,g}-[1-\frac{\mu_g}{1+\mu_g}]P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$+\sum_{g}\left[\left(\tilde{P}_{ii,g}-\frac{1}{1+\mu_g}P_{ii,g}\right)Q_{ii,g}\eta_{ii,g}\right]\left(\frac{\partial \ln E_i}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{98}$$

To be clear, holding $\tilde{P}_{ji,k}\in\mathbb{P}_i$ fixed, a change in $\tilde{P}_{ij,k}$ has no effect on the input price faced by firm located in i. That is, $\left(\partial P_{ni,g}/\partial \tilde{P}^{\mathcal{I}}_{ij,k}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0$. This point explains why the above expression is rather identical to that derived in Appendix E. Combining Equations 97 and 94, we can express the sum of all tax-revenue-related terms as

$$\left(\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=(1-\Lambda_{ij,k})\,\tilde{P}_{ij,k}Q_{ij,k}+\sum_{g}\left[\left(\tilde{P}_{ij,g}-[1-\frac{\mu_g}{1+\mu_g}]P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]+\Delta_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}, \tag{99}$$

where $\Delta_i(.)$ encompasses the terms accounting for circular income effects and is given by Equation 35. Taking note of the already-discussed distinctions between the present and baseline models and repeating the derivations performed earlier in Appendix E, the last term in right-hand side of Equation 94) can be formulated as

$$\left(\frac{\partial W_i(.)}{\partial \mathbf{w}}\right)_{\mathbb{P}_i}\cdot\left(\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right)_{\mathbb{P}_{-ij,k}}=\bar{\tau}_i(1-\Lambda_{ij,k})\tilde{P}_{ij,k}Q_{ij,k}+\sum_{g}\left[\bar{\tau}_i\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[\bar{\tau}_i\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]-\sum_{g}\sum_{n\neq i}\left[[1+\omega_{ni,g}]\bar{\tau}_iP_{ni,g}Q_{ni,g}\eta_{ni,g}\right]\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}. \tag{100}$$

Finally, plugging Equations 95, 99, and 100 back into the F.O.C. (Equation 94); and dividing by $(1+\bar{\tau}_i)$ yields the following optimality condition w.r.t. to price instrument $\tilde{P}_{ij,k}$:

$$[\text{FOC w.r.t. } \tilde{P}_{ij,k}]\qquad (1-\Lambda_{ij,k})\tilde{P}_{ij,k}Q_{ij,k}+\sum_{g}\left[\left(1-\frac{1}{(1+\bar{\tau}_i)(1+\mu_g)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\tilde{P}_{ij,g}Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_{g}\sum_{n\neq i}\left[\omega_{ni,g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial E_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0, \tag{101}$$

where $\widetilde{\Delta}_i(.)$ is defined as in Equation 40. Also, we are not unpacking the term $\bar{\tau}_i$, for the same reasons discussed earlier.

Step #3: Solving the System of F.O.C.s and Establishing Uniqueness

To determine the optimal tax schedule we need to collect the system of first order conditions and simultaneously solve them under one system. For the ease of reference, we will restate the F.O.C. w.r.t.

to each element of $\mathbb{P}_i$ below. Following Equation 93, the F.O.C. w.r.t. $\tilde{P}_{\ell i,k}$ (where $\ell = i$ or $\ell = j \neq i$), can be expressed as

$$(1)\quad \sum_{n\neq i}\sum_{g}\left[\left(1-(1+\bar{\tau}_i)(1+\omega_{ni,g})\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)e_{ni,g}\varepsilon_{ni,g}^{(\ell i,k)}\right]+$$
$$\sum_{g}\left[\left(1-\frac{1}{1+\mu_g}\frac{P_{ii,g}}{\tilde{P}_{ii,g}}\right)e_{ii,g}\varepsilon_{ii,g}^{(\ell i,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\left(\frac{\partial\ln E_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{\ell i,k}}\right)_{\mathbf{w},\mathbb{P}_{-\ell i,k}}=0.$$

where $e_{ni,g} = \tilde{P}_{ni,g}Q_{ni,g}/Y_i$ denotes the expenditure share on good ni, g. Following Equation 101, the F.O.C. w.r.t. export price $\tilde{P}_{ij,k}$ is given by

$$(2)\quad 1-\Lambda_{ij,k}+\sum_{g}\left[\left(1-\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right]$$
$$-\sum_{n\neq i}\sum_{g}\left[\omega_{ni,g}\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right]+\widetilde{\Delta}_i(\mathbb{P}_i,\mathbf{w})\frac{E_i}{E_j}\left(\frac{\partial\ln E_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{w},\mathbb{P}_{-ij,k}}=0.$$

First, note that the system of F.O.C.s (1) Appealing to above lemma, it immediately follows that the unique solution to the above equation is the trivial solution given by:

$$\frac{\tilde{P}^*_{ji,k}}{P_{ji,1}}=(1+\omega_{ji,k})(1+\bar{\tau}_i);\qquad \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}}=\frac{1}{1+\mu_g}. \tag{102}$$

With the aid of the above result, we can proceed to solving System (2), knowing that $\widetilde{\Delta}_i(\mathbb{P}^*_i,\mathbf{w}) = 0$. To this end, let us economize on the notation by defining

$$\chi_{ij,k}\equiv\frac{1}{(1+\mu_g)(1+\bar{\tau}_i)}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}.$$

Appealing to this choice of notation the F.O.C. specified by System (2) implies the following optimality condition:

$$1-\Lambda_{ij,k}+\sum_{g}\left[(1-\chi_{ij,g})\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}}\right]-\sum_{n\neq i}\sum_{g}\left[\omega_{ni,g}\frac{e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}}\right]=0. \tag{103}$$

To simplify the above expression we will use a well-know result from consumer theory, namely, the Cournot aggregation, which implies:

$$[\text{Cournot aggregation}]\qquad 1+\sum_{g}\left[\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right]=-\sum_{n\neq i}\sum_{g}\left[\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right].$$

Combining the above expression with Equation 103 and noting that by Slutsky's equation $\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)} = \varepsilon_{ij,k}^{(nj,g)}$ (if $\eta_{ni,g} = 1$ for all ni, g), yields the following:

$$-\sum_{g}\left[\chi_{ij,g}\varepsilon_{ij,k}^{(ij,g)}\right]-\sum_{n\neq i}\sum_{g}\left[(1+\omega_{ni,g})\varepsilon_{ij,k}^{(nj,g)}\right]=0\qquad\forall(ij,k).$$

We can formulate the above equation in matrix algebra as

$$-\mathbf{E}_{ij}\mathbf{X}_{ij}-\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K}+\mathbf{\Omega}_i\right)-\mathbf{\Lambda}_{ij}=0, \tag{104}$$

where $\mathbf{X}_{ij} \equiv \left[\chi_{ij,k}\right]_k$ and $\mathbf{\Lambda}_{ij} \equiv \left[\Lambda_{ij,k}\right]_k$ are $K \times 1$ vectors. The $K \times K$ matrix $\mathbf{E}_{ij} \sim \mathbf{E}_{ij}^{(ij)} \equiv \left[\varepsilon_{ij,k}^{(ij,g)}\right]$ encompasses the own- and cross-price elasticities between the different varieties sold by origin i to market j. Analogously, $\mathbf{E}_{ij}^{(-ij)} \equiv \left[\varepsilon_{ij,k}^{(nj,g)}\right]_{k,n\neq i,g}$ is a $K \times (N-1)K$ matrix of cross-price elasticities between varieties sold by i and by all other origin countries in market j. $\mathbf{\Omega}_i \equiv \left[\omega_{ni,g}\right]_{n,g}$ is a $(N-1)K \times 1$ vector of inverse export supply elasticities associated with domestic market i. To invert the system specified by Equation 104 we can use our result (from Appendix E) that $\mathbf{E}_{ij}$ is non-singular, which yields the following formulation for $\mathbf{X}_{ij}^{*} = \left[\chi_{ij,k}^{*}\right]_k$:

$$\mathbf{X}_{ij}^{*} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1}_{(N-1)K} + \mathbf{\Omega}_i\right) + \mathbf{\Lambda}_{ij}\right]. \tag{105}$$

Now, we can recover the optimal tax/subsidy rates from the optimal price wedges implies by Equations 102 and 105. Specifically, noting that

$$1 + t_{ji,k}^{*} = \frac{\tilde{P}_{ji,k}}{P_{ji,k}}; \qquad 1 + s_{i,k}^{*} = \frac{\tilde{P}_{ii,k}^{*}}{P_{ii,k}}; \qquad 1 + x_{ij,k} = \frac{\tilde{P}_{ij,k}^{*}/P_{ij,k}}{\tilde{P}_{ii,k}^{*}/P_{ii,k}};$$

country i's unilaterally optimal tax schedule can be expressed as follows:

$$\begin{aligned}
\text{[domestic subsidy]} \quad & 1 + s_{i,k}^{*} = 1 + \mu_k \\
\text{[import tariff]} \quad & 1 + t_{ji,k}^{*} = (1 + \omega_{ji,k})(1 + \bar{\tau}_i) \\
\text{[export subsidy]} \quad & \mathbf{1} + \mathbf{x}_{ij}^{*} = -\mathbf{E}_{ij}^{-1}\left[\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1} + \mathbf{\Omega}_i\right) + \mathbf{\Lambda}_{ij}\right](1 + \bar{\tau}_i).
\end{aligned}$$

The last step, is to invoke the multiplicity of optimal tax schedules. Given the multiplicity of optimal import tax and export subsidies, the term $\bar{\tau}_i$ becomes redundant. Following Lemma 1, any schedule that satisfies $1 + t_{ji,k}^{*} = (1 + \omega_{ji,k})(1 + \bar{\tau}_i)(1 + \bar{t}_i)$, where $1 + \bar{t}_i \in \mathbb{R}_+$, is also optimal. As such the exact value assigned to $\bar{\tau}_i$ is redundant. This explains why we did not unpack the term $\bar{\tau}_i$ in Step #3. There is also another dimension of multiplicity whereby any uniform shift in production subsidies (paired with a proportional adjustment to wage) preserves the equilibrium. Considering these arguments, the optimal policy schedule (accounting for all the dimensions of multiplicity) is given by:

$$\begin{aligned}
\text{[domestic subsidy]} \quad & 1 + s_{i,k}^{*} = (1 + \mu_k)(1 + \bar{s}_i) \\
\text{[import tariff]} \quad & 1 + t_{ji,k}^{*} = (1 + \omega_{ji,k})(1 + \bar{t}_i) \\
\text{[export subsidy]} \quad & \mathbf{1} + \mathbf{x}_{ij}^{*} = -\mathbf{E}_{ij}^{-1}\left(\mathbf{E}_{ij}^{(-ij)}\left(\mathbf{1} + \mathbf{t}_i^{*}\right) + \mathbf{\Lambda}_{ij}(1 + \bar{t}_i)\right),
\end{aligned}$$

where $1 + \bar{s}_i = 1 + \bar{t}_i \in \mathbb{R}_+$ are arbitrary tax shifters.

Data Appendix

I Cleaning the data on the identity/name of exporting firms

Utilizing the information on the identity of the foreign exporting firm is a critical part of our empirical exercise. Unfortunately, the names of the exporting firms in our dataset are not standardized. As a result, there are instances when the same firm is recorded differently due to using or not using the abbreviations, capital and lower-case letters, spaces, dots, other special characters, etc. To standardize

Table 5: Summary Statistics of the Colombian Import Data.

	Year						
Statistic	2007	2008	2009	2010	2011	2012	2013
F.O.B. value (billion dollars)	30.77	37.26	31.39	38.41	52.00	55.79	56.92
$\frac{\text{C.I.F. value}}{\text{F.O.B. value}}$	1.08	1.07	1.05	1.06	1.05	1.05	1.05
$\frac{\text{C.I.F. + tax value}}{\text{F.O.B. value}}$	1.28	1.21	1.14	1.19	1.15	1.18	1.15
No. of exporting countries	210	219	213	216	213	221	224
No. of imported varieties	483,286	480,363	457,000	509,524	594,918	633,008	649,561

Notes: Tax value includes import tariff and value-added tax (VAT). The number of varieties corresponds to the number of country-firm-product combination imported by Colombia in a given year.

the names of the exporting firms, we used the following procedure.[81]

1. We deleted all observations with the missing exporting names and/or zero trade values.

2. We capitalized firms names and their contact information (which is either email or phone number of the firm).

3. We eliminated abbreviation "LLC," spaces, parentheses, and other special characters (. , ; / @ ' } - & ") from the firms names.

4. We eliminated all characters specified in 3. above and a few others (# : FAX) from the contact information.

5. We dropped observations without contact information (such as, "NOTIENE", "NOREPORTA", "NOREGISTRA," etc.), with non-existent phone numbers (e.g., "0000000000", "1234567890", "1"), and with six phone numbers which are used for multiple firms with different names (3218151311, 3218151297, 6676266, 44443866, 3058712865, 3055935515).

6. Next, we kept only up to first 12 characters in the firm's name and up to first 12 characters in the firm's contact information (which is either email or phone number). In our empirics, we treat all transaction with the same updated name and contact information as coming from the same firm.

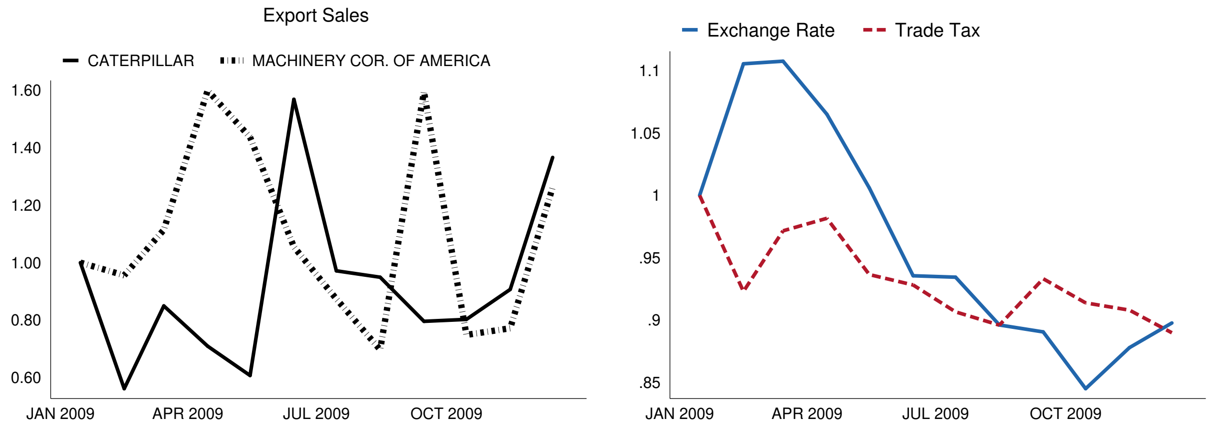
7. We also analyzed all observations with the same contact information, but slightly different name spelling. We only focused on the cases in which there are up to three different variants of the firm name. For these cases, we calculated the Levenshtein distance in the names, which is the smallest number of edits required to match one name to another. We treat all export observations as coming from the same firm if the contact phone number (or email) is the same and the Levenshtein distance is four or less.

J Illustrative Example for our Instrumental Variable

This section presents an example that highlights the workings of the shift-share instrument constructed in 6. The example corresponds to U.S. exports in product category HS8431490000 (PARTS AND ATTACHMENTS OTHER FOR DERRIKS ETC.)—a product category that features one of the most frequently imported varieties: *machine parts* from "CATERPILLAR." The left panel of Figure 3 displays how both the exchange rate and the *average* import tax rate paid by U.S. based firms varied consid-

[81] The corresponding Stata code is in the cleanFirmsNames.do.

Figure 3: *Monthly variations in national exchange rate, firm-specific exports, and import tax within a selected product-country-year category.*



Notes: The left panel plots monthly variations in exchange rate and (value-weighted) average import tax for US-based firms within product category HS8431490000-year 2009. The right panel plots monthly movements in export sales for the two biggest US firms in product HS8431490000 in year 2009— namely, Caterpillar and Machinery Corp. of America.

erably on a monthly basis in 2009. The right panel plots the monthly variation in the export sales of the two largest U.S. based firms within category HS8431490000 (namely, "CATERPILLAR" and "MACHINERY CORP. OF AMERICA"). Given that the monthly composition of exports from "CATERPILLAR" and "MACHINERY CORP. OF AMERICA" are markedly different, the two firms are affected differently by aggregate movements in the monthly exchange rate.

K Robustness Checks: *Import Demand Estimation*

This appendix reports three robustness checks that we described in Section 6. The first check addresses the possibility that firms set prices in forward-looking manner. To restate the issue, when there are lags in inventory clearances, firms' optimal pricing decisions may be forward-looking. If true, such price-setting behaviors can violate assumption (a1). To address this concern, we reconstruct our shift-share instrument using 4 lags instead of 1. If inventories clear in at most 4 years, we can deduce that pricing decisions do not internalize expected demand shocks beyond the 4 year mark. As a result, Corr $\left[\tilde{p}_{jkt-4}(\omega, m), \Delta\varphi_{\omega jkt}\right] = 0$, and this more-stringent instrument will satisfy the exclusion restriction. The *top* panel in Figure 4 compares the estimated σ_k and μ_k under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

The second check addresses the possibility that, in the presence of cross-inventory effects, $\Delta\varphi_{\omega jkt}$ may encompass omitted variables that concern firms' dynamic inventory management decisions. These decisions internalize exchange rate movements, which may violate the identifying assumption (a2), i.e., Corr $\left[\Delta\mathcal{E}_{jt}(m), \Delta\varphi_{\omega jkt}\right] \neq 0$. To address this concern, we reestimate the firm-level import demand function while directly controlling for changes on the annual exchange rate. In that case, $\mathrm{E}\left[z_{jk,t}(\omega)\ \Delta\varphi_{\omega jkt} \mid \Delta\mathcal{E}_{jt}\right]$, and the exclusion restriction will be satisfied insofar as dynamic demand optimization is a concern. The *middle* panel in Figure 4 compares the estimated σ_k and $\mu_k$under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

The third check addresses large multi-product firms that export multiple product varieties to Colombia in a given year. Suppose a multi-product firm ω exports many products including products k and g to Colombia in year t. If demand shock are correlated across the varieties supplied by this firm (i.e., $\mathrm{E}\left[\Delta\varphi_{\omega jkt}\,\Delta\varphi_{\omega jgt}\right] \neq 0$), Assumption (a2) may be violated despite each variety's market share being infinitesimally small. To address this issue, we reestimate the firm-level import demand function on a restricted sample that drops excessively large firms with a within-national market share that exceeds 0.1%. The *bottom* panel in Figure 4 compares the estimated σ_k and $\mu_k$under the new and baseline estimations. Evidently, the ordering and magnitude of the estimated elasticities is rather preserved across industries. More importantly, the new estimation retains the negative correlation between σ_k and μ_k, which is the key assumption in Proposition 1.

L Estimating the Import Demand Function in Levels

Our preferred estimates for μ_k and σ_k are obtained by estimating a firm-level import demand function in first-differences—see Section 6. The first-difference approach for estimating elasticities in this context can be traced back to the seminal work of Feenstra (1994) and Broda and Weinstein (2006)—although both studies rely on *country-level* rather than *firm-level* data. Another body of literature estimates the trade elasticity by fitting a country-level import demand function in log-levels, while controlling for appropriate fixed effects (e.g., Hummels, Lugovskyy, and Skiba (2009); Caliendo and Parro; Shapiro (2016)).

Recently, Boehm et al. (2020) have outlined the advantages and disadvantages of each approach: On the one hand, the first-difference approach performs better at handling the identification challenge poised by endogenous policy choices and omitted variable bias. On the other hand, the first difference estimator—at least when applied to country-level data—may not necessarily identify the long-run elasticity, which is the desired target for static trade models.

These issues pose a lesser problem for our firm-level estimation. We articulate this claim in two steps. First, we detail the *long-* versus *short-run* dilemma identified by Boehm et al. (2020), and explain why the same dilemma does not necessarily plague our firm-level estimation. Second, we establish our claim empirically by re-estimating our firm-level import demand function in levels. This exercise encouragingly confirms that our estimation in levels yields very similar results to our baseline estimation in differences.

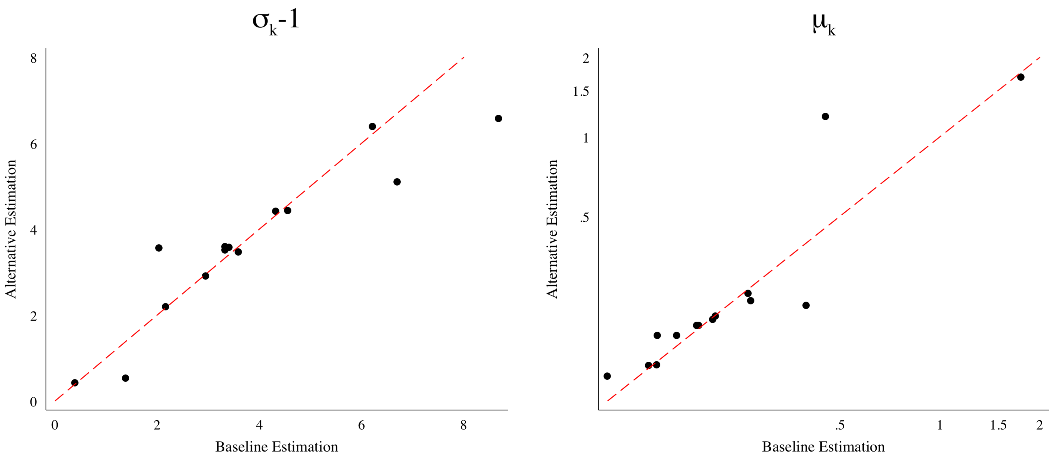
The dilemma facing country-level estimations. Country level trade flows—which are traditionally used to estimate the trade elasticity—can be decomposed as follows:

$$\tilde{X}_{ji,k} = N_{ji,k}\tilde{p}_{ji,k}q_{ji,k}$$

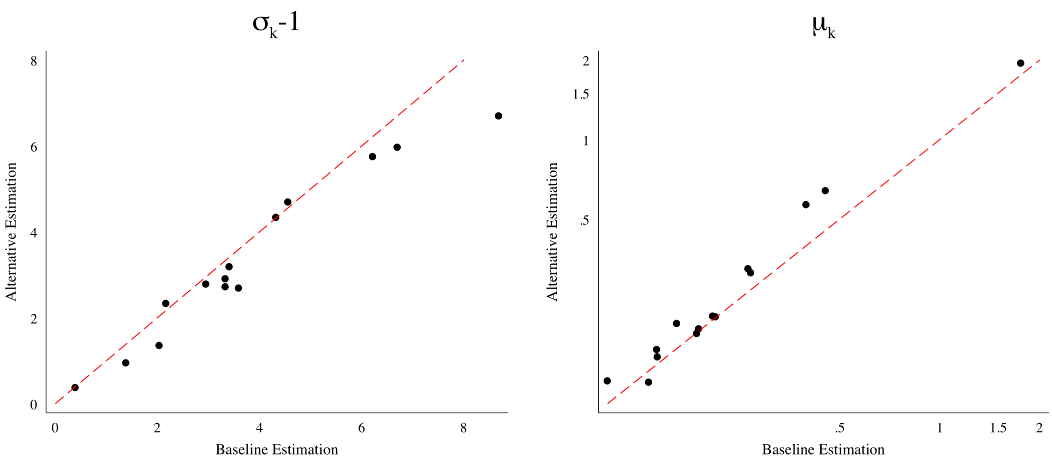
where $\tilde{X}_{ji,k}$ denotes gross sales corresponding to *origin j–destination i–industry k*; $\tilde{p}_{ji,k}q_{ji,k}$ denotes average sales per firm (i.e., the intensive margin) and $N_{ji,k}$ denotes the total mass of firms associated with transaction ji,k (i.e., the extensive margin). Accordingly, the *long-run* trade elasticity is composed of an extensive and an intensive margin component:

Figure 4: *Robustness checks to address challenges to the identification of σ_k and μ_k*

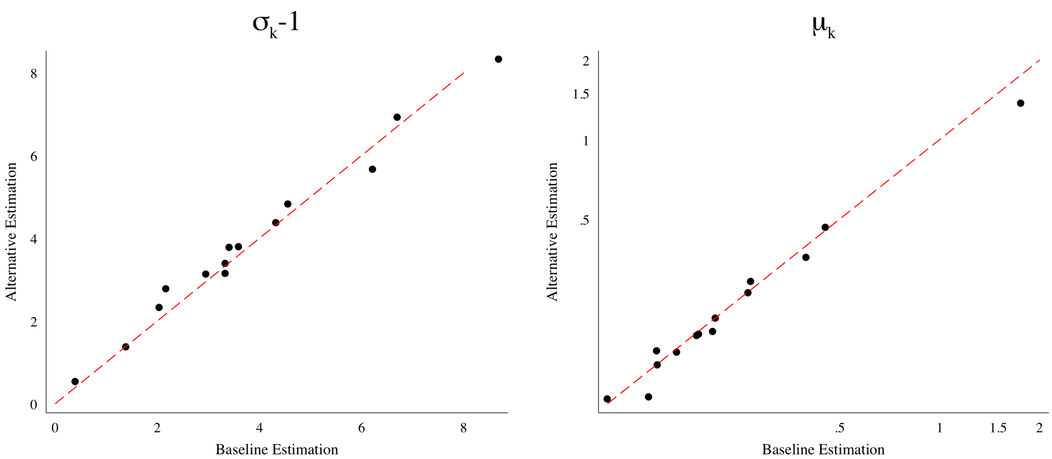
Constructing IV using 4th Lags



Dropping Large Mullti-Product Firms



Controlling for Changes in Annual Exchange Rate



$$\text{trade elasticity} \sim \frac{\partial \ln \tilde{X}_{ji,k}}{\partial \ln(1+t_{ji,k})} = \underbrace{\frac{\partial \ln N_{ji,k}}{\partial \ln(1+t_{ji,k})}}_{\varepsilon_n} + \underbrace{\frac{\partial \ln \tilde{p}_{ji,k} q_{ji}}{\partial \ln(1+t_{ji,k})}}_{\varepsilon_x}.$$

The issue raised by Boehm et al. (2020) concerns the fact that researchers do not separately observe $N_{ji,k}$ and $\tilde{p}_{ji,k} q_{ji,k}$ in country-level datasets. A standard solution to this limitation is to assume away firm-selection (i.e., set $N_{ji,k} = N_{j,k}$). Under this assumption, one can recover the trade elasticity by estimating an import demand function that controls for $N_{j,k}$ with *origin-industry* fixed effects. Crudely speaking, this solution is analogous to omitting the extensive margin component, i.e., setting $\varepsilon_n = 0$.

In practice, however, $N_{ji,k}$ may feature a bilateral element that accounts for firm-selection and which varies with the bilateral tariff rate—even after we control for a full set of origin and destination fixed effects. As noted above, traditional techniques that estimate the import demand function in *levels* with origin/destination fixed effects, are unable to account for the bilateral nature of $N_{ji,k}$. As such, traditional log-level estimators often suffer from an omitted variable bias.

Boehm et al. (2020) argue that we can overcome the omitted variable bias by estimating the country-level import demand function in differences rather than levels. Under this approach, however, one must employ long differences (over a sufficiently long time horizon) to credibly estimate the extensive margin component, ε_n. Nonetheless, the long-difference estimator may still fall short if tariff changes occur unevenly over the time-differencing horizon. In such cases, a correction must be applied to the estimated trade elasticity to account for lumpy longitudinal tariff changes.

Importantly, these limitations do not plague our firm-level estimation. We directly observe firm-level sales and need not to *infer* changes in $N_{ji,k}$ from changes in country level trade flows. Our data explicitly encompasses information on $N_{ji,k}$ and our identification strategy relies on the cross-sectional variation in firm-level variables within *importer–HS10 product–year* cells. With this level of disaggregation, our estimation is closer in spirit to the *Industrial Organization* literature on markup estimation. This literature routinely uses first difference estimators to recover markups (see, for example, equations 17-19 and related discussion in De Loecker and Warzynski (2012)). These markups estimates have been routinely used to discipline steady state models in the Macroeconomics literature (e.g., Baqaee and Farhi (2020)).

Re-estimating our firm-level import demand function in levels. Above, we presented a conceptual argument that (compared to traditional country-level estimations) firm-level estimations should yield relatively similar results whether the import demand is estimated in *levels* or in *first differences*—provided that appropriate instruments are employed to adequately handle reverse causality. To illustrate the same point empirically, we re-estimate our firm-level import demand function in levels with *two-ways fixed effects*. We then compare the *two-ways-fixed-effects* estimates for μ_k and σ_k with our baseline estimates. The estimating equation in log-levels can be expressed as follows:

$$\ln \tilde{x}_{j,kt}(\omega) = (1-\sigma_k) \ln \tilde{p}_{j,kt}(\omega) + [1-\mu_k(\sigma_k-1)] \ln \lambda_{j,kt}(\omega) + \underbrace{\delta_{kt}}_{\text{HS10-year FE}} + \underbrace{\varphi_{jk}(\omega)}_{\text{HS10-firm FE}} + \varphi_{\omega jkt}. \quad (106)$$

Recall that $\tilde{x} \equiv \tilde{p}q$ denotes gross firm-level sales value; $\tilde{p}$ denotes the consumer price which includes taxes and tariffs; $\lambda_{j,kt}(\omega)$ denotes the *within-origin* $j \times$*product* k expenditure share on firm-level variety ω; δ_{kt} accounts for product–year fixed effects, while $\varphi_{jk}(\omega)$ accounts for product-firm-origin fixed effects. The above equation differs from our baseline estimating equation in that the firm-product fixed effect, $\varphi_{jk}(\omega)$, is not differenced out. Instead the equation is estimated in levels.

As in the baseline case, we estimate the Equation (106) using a 2SLS estimator. To this end, we modify our original shift-share instrument to make it consistent with the fixed-effects estimation, which is conducted in levels. The new instrument is calculated as follows

$$\acute{z}_{j,kt}(\omega) = \sum_{m=1}^{12} \frac{\tilde{x}_{j,kt-1}(\omega;m)}{\tilde{x}_{j,kt-1}(\omega)} \mathcal{E}_{jt}(m)$$

where $\tilde{x}_{j,kt-1}(\omega;m)/\tilde{x}_{j,kt-1}(\omega)$ denotes the lagged share of Month m sales in firm ω's total annual export sales. $\mathcal{E}_{jt}(m)$, as before, denotes the exchange rate (between Origin j's currency and the Colombian Peso) in Month m of the current year. The other instrumental variables are adjusted accordingly, to be consistent with our estimation that is conducted in levels rather than in differences.

The estimation results are reported in Table 6. The estimated values for σ_k and μ_k are encouragingly similar to the baseline (first-differences) estimates. Most importantly, the new estimation quasi-maintains the ranking of industries in terms of the underlying degree of national-level market power (σ_k) and firm-level market power. Later, in Appendix P, we recalculate the gains from optimal policy using the newly-estimated μ_k's and σ_k's. Encouragingly, the implied gains are starkly similar to those implied by our baseline estimates.

M Examining the Plausibility of Estimates

In this Appendix we examine the plausibility of our estimated parameters from a different angle. We show that when our estimated parameters are plugged into a workhorse trade model, they resolve the *income-size* elasticity puzzle. This puzzle, as noted by Ramondo et al. (2016), concerns the fact that a large class of quantitative trade models—including Krugman (1980), Eaton and Kortum (2001), and Melitz (2003)—predict a counterfactually high income-size elasticity (i.e., the elasticity at which real per capita income increases with population size). One straightforward remedy for this counterfactual prediction is introducing domestic trade frictions into the aforementioned models. This treatment, however, is only a partial remedy. As shown by Ramondo et al. (2016), even after controlling for direct measures of internal trade frictions, the predicted income-size elasticity remains counterfactually strong.

To test macro-level predictions, we first produce economically-representative estimates for σ_k and μ_k. We do so by pooling data across all manufacturing and non-manufacturing industries and estimating Equation 17 on theses two pooled samples.The estimation results are reported in Table 7, and imply that $\sigma - 1 \approx 3.6$ and $\frac{\sigma-1}{\gamma-1} \approx 0.66$ across manufacturing industries. For the sake of comparison, the same table also reports estimates produced using the standard OLS estimator.

To understand the income-size elasticity puzzle, consider a single-industry version of the model presented in Section 2. Such a model implies the following expression relating country i's real income per worker or TFP ($W_i = w_i/P_i$) to its structural efficiency, A_i, population size, L_i, trade-to-GDP ratio, λ_{ii}, and a measure of internal trade frictions, τ_{ii}:

$$W_i = \gamma \, A_i \, L_i^{\mu} \, \lambda_{ii}^{-\frac{1}{\sigma-1}} \, \tau_{ii}^{-1}. \tag{107}$$

The standard Krugman model assumes extreme love-of-variety (or extreme scale economies), which

Table 6: Two-ways fixed effects estimation results

Sector	ISIC4 codes	Estimated Parameter $\sigma_k - 1$	$\frac{\sigma_k-1}{\gamma_k-1}$	μ_k	Obs.	Weak Ident. Test
Agriculture & Mining	100-1499	4.563 (1.739)	0.698 (0.132)	0.153 (0.089)	10,762	3.07
Food	1500-1699	2.476 (0.818)	0.927 (0.050)	0.374 (0.284)	17,594	5.01
Textiles, Leather & Footwear	1700-1999	3.256 (0.297)	0.685 (0.023)	0.210 (0.024)	110,925	59.94
Wood	2000-2099	2.093 (1.196)	0.893 (0.191)	0.427 (0.801)	5,282	2.12
Paper	2100-2299	7.858 (3.953)	0.895 (0.154)	0.114 (0.177)	35,058	2.65
Petroleum	2300-2399	0.397 (0.342)	0.698 (0.081)	1.758 (1.584)	3,675	2.53
Chemicals	2400-2499	4.738 (0.496)	0.913 (0.031)	0.193 (0.071)	127,946	29.71
Rubber & Plastic	2500-2599	4.025 (0.791)	0.664 (0.062)	0.165 (0.045)	101,730	9.95
Minerals Basic & Fabricated Metals	2600-2699 2700-2899	3.390 (0.453)	0.681 (0.036)	0.201 (0.035)	173,432	20.03
Machinery	2900-3099	4.402 (1.765)	0.710 (0.080)	0.161 (0.065)	257,788	19.88
Electrical & Optical Equipment	3100-3399	0.756 (0.300)	0.609 (0.015)	0.806 (0.099)	246,597	19.25
Transport Equipment N.E.C. & Recycling	3400-3599 3600-3800	2.156 (0.462)	0.514 (0.032)	0.238 (0.053)	147,772	11.37

Notes. Estimation results of Equation (17). Standard errors in parentheses. The estimation is conducted with HS10 product-year fixed effects. All standard errors are simultaneously clustered by product-year and by origin-product, which is akin to the correction proposed by Adao et al. (2019). The weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist et al. (1996).

implies $\mu = 1/(\sigma - 1)$ and precludes internal trade frictions, which results in $\tau_{ii} = 1$. Given these two assumptions, we can compute the real income per worker predicted by the standard Krugman model and contrast it to actual data for a cross-section of countries.

For this exercise, we use data on the trade-to-GDP ratio, real GDP per worker, and population size for 116 countries from the PENN WORLD TABLES in the year 2011. Given our micro-estimated trade elasticity, $\sigma - 1$, and plugging $\tau_{ii} = 1$ as well as $\mu = 1/(\sigma - 1)$ into Equation 107, we can compute the real income per worker predicted by the Krugman model. Figure 5 (top panel) reports these predicted values and contrasts them to factual values. Clearly, there is a sizable discrepancy between the income-

Table 7: Pooled estimation results

	Manufacturing		Non-Manufacturing	
Variable (log)	2SLS	OLS	2SLS	OLS
Price, $1-\sigma$	-3.588***	0.202***	-4.658***	0.104***
	(0.221)	(0.002)	(0.504)	(0.004)
Within-national share, $1-\mu(\sigma-1)$	0.333***	0.814***	0.190***	0.802***
	(0.012)	(0.001)	(0.028)	(0.003)
Weak Identification Test	94.84	...	24.16	...
Under-Identification P-value	0.00	...	0.00	...
Within-R^2	...	0.77	...	0.73
N of Product-Year Groups	23,683		10,461	
Observations	1,126,976		205,580	

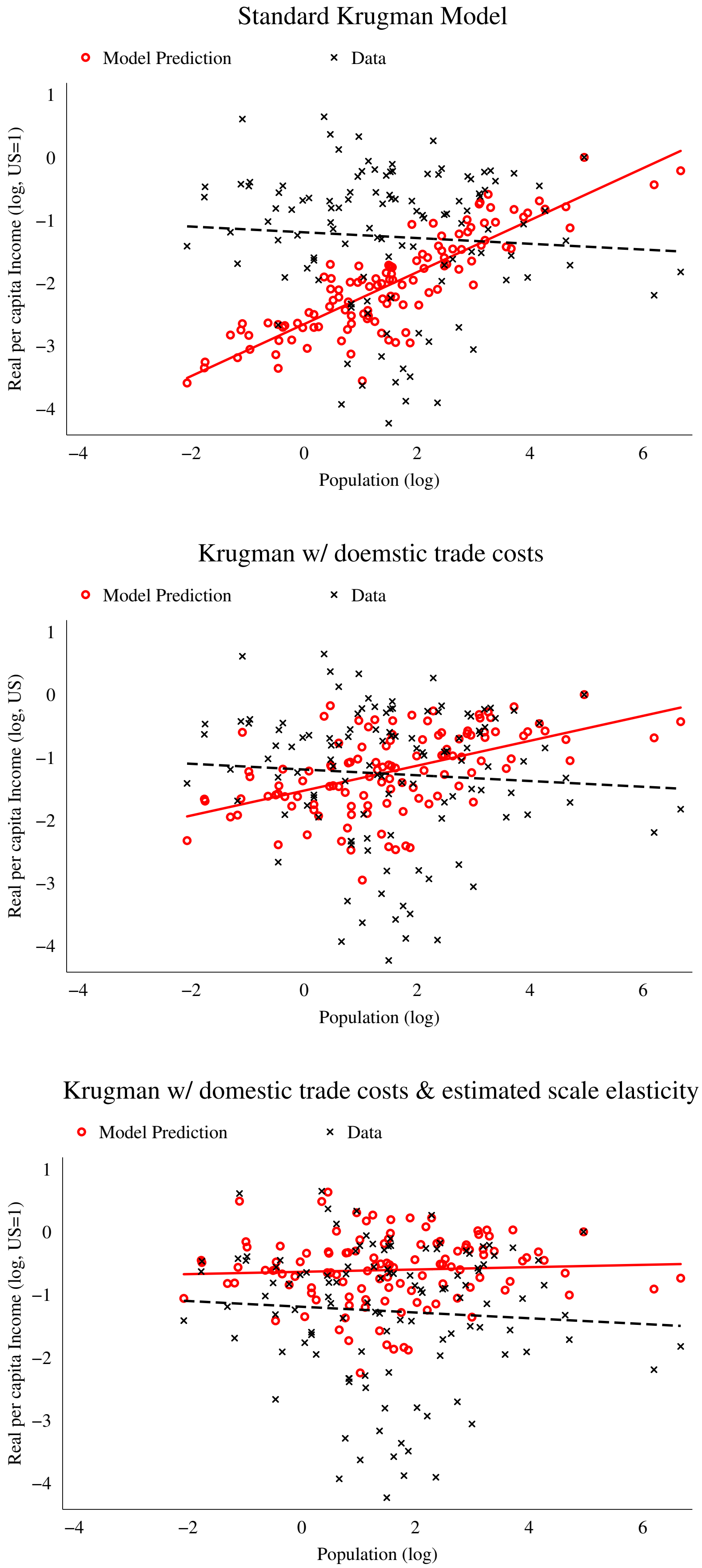
Notes: *** denotes significant at the 1% level. The Estimating Equation is (17). Standard errors in brackets are robust to clustering within product-year. The estimation is conducted with HS10 product-year fixed effects. The reported R^2 in the OLS specifications correspond to within-group goodness of fit. Weak identification test statistics is the F statistics from the Kleibergen-Paap Wald test for weak identification of all instrumented variables. The p-value of the under-identification test of instrumented variables is based on the Kleibergen-Paap LM test. The test for over-identification is not reported due to the pitfalls of the standard over-identification Sargan-Hansen J test in the multi-dimensional large datasets pointed by Angrist et al. (1996).

size elasticity predicted by the standard Krugman model (0.36, standard error 0.03) and the factual elasticity (-0.04, standard error 0.06). To gain intuition, note that small countries import a higher share of their GDP (i.e., posses a lower λ_{ii}), which partially mitigates their size disadvantage. However, even after accounting for observable levels of trade openness, the scale economies underlying the Krugman model are so strong that they lead to a counterfactually high income-size elasticity.

One solution to the income-size elasticity puzzle is introducing internal trade frictions into the Krugman model (i.e., relaxing the $\tau_{ii} = 1$ assumption). Ramondo et al. (2016) perform this task using direct measures of domestic trade frictions. Their calibration is suggestive of $\tau_{ii} \propto L_i^{0.17}$. Plugging this implicit relationship into Equation 107 and using data on population size and trade openness, we compute the model-predicted real income per worker and contrast it with actual data in Figure 5 (middle panel). Expectedly, accounting for internal frictions shrinks the income-size elasticity. However, as pointed out by Ramondo et al. (2016), the income-size elasticity remains puzzlingly large.

We ask if our micro-estimated scale elasticity can help resolve the remaining income-size elasticity puzzle. To this end, in Equation 107, we set the scale elasticity to $\mu = \alpha/(\sigma-1)$ where α is set to 0.65 as implied by our micro-level estimation. Then, using data on population size and trade-to-GDP ratios, we compute the real income per capita predicted by a model that features both domestic trade frictions *and* adjusted scale economies. Figure 5 plots these predicted values, indicating that this adjustment indeed resolves the income-size elasticity puzzle. In particular, the income-size elasticity predicted by the Krugman model with adjusted scale economies is statistically insignificant (0.02, standard error 0.03), aligning very closely with the factual elasticity.

Figure 5: *Resolving the income-size elasticity puzzle*



N Mapping Second-Best Tax Formulas to Data

In this appendix, we present an analog to Proposition 3, but for second-best trade taxes under restricted entry (as specified by Theorem 2). As in Section 7, we assume that preferences have a CES-Cobb-Douglas parametrization. We use the "$\star$" superscript indicates that a variable is being evaluated in the counterfactual *second-best* optimal policy equilibrium. We assume hereafter that countries do not apply domestic subsidies in the factual equilibrium, i.e., $s_{n,k} = 0$ for all $n \in \mathbb{C}$. Using the hat-algebra notation and the expression of the good-specific supply elasticity, $\omega_{ji,k}$ (Equation 10), we can write the second-best tax formulas in changes as follows:

$$\text{[optimal import tax]} \quad 1 + t^*_{ji,k} = \frac{1 + (\sigma_k - 1)\hat{\lambda}_{ii,k}\lambda_{ii,k}}{1 + \frac{1+\overline{\mu}^*_i}{1+\mu_k}(\sigma_k - 1)\hat{\lambda}_{ii,k}\lambda_{ii,k}}$$

$$\text{[optimal export subsidy]} \quad 1 + x^*_{ij,k} = \frac{(\sigma_k - 1)\sum_{n\neq i}\left[(1 + t^*_{ni,g})\hat{\lambda}_{nj,k}\lambda_{nj,k}\right]}{1 + (\sigma_k - 1)(1 - \hat{\lambda}_{ij,k}\lambda_{ij,k})}\left(\frac{1 + \mu_k}{1 + \overline{\mu}^*_i}\right),$$

$$\text{[change in taxes]} \quad \widehat{1 + s_{i,k}} = 1; \qquad \widehat{1 + t_{ji,k}} = \frac{1 + t^*_{ji,k}}{1 + t_{ji,k}}; \qquad \widehat{1 + x_{ij,k}} = \frac{1 + x^*_{ij,k}}{1 + x_{ij,k}}. \tag{108}$$

Since the rest of world is passive, $\widehat{1 + s_{n,k}} = \widehat{1 + t_{jn,k}} = \widehat{1 + x_{nj,k}} = 1$ for all $n \neq i$. To determine the change in expenditure shares, $\hat{\lambda}_{ji,k}$, we need to determine the change in consumer price indexes. Invoking the CES structure of within-industry demand, we can express the change in *market i–industry k*'s consumer price index as

$$\text{[price indexes]} \quad \hat{\tilde{P}}_{i,k} = \sum_{n=1}^{N}\left(\lambda_{ni,k}\left[\frac{\widehat{1 + t_{ni,k}}}{\widehat{1 + x_{ni,k}}}\widehat{w}_n\right]^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}. \tag{109}$$

Given $\hat{\tilde{P}}_{i,k}$, we can calculate the change in expenditure shares $\hat{\lambda}_{ji,k}$ and revenue shares $\hat{r}_{ji,k}$ as

$$\text{[expenditure shares]} \quad \hat{\lambda}_{ji,k} = \left[\frac{\widehat{1 + t_{ji,k}}}{\widehat{1 + x_{ji,k}}}\widehat{w}_j\right]^{1-\sigma_k}\hat{\tilde{P}}^{\sigma_k - 1}_{i,k}$$

$$\text{[revenue shares]} \quad \hat{r}_{ji,k} = \left(\frac{\widehat{1 + t_{ji,k}}}{\widehat{1 + x_{ji,k}}}\hat{\lambda}_{ji,k}\hat{Y}_i\right)\left(\sum_{n=1}^{N}\frac{\widehat{1 + t_{jn,k}}}{\widehat{1 + x_{jn,k}}}\hat{\lambda}_{jn,k}\hat{Y}_n\right)^{-1}. \tag{110}$$

The change in wage rates, $\hat{w}_i$, and labor shares, $\hat{\rho}_{i,k}$, are dictated by the labor market clearing (LMC) condition, which ensures that industry-level sales match wage payments:

$$\text{[LMC]} \quad (1 + \bar{\mu}^*_i)\hat{w}_i w_i L_i = \sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1 + x^*_{ji,k}}{1 + t^*_{ji,k}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]. \tag{111}$$

where the output-weighted average markup in the counterfactual equilibrium is given by

$$1 + \bar{\mu}^*_i = \frac{\sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1+x^*_{ji,k}}{1+t^*_{ji,k}}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]}{\sum_{j\in\mathbb{C}}\sum_{k\in\mathbb{K}}\left[\frac{1+x^*_{ji,k}}{(1+\mu_k)(1+t^*_{ji,k})}\hat{\lambda}_{ij,k}\lambda_{ij,k}e_{j,k}\hat{Y}_j Y_j\right]}. \tag{112}$$

The change in national expenditure, $\hat{Y}_i$, is governed by the balanced budget (BB) condition, which ensures that total expenditure matches total income from wage payments and tax revenues:

$$[\text{BB}] \quad \hat{Y}_i Y_i = +(1+\bar{\mu}_i^*)\hat{w}_i w_i L_i + \sum_{j \neq i} \left(\frac{t_{ji,k}^*}{1+t_{ji,k}^*} \lambda_{ji,k} \hat{\lambda}_{ji,k} e_{i,k} \hat{Y}_i Y_i + \frac{1-(1+x_{ij,k}^*)}{1+t_{ij,k}^*} \lambda_{ij,k} \hat{\lambda}_{ij,k} e_{j,k} \hat{Y}_j Y_j \right). \tag{113}$$

Equations 108-113 represent a system of $2N + NK + 2(N-1)K$ independent equations and unknowns. The independent unknowns are, namely, $\hat{w}_i$ (N unknowns), $\hat{Y}_i$ (N unknowns), $\hat{\rho}_{i,k}$ (NK unknowns), $\widehat{1+t_{ji,k}}$ ($(N-1)K$ unknowns), and $\widehat{1+x_{ij,k}}$ ($(N-1)K$ unknowns). Solving the aforementioned system is possible with information on observable data points, $\mathbb{D}$, and estimable parameters, $\Theta \equiv \{\mu_k, \sigma_k\}$. Once we solve this system, the welfare consequences of country i's optimal policy are also fully determined. The following proposition outlines this result.

Proposition 4. *Suppose we have data on observable shares, national accounts, and applied taxes,* $\mathbb{D}=\{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, Y_i, w_i L_i,$ *and information on structural parameters,* $\Theta \equiv \{\mu_k, \sigma_k\}$. *We can determine the economic consequences of country i's second-best optimal policy by calculating* $\mathbb{X} = \left\{\hat{Y}_i, \hat{w}_i, \hat{\rho}_{i,k}, \widehat{1+t_{ji,k}}, \widehat{1+x_{ij,k}}\right\}$ *as the solution to the system of Equations 108-113. After solving for* $\mathbb{X}$, *we can fully determine the welfare consequence of country i's optimal policy as*

$$\hat{W}_n = \hat{Y}_n / \prod_{k \in \mathbb{K}} \hat{\tilde{P}}_{n,k}^{e_{n,k}}, \qquad (\forall n \in \mathbb{C})$$

where $\hat{\tilde{P}}_{n,k}$ *can be computed as function of* $\mathbb{X}$ *and data,* $\mathbb{D}$, *using Equation 109.*

O Additional Details about the World Input-Output Database

This appendix presents additional details about the World Input-Output Database analyzed in Section 7. Table 8 describes our aggregation of WIOD industries into 16 industries. To summarize the information in this table, we aggregate the 'Agriculture' and Mining' industries into one non-manufacturing industry. We also follow Costinot and Rodríguez-Clare (2014) in two details: First, we aggregate the 'Textile' and 'Leather' industries into one industry. Second, we lump all service-related industries together treating them as one semi-non-tradable sector.

Following Proposition 3 in Section 7, we need data on observable shares, national accounts, and applied taxes ($\mathbb{D} = \{\lambda_{ji,k}, r_{ji,k}, e_{i,k}, \rho_{i,k}, Y_i, w_i L_i, x_{ij,k}, t_{ji,k}, s_{i,k}\}_{j,i,k}$) to compute the gains from policy. The WIOD reports data on trade values, $X_{ji,k} \equiv P_{ji,k} Q_{ji,k}$, for each origin j–destination i–industry k. The aggregated version of the data covers $N = 33$ countries (including the rest of the world) and $K = 16$ industries. Below, we describe how each element in $\mathbb{D}$ is computed based on $X_{ji,k}$ and our estimated values for μ_k. Assuming that countries impose no taxes under the status-quo, we can compute national income and the wage bill in each country i as follows:

$$Y_i = \sum_{k=1}^{K} \sum_{n=1}^{N} X_{ni,k}; \qquad w_i L_i = \begin{cases} \sum_{k=1}^{K} \sum_{n=1}^{N} X_{in,k} & \text{if entry is free} \\ \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{1}{1+\mu_k} X_{in,k} & \text{if entry is restricted} \end{cases}$$

Next, we can compute the within-industry and industry-level expenditure shares for each market i based on the following calculations:

$$\lambda_{ji,k} = \frac{X_{ji,k}}{\sum_{n=1}^{N} X_{ni,k}}; \qquad e_{i,k} = \frac{\sum_{n=1}^{N} X_{ni,k}}{\sum_g \sum_{n=1}^{N} X_{ni,g}} = \frac{\sum_{n=1}^{N} X_{ni,k}}{Y_i}.$$

Lastly, we can compute the within-industry revenue share and the industry-level labor share in each country using the following equations:

$$r_{in,k} = \frac{X_{in,k}}{\sum_{n=1}^{N} X_{in,g}}; \qquad \rho_{i,k} = \frac{\sum_{n=1}^{N} X_{in,k}}{\sum_{g=1}^{K}\sum_{n=1}^{N} X_{in,g}}.$$

P Gains from Policy Under Alternative Assumptions

In this appendix we quantify the gains from optimal policy under three alternative scenarios, comparing them to the baseline gains reported in Section 7. In each case, we contrast the new policy gains with the baseline gains along the two dimensions: First, in terms of the gains from first-best trade and industrial policies. Second, in terms of the effectiveness of second-best trade taxes at replicating the first-best outcome.

P.1 Gains Implied by the Melitz-Pareto Model

Suppose the data generating process is consistent with a Melitz-Pareto model that accommodates firm-selection effects. In that case, Theorem 1 characterizes the optimal policy under the following reinterpretation of parameters—see Appendix D:

$$\mu_k^{\text{Melitz}} = \begin{cases} \frac{\vartheta_k+1}{\mu_k\vartheta_k-1} & \text{if entry is restricted} \\ \vartheta_k & \text{if entry is free} \end{cases}; \qquad \sigma_k^{\text{Melitz}} = 1 + \frac{\vartheta_k}{1+\vartheta_k\mu_k\left(\frac{1}{\mu_k(\sigma_k-1)}-1\right)}$$

To compute the gains from policy we, therefore, need estimates for σ_k, μ_k, and ϑ_k. We have already produced estimates for the former two parameters. To estimate ϑ_k, we can first recover σ_k^{Melitz} using a standard gravity estimation à la Caliendo and Parro (2015). To explain the estiation procedure, suppose tariffs are applied before markups and industrial and export subsidies are zero ($x_{ji,k} = s_{j,k} = 0$ for all i, j, k). In that case, the national-level import demand function transforms into the following industry-level gravity equation:[82]

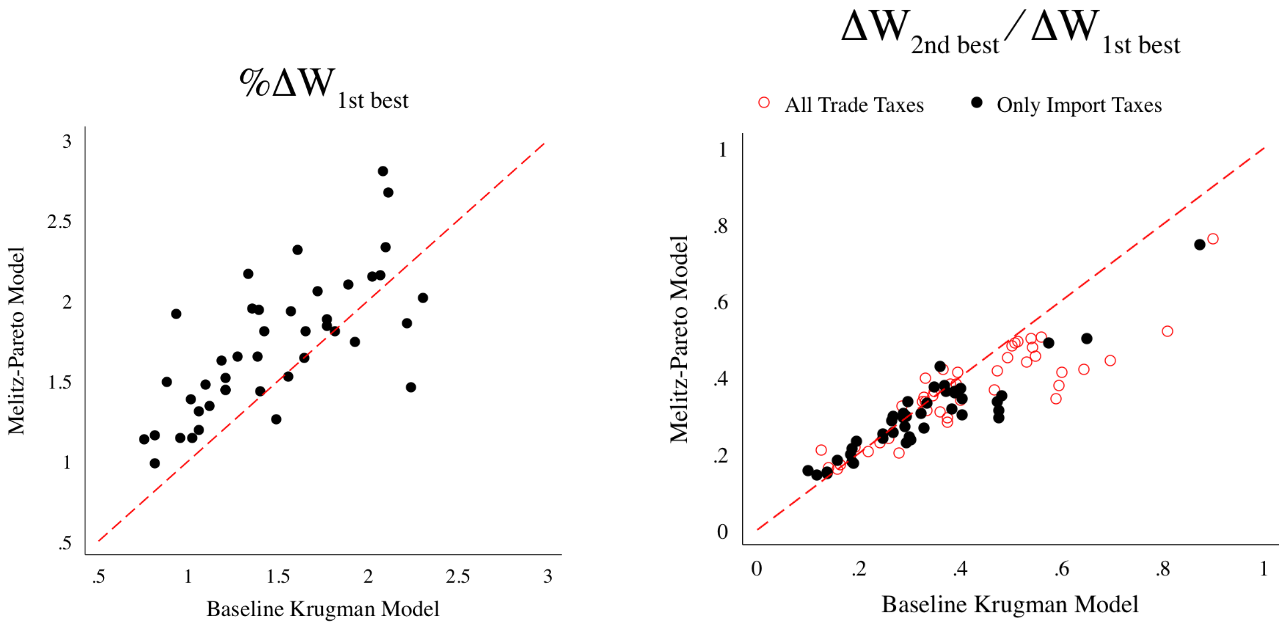
$$\tilde{X}_{ji,k} \equiv \tilde{P}_{ji,k}Q_{ji,k} = \Phi_{j,k}\Omega_{i,k}\tau_{ji,k}^{1-\sigma_k^{\text{Melitz}}}(1+t_{ji,k})^{1-\sigma_k^{\text{Melitz}}},$$

where $\Phi_{j,k} \equiv L_{j,k}^{\mu_k^{\text{Melitz}}}\bar{a}_{j,k}^{1-\sigma_k^{\text{Melitz}}}w_{j,k}^{1-\sigma_k^{\text{Melitz}}}$ and $\Omega_{i,k} \equiv \sum_n \left[\bar{a}_{n,k}w_{n,k}^{1-\sigma_k^{\text{Melitz}}}\tau_{ni,k}^{1-\sigma_k^{\text{Melitz}}}(1+t_{ni,k})^{1-\sigma_k^{\text{Melitz}}}\right]e_{i,k}Y_{i,k}$ can be viewed as the exporter and importer fixed effects in the standard gravity estimation sense. To produce our final estimating equation, we assume that iceberg trade costs are given by $\ln\tau_{ji,k} = \ln d_{ji,k} + \varepsilon_{ji,k}$, where (i) $d_{ji,k} = d_{ij,k}$ is a systematic and symmetric cost component that accounts for the effect of distance, common language, and common border, while (ii) $\varepsilon_{ji,k}$ is a random disturbance term that represents any deviation from symmetry. Invoking this decomposition, we can produce the following estimating equation for any triplet (j, i, n):

$$\ln\frac{\tilde{X}_{ji,k}\tilde{X}_{in,k}\tilde{X}_{nj,k}}{\tilde{X}_{ij,k}\tilde{X}_{ni,k}\tilde{X}_{jn,k}} = -\left(\sigma_k^{\text{Melitz}}-1\right)\ln\frac{(1+t_{ji,k})(1+t_{in,k})(1+t_{nj,k})}{(1+t_{ij,k})(1+t_{ni,k})(1+t_{jn,k})} + \varepsilon_{jin,k}.$$

[82] The assumption that tariffs are applied before markups, amounts to saying that tariffs act as a cost-shifter. Alternatively, if tariffs are applied after markups, they act as a demand shifter. In the latter case, the elasticity of trade with respect to tariffs diverges from the trade elasticity in its standard definition—see Costinot and Rodríguez-Clare (2014) for more details.

Figure 6: *The gains from policy under the Melitz-Pareto model*



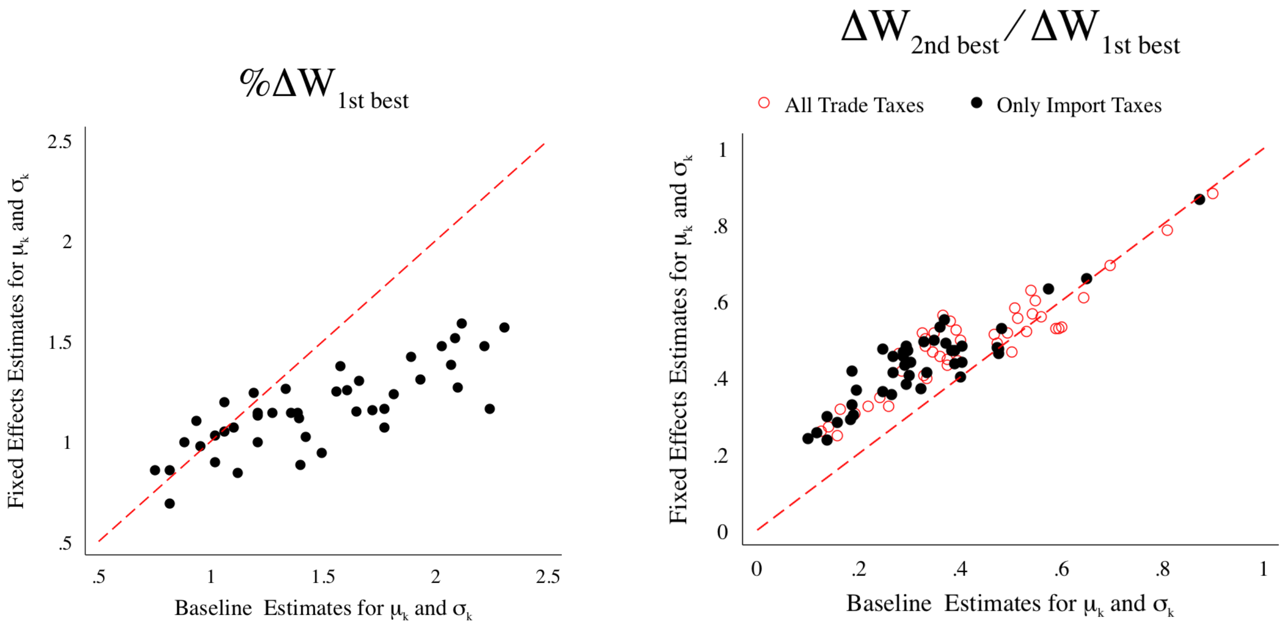
The left-hand side variable, in the above equation, is composed of observable national-level trade values in industry k. The right-hand side variable is composed of observable industry-level tariff rates. The error term $\varepsilon_{jin,k} \equiv \theta_k(\varepsilon_{ij,k} - \varepsilon_{ji,k} + \varepsilon_{in,k} - \varepsilon_{ni,k} + \varepsilon_{nj,k} - \varepsilon_{jn,k})$ encompasses any idiosyncratic variation in non-tariff barriers. Under the identifying assumption that applied tariff rates are orthogonal to $\varepsilon_{jin,k}$, *i.e.*, $\text{Cov}(t_{ji,k}, \varepsilon_{ji,k}) = 0$, we can estimate σ_k^{Melitz} by estimation the above equation with data on trade values, $\tilde{X}_{ji,k}$, and applied tariffs, $t_{ji,k}$, from the WIOD and TRAINS-UNCTAD datasets. After estimating σ_k^{Melitz}, we can recover ϑ_k for our previously-estimated values for σ_k and μ_k (which are reported in Table 2):

$$\vartheta_k = \frac{\hat{\sigma}_k^{\text{Melitz}} - 1}{1 - \mu_k\left(\hat{\sigma}_k^{\text{Melitz}} - 1\right)\left(\frac{1}{\mu_k(\sigma_k - 1)} - 1\right)}.$$

For the analysis that follows, we borrow the estimated values for σ_k^{Melitz} from Lashkaripour (2020a), which is based on the 2014 WIOD and TRAINS-UNCTAD datasets. After pinning down all the necessary parameters, we simply evaluate and plug σ_k^{Melitz} and μ_k^{Melitz} into our optimal tax formulas to compute the gains from optimal policy. The process is akin to that outlined in Section 7. Importantly, one should note that without our micro-level estimates for σ_k and μ_k, it is impossible to recover both σ_k^{Melitz} and μ_k^{Melitz} from macro-level trade and tariff data.

The optimal policy gains implied by the Melitz-Pareto model are reported under Figure 6. The results indicate that accounting for firm-selection (à la Melitz-Pareto) magnifies the gains from the first-best trade and industrial policy schedule. Moreover, accounting for firm-selection dampens the efficacy of second-best trade taxes at replicating the first-best policy gains. If anything, these results indicate that our baseline claim that trade taxes are an ineffective second-best substitute for industrial subsidies is strengthened once we account for firm-selection effects.

Figure 7: The gains from policy under alternative estimates for σ_k and μ_k



P.2 Gains Implied by the Fixed-Effect Estimates for μ_k and σ_k

Our baseline estimation of the gains from policy in Section 7 utilized the first-difference estimates for μ_k and σ_k—these estimates were reported under Table 2. In Appendix L (under Table 6), we reported alternative estimates for μ_k and σ_k based on a two-ways fixed-effects estimation of the firm-level import demand function. In this appendix, we recompute the gains from policy using these alternative estimates for μ_k and σ_k.

The implied gains from optimal policy are reported under Figure 7. The fixed-effects estimates for σ_k and μ_k imply (on average) smaller gains from first-best trade and industrial policies. This outcome drives from two main factors: First, the fixed-effects estimates for μ_k exhibit smaller heterogeneity across industries. As such, they imply a small degree of misallocation in the economy compared to the baseline estimates. Second, the fixed-effects estimates for σ_k are generally smaller and imply larger unilateral gains from terms-of-trade manipulation.

Another takeaway from Figure 7 is that second-best trade taxes exhibit a greater degree of efficacy compared to the baseline case. This outcome reflects two issues: First, the corrective gains from policy are a smaller fraction of the overall first-best policy gains, once we plug the fixed-effects-estimated values for σ_k and μ_k. Second, the fixed-effects-estimated values for σ_k and μ_k exhibit a smaller negative correlation relative to the baseline estimates. As explained in Section 5, the less negative $\text{Cov}_k(\sigma_k, \mu_k)$, the smaller the implicit tensions between the terms-of-trade-improving and corrective gains from trade taxation—hence, the greater efficacy of second-best trade taxes.

P.3 Assigning Alternative Values to μ_k and σ_k for the Service Sector

Our estimation of σ_k and μ_k in Section 6 relied on transaction-level trade data, which is scarce for (semi-non-traded) service industries. To address this issue, our baseline estimation of the gains from policy normalized the aforementioned parameters in service-related industries as follows:

$$\sigma_k = 11; \qquad \mu_k = 0 \qquad \text{if } k \in \text{Service}$$

The value assigned to σ_k for service-related industries is less consequential for our estimated welfare gains. The reason is that σ_k governs the gains from terms-of-trade manipulation. However, under the status quo, there is little-to-no trade occurring in service industries. With little-to-no service trade under the status quo equilibrium, the scope for terms-of-trade manipulation is limited in service industries—all irrespective of the value assigned to σ_k.[83]

The value assigned to μ_k, however, can have a profound effect on the estimated gains from optimal policy. To elaborate on this point, recall that one function of optimal policy (in our framework) is to correct misallocation due to markup heterogeneity. The degree of misallocation can be crudely measured as the cross-industry variance in markups, i.e., $\text{Var}_k(\mu_k)$. Data indicate that the service sector constitutes a non-trivial fraction of total output in each country. So, the value assigned to the service sector's μ_k is a non-trivial determinant of misallocation, as measured by $\text{Var}_k(\mu_k)$.

As indicated above, our baseline analysis assumed that the service sector is perfectly competitive. This assumption, which is rather standard in the quantitative trade literature, amounts to setting $\mu_k = 0$ for any service-related industry, k. In this appendix, we contrast our baseline results with those obtained under the alternative but extremely conservative assumption that μ_k in services equals the average μ_k in traded (non-service) industries. This assumption is conservative because it artificially deflates $\text{Var}_k(\mu_k)$ and, accordingly, dampens the corrective gains from optimal policy.

The gains computed under our conservative treatment of the service sector are reported under Figure 8. As expected, the gains from first-best policies are relatively lower under the conservative treatment—simply because the conservative value assigned to the service sector markup artificially lowers the degree of misallocation and the scope for policy intervention. Relatedly, second-best trade taxes are also more successful at replicating the gains obtainable under the first-best policy schedule. The intuition is that the *corrective* gains from policy constitute a smaller fraction of the first-best policy gains under the conservative model. Hence, the inability of trade taxes to replicate corrective gains becomes less consequential.

Q The Gains from Policy Under Artificial Parameter Values

Under what parameter values will our framework predict larger gains from policy? To answer this question, we simulate an artificial economy (with artificial values assigned to σ_k and μ_k) to examine the degree to which the gains from policy inflate under more extreme parameter values. Our theory indicates that the gains from optimal policy are regulate by two key statistics:

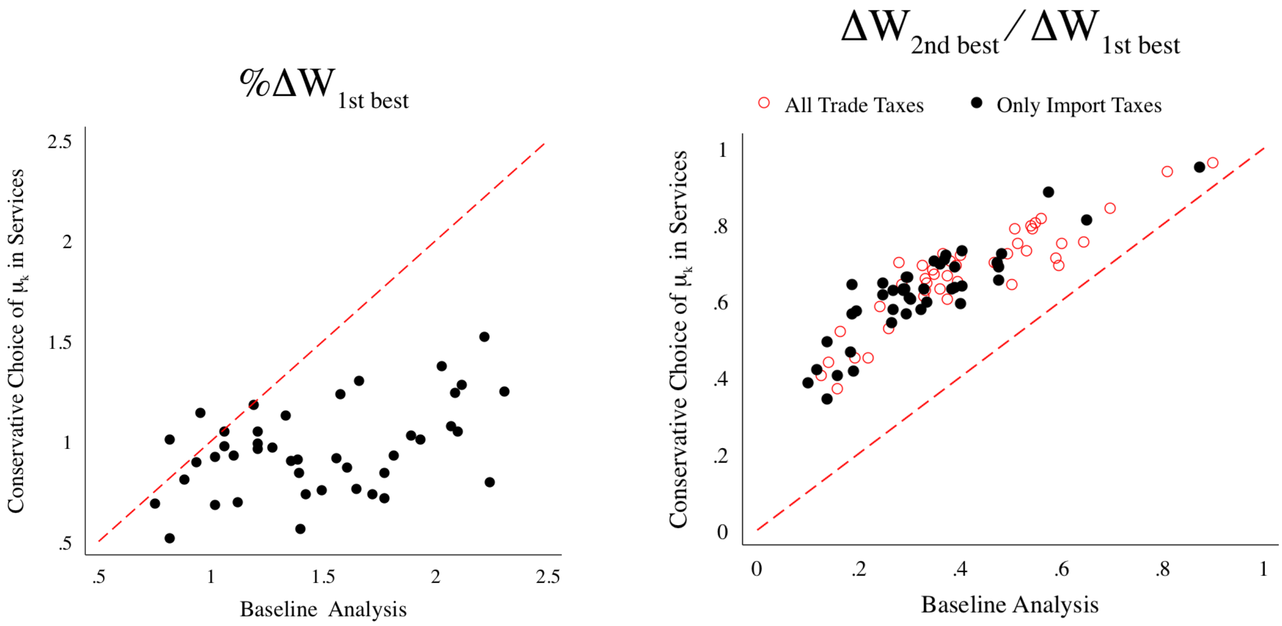
a) The variance in the industry-level scale elasticities, $\text{Var}_k(\mu_k)$.

[83] This outcome is an artifact of the CES parametrization of import demand. Specifically, in response to a change, $\hat{\boldsymbol{\tau}}$, in trade taxes, the post-tax-change expenditure shares remain zero if start as zero in the initial equilibrium—all irrespective of the trade elasticity values. Stated in mathematical terms,

$$\lim_{\lambda_{ji,k}\to 0} \hat{\lambda}_{ji,k} = \frac{\lambda_{ji,k}\left(\hat{\tau}_{ji,k}\hat{w}_j\right)^{1-\sigma_k}}{\sum_n \lambda_{ni,k}\left(\hat{\tau}_{ni,k}\hat{w}_n\right)^{1-\sigma_k}} = 0 \qquad \forall \sigma_k \geq 1.$$

Since $\lambda_{ji,k} \approx 0$ in services, trade taxes have little-to-no ability at improving the terms-of-trade, as doing so requires policy to shrink exports/imports in the service sector away from their factual level.

Figure 8: The gains from policy when the service sector is modeled more conservatively



b) The average level of the (inverse) industry-level trade elasticities, $\text{Avg}_k(\frac{1}{\sigma_k - 1})$.

The first statistic governs the extent to which countries can gain from restoring allocative efficiency. To explain this statistic, we can appeal to the Hsieh and Klenow (2009) exact formula for distance from the efficient frontier. Suppose $\sigma_k = \sigma$ is uniform across industries, and that industry-wide productivity levels and μ_k's have a joint log-normal distribution. The distance from the frontier can be approximated to a first-order as

$$\text{Distance from efficient frontier} \approx \frac{1}{2}(\sigma - 1)\text{Var}_k(\ln \mu_k).$$

The average level of μ_k is, however, inconsequential. To convey this point, suppose we multiply all the markups by some number $a \in \mathbb{R}_+$. Since this change is akin to offering a uniform industrial subsidy a to all industries, then it preserves real welfare based on Lemma 1.

The second statistic determines the degree of national-level market power and, thus, governs the degree to which countries can gain from ToT manipulation. To explain this statistic succinctly, consider a country that is sufficiently small in relation to the rest of the world. Following Theorem 1, the average optimal trade tax for this country is given by

$$\text{Avg. optimal trade tax} \;\approx\; \text{Avg}_k\left(\frac{1}{\sigma_k - 1}\right).$$

If $\sigma_k \to \infty$ for all k, the average optimal trade tax approaches zero, leaving no room for unilateral ToT improvements. Conversely, as σ_k approaches 1 the average optimal trade tax increases and so do the implicit gains from unilateral trade restrictions.[84]

Noting the above background, we recompute the gains from policy by artificially increasing $\text{Var}_k(\mu_k)$ and decreasing $\text{Avg}(\sigma_k)$, starting from our estimated vectors of $\{\sigma_k\}$ and $\{\mu_k\}$. The results are reported

[84] Since there is no choke price in our setup, the optimal export tax can approach infinity in the limit where $\sigma_k \to 1$. Introduce a choke price, then the optimal export tax will exhibit a limit-pricing formulation—see Costinot et al. (2015).

in 9 for a select set of countries—namely, the United States, China, Indonesia, and Korea. The graph indicates that the gains from policy nearly double if we artificially raise $\text{Var}_k(\mu_k)$ by a factor of two. A similar effect is borne out if we artificially lower $\text{Avg}(\sigma_k)$ by a factor of about two. An apparent pattern, here, is that the gains from policy exhibit similar sensitivity levels to $\text{Var}_k(\mu_k)$ across all countries, but different sensitivity levels to $\text{Avg}(\sigma_k)$. This pattern is expected, because $\text{Avg}(\sigma_k)$ governs the gains from ToT-improvement which are smaller (by design) for larger economies like the United States or China. The gains for restoring allocative efficiency, however, depend less on size and more on a country's industrial pattern of specialization under the status quo—see Kucheryavyy et al. (2016) for the role of specialization patterns.

These findings provide a platform to compare our estimated gains with alternatives in the literature. Our finding that the gains from restoring allocative efficiency are large sits well with the findings in Baqaee and Farhi (2017) that eliminating sectoral markup-heterogeneity in the U.S. economy can raise real GDP by 2.3%.[85] Bartelme et al. (2019), however, estimate smaller gains from similar policies. To understand these differences, note the formula for distance from the efficient frontier. Also note that *true* value for the scale elasticity, $\mu_k^{\text{True}} = \mu_k + \psi_k$, where ψ_k denotes the elasticity of Marshallian externalities. Accordingly, the *true* distance from the frontier can be approximated as follows:

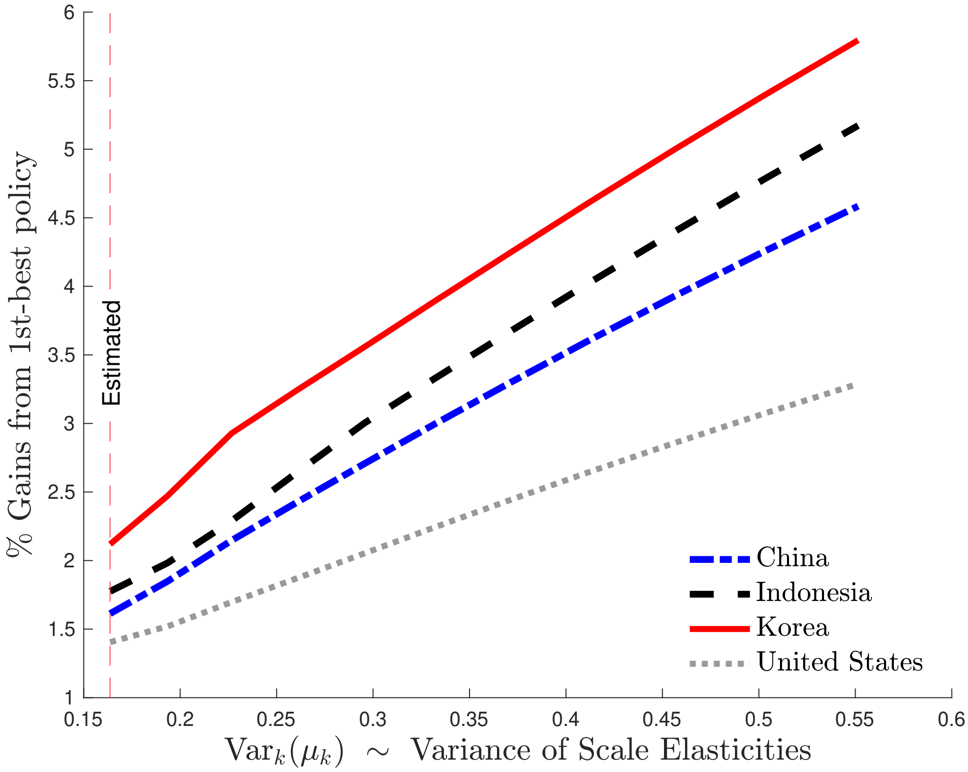
$$\mathcal{L}_{\text{True}} \approx \frac{1}{2}(\sigma - 1)\text{Var}_k(\ln(\psi_k + \mu_k))$$

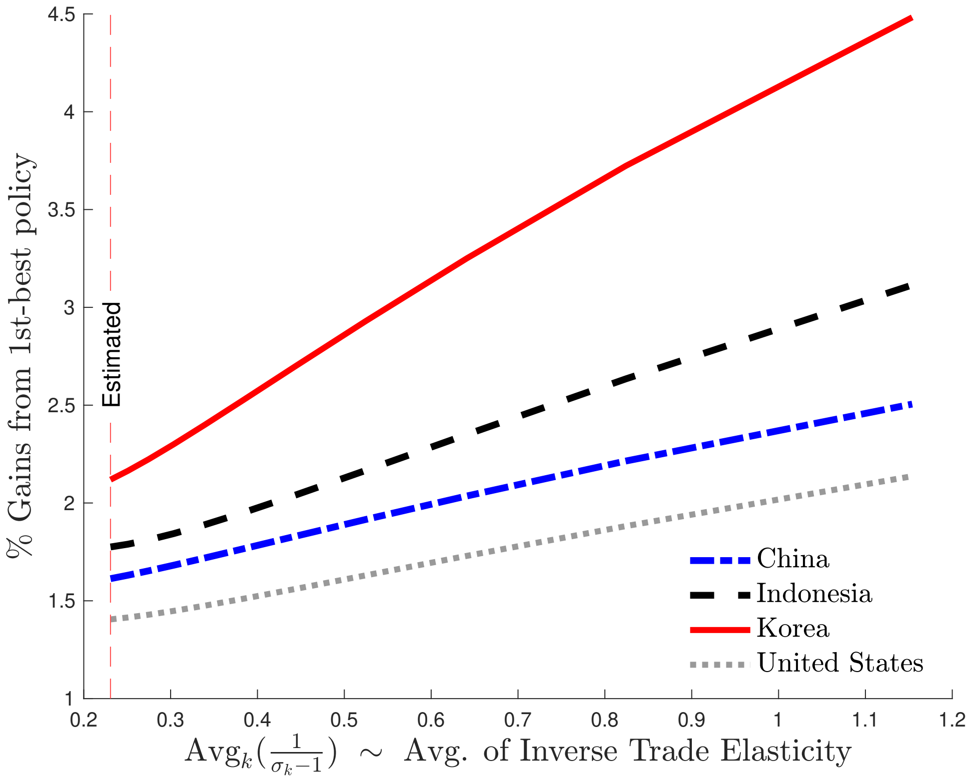
Our analysis like Baqaee and Farhi (2017) sets $\psi_k = 0$, and measures the degree of allocative inefficiency as $\mathcal{L}_{LL} \approx \frac{1}{2}(\sigma - 1)Var(\ln \mu_k)$. This approach can lead to an overstatement of $\mathcal{L}$ if ψ_k is negatively correlated with firm-level market power, μ_k.[86] In comparison and as noted in Section 6.4, the degree of misallocation in BDCR's analysis is measured as $\mathcal{L}_{BCDR} \approx \frac{1}{2}(\sigma - 1)Var(\ln(\mu_k + \psi_k - \frac{\beta_k}{\sigma - 1}))$, where β_k is the share of industry-specific factors in production. This approach can understate $\mathcal{L}$ when there are significant diseconomies of scale due to a high β_k.

[85] This number corresponds to the average of the numbers reported in the last column of Table 2 in Baqaee and Farhi (2017).

[86] Another issue is that we are assuming away selection effects in our quantitative analysis. In the presence of selection effects, we can still use our estimates for σ_k and μ_k to identify the scale elasticity up-to an externally chosen trade elasticity. Doing so, however, may lead to a lower or higher $\mathcal{L}$.

Figure 9: *The gains from policy under artificially higher* $Var_k(\mu_k)$ *and* $Avg_k(\frac{1}{\sigma_k - 1})$





Note: The data source is the 2014 World Input-Output Database (WIOD, Timmer et al. (2012)). The *1st best* policy is characterized by Theorem 1 for the case of restricted entry.

__Table 8:__ List of industries in the World Input-Output Database

WIOD Sector	Sector's Description	Trade Ealsticity	Scale Ealsticity
1	Agriculture, Hunting, Forestry and Fishing	6.212	0.141
2	Mining and Quarrying	6.212	0.141
3	Food, Beverages and Tobacco	3.333	0.265
4	Textiles and Textile Products Leather and Footwear	3.413	0.207
5	Wood and Products of Wood and Cork	3.329	0.270
6	Pulp, Paper, Paper , Printing and Publishing	2.046	0.397
7	Coke, Refined Petroleum and Nuclear Fuel	0.397	1.758
8	Chemicals and Chemical Products	4.320	0.212
9	Rubber and Plastics	3.599	0.162
10	Other Non-Metallic Mineral	4.561	0.186
11	Basic Metals and Fabricated Metal	2.959	0.189
12	Machinery, Nec	8.682	0.100
13	Electrical and Optical Equipment	1.392	0.453
14	Transport Equipment	2.173	0.133
15	Manufacturing, Nec; Recycling	6.704	0.142
16	Electricity, Gas and Water Supply Construction Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods Hotels and Restaurants Inland Transport Water Transport Air Transport Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies Post and Telecommunications Financial Intermediation Real Estate Activities Renting of M&Eq and Other Business Activities Education Health and Social Work Public Admin and Defence; Compulsory Social Security Other Community, Social and Personal Services Private Households with Employed Persons	11	0