



Full length articles

Discrete trade

Ahmad Lashkaripour

Indiana University, Wylie Hall 305, 100 S Woodlawn Ave, Bloomington, IN 47405, USA



ARTICLE INFO

Article history:

Received 25 February 2019

Received in revised form 23 June 2020

Accepted 24 June 2020

Available online 12 July 2020

Research data related to this submission:

https://alashkar.pages.iu.edu/Replication_Page_Discrete_Trade.html

Keywords:

Discrete

Indivisible

Quality specialization

Pricing to market

Non-traded

Services, Gains from trade

Scale effects

Big push

Home-market effect

ABSTRACT

International quality specialization (IQS) and pricing-to-market (PTM) are two of the most studied phenomena in international trade. I present new evidence that PTM and IQS are significantly more pronounced in discrete industries that involve indivisible consumption goods. In light of this observation, I develop a generalized model of international trade that accommodates both discrete and continuous industries. I argue that IQS and PTM in discrete industries are magnified by (i) affordability constraints, and (ii) cross-country differences in the price of non-traded services—both of which are less relevant in continuous industries. This prediction finds support from micro-level trade data. I then map the discrete model to macro-level data and show that the unique forces underlying the model shed fresh light on the gains from trade and the “big-push” effects of globalization.

© 2020 Elsevier B.V. All rights reserved.

1. Introduction

Around 50% of all U.S. import transactions from 1989 to 2015 involved *discrete* goods such as “Transport Equipment”, “Electronics & Appliances,” and “Machinery.” A defining feature of these goods is that they are *indivisible* and durable. That is, consumers can purchase discrete goods, like cars, smartphones, or washer-dryers, in only integer quantities. However, once purchased, these goods supply a continuous flow of transportation, communication, or laundry services.¹

Indivisibility of this kind has sharp implications for consumer choice. Most importantly, consumers cannot buy a fraction of multiple discrete goods. They are instead left with two options: (a) pay the price of one full unit, which is akin to paying a fixed acquisition cost; or (b) purchase a service that can substitute for such goods. They can also combine these two options. For instance, a consumer may purchase a personal vehicle for day-to-day use and spend the rest of their transportation budget on more-specialized transportation services. Either way and unlike what standard models assume, consumers cannot just buy a small fraction of different varieties of cars for different transportation demands.²

E-mail address: .

¹ Researchers often think of discrete goods as providing a flow of services: Cars, for instance, provide a flow of transportation services, or washer-dryer units provide a flow of dry-cleaning services—see Benhabib et al. (1991), Cooley and Prescott (1995), and Fernandez-Villaverde and Krueger (2011), among many others, for this analogy.

² There is always the option of renting or leasing multiple varieties of a discrete good. Here, I treat that option as a type of service. But even then, one should bear in mind that in most low-income countries this option is not widely-accessible (see Banerjee (2003)). Also, this option is less plausible for non-portable discrete goods such as home appliances.

These practical issues have received formal attention in several fields, including *industrial organization* and *macroeconomics*. Given the prevalence of discrete trade, one may also expect careful modeling of these same issues in mainstream theories of international trade. However, on the contrary, the issue of *discreteness* remains largely untreated in mainstream trade models.

A possible argument for this lack of attention is that discreteness plays a less critical role when studying macro-level trade patterns. But a basic analysis of product-level US trade data challenges this argument. In particular, comparing discrete industries to continuous (non-discrete) counterparts, such as “Food,” “Chemicals,” or “Textile & Apparel,” indicates that

- (i) There is significantly more cross-national quality/price heterogeneity within discrete industries;
- (ii) There is a greater degree of international quality specialization (IQS) in discrete industries; and
- (iii) There is a greater degree of pricing-to-market (PTM) in discrete industries.

These differences remain significant, even after we control for other characteristics like the intended final use or degree of product differentiation. At face value, the above regularities suggest that in discrete industries, trade patterns such as IQS and PTM are amplified by forces that are dormant in continuous industries. Such forces are also inevitably overlooked by mainstream theories that treat all industries or goods as continuous.

Motivated by this evidence, I develop a simple model of discrete trade that pursues two objectives. First, I use the model to elucidate how *discreteness* may affect macro-level trade patterns (like IQS and PTM) in ways that are difficult to understand from the lens of standard trade models. Second, I use the discrete model to determine how accounting for discrete trade revises our understanding of the gains from trade—an issue that has received considerable attention in the contemporary trade literature.

My theory accommodates both continuous and discrete industries. Continuous industries are modeled in a fairly standard fashion. My modeling of discrete industries, however, is grounded in three basic assumptions. (i) Discrete goods are indivisible, wherein purchasing them involves paying a unit price that is akin to a fixed acquisition cost. (ii) The consumption flow derived from a personally-acquired discrete good is increasing in its quality and can be substituted with analogous services. (iii) Quality and quantity are not isomorphic: a firm cannot produce a lower quality version of their product by employing fewer inputs under the same production technology. Instead, any supply-side adjustment to output quality involves paying a fixed R&D cost.

Using the above model, I show that in *discrete* industries, IQS and PTM are magnified by two forces that are less relevant in *continuous* industries:

- (i) Cross-national heterogeneity in the price of non-traded services, and
- (ii) Affordability constraints, whereby consumers with a limited budget cannot afford certain discrete varieties.

Both of these forces operate in a somewhat similar fashion. They lead to the *market-level* demand elasticity and demand for quality to diverge across rich and poor markets, even when *individual-level* preferences are homothetic and identical across rich and poor consumers.

The role of force (i) in prompting cross-national demand differences can be explained as follows. In each national market, discrete goods compete head-to-head with services; but services are significantly more expensive in high-wage economies (i.e., the Balassa-Samuelson effect). As a result, the opportunity cost of acquiring a discrete good is lower in high-wage markets, and this is true across all income groups. These considerations give rise to two macro-level demand patterns. First, the overall demand for discrete goods is *less* price-elastic in high-wage markets. Second, there is also more demand for high-quality (high-cost) discrete varieties in these markets.

To give an example, consider a premium hair-clipper that is a discrete tradable good. In Mumbai, India, the supplier of the hair-clipper competes with \$5 haircut services, whereas in Oslo, Norway, it competes with \$80 haircut services.³ As a result, Indian consumers (across all income groups) will exhibit a higher degree of price sensitivity when purchasing the hair-clipper. The supplier will internalize this when setting their price or choosing their output quality for these two markets.

Contrast this with continuous goods like shampoo or hair products. Since these goods perish upon consumption, they cannot be directly substituted with services. Instead, one has to (indirectly) pay for them even if they use the barber service. The same goes for the gas used in transportation services or the food used in restaurant services. One can circumvent paying for the full price of a vehicle or a premium grill, by taking a cab or going to a restaurant. But irrespective of whether they use such services or operate their personal vehicle/grill, they have to pay for the continuous perishable input, which is food or gas in this example.

Force (ii) prompts cross-national demand differences in a different way. Since discrete goods cannot be purchased in fractional quantities and come with a pre-customized quality, they can be unaffordable to low-budget consumers. Consider, for example, the Toyota Corolla which, in 2017, came with a price tag that was 8-times the annual wage rate in India. In such circumstances, *one* TOYOTA COROLLA can be prohibitively expensive for a significant portion of Indian consumers—especially given that most low-income consumers do not have access to financing options (Banerjee, 2003). Correspondingly, a marginal increase in price can rule out a non-trivial measure of potential customers.

The elasticity of demand in discrete industries, therefore, depends on the hazard rate at which a price increase renders the product unaffordable to marginal, low-budget consumers. This hazard rate is itself decreasing in the discrete good's overall *degree of affordability*, i.e., its price relative to the wage rate in the economy.⁴ These considerations, once again, give rise to two macro-

³ Source: <http://www.businessinsider.com/how-expensive-haircuts-are-around-the-world-2015-9>

⁴ This relationship holds for any empirically-relevant income distribution, since they exhibit an Increasing Generalized Failure Rate—see Larivière and Porteus (2001) and Larivière (2006).

level demand patterns. First, the overall demand for discrete goods is *less* price-elastic in high-wage markets, where the good is on average more affordable. Second, there is more demand for high-quality (high-cost) varieties in these markets.

The cross-national demand differences triggered by Forces (i) and (ii), can magnify the extent of PTM and IQS in *discrete* industries quite significantly. First, they prompt the suppliers of discrete goods to set higher markups in high-wage markets where their good is more affordable and also competes with expensive services. Second, the *affordability-driven* and *service price-driven* local demand for quality in high-wage economies prompts high-quality specialization in these economies through scale effects.

To test the relevance of these forces, I combine data on export prices, export sales, and non-traded service prices for 147 countries across 5231 6-digit (HS6) industries in the year 2011. As in the prior literature, I document that in all industries export price levels depend systematically on market size, income per capita, and geography. In discrete industries, however, export prices increase with three additional factors that are less-relevant in continuous industries: (i) the price of non-traded services in the importing market, (ii) the price of non-traded services in the exporting economy, and (iii) the exporting economy's overall cost efficiency in the industry of interest. While the empirical significance of these additional factors confirms the predictions of the model, the estimated effects are also quite profound. For example, a 10% increase in the price of services in an importing market is associated with a 2.5% increase in the price of *discrete* imports.

On a broader level, accounting for discrete trade can revise our estimates for the gains from trade. When the traded sector is modeled as discrete rather than continuous, the predicted gains from trade are on average 28% larger but more unequally distributed across countries. These differences are driven by two factors. The first factor is somewhat mechanical: The gains from discrete trade depend on the *quantity* share of foreign goods in national consumption, whereas the gains from continuous trade depend on the *value* share of foreign goods in national expenditure (Arkolakis et al., 2012).

The second factor is more profound. The discrete model predicts scale-driven gains from trade that are absent in the continuous model. To elaborate, in the continuous model, the scale of national production in the traded sector is invariant to trade.⁵ In the discrete model, however, opening to trade induces firm-relocation from large to small and low-income economies. This type of relocation allows firms to collect a higher markup on marginal cost when selling to large and rich international markets. It also expands the scale of production in small and low-income economies at the expense of others.

Consider Canada, for instance, which is located close to the richer and larger U.S. market. From the lens of the discrete model, trade-induced scale expansion accounts for 5.6% of Canada's real GDP. Intuitively, trade openness induces firms to relocate from the U.S. to Canada, using it as a platform to export to the U.S. at a higher markup. Given the sheer size of the U.S. economy, these developments provide a sizable boost for Canada's real GDP. They also relate to the *"big-push"* effects of economic development, wherein the growth of an economy may be hindered by its small local market (Rosenstein-Rodan, 1943; Lewis, 1954; Murphy et al., 1989). But through firm-relocation, international trade can push such an economy out of this supposed growth trap.

Aside from contributing to ongoing debates about the size and nature of the gains from trade, the present paper contributes to our understanding of IQS and PTM. Existing theories of IQS and PTM often model industries as continuous.⁶ As a result, they naturally emphasize non-homothetic preferences and cross-national technology differences as the drivers of PTM and IQS. Prior theories of IQS and PTM that admit discrete goods (e.g., Flam and Helpman, 1987; Murphy and Shleifer, 1997; Verhoogen, 2008; Khandelwal, 2010; Fajgelbaum et al., 2011; Dingel, 2017; Auer et al., 2018), have often (i) abstracted from affordability constraints, and (ii) do not include a non-traded service sector.⁷ As a result, from the lens of these models, discreteness in itself does not contribute to IQS or PTM. Accordingly, these models cannot explain why IQS and PTM are more pronounced in discrete industries.

2. Suggestive evidence

I uncover four new regularities, which suggest that discrete goods account for a significant portion of global trade; and that macro-level trade patterns such as international quality specialization (IQS) and pricing-to-market (PTM) occur more intensively in discrete product categories. These regularities are documented using the publicly-available US import and export databases compiled and updated by Feenstra et al. (2002). The data covers 36 years of US trade, reporting the f.o.b. (free on board) value and physical quantity of district-level imports (exports) from (to) various countries during the 1989–2015 period. Each observation classifies the goods based on the 10-digit Harmonized System (HS10) classification and reports the "unit" in which quantity is measured.⁸ The import-side data, for instance, reports quantity in 48 different units, with well over 70% of the observations reporting quantity in terms of either "count" or "kilogram."

⁵ More specifically, the national scale of production in the traded sector is determined exclusively by population size. This result will be revised if we model the traded sector as a blend of multiple industries with different scale elasticities—see Costinot and Rodriguez-Clare (2014) and Kucheryavy et al. (2016).

⁶ See Schott (2004); Hallak (2006); Choi et al. (2009); Baldwin and Harrigan (2011); Lugovskyy and Skiba (2015); Feenstra and Romalis (2014); Antoniadis (2015); Jaimovich and Merella (2015); Alcalá (2016) for papers studying IQS using continuous good frameworks, as well as Atkeson and Burstein (2008); Hummels and Lugovskyy (2009); Bekkers and Simonovska (2015) Chen and Juvenal (2014); Simonovska (2015); Bertolotti et al. (2018) for papers studying PTM using continuous good frameworks.

⁷ For instance, in the seminal work of Fajgelbaum et al. (2011), IQS is driven solely by non-homothetic preferences for quality, through the home-market effect. Another closely related framework by Matsuyama (2000) emphasizes a hierarchical demand system where different sectors produce goods of different *priority* as opposed to different quality.

⁸ There is a small subset of observations for which the unit of measurement is missing. Also, during the 1990–2006 period, the HS10 codes underwent multiple revisions. So, to concord HS10 codes across revisions, I use the mapping developed by Pierce and Schott (2012).

My analysis in this section is complicated by two basic challenges: First, *discreteness* may be correlated with other product characteristics such as degree of differentiation or intent of final use. Second, *discreteness* may be correlated with hidden firm-level characteristics that are impossible to control for in the aggregate-level US data. To address the former issue, I use the classification by Rauch (1999) to control for product differentiation and the classification of Broad Economic Categories (BEC) to control for the intent of final use. To address the latter concern, I cross-check all the benchmark results with firm-level evidence. This evidence is documented using firm-level Colombian import and export data, which report trade statistics corresponding to individual importing and exporting firms at the HS10 level of product aggregation during the 2007–2013 period—see Appendix A for a more detailed description.

Regularity 1. A significant portion of foreign trade involves discrete goods.

I classify an HS10 category as *discrete* if it involves countable goods.⁹ As reported in Table 1, discrete HS10 products typically belong to the “Machinery,” “Electrical & Optical Equipment,” and “Transport Equipment” industries. More than 90% of import transactions in these industries involve discrete goods like passenger cars, television sets, or washer/dryers. In many widely-used classifications, these industries are also often classified as durable.

In comparison, industries like “Food,” “Chemicals”, and “Textiles” consist of mostly non-discrete HS10 categories. They feature products such as coffee beans, rubbing alcohol, or yarns that typically report quantity in kilograms. Considering that weight is a relatively continuous unit of measurement and to fix ideas, I refer to such HS10 categories as *continuous*.

A possible concern with the above classification is that some HS10 product codes involve countable but inexpensive goods such as flashlights. The low unit price in these categories can render indivisibility or discreteness redundant. So, when documenting the subsequent regularities, I check robustness under a more conservative classification, wherein HS10 categories are coded as discrete if they are both (i) countable and (ii) exhibit an average unit price above \$1000.¹⁰

Regularity 2. There is significantly more international price heterogeneity within discrete product categories.

I establish the above regularity using *quantity-weighted* unit prices, which I denote by $\bar{p}_{ji,kt}$ (origin country j –US district i –HS10 product k –year t). Table 2 reports basic statistics describing the cross-national heterogeneity in $\bar{p}_{ji,kt}$ within *HS10 product* × *district* × *year* cells. For discrete product categories, the median cell exhibits a 75/25 percentile price spread of 12.0. For *continuous* product categories, however, the median cell exhibits a 75/25 price spread of only 2.1. The difference is even greater if we adopt a more conservative definition of discreteness, whereby product categories are classified as discrete if they are not only countable but also exhibit an average price per good of at least \$1000. In that case, the median 75/25 price spread stands at 29.3. The same exact ranking is borne out if we use the within-product coefficient of variation as our measure of price heterogeneity.

The lower panel in Table 2 demonstrates that similar patterns hold even after we control for the importing firm’s characteristics. This panel reports the (within cell ikt) heterogeneity in $\bar{p}_{ji,kt}$, where i now indexes a Colombian importing firm. To elaborate, firm i may purchase HS10 product k in year t from various origin countries indexed by j . The discrete products purchased by the same firm in the same year exhibit significantly more cross-supplier price heterogeneity than non-discrete products.

Trade economists generally attribute such within-product price heterogeneity to international quality and markup differences (Schott, 2004; Hallak, 2006; Baldwin and Harrigan, 2011; Hallak and Schott, 2011). Under this interpretation, Regularity 2 suggests that there is significantly more international quality and markup heterogeneity in discrete product categories. The following two regularities confirm that the greater quality/markup heterogeneity in discrete product categories has a systematic nature, and may be due to the greater degrees of IQS and PTM.

Regularity 3. North-South quality specialization is more pronounced in discrete product categories.

To analyze quality specialization, I adopt the perspective in Schott (2004) and Hummels and Klenow (2005), whereby the cross-national variation in export prices to the same market is attributed to quality differences.¹¹ Correspondingly, I define the degree of North-South quality specialization as $\frac{\partial \ln \text{export unit price}}{\partial \ln \text{origin's GDP}/c}$. To determine how product discreteness influences North-South quality specialization, I run the following regression on the US import data¹²:

$$\log \bar{p}_{ji,kt} = \left(\beta_w + \beta_w^D \times 1\{\text{Discrete}\} \right) \log \text{GDP}/c_{j,t} + \text{Controls}_{j,t} + \delta_{ikt} + \varepsilon_{jikt}, \quad (1)$$

⁹ A very small minority of HS10 codes report quantity in multiple units. These observations are dropped throughout my analysis.

¹⁰ In the interest of space, Regularities 3 and 4 are documented and reported only under the benchmark classification. However, they can be stated as is under my more conservative classification of discrete products.

¹¹ Recent theoretical advances have corroborated this interpretation. Most notably, Arkolakis et al. (2019) show that (if the firm-level productivity distribution is Pareto), the distribution of markups from country j to i is invariant to country j 's characteristics. That being the case, the cross-exporter variation in prices can be entirely attributed to export quality differences.

¹² Manova and Zhang (2012) adopt a similar estimating equation to analyze the within-product heterogeneity in unit prices, but with firm-level data.

Table 1

Discrete versus continuous industries in data.

Industry	US Imports		US Exports	
	% Share of discrete goods	% Share of industry in total imports	% Share of discrete goods	% Share of industry in total exports
Machinery	97.8	12.3	96.0	14.2
Electrical & Optical Equipment	96.8	13.6	94.3	14.4
Transport Equipment	93.0	19.2	78.8	19.3
N.E.C. & Recycling	51.0	2.8	52.2	0.8
Rubber & Plastic	38.0	1.6	24.3	2.1
Minerals	17.5	1.1	20.7	0.8
Paper	16.6	2.5	27.4	4.3
Textiles, Leather & Footwear	12.5	10.1	5.0	3.4
Wood	9.3	1.5	7.1	0.9
Basic & Fabricated Metals	6.7	6.6	6.2	6.4
Agriculture & Mining	2.3	13.5	2.4	8.7
Chemicals	2.1	7.9	1.6	15.6
Food	0.0	4.0	1.0	6.7
Petroleum	0.0	3.4	0.0	2.3
Total	47.9	100.0	45.1	100.0

Note: All percentage shares are value-weighted. The sectoral classification is from the WIOD. Observations that report quantity in units of “count” are classified as discrete.

Table 2

Price heterogeneity: discrete vs. continuous HS10 categories.

	Discrete		Continuous
	All	$P_{avg} > \$1000$	
District-level data (United States)			
75/25 price spread (median)	12.00	29.32	2.06
CV of unit price (median)	221%	267%	131%
Observations	9,069,000	3,613,000	7,520,000
Firm-level data (Colombia)			
	All	$P_{avg} > \$1000$	Continuous
75/25 price spread (median)	4.35	4.88	1.16
CV of unit price (median)	89%	93%	10%
Observations	2,407,000	1,304,000	881,000

Note: All statistics are value-weighted. The statistics in the top panel are documented using district-level US import data from 1989 to 2015. The statistics in the bottom panel are documented using firm-level Colombian import data from 2007 to 2013. Continuous goods correspond to observations that report quantity in unit other than count.

The dependent variable, $\bar{p}_{j,i,kt}$ denotes the unit price of country j 's exports to US district i in HS10 product category k , in year t . On the right-hand side, $GDPp/c_{j,t}$ controls for the origin country's GDP per capita; $1\{\text{Discrete}\}$ is a dummy variable that assumes a value of 1 if product category k is discrete and is zero otherwise; $\text{Controls}_{j,t}$ is composed of a set of standard gravity controls like total GDP, geo-distance, and Free Trade Agreements; and finally, δ_{ikt} controls for district-product-year fixed effects.

In addition to estimating Eq. (1) on the entire sample, I perform the same estimation while restricting the sample to (i) only final consumption goods, and (ii) only differentiated goods as classified by Rauch (1999). Doing so removes the possible concern that discreteness is correlated with other characteristics like the intended use or the degree of product differentiation. Also, since each observation in my country-level data represents a quantity-weighted average of firm-level variables, the above estimation is subject to the classic heteroskedasticity problem. To attain consistent estimates, I follow Kmenta (1997) and weigh each observation by the underlying trade quantity.¹³ To account for sample selection, I use the two-step correction developed by Wooldridge (1995).¹⁴

¹³ When handling grouped data, the weighted estimator where each observation is weighted by the square root of group size is BLUE. Here, trade quantity serves as a theory-consistent proxy for group size. Adopting such a weighting scheme delivers consistent estimates even if group size is correlated with ε_{jikt} —see Solon et al. (2015).

¹⁴ Specifically, I estimate an analog of Regression 1 by Tobit, but with trade values as the dependent variable. The residual from this first regression is then used as an additional control when estimating Eq. (1)—see Harrigan et al. (2015) for further details.

Table 3
Discreteness and north-south quality specialization (dependent: $\log \bar{p}_{jikt}$).

	All goods		Final goods	Differentiated goods		
	District-level data (United States)					
$\log \text{GDPp}/c_{j,t}$	0.14*** (0.010)	0.02*** (0.009)	0.11*** (0.007)	0.09*** (0.010)	0.13*** (0.010)	0.01 (0.015)
$\log \text{GDPp}/c_{j,t} \times \text{Discrete}_k$...	0.18*** (0.014)	...	0.04*** (0.011)	...	0.17*** (0.017)
Observations	16,636,000	6,647,000	13,107,000			
Fixed effects	Importing district \times HS10 prdouct \times year					
	All goods		Final goods	Differentiated goods		
	Firm-level data (Colombia)					
$\log \text{GDPp}/c_{j,t}$	0.16*** (0.010)	0.12*** (0.008)	0.16*** (0.010)	0.12*** (0.008)	0.17*** (0.010)	0.13*** (0.009)
$\log \text{GDPp}/c_{j,t} \times \text{Discrete}_k$...	0.05*** (0.007)	...	0.06*** (0.008)	...	0.04*** (0.008)
Observations	3,288,000	2,533,000	2,474,000			
Fixed effects	Importing firm \times HS10 prdouct \times year					

Note: The estimating equation is Eq. (1). The top panel is produced using district-level US import data from 1989 to 2015. The bottom panel is produced using firm-level Colombian import data from 2007 to 2013. All estimations are conducted with additional controls for total GDP, distance, and free trade agreements. Each observation is weighted by the corresponding trade quantity. Standard errors are reported in parentheses and clustered by exporting country and year. ** $p < 0.05$, *** $p < 0.01$.

In the above regression, the coefficient β_w reflects the baseline degree of North-South quality specialization, while β_w^D reflects the extent to which this kind of specialization is more pronounced in discrete product categories. The estimation results reported in Table 3 point to a positive and statistically significant β_w^D , indicating that IQS is noticeably more pronounced in discrete product categories. This remains to be true if we confine our sample to only final consumption goods or differentiated goods.¹⁵

The bottom panel in Table 3 documents the same pattern while controlling for firm-level characteristics. The results in this panel correspond to an estimation of Eq. (1) on the firm-level Colombian import data, in which case index i indexes an importing firm rather than a district. These results indicate that IQS is more pronounced in discrete product categories even after we control for *importing firm-product-year* fixed effects and divide goods by degree of differentiation and intent of final use.

The above claims rest on an implicitly-assumed link between quality and unit price, which can be complicated by cross-national productivity differences. Specifically, the results in Table 3 can be alternatively interpreted as high-income countries having a comparative cost advantage in continuous industries. I discuss this issue more elaborately in Section 5, where I analyze the role of discreteness while controlling for cross-country productivity differences as well as other confounding factors.

Regularity 4. Pricing-to-market is more pronounced in discrete product categories.

To analyze PTM, I adopt the perspective in Alessandria and Kaboski (2011) and Simonovska (2015), whereby the cross-market variation in prices from the same exporter is attributed to markup differences.¹⁶ Accordingly, I define the degree of PTM as $\frac{\partial \ln \text{export unit price}}{\partial \ln \text{destination's GDPp}/c}$. To determine how discreteness influences PTM, I run the following regression on the US export data:

$$\log \bar{p}_{jikt} = \left(\tilde{\beta}_w + \tilde{\beta}_w^D \times 1\{\text{Discrete}\} \right) \log \text{GDPp}/c_{i,t} + \text{Controls}_{i,t} + \delta_{jkt} + \varepsilon_{jikt} \quad (2)$$

The dependent variable, \bar{p}_{jikt} , denotes the unit price of US district j 's exports to country i in HS10 category k , in year t . On the right-hand side, $\text{GDPp}/c_{i,t}$ controls for the *destination* country's GDP per capita, $1\{\text{Discrete}\}$ is a discrete product dummy; $\text{Controls}_{i,t}$ is composed of a set of standard gravity controls like total GDP, geo-distance, and Free Trade Agreements; and finally, δ_{jkt} controls for district- product-year fixed effects. Also, as before, I estimate Eq. (2) separately for all goods as well the subset of final consumption goods and differentiated goods.

In the above regression, $\tilde{\beta}_w$ reflects the baseline degree of PTM, while $\tilde{\beta}_w^D$ reflects the extent to which PTM is more pronounced in discrete product categories. The estimation results reported in Table 4 point to a positive and statistically significant $\tilde{\beta}_w^D$,

¹⁵ The Weighted OLS estimator may not correct for the overrepresentation of observations with extreme values of the explanatory variables (Solon et al., 2015). Nonetheless, dropping observations with explanatory variables that fall above/below the 95/5 percentile does not qualitatively change any of the conclusions deduced from my analysis.

¹⁶ Alternatively, the same pattern can be attribute to firm varying their quality across markets. In that case, Regularity 4 can be stated as firms vary their product quality more intensively discrete industries.

Table 4
Discreteness and pricing-to-market (dependent: $\log \bar{p}_{jikt}$).

	All goods		Final goods		Differentiated goods	
	District-level data (United States)					
$\log \text{GDPp}/c_{i,t}$	0.04*** (0.007)	0.02*** (0.004)	0.05*** (0.007)	0.01*** (0.004)	0.09*** (0.015)	0.05*** (0.011)
$\log \text{GDPp}/c_{i,t} \times \text{Discrete}_k$...	0.06*** (0.018)	...	0.13*** (0.026)	...	0.06*** (0.024)
Observations	7,855,000	1663,00	5,832,608			
Fixed effects	Exporting district \times HS10 prduct \times year					
	All goods		Final goods		Differentiated goods	
	Firm-level data (Colombia)					
$\log \text{GDPp}/c_{i,t}$	0.04*** (0.007)	0.02*** (0.004)	0.05*** (0.007)	0.01*** (0.004)	0.09*** (0.015)	0.05*** (0.011)
$\log \text{GDPp}/c_{i,t} \times \text{Discrete}_k$...	0.06*** (0.018)	...	0.13*** (0.026)	...	0.06*** (0.024)
Observations	668,000	387,000	567,000			
Fixed effects	Exporting firm \times HS10 prduct \times year					

Note: The estimating equation is Eq. (2). The top panel is produced using district-level US export data from 1989 to 2015. The bottom panel is produced using firm-level Colombian export data from 2007 to 2013. All estimations are conducted with additional controls for total GDP, distance, and free trade agreements. Each observation is weighted by the corresponding trade quantity. Standard errors are reported in parentheses and clustered by importing country and year. ** $p < 0.05$, *** $p < 0.01$.

indicating that PTM is in fact noticeably more pronounced in discrete product categories. This also remains to be true if we confine our sample to only final consumption goods or differentiated goods.

The bottom panel in Table 4 documents the same pattern while controlling for firm-level characteristics. The results in this panel are derived by estimating Eq. (2) on the firm-level Colombian export data, in which case index j identifies an exporting firm rather than a district.¹⁷ They indicate that PTM is more pronounced in discrete product categories even after we control for *exporting firm-product-year* fixed effects and divide goods by degree of differentiation and intent of final use.

To take stock, the evidence presented in this section indicate that IQS and PTM are more pronounced in discrete industries. One possible reason for this is that the standard drivers of IQS and PTM operate more forcefully in discrete industries. Another possibility is that *discreteness* gives rise to alternative driving forces behind IQS and PTM that are otherwise irrelevant. In what follows, I present a theoretical framework that highlights two such alternative forces.

3. Theoretical framework

My main objective in this section is to highlight two alternative drivers of IQS and PTM in discrete industries. Considering this objective, my theory purposely abstracts from *taste-driven* or *cost-driven* IQS and PTM, which have little to do with discreteness. Accordingly, I model continuous industries in a rather standard fashion. My modeling of discrete industries and the insights that derive from it, however, rests on three distinctive assumptions:

- (i) Discrete goods are indivisible: purchasing them requires paying the price of one full unit, which is akin to paying a fixed acquisition cost.
- (ii) The consumption flow derived from a personally-acquired discrete good can be substituted with analogous services—e.g., consumers can forgo buying a truck for moving purposes by renting a moving service.¹⁸
- (iii) Quality and quantity are not isomorphic. In particular, from the perspective of a firm, supplying higher quality is different from supplying more quantity. The former requires incurring a sunk R&D cost, whereas the latter only requires employing more inputs without necessarily altering one's production technology.

For the sake of composition, I also make various parametric assumptions throughout this section; but these assumptions are less consequential to the theoretical results that follow—Appendix B shows that the main predictions of the model hold equally in a more general (less-parametric) environment.

¹⁷ In the Colombia sample numerous firms supply both discrete and continuous goods. To give an example, firm #900500347 exports both durometers (HS9024800000) that are discrete and priced over \$500 per unit as well as continuous aluminum sheets (HS7616999000).

¹⁸ This feature can be attributed to the strong overlap between discreteness and durability (see Section 2). Unlike perishable consumption goods, durable consumption goods are non-rival. That is, one consumer's usage of the good does not exclude others from using it later. Accordingly, one can use a taxi service without (effectively) paying for the car but by only paying for the labor and the perishable fuel used in providing the service. This feature of durable goods has been widely-invoked in macro-economics literature—see Benhabib et al. (1991), Cooley and Prescott (1995), and Fernandez-Villaverde and Krueger (2011).

3.1. Economic environment

3.1.1. Countries

The global economy consists of $i = 1, \dots, I$ countries, with \mathbb{C} denoting the set of countries. Country i is populated by L_i individuals, each of whom is endowed with $y \in \mathbb{R}_+$ units of effective labor that is the sole factor of production. All individuals are perfectly mobile across the production of different goods but are immobile across countries; and are paid an economy-wide wage, w_i , per unit of effective labor.

3.1.2. Income distribution

I assume that the distribution of effective labor, y , across individuals within country i is given by

$$G_i(y) = 1 - e^{-\zeta_i(y - \bar{y}_i)}.$$

Since each individual residing in country i collects an income equal to yw_i , then $G_i(\cdot)$ also governs the distribution of income in that country. The exponential parametrization of the income distribution is motivated by evidence in Drăgulescu and Yakovenko (2001) and Gabaix (2009). I allow parameters ζ_i and \bar{y}_i to vary arbitrarily across countries; but I impose that the mean of the distribution equals one in all countries, i.e., $\mathbb{E}_i(y) = \bar{y}_i + \zeta_i = 1$ for all y .

3.1.3. Industries

The economy is composed of multiple industries (indexed by k) that fall under three general categories:

- (i) Continuous industries, which involve traded and *infinitely-divisible* goods, with \mathbb{K}_C denoting the set of all continuous industries
- (ii) Discrete industries, which involve traded but *indivisible* goods, with \mathbb{K}_D denoting the set of all discrete industries.
- (iii) Non-traded service industries that supply a homogeneous output, which serves as a substitute for discrete goods.

I hereafter use the *tilde* notation to distinguish variables corresponding to the service sector. For instance, based on this choice of notation, $\tilde{p}_{i,k}$ denotes the price of industry k -related services in market i .

3.1.4. Product space

Traded goods (whether continuous or discrete) are horizontally and vertically differentiated. That is, they come in different firm-level varieties (indexed by ω) and different quality tiers (indexed by φ). The set of admissible quality tiers in each industry is predetermined and denoted by Φ_k . As in Fajgelbaum et al. (2011), firms sort into (only one) of these predetermined quality tiers and supply their differentiated variety in that particular quality level.

In such a setup, the firm index ω uniquely identifies not only the supplying firm but also the quality of the variety supplied by that firm. To be more specific, let $\Omega_{j,k}(\varphi)$ denote the set of all firms supplying quality φ from country j in industry k . Then, if $\omega \in \Omega_{j,k}(\varphi)$ it is automatically implied that $\omega \notin \Omega_{j,k}(\varphi')$ for any $\varphi' \neq \varphi$. In that regard, the index ω implicitly conveys full information about quality as well as industry and country of origin.

3.2. Demand and supply in continuous industries

I start by describing the supply and demand structure in continuous industries. Given its standard underpinning, the continuous model can also serve as a useful benchmark to better understand my less-standard model of discrete industries.

3.2.1. Demand

The demand structure is similar to Krugman (1980), except that there are multiple quality tiers to choose from. In particular, each individual spends a constant fraction, e_k , of their income on industry k goods.¹⁹ The budget allocated to *continuous* industry $k \in \mathbb{K}_C$ can be spent on buying a fractional quantity of various firm-level varieties. An individual's utility from consuming a basket $\mathbf{q} = \{q_{ji,k}(\omega)\}$ of firm-level varieties is given by a nested-CES function,²⁰

$$U_k(\mathbf{q}) = \left(\sum_{\varphi \in \Phi_k} \left[\sum_{j \in \mathbb{C}} \sum_{\omega \in \Omega_{j,k}(\varphi)} \varphi^{1+\theta_k} q_{ji,k}(\omega)^{\frac{\theta_k}{1+\theta_k}} \right]^{\frac{1+\theta_k}{\theta_k} \frac{\gamma_k}{1+\gamma_k}} \right)^{\frac{1+\gamma_k}{\gamma_k}}, \quad k \in \mathbb{K}_C$$

where γ_k and $\theta_k > 0$ reflect the degree of within- and cross-quality substitutability between different firm-level varieties. Utility maximization subject to the budget constraints, $\sum_{\varphi} \sum_j \sum_{\omega} p_{ji,k}(\omega) q_{ji,k}(\omega) \leq e_k w_i y_i$ implies that the share of spending on variety ω

¹⁹ To be specific, the welfare of individual ι is a Cobb-Douglas aggregate of the industry specific utility levels. Namely, $W_{\iota} = \prod_{k \in \mathbb{K}} U_{\iota,k}^{\alpha_k}$.

²⁰ Implicit in the above formulation is that markets are segmented. Without this assumption, PTM and the possibility of international arbitrage may lead to zero trade between countries—see Foellmi et al. (2017).

is independent of personal income and given by,

$$\lambda_{ji,k}(\omega) = \frac{p_{ji,k}(\omega)^{-\theta_k}}{\sum_{i \in C} \sum_{\omega' \in \Omega_{i,k}(\varphi)} p_{i,k}(\omega')^{-\theta_k}} \times e_{i,k}(\varphi), \quad \forall \omega \in \Omega_{j,k}(\varphi) \tag{3}$$

where $e_{i,k}(\varphi)$ is the total share of spending on quality φ :

$$e_{i,k}(\varphi) = \frac{\left(\varphi \sum_{i \in C} \sum_{\omega \in \Omega_{i,k}(\varphi)} p_{i,k}(\omega)^{-\theta_k}\right)^{\gamma_k/\theta_k}}{\sum_{\varphi' \in \Phi_k} \left(\varphi' \sum_{i \in C} \sum_{\omega \in \Omega_{i,k}(\varphi')} p_{i,k}(\omega)^{-\theta_k}\right)^{\gamma_k/\theta_k}}, \quad \forall \varphi \in \Phi_k.$$

Accordingly, total demand for variety ω in market i can be expressed as,

$$q_{ji,k}(\omega) = \lambda_{ji,k}(\omega) e_k w_i L_i / p_{ji,k}(\omega), \tag{4}$$

where $w_i L_i$ is total income in market i —the fact that total income equals $w_i L_i$ derives from my earlier choice of normalization, $\mathbb{E}_i(y) = 1$.

3.2.2. Supply

The supply-side of the economy is similar to Fajgelbaum et al. (2011) extension of Krugman (1980). That is, labor is the sole factor of production. Firms supplying goods of the same quality (in industry k) are symmetric within each country, and compete under monopolistic competition—as shown in Appendix D the model can be easily extended to account for firm heterogeneity. A typical firm ω supplying quality φ from country j has to incur a sunk entry cost, $w_j f_k^e(\varphi)$. It also incurs a marginal cost for each unit of output produced and transported to market i that is given by

$$c_{ji,k}(\omega) = \tau_{ji,k} a_{j,k} c(\varphi) w_j. \quad \omega \in \Omega_{j,k}(\omega)$$

In the above cost specification $\tau_{ji,k}$ denotes the iceberg transport cost; $a_{j,k}$ is a quality-neutral (*country*×*industry*-specific) cost shifter; and $c(\varphi)$ is a universal cost shifter that accounts for high-quality output being more costly to produce in every country. An important feature of the above parametrization is that it precludes any Ricardian comparative advantage along the quality margin.

The variable profit collected by a typical firm ω is, thus, given by

$$\pi_{ji,k}(\omega) = [p_{ji,k}(\omega) - c_{ji,k}(\omega)] q_{ji,k}(\omega),$$

where $q_{ji,k}(\omega)$ is determined by Eq. (4). Firms choose their optimum price, $p_{ji,k}(\omega)$, by maximizing $\pi_{ji,k}(\omega)$, which implies a standard monopoly markup over marginal cost. The number of firms supplying quality φ from country j (namely, $N_{j,k}(\varphi)$) is then determined by the *free-entry* condition, wherein firms enter until net profits are drawn to zero in every quality tier of industry k :

$$\sum_{i \in C} \pi_{ji,k}(\omega) = w_j f_k^e(\varphi), \quad \forall \omega \in \Omega_{j,k}(\varphi).$$

3.3. Demand and supply in discrete industries

Discrete industries involve goods that are *indivisible* and substitutable with analogous services. I use a discrete choice framework to model consumer demand in these industries. On the supply-side, though, I assume that discrete industries are identical to continuous counterparts.

3.3.1. Demand

To draw consumption utility from a discrete variety $\omega \in \Omega_{j,k}(\varphi)$, consumers have to pay the unit price, $p_{ji,k}(\omega)$, which is akin to paying a fixed acquisition cost. After the good is acquired, it delivers a flow of consumption quantity, $q_k(\varphi)$, that is increasing in the good's quality, φ . The flow of personal consumption provided by a discrete good (like a car) is substitutable with analogous services (like a taxi service). Consumers can, therefore, circumvent paying the acquisition cost altogether, or they can purchase a cheaper (lower- φ) discrete variety and complement it with a range of specialized services.

Suppose individual ι purchases discrete variety $\omega \in \Omega_{j,k}(\varphi)$ (a firm-level variety of quality φ originating from country j) for personal use and combines it with \tilde{q}_k hours of service. The utility they derive from this choice is given by

$$U_\iota(\omega, \tilde{q}) = \tilde{q}_k + q_k(\varphi) + \xi_\iota(\omega), \quad \omega \in \Omega_{j,k}(\varphi)$$

where $q_k(\varphi)$ (as defined earlier) denotes the consumption quantity derived from a personally-acquired unit of good ω ; and $\xi_\iota(\omega)$ is an individual \times variety-specific utility shifter that accounts for individual ι 's personal taste for variety ω . Implicit in the above parametrization is the fact that the consumption derived from a discrete good is perfectly substitutable with analogous services. Without loss of generality, I henceforth impose that $q_k(\varphi) = \varphi$.

As is common in the literature, I assume that the vector $\xi_{\iota,k} \equiv \{\xi_\iota(\omega)\}$, which describes individual ι 's taste with respect to various industry k varieties, is drawn independently from a General Extreme Value distribution: $F_k(\xi_{\iota,k}) = \exp(-\sum_{\varphi \in \Phi_k} (\sum_{\omega \in \Omega_{j,k}(\varphi)} \exp(\xi_\iota(\omega))^{-\theta_k})^{\gamma_k/\theta_k})$, where $\theta_k \geq \gamma_k \geq 0$ are parameters governing the heterogeneity of consumer taste within and across quality levels in industry k . This assumption allows me to obtain closed-form demand functions.

An individual (with endowment y) maximizes their utility by choosing their preferred variety, ω , within industry k subject to a personal budget constraint. Recall that a fraction e_k of an individual's income is spent on industry k goods/services (e.g., transportation goods and services). The budget constraint imposes that the price of variety ω not exceed one's budget, $e_k w_i y$. Moreover, given the choice of ω , the budget constraint implicitly determines their choice of service consumption, \tilde{q}_k . That is, if an individual chooses variety $\omega \in \Omega_{j,k}(\varphi)$, their budget constraint,

$$\tilde{p}_{i,k} \tilde{q}_k + p_{ji,k}(\omega) = e_k w_i y,$$

implicitly pins down $\tilde{q}_k = [e_k w_i y - p_{ji,k}(\omega)] / \tilde{p}_{i,k}$. Taking into account the individual's budget constraint and appealing to the Theorem of Extreme Value, utility maximization then implies that the share of individuals with endowment y who choose variety ω is given by

$$\lambda_{ji,k}(\omega; y) = \frac{\exp(p_{ji,k}(\omega))^{-\theta_k/\tilde{p}_{i,k}}}{\sum_{\ell \in \mathbb{C}} \sum_{\omega' \in \Omega_{\ell,k}(\varphi, y)} \exp(-p_{\ell i,k}(\omega'))^{-\theta_k/\tilde{p}_{i,k}}} \times e_{i,k}(\varphi; y), \tag{5}$$

if $y \geq \frac{p_{ji,k}(\omega)}{e_k w_i}$; and is zero otherwise (see Appendix C.1 for proof). The share of individuals with endowment y who choose a variety with quality φ is similarly given by

$$e_{i,k}(\varphi; y) = \frac{\left[\varphi \sum_{\ell \in \mathbb{C}} \sum_{\omega' \in \Omega_{\ell,k}(\varphi, y)} \exp(-p_{\ell i,k}(\omega'))^{-\theta_k/\tilde{p}_{i,k}} \right]^{\frac{\gamma_k}{\theta_k}}}{\sum_{\varphi' \in \Phi_k} \left[\varphi' \sum_{\ell \in \mathbb{C}} \sum_{\omega' \in \Omega_{\ell,k}(\varphi', y)} \exp(-p_{\ell i,k}(\omega'))^{-\theta_k/\tilde{p}_{i,k}} \right]^{\frac{\gamma_k}{\theta_k}}},$$

where $\Omega_{j,k}(\varphi, y) \subset \Omega_{j,k}(\varphi)$ denotes the subset of varieties in set $\Omega_{j,k}(\varphi)$ that are affordable to individuals with endowment y . The total demand facing firm ω can, therefore, be calculated as the sum of the demand across all individuals in the economy as follows:

$$q_{ji,k}(\omega) = \left(\int_{y=\frac{p_{ji,k}(\omega)}{e_k w_i}}^{\infty} \frac{p_{ji,k}(\omega)}{e_k w_i} \lambda_{ji,k}(\omega; y) dG_i(y) \right) L_i. \tag{6}$$

Before moving forward let me draw a brief comparison between the demand structure in discrete and continuous industries. In both sets of industries, individual-level preferences are homothetic in the following sense. If two individuals have different income levels (namely, y' and y) but can choose from the same set of varieties, then

$$\lambda_{ji,k}(\varphi; y) = \lambda_{ji,k}(\varphi; y').$$

Moreover, if the above condition is satisfied, it automatically follows that the two individuals also exhibit the same demand for quality: $e_{i,k}(\varphi, y) = e_{i,k}(\varphi, y')$.

3.3.2. Supply

On the supply side, discrete industries are assumed to be identical to continuous industries. That is, labor is the only factor of production and firms compete under monopolistic competition. Each firm in country j incurs a sunk cost, $w_j f_k^e(\varphi)$, to enter

industry k in quality tier φ . Upon entry, it also incurs a marginal cost equal to $c_{j,i,k}(\omega) = \tau_{j,i,k} a_{j,k} c(\varphi) w_j$ in order to supply each unit of output to market i . Recall that this cost structure rules out Ricardian comparative advantage along the quality margin. Each firm sets their optimum price by maximizing their variable profit,

$$\pi_{j,i,k}(\omega) = [p_{j,i,k}(\omega) - c_{j,i,k}(\omega)] q_{j,i,k}(\omega),$$

where $q_{j,i,k}(\omega)$ is now given by Eq. (6). Profit maximization simply entails a standard monopoly markup over marginal cost. The number of firms in every quality tier of industry k is determined by the *free-entry* condition, wherein firms enter until net profits are drawn to zero: $\sum_{i \in \mathbb{C}} \pi_{j,i,k}(\omega) = w_j f_k^e(\varphi)$.

It is important to note that the production of quality is subject to scale economies in all industries. That is, the average cost of supplying quality is decreasing in total output. As a result, to circumvent trade costs, high-quality firms agglomerate in economies where there is a relatively large local demand for high-quality varieties, leading to the well-known *home-market* effect.

3.4. Demand and supply of non-traded services

The demand for non-traded services is implicitly determined by the consumption choice in discrete industries.²¹ I also assume that the service sector is perfectly competitive. In principle, services may employ discrete goods as an intermediate input (e.g., taxi services use cars as an input), but I assume that due to repeated use, the fixed cost paid for acquiring the discrete good constitutes a negligible fraction of the total cost.²² Consequently, the average cost, effectively, equals the marginal cost, $\bar{a}_{i,k} w_i$, for service suppliers.

I also assume that the marginal labor cost of supplying services is the same across all industries, i.e., $\bar{a}_{i,k} = \bar{a}_{i,g}$ for all g and $k \in \mathbb{K}_D$. Altogether, these assumptions imply that the competitive price of services in market i equals:

$$\bar{p}_{i,k} = \bar{p}_i = \bar{a}_i w_i \quad \forall k.$$

The above equation indicates that \bar{p}_i and the local wage rate, w_i , are positively but not perfectly correlated.²³ We can actually construct many theory-consistent examples where the price of non-traded services differ across two markets with the same w_i . This distinction plays an important role when testing the model, later in Section 5.

3.5. Equilibrium

In equilibrium, each firm ω that supplies quality φ from country j in industry k (i.e., $\omega \in \Omega_{j,k}$) charges the same price and sells the same quantity of output. Considering this, I can express all variables as a function of quality rather than the firm-level index, ω . In that case, $p_{j,i,k}(\varphi)$ denotes the price charged in market i by a typical firm $\omega \in \Omega_{j,k}(\varphi)$. With this choice of notation, I now present the main equilibrium outcomes and formally define the equilibrium.

Following Eqs. (3) and (5), the market share of origin j varieties in country i can be expressed as²⁴

$$\lambda_{j,i,k}(\varphi) = \begin{cases} \frac{N_{j,k}(\varphi) p_{j,i,k}(\varphi)^{-\theta_k}}{\sum_{i \in \mathbb{C}} N_{i,k}(\varphi) p_{i,i,k}(\varphi)^{-\theta_k}} \times e_{i,k}(\varphi; y) & \text{if } k \in \mathbb{K}_C \\ \int_{y=\frac{c_{j,i,k}^D(\varphi)}{e_k w_i}}^{\infty} \frac{N_{j,k}(\varphi) \exp\left(p_{j,i,k}(\varphi) \frac{\theta_k}{\bar{p}_i}\right)}{e_k w_i \sum_{i \in \mathbb{C}_i(\varphi, y)} N_{i,k}(\varphi) \exp\left(p_{i,i,k}(\varphi) \frac{\theta_k}{\bar{p}_i}\right)} \times e_{i,k}(\varphi; y) dG_i(y) & \text{if } k \in \mathbb{K}_D \end{cases} \quad (7)$$

where \mathbb{K}_C denotes the *set of continuous industries*; \mathbb{K}_D denotes the *set of discrete industries*; and $\mathbb{C}_i(\varphi, y) \in \mathbb{C}$ denotes the subset of countries whose quality φ varieties are affordable to income group y in market i . The share of quality φ in total consumption is,

²¹ This is a simplifying but innocuous assumption. One can always assume that the economy features an extra set of non-traded service industries that are unrelated to discrete industries. Adding this feature, as long as we maintain the Cobb-Douglas assumption on industry-level expenditure shares, will not change any of the results in the paper.

²² Specifically, consider an incumbent firm supplying *industry k-related* services in market i . Suppose such a firm spends $\bar{c}_{i,k}^D$ on purchasing discrete input varieties and sells in total $\bar{Q}_{i,k}$ units of services. The implicit assumption here is that $\bar{c}_{i,k}^D / \bar{Q}_{i,k} \approx 0$, so that the average cost for this supplier is approximately equal to the marginal cost:

$$\bar{A}C_i = \bar{a}_{i,k} w_i + \frac{\bar{c}_{i,k}^D}{\bar{Q}_{i,k}} \approx \bar{a}_{i,k} w_i.$$

²³ The positive relation between \bar{p}_i and w_i is contingent on \bar{a}_i 's being sufficiently homogeneous across countries. This condition serves as the foundation of the celebrated *Blassa-Samuels effect* (Bhagwati (1984); Samuelson (1994)).

²⁴ Note that in the case of continuous industries, $\lambda_{j,i,k}(\varphi)$ denotes the *value-share* of spending; whereas in the case of discrete industries, it denotes the *quantity-share*.

meanwhile, given by:

$$e_{i,k}(\varphi; y) = \begin{cases} \frac{\left(\varphi \sum_{\ell \in \mathbb{C}} N_{\ell,k}(\varphi) p_{\ell i,k}(\varphi)^{-\theta_k}\right)^{\gamma_k/\theta_k}}{\sum_{\varphi' \in \Phi_k} \left(\varphi' \sum_{\ell \in \mathbb{C}} N_{\ell,k}(\varphi') p_{\ell i,k}(\varphi')^{-\theta_k}\right)^{\gamma_k/\theta_k}} & \text{if } k \in \mathbb{K}_C \\ \frac{\left(\varphi \sum_{\ell \in \mathbb{C}(\varphi, y)} N_{\ell,k}(\varphi) \exp\left(\frac{p_{\ell i,k}(\varphi)}{\bar{p}_i}\right)\right)^{\theta_k/\theta_k}}{\sum_{\varphi' \in \Phi_k} \left(\varphi' \sum_{\ell \in \mathbb{C}(\varphi, y)} N_{\ell,k}(\varphi') \exp\left(\frac{p_{\ell i,k}(\varphi')}{\bar{p}_i}\right)\right)^{\theta_k/\theta_k}} & \text{if } k \in \mathbb{K}_D \end{cases}$$

The above expressions immediately imply that the equilibrium demand elasticity facing individual firms can be stated as (see Appendix C.3 for derivation):

$$\varepsilon_{j i,k}(\varphi) = \begin{cases} \theta_k + 1 & \text{if } k \in \mathbb{K}_C \\ \theta_k \frac{p_{j i,k}(\varphi)}{\bar{p}_i} + \zeta_i \frac{p_{j i,k}(\varphi)}{e_k w_i} & \text{if } k \in \mathbb{K}_D \end{cases} \tag{8}$$

The difference between the demand elasticity faced by firms in discrete versus continuous industries plays a key role in my theory. So, I will discuss these differences more elaborately in the following Subsection. Finally, given the expression for $\lambda_{j i,k}(\varphi)$, aggregate trade values, $X_{j i,k}(\varphi) \equiv N_{j,k}(\varphi) p_{j i,k}(\varphi) q_{j i,k}(\varphi)$, are given by

$$X_{j i,k}(\varphi) = \begin{cases} \lambda_{j i,k}(\varphi) e_k Y_i & \text{if } k \in \mathbb{K}_C \\ \lambda_{j i,k}(\varphi) \frac{p_{j i,k}(\varphi)}{w_i} Y_i & \text{if } k \in \mathbb{K}_D \end{cases}, \tag{9}$$

where $Y_i \equiv w_i L_i$. Equilibrium corresponds to a vector of prices, $\mathbf{p} \equiv \{p_{j i,k}(\varphi)\}$, wage rates, $\mathbf{w} \equiv \{w_j\}$, and the number of firms, $\mathbf{N} \equiv \{N_{j,k}(\varphi)\}$, that satisfy (i) the optimal monopoly pricing condition (MP), (ii) the balanced trade condition (BT), and (iii) the free entry condition (FE):

$$\begin{cases} p_{j i,k}(\varphi) = \frac{\varepsilon_{j i,k}(\varphi)}{\varepsilon_{j i,k}(\varphi) - 1} \tau_{j i,k} a_{j i,k}^C(\varphi) w_j & \forall \varphi \in \Phi_k, k \in \mathbb{K} \quad (\text{MP}) \\ \sum_{j \in \mathbb{C}} \sum_{k \in \mathbb{K}} \sum_{\varphi \in \Phi_k} X_{j i,k}(\varphi) = \sum_{\ell \in \mathbb{C}} \sum_{k \in \mathbb{K}} \sum_{\varphi \in \Phi_k} X_{\ell i,k}(\varphi) & \forall k \in \mathbb{K} \quad (\text{BT}) \\ N_{j,k}(\varphi) w_j f_k^e(\varphi) = \sum_{i \in \mathbb{C}} X_{j i,k}(\varphi) / \varepsilon_{j i,k}(\varphi) & \forall k \in \mathbb{K} \quad (\text{FE}) \end{cases},$$

By Walras' law, the satisfaction of the BT and FE conditions ensures that labor markets also clear in each country.

3.6. Discussion: the demand elasticity facing discrete goods

While I assumed that individual preferences for traded goods are homothetic, the market-level demand elasticity facing discrete goods varies systematically across markets. This variation is driven by (i) cross-market heterogeneity in the price of non-traded services, and (ii) affordability constraints being more-or-less binding in different markets. To elaborate on these two channels, we can decompose the demand elasticity facing discrete goods in market i as follows:

$$\varepsilon_{j i,k}(\varphi) = \underbrace{\theta_k \frac{p_{j i,k}(\varphi)}{\bar{p}_i}}_{\text{NT-driven}} + \underbrace{\zeta_i \frac{p_{j i,k}(\varphi)}{e_k w_i}}_{\text{affordability-driven}}.$$

In the above expression, the *NT-driven* component accounts for cross-country differences in the price of non-traded services. This component depends on (a) the degree of product differentiation θ_k , the role of which is well understood, as well as (b) $p_{j i,k}(\omega) / \bar{p}_i$, which is the amount of service consumption one forgoes to acquire a full unit of variety ω . Importantly, the demand elasticity is lower the higher the price of services in market i .

²⁵ As shown in Appendix B, even when $G_i(y)$ does not exhibit a constant hazard rate, the propositions that follow hold as long as $G_i(y)$ satisfies the IGFR property.

The *affordability-driven* component is driven by the fact that an incremental increase in $p_{ji,k}(\omega)$ can make variety ω prohibitively expensive for a fraction of existing consumers in market i . The extent of demand loss due to this channel, depends on (a) variety ω 's *degree of affordability* in market i , $p_{ji,k}(\omega)/e_k w_i$, as well as (b) the hazard rate of the income distribution, ζ_i .²⁵

4. Quality specialization and pricing-to-market

In this section, I use my theoretical model to outline two alternative drivers of pricing-to-market (PTM) and international quality specialization (IQS) in discrete industries. This section, therefore, sheds light on why (as documented in Section 2) PTM and IQS occur more intensively in discrete industries.

4.1. Pricing-to-market (PTM)

Using the demand elasticity specified by Eq. (8), we can immediately calculate the optimal monopoly price for each traded variety. Doing so, implies the following price formulation in continuous ($k \in \mathbb{K}_C$) and discrete ($k \in \mathbb{K}_D$) industries:

$$p_{ji,k}(\varphi) = \begin{cases} (1 + 1/\theta_k)c_{ji,k}(\varphi) & \text{if } k \in \mathbb{K}_C \\ c_{ji,k}(\varphi) + \vartheta_{i,k} w_i & \text{if } k \in \mathbb{K}_D \end{cases}$$

with $1/\vartheta_{i,k} \equiv \frac{\theta_k}{\bar{a}_i} + \frac{\zeta_i}{e_k}$ denoting a market-specific *structural* parameter. The above equation indicates that (absent non-homothetic preferences) there is no scope for PTM in continuous industries. In discrete industries, however, PTM occurs even though preferences are homothetic; as suppliers charge systematically higher prices in markets where (a) they compete with higher-priced services; and (b) their product is more affordable to the average consumer.

These results can perhaps explain why PTM is more pronounced in discrete industries. In both discrete and continuous industries, PTM may be driven by non-homothetic preferences. But in discrete industries, PTM is magnified by (a) cross-national heterogeneity in the price of non-traded services, and (ii) affordability constraints; both of which are less relevant in continuous industries.

4.2. International quality-specialization (IQS)

To fix ideas, allow me to first define IQS formally from the lens of the present model. Let $Q_{ji,k}(\varphi) \equiv N_{j,k}(\varphi)q_{ji,k}(\varphi)$ denote aggregate export quantity and suppose φ_H and φ_L respectively denote a *high* and a *low*-quality tier in industry k , i.e., $\varphi_H > \varphi_L$. As a matter of definition, an increase in country-level characteristic x prompts high-quality-specialization if and only if

$$\frac{\partial}{\partial x_j / x^\ell} \left\{ \frac{Q_{ji,k}(\varphi_H)/Q_{ii,k}(\varphi_H)}{Q_{ji,k}(\varphi_L)/Q_{ii,k}(\varphi_H)} \right\} > 0,$$

for (nearly) all j, ℓ , and $i \in \mathbb{C}$. So, to determine patterns of IQS based on this definition, we need to derive an expression for the relative exports of φ_H to φ_L . Doing so in Appendix C.4, yields the following expression:

$$\frac{Q_{ji,k}(\varphi_H)/Q_{ii,k}(\varphi_H)}{Q_{ji,k}(\varphi_L)/Q_{ii,k}(\varphi_L)} \approx \begin{cases} 1 & \text{if } k \in \mathbb{K}_C \\ \frac{N_{j,k}(\varphi_H)/N_{\ell,k}(\varphi_H)}{N_{j,k}(\varphi_L)/N_{\ell,k}(\varphi_L)} e^{(\tau_{ii} w^{a_{i,k}} - \tau_{ji} w_j a_{j,k}) \frac{\vartheta_{i,k}}{w_i}} & \text{if } k \in \mathbb{K}_D \end{cases} \quad (10)$$

The above expression clearly indicates that there is no scope for IQS in continuous industries. That is because IQS in these industries is either cost-driven or taste-driven, both of which have been ruled out by assumption. By contrast, in the case of discrete industries, the above expression identifies two alternative drivers of IQS that are neither cost-driven nor taste-driven.

The first of these are affordability constraints, which operate (primarily) through the exponential term in Eq. (10). The exponential terms account for the fact that, due to affordability constraints, an across-the-board increase in the cost of all varieties supplied by country j has a disproportionately higher effect on demand for high-quality (high-cost) varieties. To demonstrate this, we

²⁶ The above result offers a possible micro-foundation for the assumption underlying Alcalá (2016) that efficiency or productivity is quality-biased.
²⁷ A higher \bar{p}_i lowers the demand elasticity for discrete goods, which in turn creates relatively more demand for high-quality (high-cost) product varieties—see Fajgelbaum et al. (2011); Matsuyama (2015); Dingel (2017) for a similar argument, but in the case where the home-market effect is driven by non-homothetic preferences over quality.

can take a basic derivative from Eq. (10) with respect to the national level cost-shifters, $a_{j,k}$, which implies that:

$$\frac{\partial}{\partial a_{\ell,k}/a_{j,k}} \left\{ \frac{Q_{j\ell,k}(\varphi_H)/Q_{i\ell,k}(\varphi_H)}{Q_{j\ell,k}(\varphi_L)/Q_{i\ell,k}(\varphi_L)} \right\} > 0 \quad \text{if } k \in \mathbb{K}_D.$$

Based on the above expression, countries tend to specialize in higher-quality varieties in industries where they are more cost-efficient. The intuition is that affordability constraints are more binding for high-quality varieties. So, an improvement in cost-efficiency (keeping all else the same) increases market access disproportionately more for high-quality varieties. Such a link between within-industry IQS and industry-level cost efficiency is quite unique and has little precedent in the literature.²⁶

The second driver of IQS in discrete industries is the *home-market effect*, which operates through the number of firms in Eq. (10). Unlike standard theories, though, the home-market effect here is not driven by non-homothetic taste for quality. Instead, it is driven (primarily) by cross-market differences in \tilde{p}_i . That is, a higher price of services increases the home-market demand for quality, leading to high-quality specialization due to scale economies in production.²⁷ To demonstrate this point formally, we can take a simple derivative from the free-entry condition with respect to \tilde{p}_i , noting that $\partial \varepsilon_{j\ell,k}(\varphi) / \partial \tilde{p}_i < 0$ for all j and i . Doing so, implies the following, provided that international trade costs are sufficiently high:

$$\frac{\partial}{\partial \tilde{p}_j / \tilde{p}_\ell} \left\{ \frac{N_{j,k}(\varphi_H) / N_{\ell,k}(\varphi_H)}{N_{j,k}(\varphi_L) / N_{\ell,k}(\varphi_L)} \right\} > 0 \Rightarrow \frac{\partial}{\partial \tilde{p}_j / \tilde{p}_\ell} \left\{ \frac{Q_{j\ell,k}(\varphi_H) / Q_{i\ell,k}(\varphi_H)}{Q_{j\ell,k}(\varphi_L) / Q_{i\ell,k}(\varphi_H)} \right\} > 0. \tag{11}$$

Let me reiterate that the *home-market effect* in itself is not specific to this model. In fact, Costinot et al. (2016) have already characterized this result under very general conditions.²⁸ What is instead novel, is that the *home-market effect* is not driven by non-homothetic taste but by cross-country differences in the price of non-traded services. Along similar lines, affordability constraints can also trigger IQS through the home-market effect, given that high-quality varieties are more affordable and, therefore, more-demanded in high-wage markets. The following proposition summarizes the above results.

Proposition 1. *Absent Ricardian comparative cost advantage and non-nomothetic preferences: (a) There is no scope for either quality specialization nor pricing-to-market in continuous industries; but (b) quality specialization and pricing-to-market can still occur in discrete industries due to cross-national differences in the price of non-traded services and affordability constraints.*

Fig. 1 elucidates the above arguments using a simulated economy that features multiple countries and quality tiers.²⁹ It compares two countries in this simulated economy (namely, North, n , and South, s) that differ only in a Hicks-neutral cost shifter, $a_s/a_n = 2$. The left panel assumes all varieties are universally affordable and IQS is driven by the combination of scale economies and cross-national differences in \tilde{p}_i . The right panel shuts down scale economies, so that IQS is driven solely by affordability constraints.

A natural follow-up question to Proposition 1 is whether the alternative forces that drive PTM and IQS in discrete industries are empirically distinguishable from cost-driven or taste-driven IQS and PTM? To answer this question, it is useful to define two aggregate trade statistics that are often used to describe standard theories of IQS and PTM. The first statistic is the industry-level export unit price, which can be defined as

$$\bar{p}_{j\ell,k} \equiv \frac{\sum_{\varphi \in \Phi_k} p_{j\ell,k}(\varphi) Q_{j\ell,k}(\varphi)}{\sum_{\varphi \in \Phi_k} Q_{j\ell,k}(\varphi)}.$$

The second statistic is the industry-level export quality, which can be analogously defined as:

$$\bar{\varphi}_{j\ell,k} \equiv \frac{\sum_{\varphi \in \Phi_k} \varphi Q_{j\ell,k}(\varphi)}{\sum_{\varphi \in \Phi_k} Q_{j\ell,k}(\varphi)}.$$

As proven in Appendix C.5, combining the above definition with our prior results on PTM and IQS, yields the following proposition.

²⁸ In the present model, the scale elasticity (defined as $\partial \ln Q_{j\ell,k}(\varphi) / \partial \ln N_{j\ell,k}(\varphi)$) is equal to “one.” So, scale effects are strong enough to trigger a *strong* home-market effect (see Costinot et al. (2016)). That is, all else equal, high-wage countries are not only gross exporters of high-quality but also net exporters of high-quality varieties.
²⁹ The example involves 5 economies producing/consuming only discrete varieties in 50 different quality tiers: $\varphi = 1, \dots, 50$. Population size in all countries is normalized to one: $L_j = 1, \forall j$; but countries differ in their productivity: $a_j = 1 + 0.25(j - 1)$. The region with the lowest productivity is labeled the *South* (s) and the highest productivity region is labeled the *North* (n); $a_s/a_n = 2$. I also assume $c(\varphi) = 0.08\varphi$, $f(\varphi) = \varphi^{0.25}$, $\theta = 4$, and $\tau_{ji} = 2$ for all $j \neq i$.

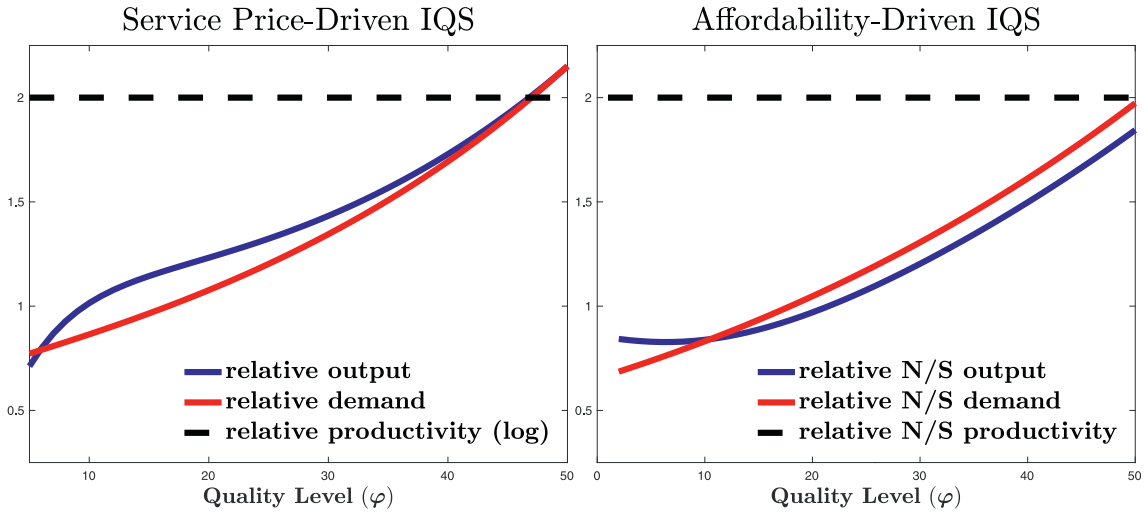


Fig. 1. Numerical example: north-south quality specialization. Note: This graph compares demand and output across different quality levels ($\varphi = 1, \dots, 50$) for two economies (North and South) that differ only in their Hicks-neutral productivity, i.e., $a_N/a_S = 2$. In the left panel, service prices vary between countries but affordability constraints are non-binding. In the right panel, affordability constraints are bindings but there are no scale economies in production. More details about the simulated economy are provided in Footnote 29.

Proposition 2. (i) Export unit prices are increasing in the price of non-traded services in the importing market, but only so in discrete industries:

$$\begin{cases} \frac{\partial \bar{p}_{ji,k}(\cdot)}{\partial \bar{p}_i} = 0 & \text{if } k \in \mathbb{K}_C \\ \frac{\partial \bar{p}_{ji,k}(\cdot)}{\partial \bar{p}_i} > 0 & \text{if } k \in \mathbb{K}_D \end{cases};$$

(ii) Export unit prices are increasing in the price of non-traded services in the exporting market, but only so in discrete industries.

$$\begin{cases} \frac{\partial \bar{p}_{ji,k}(\cdot)}{\partial \ln \bar{p}_j} = 0 & \text{if } k \in \mathbb{K}_C \\ \frac{\partial \bar{p}_{ji,k}(\cdot)}{\partial \ln \bar{p}_j} > 0 & \text{if } k \in \mathbb{K}_D \end{cases};$$

(iii) Export quality is increasing in a country's industry-level degree of comparative cost advantage, but only so in discrete industries:

$$\begin{cases} \frac{\partial \bar{\varphi}_{ji,k}(\cdot)}{\partial a_{j,k}} = 0 & \text{if } k \in \mathbb{K}_C \\ \frac{\partial \bar{\varphi}_{ji,k}(\cdot)}{\partial a_{j,k}} < 0 & \text{if } k \in \mathbb{K}_D \end{cases}$$

The above proposition presents a set of comparative static results indicating that *the price of non-traded services and cost-efficiency* have differential effects on aggregate export price/quality levels in discrete versus continuous industries. In the following section, I will use the above proposition to empirically test my theory. Before doing so, let me note that $\bar{p}_i, \bar{p}_j, a_{j,k}$ affect the price of exports/imports through both the quality and markup channels. These two channels are independent, but (in the present setup) they always reinforce one another's effect on prices. That being the case, I can just rely on readily observable price data to test the model's predictions. Encouragingly, as I will show next, the above predictions are all borne out.

³⁰ The ICP data has been released for years 1970, 1973, 1975, 1980, 1985, 1993, 2005, and 2011. Here, I choose 2011 as the benchmark year for my analysis.

³¹ I treat the following basic headings in the ICP as non-tradable: 'Housing', 'Education', 'Health', and 'Restaurants and Hotels'. In the main analysis 'Communication' is not treated as a non-tradable heading, but treating it as one does not alter any of the results qualitatively. Moreover, similar Sposi (2015), I treat "Change in inventories and valuables," as unclassified.

5. Testing the predictions of the model

Proposition 2 will serve as the basis for testing my theory. In particular, I test (a) if the export unit price, $\bar{p}_{j,k}$, is positively related to \tilde{p}_i , \tilde{p}_j , and $a_{j,k}$; and also (b) if these relationships are amplified by *discreteness*. To conduct my analysis, I merge two different datasets. The first of which is from the 2011 International Comparison Program (ICP) compiled by the World Bank Development Data Group.³⁰ The ICP data reports national expenditures and PPPs for more than 150 “Basic Headings” categories. This information allows me to construct the aggregate price of non-traded services, \tilde{p}_i for 182 countries.³¹

I merge the ICP data with the COMTRADE-BACI database, which is the most comprehensive database on export unit prices. The BACI data is compiled by the CEPII based on the United Nations COMTRADE database but uses a harmonization methodology to reconcile mirror flows. This harmonization approach provides a more complete geographical coverage than if only a single direction of COMTRADE statistics were to be used—Gaulier and Zignago (2010) describe the BACI approach to harmonization, which involves computing a weighted average of mirror flows. For the year 2011, the COMTRADE-BACI database reports physical quantities and values of bilateral trade flows by 6-digit HS product category, allowing me to construct the corresponding unit price levels.

Combining the ICP and the BACI datasets, leaves me with a sample of bilateral trade statistics for 147 importers and 147 exporters across 5231 HS6 product categories in the year 2011. For each country in the sample, Head et al. (2010) report matching data for GDP, distance, and population size. So, altogether, for every exporting country $j \times$ importing country $i \times$ HS6 industry k (observation ji,k), I can construct a quantity-weighted unit price, $\bar{p}_{j,k}$, as well as exporter/importer characteristics such as total GDP, GDP per capita, geo-distance, \tilde{p}_j , and \tilde{p}_i . To test the prediction (iii) from Proposition 2, I also need data on the degree of industry-level comparative cost advantage. Unfortunately, the technology parameters, $a_{j,k}$, are not directly observable. So, as is common in the literature, I infer them from sales data. To this end, I construct the Balassa index of revealed comparative advantage for each country j in industry k , as follows³²:

$$RCA_{j,k} = \frac{\sum_i X_{ji,k} / \sum_i \sum_g X_{ji,g}}{\sum_\ell \sum_i X_{\ell i,k} / \sum_\ell \sum_i \sum_g X_{\ell i,g}}$$

The above measure provides a theory-consistent proxy for $a_{j,k}$; since from the lens of the model, a lower $a_{j,k}$ (ceteris paribus) is associated with a higher $RCA_{j,k}$.

The BACI data does not explicitly report the units in which quantity is measured.³³ So, I use the U.S. import and export data to identify the discrete product categories, with $1_{k \in \mathbb{K}_D}$ denoting the corresponding discrete product dummy.³⁴ This approach can classify 90% of observations in the BACI. Alternatively, to preserve all observations, I can classify a HS6 product category as discrete if it belongs to either the “Machinery,” “Electrical & Optical Equipment,” or “Transport Equipment” sectors. Both approaches deliver qualitatively similar results.

To test the model, I run the following regression on the final sample that includes 5,249,424 observations³⁵:

$$\begin{aligned} \ln \bar{p}_{j,k} = & \left(\beta_1 + \beta_1^D \cdot 1_{k \in \mathbb{K}_D} \right) \ln \tilde{p}_i + \left(\beta_2 + \beta_2^D \cdot 1_{k \in \mathbb{K}_D} \right) \ln \tilde{p}_j \\ & + \left(\beta_3 + \beta_3^D \cdot 1_{k \in \mathbb{K}_D} \right) \ln RCA_{j,k} + Controls_{ji} + \delta_k + \varepsilon_{j,k}, \end{aligned} \quad (12)$$

where δ_k controls for HS6 fixed effects; and $Controls_{ji}$ is composed of additional controls including total GDP, GDP per capita, and geo-distance. In addition to the baseline specification, I run the above regression under alternative specifications, controlling for exporter, importer, and exporter \times importer fixed effects.³⁶

Based on Proposition 2, the estimated coefficients β_1^D , β_2^D , and β_3^D should be positive and statistically significant.³⁷ Encouragingly, the estimation results presented in Table 5 support these predictions.

In line with prediction (i) from Proposition 2, the elasticity of export prices with respect to the price of non-traded services in the destination market is significantly higher in discrete industries. To provide numbers, consider Column 3 that controls for

³² The results of my analysis are qualitatively robust to adopting alternative measures of revealed comparative advantage, such as that proposed by Proudman and Redding (2000).

³³ Unlike the U.S. data which identify the unit in which physical quantity is measured, the Baci data only reports the weight-equivalent of physical quantity for all observations.

³⁴ By this classification, discrete goods account for 36% of global trade in the BACI dataset.

³⁵ Using a linear regression to test the comparative static results outlined by Proposition 2, requires some clarification. In the words of Reiss and Wolak (2007), the standard reasoning behind this choice is that “When the conditional expectation of y is nonlinear in x , statistical theory tells us (under certain sampling assumptions) that a regression provides a best (minimum expected squared prediction error) linear approximation to the nonlinear conditional expectation function.” However, as Reiss and Wolak (2007) point out, there are basic caveats associated with using a linear reduced-form specification to test comparative static results.

³⁶ As in Section 2, since I am dealing with grouped data each observation is weighted by trade quantity, which is a theory-consistent measure of the number of firms grouped into each observation—refer to Section 2 for a more elaborate discussion.

³⁷ The estimated coefficients β_1 , β_2 , and β_3 may be positive given that $1_{k \in \mathbb{K}_D}$ does not perfectly identify discrete product categories. More specifically, an industry where $1_{k \in \mathbb{K}_D} = 0$ may still contain a small fraction of observations that correspond to discrete goods.

³⁸ Adding $\ln \text{GDP per capita}_i \times 1_{k \in \mathbb{K}_D}$ as an additional control does not change the sign of the coefficients reported in Table 5. However, it increases the standard errors, which can be indicative of multicollinearity between $\ln \text{GDP per capita}_i \times 1_{k \in \mathbb{K}_D}$ and $1_{k \in \mathbb{K}_D} \times \ln \tilde{p}_i$.

³⁹ That $RCA_{j,k}$ matters in continuous industries may be driven by two factors. First, industries coded as continuous in my analysis may feature a small number of discrete categories. Second, that $RCA_{j,k}$ is inherently related to export prices as it reflects either a comparative quality advantage or a comparative cost advantage. In the former case, $RCA_{j,k}$ is positively related to $\bar{p}_{j,k}$, while in the latter case it is negatively related to $\bar{p}_{j,k}$.

Table 5
The determinants of IQS and PTM (Dependent Variable: $\ln \bar{p}_{i,k}$).

Regressor (log)	(1)	(2)	(3)	(4)
\bar{p}_i	0.074* (0.039)	...	0.006 (0.022)	...
$\bar{p}_i \times$ Discrete	0.015 (0.108)	...	0.153*** (0.021)	0.161*** (0.021)
\bar{p}_j	0.070 (0.051)	0.073** (0.033)
$\bar{p}_j \times$ Discrete	0.061* (0.033)	0.067** (0.029)	...	0.104*** (0.029)
$RCA_{j,k}$	0.032*** (0.005)	0.021*** (0.004)	...	0.029*** (0.005)
$RCA_{j,k} \times$ Discrete	0.091*** (0.015)	0.055*** (0.011)	...	0.071*** (0.012)
Controls for GDP & GDP p/c	Y	Y	Y	...
Controls for Distance	Y	Y	Y	...
HS6 FE	Y	Y
Exporter \times HS6 FE	N	N	Y	N
Importer \times HS6 FE	N	Y	N	N
Exporter \times Importer FE	N	Y
Observations	4,734,290	4,734,290	4,734,290	4,734,290

Note: this table corresponds to estimating Eq. (12). Standard errors are reported in parentheses and clustered by destination and origin country. Unit prices are constructed using the BACI database in 2011; the price of non-traded services are constructed using the ICP database in the same year; and each observation in the sample is weighted by trade quantity. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

exporter \times HS6 fixed effects. A 10% increase in \bar{p}_i is associated with a 1.53% increase in the price of exports to market i in discrete industries. But in continuous industries, the effect is only 0.06% and statistically insignificant. To put this number in perspective, the same estimation indicates that a 10% increase in the GDP per capita of country i is associated with an only 1.6% increase in the price of exports to that market.³⁸

Prediction (ii) is also borne out, albeit less decisively. Discreteness has a positive and statistically significant effect on how export prices vary with the price of non-traded services in the origin country. In general, this outcome may be driven by services being used as an input in production. This channel perhaps explains why $\bar{p}_{i,k}$ and \bar{p}_j are positively correlated, even in continuous industries. However, in line with the discrete model's prediction, this positive correlation is significantly more pronounced in discrete industries; standing around 0.67 percentage points higher (Table 5, Column 2).

Finally, prediction (iii) is also borne out: In discrete industries, a 10% increase in the origin country's degree of comparative advantage, $RCA_{j,k}$, is associated with a 0.77% increase in export prices. In continuous industries, by comparison, $RCA_{j,k}$ is associated with a mere 0.21% increase in export prices (Table 5, Column 2).³⁹ One caveat here, though, is that I am not testing the model's predictions directly with information on quality and markups. The above analysis, therefore, cannot rule out the case where \bar{p}_j , \bar{p}_i , and $RCA_{j,k}$ have countervailing effects on export markups and qualities. Instead, loosely speaking, the analysis can only identify the joint effect of these variables on *quality* \times *markup*. This limitation is, however, hard to overcome as estimates for quality and markup are difficult to attain at the scale in which my analysis is conducted. This limitation notwithstanding, the above results plus those presented in Section 2 corroborate the assertion that IQS and PTM are magnified by additional forces in discrete industries.

5.1.1. Restricting sample to differentiated or final goods

In the above analysis, discreteness may be accounting for other omitted product characteristics such as the degree of differentiation or intent of final use. To address this issue, I reestimate Eq. (12) on two restricted samples: First, a sample that only includes differentiated HS6 products as classified by Rauch (1999). Second, a sample that includes only HS6 products that are intended for final consumption, based on the Broad Economic Categories (BEC) classification.

The results derived from these restricted samples are reported in Table 6. They indicate that, even within differentiated goods, discreteness has a significant effect on how export prices, $\bar{p}_{i,k}$, vary with the price of services in the origin and destination economies, \bar{p}_j , and \bar{p}_i . Likewise, discreteness has a significant effect on how export prices vary with the origin country's degree of comparative advantage as measured by $RCA_{j,k}$. The same applies when we restrict attention to only final consumption goods. Appendix E presents an additional test in which HS6 products are grouped into *more-* or *less-differentiated* bins according to the demand elasticity estimates in Broda and Weinstein (2006). Estimating Eq. (12) separately for these product groups produces similar results.

6. The big picture: the gains from discrete trade

Does accounting for discrete trade revise our understanding of the gains from trade? To answer this question, we can appeal to the result in McFadden et al. (1978) that the expected welfare of each individual in country i with respect to discrete industry k

Table 6
Testing the model on only differentiated and final goods (Dependent: $\ln \bar{p}_{j,k}$).

Regressor (log)	Final goods			Differentiated goods		
	(1)	(2)	(3)	(1)	(2)	(3)
\bar{p}_i	...	0.074** (0.034)	0.056* (0.034)	...
$\bar{p}_i \times \text{Discrete}$...	0.179*** (0.019)	0.183*** (0.019)	...	0.148*** (0.021)	0.156*** (0.016)
\bar{p}_j	-0.087 (0.086)	0.079*** (0.039)
$\bar{p}_j \times \text{Discrete}$	0.186*** (0.025)	...	0.155*** (0.021)	0.052*** (0.005)	...	0.097*** (0.019)
$RCA_{j,k}$	0.036*** (0.006)	...	0.050*** (0.005)	0.017*** (0.007)	...	0.029*** (0.006)
$RCA_{j,k} \times \text{Discrete}$	0.004*** (0.001)	...	0.012 (0.008)	0.057*** (0.011)	...	0.070*** (0.010)
Controls for GDP & GDP p/c	Y	Y	...	Y	Y	...
Controls for Distance	Y	Y	...	Y	Y	...
HS6 FE	Y	Y
Exporter \times HS6 FE	N	Y	N	N	Y	N
Importer \times HS6 FE	Y	N	N	Y	N	N
Exporter \times Importer FE	Y	Y
Observations	1,413,000		3,589,000			

Note: this table estimates Eq. (12) separately for differentiated and final good categories. Differentiated goods are identified based on Rauch (1999). Final goods are identified using the BEC classification. Standard errors are reported in parentheses and clustered by origin and destination country. Unit prices are constructed using the BACI database in 2011; the price of non-traded services are constructed using the ICP database in the same year; and each observation in the sample is weighted by trade quantity. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

$\in \mathbb{K}_D$ can be expressed as follows:

$$W_{i,k}(y) = \exp(y) \left(\sum_{\varphi \in \Phi_k} \left[\varphi \sum_{j \in \mathcal{J}_{i,k}(y, \varphi)} N_{j,k}(\varphi) \exp(p_{j,i,k}(\varphi)) \frac{\theta_k}{\bar{p}_i} \frac{\gamma_k}{\theta_k} \right]^{\gamma_k} \right)^{\frac{1}{\gamma_k}}$$

where $\mathcal{J}_{i,k}(y, \varphi) = \{j \in \mathbb{C} | p_{j,i,k}(\varphi) \leq e_k w_i y\}$ denotes the set of countries from which an individual with endowment y can affordably buy quality φ varieties. Correspondingly, the expected welfare of the same individual under autarky (denoted by A) is given by

$$W_{i,k}^A(y) = \exp(y) \left(\sum_{\varphi \in \Phi_k(y)} \left[\varphi N_{i,k}(\varphi) \exp(p_{i,i,k}(\varphi)) \frac{\theta_k}{\bar{p}_i} \frac{\gamma_k}{\theta_k} \right]^{\gamma_k} \right)^{\gamma_k}, \quad k \in \mathbb{K}_D$$

where $\Phi_k(y) = \{\varphi \in \Phi_k | p_{i,i,k}(\varphi) \leq e_k w_i y\}$ denotes the set of quality levels that can be affordably purchased from domestic suppliers. Combining the above two equations; using the exact hat-algebra notation, $\hat{x} = x/x^A$; and noting that the gains from trade in continuous industries are governed by the Arkolakis et al. (2012) (ACR) formula, yields the following proposition.

Proposition 3. [The Gains from Trade] The gains from trade, $GT_i(y) \equiv \Delta W_i(y)/W_i(y)$, for income group y residing in country i are given by

$$GT_i(y) = 1 - \prod_{k \in \mathbb{K}} \left(\sum_{\varphi \in \Phi_k} \lambda_{i,i,k}(\varphi, y)^{\frac{\gamma_k}{\theta_k}} \bar{N}_{i,k}(\varphi)^{-\frac{\gamma_k}{\theta_k}} e_{i,k}(\varphi, y) \right)^{\frac{e_k}{\gamma_k}}$$

where $e_{i,k}(\varphi, y)$ denotes the probability of choosing quality level $\varphi \in \Phi_k$, while $\lambda_{i,i,k}(\varphi, y)$ denotes the probability of purchasing a domestic variety.

Based on the above proposition, the gains from trade are materialized through two distinct channels: (i) the scale-driven gains that depend on $\bar{N}_{i,k}(\varphi)$; and (ii) the import variety gains that depend on $\lambda_{i,i,k}(\varphi, y)$. Beholding this decomposition, let me first qualitatively discuss the distributional consequences of trade. Then, I will formally quantify the aggregate gains from discrete trade.

6.1. The distributional gains from trade

A straightforward corollary of Proposition 3 is that the gains from trade favor consumers that are more likely to purchase foreign varieties, i.e., consumers who exhibit a low $\lambda_{ii, k}(\varphi, y)$. In discrete industries, if trade costs are sufficiently large, low-income individuals cannot afford many imported varieties. Hence, even if individual-level preferences are homothetic, $\lambda_{ii, k}(\varphi, y)$ can be decreasing in y , and the gains from trade can favor high-income (high- y) individuals.

So, how can we reconcile the above claim with the claim in Fajgelbaum and Khandelwal (2014) that the gains from trade favor low-income consumers who spend a greater fraction of their income on tradables? The answer lies in the different interpretations of $\lambda_{ii, k}(\varphi, y)$ in discrete versus continuous industries. In continuous industries, $\lambda_{ii, k}(\varphi, y)$ perfectly coincides with the share of expenditure on domestic varieties. In discrete industries, however, $\lambda_{ii, k}(\varphi, y)$ can be strictly unrelated to the expenditure share. In particular, high-income individuals may be more likely to purchase imported varieties; but since they buy only one unit of each good, such a purchase may exhaust a smaller fraction of their income. Quantifying these distributional gains, though, requires detailed international expenditure data across various income groups. That type of data is largely inaccessible to researchers at this point. Given this underlying challenge, I turn my attention instead to quantifying the aggregate gains from trade.

6.2. The aggregate gains from trade

My discussion above indicated that affordability constraints matter for the distributional consequences of trade. Now, I purposely abstract from these constraints to quantify the representative gains from trade for the case where affordability constraints are non-binding and preferences are Gorman representable. In this special case, unobservable quality differentiation is also irrelevant for the aggregate gains from trade, and we can disregard it hereafter.⁴⁰ The representative gains from trade can, therefore, be expressed as follows:

$$GT_i = 1 - \prod_{k \in \mathbb{K}} \left(\lambda_{ii, k} / \hat{N}_{i, k} \right)^{\frac{e_{i, k}}{\theta_k}} \tag{13}$$

From the lens of the above formula, *discrete* and *continuous* industries are distinguished by (i) the different interpretations of $\lambda_{ii, k}$, and (ii) the different characterizations of $\hat{N}_{i, k}$ in discrete versus continuous industries. If we treat all industries as *continuous*, the above formula can be simply interpreted as the celebrated ACR formula.

6.3. Computing the gains from trade

To transparently highlight the role of discreteness, I use Eq. (13) to quantify the gains from trade in two extreme cases. First, as is common in the literature, I treat the traded sector of the economy as one integrated *continuous* industry. Second, I opt for the other extreme, treating the traded sector as one integrated *discrete* industry. Accordingly, I hereafter drop the industry subscript, k , and use \mathcal{C} and \mathcal{D} to respectively indicate if the traded sector is modeled as continuous or as discrete.⁴¹

When applying Eq. (13) under the presumption that the traded sector is continuous ($k = \mathcal{C}$), $\lambda_{ii, \mathcal{C}}$ can be calculated as the *value* share of domestic goods in tradable consumption. $\hat{N}_{i, \mathcal{C}}$ can be derived using the free entry condition. In particular, free entry implies that $N_{i, \mathcal{C}} = r_{i, \mathcal{C}} L_i / (1 + \theta) f^e$, with $r_{i, \mathcal{C}}$ denoting the share of economy i 's total revenue generated in the traded sector. From this expression we can conclude that $\hat{N}_{i, \mathcal{C}} = 1$ since $r_{i, \mathcal{C}}^A = r_{i, \mathcal{C}} = e_{i, \mathcal{C}}$, where $e_{i, \mathcal{C}}$ denotes the share of expenditure on traded goods.⁴² Plugging $\hat{N}_{i, \mathcal{C}} = 1$ back in to Eq. (13), yields the following formula for the gains from trade under the continuous interpretation of the economy:

$$GT_i^{ACR} = 1 - \lambda_{ii, \mathcal{C}}^{\frac{e_{i, \mathcal{C}}}{\theta}} \tag{14}$$

In the other extreme where the traded sector is treated as entirely discrete (\mathcal{D}), $\lambda_{ii, \mathcal{D}}$ can be calculated as the *quantity* share of domestic goods in tradable consumption. $\hat{N}_{i, \mathcal{D}}$ can be once again derived from the free entry condition. As demonstrated in Appendix C.6, free entry implies that $N_{i, \mathcal{D}} = \sum_{\ell \in \mathcal{C}} (\lambda_{ii, \mathcal{D}} Y_{\ell} / Y_i) L_i / \theta f^e$ and, correspondingly, $N_{i, \mathcal{D}}^A = L_i / \theta f^e$.⁴³ Combining these two

⁴⁰ To elaborate, if preferences are Gorman representable, then $\lambda_{ii, k}(y, \varphi) = \lambda_{ii, k}(y, \varphi')$ for all φ and $\varphi' \in \Phi_k$. Consequently, the quality dimension disappears from the gains from trade equation in this case. This outcome, though, is an artifact of assuming that θ_k is uniform across different quality tiers within the same industry.

⁴¹ An alternative approach would be to use industry-level data and treat select industries as discrete. Under this approach the resulting intuitions would remain qualitatively the same.

⁴² $r_{i, \mathcal{C}}^A = r_{i, \mathcal{C}} = e_{i, \mathcal{C}}$ results from assuming one traded sector and balanced trade.

⁴³ These expressions are derived under the implicit assumption that productivity in the traded sector is sufficiently high, so that domestic discrete varieties are affordable to local consumers irrespective of international trade costs. This assumption ensures that the demand function facing domestic varieties is continuous and well-defined.

⁴⁴ See Appendix 10 for a descriptive list of consumption categories that fit this bill.

expressions, $\hat{N}_{i,D} = N_{i,D}/N_{i,D}^A$ is given by

$$\hat{N}_{i,D} = \sum_{\ell \in \mathbb{C}} \lambda_{i,\ell,D} \frac{Y_\ell}{Y_i} \quad (15)$$

Plugging the above expression back into Eq. (13), yields the following formula for the gains from trade under the discrete interpretation of the economy:

$$GT_i^{\text{Discrete}} = 1 - \lambda_{ii,D}^{\frac{e_{i,D}}{\theta}} \left(\sum_{\ell \in \mathbb{C}} \lambda_{i,\ell,D} Y_\ell / Y_i \right)^{\frac{e_{i,D}}{\theta}}, \quad (16)$$

where $e_{i,D}$ denotes the share of expenditure on categories where discrete goods are bundled with non-traded services in the consumption basket.⁴⁴

Evaluating Eq. (14) is rather straightforward, but mapping Eq. (16) to data involves a couple of auxiliary steps. First, we need to evaluate $\lambda_{ii,D}$, which corresponds to the *quantity* share of domestic consumption. The COMTRADE-BACI database reports import quantities, but measuring $\lambda_{ii,D}$ also requires information on the quantity of domestic consumption. One way to overcome this measurement challenge is to infer $\lambda_{ii,D}$ from the domestic expenditure shares and prices. To demonstrate this, recall that X_{ji} denotes country i 's total spending on tradable varieties from country j , with p_{ji} denoting the corresponding price of these varieties. Given information on $\{X_{ji}\}$ and $\{p_{ji}\}$, we can express $\lambda_{ii,D} = Q_{ii}/\sum_j Q_{ji}$ as follows

$$\lambda_{ii,D} = \frac{X_{ii}/p_{ii}}{X_{ii}/p_{ii} + \sum_{j \neq i} X_{ji}/p_{ji}} = \frac{X_{ii}/X_i}{X_{ii}/X_i + (1 - X_{ii}/X_i) \bar{p}_{-ii}},$$

where $X_i = \sum_j X_{ji}$, while $\bar{p}_{-ii} \equiv \sum_{j \neq i} \frac{X_{ji}}{X_i} p_{ji}$ denotes the (trade-weighted average) unit price of country i 's imports from the rest of the world. To evaluate the above equation, we need data on p_{ii}/\bar{p}_{-ii} and $\{X_{ji}\}$. Data on the former variable is obtainable from the Penn World Tables (PWT), which report aggregate price levels for imports and domestic expenditure. The data on tradable expenditure values, $\{X_{ji}\}$ are taken from the 2008 version of the World Input-Output Database (WIOD, Timmer et al. (2012)).⁴⁵ Combining the aforementioned data, I can measure $\lambda_{ii,D}$ for 31 countries. Likewise, measures for $e_{i,C}$ and $e_{i,D}$ can be constructed with data on $\{X_{ji}\}$ by grouping industries into tradable and non-traded categories—see Table E in the appendix for details.⁴⁶

The second term in Eq. (16), $(\sum_{\ell \in \mathbb{C}} \lambda_{i,\ell,D} Y_\ell / Y_i)^{\frac{e_{i,D}}{\theta}}$, is computed using bilateral trade quantities, $Q_{j\ell}$, from the COMTRADE-BACI database described in Section 5. Specifically, the export quantity share $\lambda_{i,\ell,D}$ is computed as,

$$\lambda_{i,\ell,D} = \frac{Q_{i\ell}}{\sum_{j \neq \ell} Q_{j\ell}} (1 - \lambda_{\ell,\ell,D}).$$

Data on total domestic absorption, Y_i , are taken from the PWT. Plugging values for $\lambda_{i,\ell,D}$ and Y_i into Eq. (16) and combining it with the previously-constructed data for $\lambda_{ii,D}$, allows me to compute the gains from discrete trade for each of the 31 countries in the 2008 WIOD sample. For the same set of countries, I can compute the gains implied by the ACR formula by plugging data values for $e_{i,C}$, $e_{i,D}$, and $\lambda_{ii,C} = X_{ii}/X_i$ into Eq. (14).

6.3.1. Estimating θ

I estimate θ using the triple difference approach developed by Caliendo and Parro (2015). This approach is fully described in Appendix F, and requires data on bilateral trade shares and tariff rates. Trade shares are taken directly from the WIOD. Data on bilateral tariff rates are taken from the United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAITS, see Appendix F). The estimated θ is reported in Table 7. The main difference between the continuous and discrete cases is that under the former the estimating equation reduces to the familiar log-log specification implied by CES-based models. Under the latter, the estimating equation adopts a log-level specification—see Appendix F for further details. These differences notwithstanding, both approaches estimate a relatively similar value for θ , which is in line with exiting estimates from the literature.⁴⁷

⁴⁵ To make the data consistent with theory, I purge it from trade imbalances following the methodology in Costinot and Rodriguez-Clare, (2014).

⁴⁶ Table 8 reports the corresponding values for $e_{i,C}$ and $e_{i,D}$ for each of the 31 countries. Unlike the continuous model that is calibrated to the share of tradable expenditure, the discrete model is calibrated to $e_{i,D}$, which is the sum of expenditure on discrete tradables and matching services. In Appendix G, I check if the discrete model can replicate the (out-of-sample) share of tradable expenditure in isolation. Reassuringly, the model performs relatively well on this front.

⁴⁷ Broda and Weinstein (2006) estimate an import demand elasticity that is analogous to $\theta + 1$. Their product-level estimates exhibit an average value of 6.

Table 7
Estimated θ under discrete and continuous models.

	Discrete model	Continuous model
Estimated θ	4.30 (0.53)	3.63 (0.52)
observations	163,058	

Note: This table reports the estimated θ using the triple differences methodology described in Appendix F. The estimation uses (a) pooled data on trade shares from the 2008 version of the WIOD that span 15 traded industries, and (b) matched tariff data from the UNCTAD-TRAINS database. The first column treats traded industries as discrete. The second columns treats traded industries as continuous. Standard errors are reported in parentheses.

6.4. Quantitative results

The computed gains from trade are reported in Table 8. In summary, the discrete interpretation of the data predicts gains from trade that are on average larger (7.6% versus 5.9%) and more heterogeneous than those implied by the continuous interpretation. These differences are driven by two factors: First, from the view of the discrete model, the gains from trade can be large even if a country spends a small fraction of its income on tradable goods. To elaborate, consider the US and Mexican economies that spend a similar fraction of their income on categories where traded goods are bundled with services. The US economy, though, spends only 27% of its total expenditure on *specifically* traded goods compared to 44% in Mexico. The continuous interpretation of the economy attributes this pattern to the US consumers assigning a lower weight to traded goods in their preferences. This interpretation inevitably predicts relatively small gains for the US. The discrete model, however, attributes the same pattern to the income-inelastic nature of discrete goods and the higher purchasing power of US consumers relative to Mexican Consumers.

Second, the discrete model admits *scale-driven* gains that are highly heterogeneous across countries. They favor small countries (e.g., Belgium and Canada) or low-income countries (e.g., Brazil and Indonesia) that are located in the proximity of larger and richer markets. For instance, the ACR formula predicts that trade increases Brazil's real GDP by only 1.8%. The discrete model, meanwhile, predicts gains of around 6.3% for Brazil, of which 3.6% is purely scale-driven. For Canada, the ACR formula predicts gains of around 5.4%. The discrete model predicts 15% in gains, of which 5.6% is scale-driven. As I will elaborate below, these gains are driven by firm-relocation into these countries from larger and richer trading partners like the US. Correspondingly, the scale-driven gains from trade are negative for the US and stand around -3%.

6.5. Elucidating the scale-driven gains from discrete trade

The scale-driven gains from discrete trade operate through a unique channel that is akin to a *reverse home-market effect*. After opening to trade, firms relocate from richer and larger economies that host their main customer-base to lower-income countries. Doing so exposes them to higher trade costs but allows them to collect a higher markup over marginal cost. To better understand these effects, consider a world economy with two countries: $i = 1, 2$. Suppose the two countries have identical productivity levels and face similar trade costs, but differ in their population sizes:

$$a_{1,D} = a_{2,D} = a; \quad \tilde{a}_1 = \tilde{a}_2 = 1; \quad \tau_{12} = \tau_{21} = \tau > 1; \quad L_2 > L_1.$$

In this basic setup, the free entry condition implies that (see Appendix C.6)

$$N_{i,D} = \sum_{\ell=1,2} \left(\lambda_{i\ell,D} \frac{w_\ell L_\ell}{w_i L_i} \right) \frac{L_i}{\theta f^{\bar{e}}}, \quad i = 1, 2. \quad (\text{Free Entry})$$

Moreover, given that $p_{ij,D} = \tau a w_i + w_j / \theta$, the balanced trade condition can be expressed as follows:

$$\left(\tau a \frac{w_2}{w_1} + \frac{1}{\theta} \right) \lambda_{21,D} w_1 L_1 = \left(\tau a \frac{w_1}{w_2} + \frac{1}{\theta} \right) \lambda_{12,D} w_2 L_2. \quad (\text{Balanced Trade})$$

⁴⁸ We can see the same effect by reformulating Eq. (15) as follows:

$$\hat{N}_{i,D} = \frac{\sum_{\ell \in C} (1 - \mu_{i\ell,D}^{-1}) \omega_{i\ell,D}}{\sum_{\ell \in C} (1 - \mu_{i\ell,D}^{-1}) \lambda_{i\ell,D}},$$

where $\mu_{i\ell,D} \equiv p_{ij,D} / c_{ji,D}$ denotes the multiplicative markup over marginal cost, and $\omega_{i\ell,D}$ denotes the share of market ℓ in country i 's total exports ($\sum_{\ell \in C} \omega_{i\ell,D} = 1$).

Table 8
The gains from trade.

Country	$e_{i,C}$	$e_{i,D}$	GDP	L_i	GT_i continuous	GT_i Discrete	
						Total	Scale-driven
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
AUS	0.25	0.56	0.07	0.07	3.1%	8.4%	3.1%
AUT	0.32	0.64	0.03	0.03	8.1%	8.2%	-5.2%
BEL	0.32	0.63	0.04	0.04	12.7%	25.0%	4.1%
BRA	0.42	0.76	0.63	0.11	1.8%	6.3%	3.6%
CAN	0.31	0.68	0.11	0.10	5.4%	15.0%	5.6%
CHN	0.63	0.79	4.36	0.31	3.2%	2.5%	-0.8%
CZE	0.42	0.69	0.03	0.02	8.6%	13.0%	0.6%
DEU	0.35	0.67	0.27	0.26	6.3%	5.3%	-4.6%
DNK	0.27	0.63	0.02	0.02	7.3%	8.7%	-5.5%
ESP	0.31	0.60	0.15	0.11	4.1%	3.1%	-3.8%
FIN	0.35	0.64	0.02	0.02	4.9%	7.2%	-0.6%
FRA	0.28	0.59	0.21	0.20	4.2%	3.3%	-4.2%
GBR	0.21	0.62	0.20	0.19	4.4%	4.0%	-6.3%
GRC	0.33	0.65	0.04	0.02	5.3%	2.1%	-6.9%
HUN	0.45	0.73	0.03	0.01	11.5%	12.4%	-4.1%
IDN	0.51	0.73	0.77	0.03	3.5%	8.5%	4.3%
IND	0.51	0.76	3.86	0.08	3.2%	2.7%	-1.2%
IRL	0.27	0.58	0.01	0.02	5.8%	5.0%	-5.5%
ITA	0.34	0.65	0.19	0.16	3.7%	2.6%	-3.5%
JPN	0.36	0.66	0.42	0.33	2.2%	0.6%	-2.5%
KOR	0.52	0.77	0.16	0.07	5.4%	3.3%	-3.3%
MEX	0.44	0.70	0.38	0.07	4.9%	6.8%	0.1%
NLD	0.27	0.62	0.05	0.06	8.8%	16.0%	-0.8%
POL	0.39	0.69	0.13	0.04	6.0%	7.5%	-1.9%
PRT	0.33	0.65	0.03	0.02	6.4%	5.9%	-5.2%
RUS	0.42	0.69	0.47	0.11	5.9%	9.1%	5.1%
SVK	0.41	0.71	0.02	0.01	11.5%	17.5%	0.0%
SVN	0.38	0.64	0.01	0.00	11.4%	12.5%	-4.2%
SWE	0.30	0.64	0.03	0.03	6.0%	8.8%	-2.3%
TUR	0.42	0.73	0.23	0.05	3.9%	3.0%	-2.9%
USA	0.27	0.67	1.00	1.00	2.3%	1.5%	-3.0%

Note: $e_{i,C}$ denotes the share of expenditure on traded goods in country i . $e_{i,D}$ denotes the share of expenditure on categories where traded goods are bundled with matching services. See Table 1 in the appendix for details.

Combining the balanced trade and free entry conditions yields the following expression for the mass of firms located in country $i = 1, 2$ (see Appendix C.7):

$$N_{i,D} = \left(1 + \frac{\frac{w_{-i} - w_i}{w_i} - \frac{w_i}{w_{-i}} \lambda_{-ii,D}}{\frac{w_{-i}}{w_i} + \frac{1}{a\tau\theta}} \right) \frac{L_i}{\theta f^e}. \quad (17)$$

Since $N_i^A = L_i/\theta f^e$ denotes the mass of firms under autarky, the above equation states that opening to trade preserves the total mass of firms, globally; but induces firm-relocation from the high- to low-wage country. The intuition is that by relocating to the low-wage country, firms can collect a higher multiplicative markup over marginal cost, as is evident from the optimal price equation, $p_{ij,D} = a\tau w_i (1 + \frac{1}{a\tau\theta} \frac{w_j}{w_i})$.⁴⁸ Furthermore, we can invoke the balanced trade condition to show that the larger economy, in our example, pays a higher wage, i.e., $w_2 > w_1$. In that case, Eq. (17) implies that opening to trade induces firm-relocation from the larger to the smaller country:

$$\hat{N}_{1,D} > 1; \quad \hat{N}_{2,D} < 1.$$

These effects resemble the “big-push” effects of economic development, as formalized by Rosenstein-Rodan (1943), Lewis (1954), and Murphy et al. (1989). To outline this connection, note that the scale of production in small or poor economies is constrained by their inadequate local demand. Opening to trade, however, relocates production activity towards these economies. In doing so, it provides them with the necessary pig-push platform to expand.

7. Concluding remarks

“Why nations trade” and “how much they gain from trade” are perhaps two of the most central questions in the history of economic thought. Economists have historically approached these two questions with trade models that treat all goods as infinitely divisible. This paper asked how our answers to these old questions are revised if we account for the discrete nature of traded goods. In this process, I identified two new drivers of international specialization and pricing-to-market. I further demonstrated that these previously-overlooked forces are empirically significant. Perhaps most importantly, I illustrated that accounting for the discrete or lumpy nature of trade can greatly modify our estimates for the gains from trade.

In the quest to understand discrete trade, this paper only scratched the surface. Given the fruitful nature of this preliminary step, several extensions of the discrete model are in order. First, the model provides a new perspective on the determinants of market power. In that regard, it can potentially shed light on the documented rise of global market power in import-competing industries (see De Loecker and Eeckhout, 2017). Relatedly, the discrete model can provide a fresh perspective on the trade slowdown and zeros. To elaborate on the latter subject, there is a withstanding puzzle as to why the incidence of zero trade is related to income per capita. Appealing to affordability constraints can perhaps help resolve this puzzle. Finally, in the interest of space, the present paper resorted to quantifying the aggregate gains from discrete trade. Applying the discrete trade framework to micro-level expenditure data presents a promising avenue for future research on the distributional consequences of trade.

Appendix A. The firm-level colombian export and import data

The firm-level data used in Section 2 covers the universe of Colombian import and export transactions for the 2007–2013 period. The data has been collected and made available by the National Tax Agency. For each import transaction, it identifies the exporting and importing firm's id, the 10-digit Harmonized System (HS10) classification to which the imported or exported goods belong, as well as the f.o.b. value in US dollars, quantity, and the unit in which quantity is reported.

The import database features 7296 distinct HS10 product categories, and 226,288 firms from 251 different countries. Each of these import transactions is conducted by Colombian firm that is identified by their unique 9-digits tax id. In total there are 95,071 of such importers, most of which import a wide range of discrete and non-discrete HS10 products. The export database features 6590 distinct HS10 product categories sold to 231 different markets. Each of these export transactions is conducted by Colombian firm that is identified by their unique 9-digits tax id. In total there are 33,075 exporting firms in the database, many of which export a wide range of discrete and non-discrete HS10 products to various markets.

The Colombia import and export data report quantity in 10 different units. The vast majority of entries report quantity either in counts, “UNIDADES O ARTICULOS”, or in kilograms, “KILOGRAMO.” 72% of all observations in the import data (51% in value terms) involve goods that report quantity in counts. In comparison, 63% of all observations in the export data (12% in value terms) involve goods that report quantity in counts. I classify such goods as *discrete* and classify all other goods as *continuous*.

Appendix B. Non-parametric model

In this appendix, I present a general non-parametric version of the model, which features the following elements: (i) the demand-side is governed by a general non-parametric discrete choice framework that can reproduce an important class of continuous and discrete demand functions, (ii) income heterogeneity is governed by a non-parametric endowment distribution, $G_i(y)$, and (iii) the supply-side is identical to the baseline model.

B.1. General discrete choice preference structure

Demand in all industries is characterized by a general discrete choice framework that can reproduce an important class of well-known demand systems. As in the baseline model, the economy is populated with $k = 1, \dots, K$ industries, with e_k denoting the constant share of expenditure on industry k . Unless stated otherwise, the notation is similar to the one used in Section 3. The utility individual ι extracts from choosing variety $\omega \in \Omega_{j,k}(\varphi)$ and consuming $q_{j,k}(\omega)$ units of it plus \tilde{q} hours of industry k -related services is given by:

$$U_{\iota}(q(\omega), \tilde{q}; \varphi) = \mathcal{U}_{\iota}(\tilde{q}, q(\omega); \varphi) + \xi_{\iota}(\omega), \quad \omega \in \Omega_{j,k}(\varphi, y)$$

where $\xi_{\iota}(\omega)$ is an *individual* × *variety*-specific utility shifter that accounts for individual ι 's personal taste for variety ω . $\Omega_{j,k}(\varphi, y) \subset \Omega_{j,k}(\varphi)$ denotes the subset of varieties affordable to an individual with endowment y . An individual residing in country i with endowed y_{ι} has a budget equal to $e_k y_{\iota} w_i$. The discrete choice problem facing them, therefore, can be stated as

$$\max_{q(\omega), \tilde{q}} \mathcal{U}_{\iota}(\tilde{q}, q(\omega); \varphi) + \xi_{\iota}(\omega)$$

⁴⁹ Proof provided in Appendix C.1.

$$\text{s.t. } \tilde{p}_{i,k} \tilde{q} + p_{ji,k}(\omega) q(\omega) \leq e_k y_i w_i .$$

The above problem can be solved in two separate steps. First, conditional on choice ω , the optimal choice for $q(\omega)$ and \tilde{q} can be attained by maximizing $\mathcal{U}_k(\cdot)$ subject to the budget constraint. This stage delivers an indirect utility function $V_k(yw_i, \tilde{p}_{i,k}, p_{ji,k}(\omega))$, which can be used to solve the second stage, which is choosing the optimal variety ω given the menu of prices and individual taste. Namely,

$$\max_{\omega} V_k(yw_i, \tilde{p}_{i,k}, p_{ji,k}(\omega); \varphi) + \xi_k(\omega). \tag{B.1}$$

To simplify the notation, I define $\mathcal{V}_k(\cdot) \equiv \exp(V_k(\cdot))$. Moreover, to decouple the role of *horizontal* and *vertical* differentiations, I assume that $\mathcal{V}_k(\cdot)$ is separable in quality:

$$\mathcal{V}_k(yw_i, \tilde{p}_{i,k}, p_{ji,k}(\omega); \varphi) = v_k(yw_i, \tilde{p}_{i,k}, p_{ji,k}(\omega)) f_k(\varphi)$$

As in the baseline model, $\xi_{k, k} \equiv \{\xi_k(\omega)\}$ is drawn independently from a General Extreme Value distribution, $F_k(\xi_k) = \exp(-\sum_{\varphi \in \Phi_k} (\sum_{\omega} \exp(\xi(\omega))^{-\theta_k})^{\gamma_k/\theta_k})$, where $\theta_k \geq \gamma_k \geq 0$. With this assumption, we can invoke the theorem of General Extreme Value (McFadden et al. (1978); Ben-Akiva et al. (1985)) to produce the following lemma about individual-level and market-level demand functions.⁴⁹

Lemma 1. *The probability of choosing variety $\omega \in \Omega_{i,k}(\varphi, y)$ by an individual with is*

$$\lambda_k(y, w_i, p_{ji,k}(\omega), \varphi, \tilde{p}_{i,k}, \mathbf{P}_i(y)) = v_k(yw_i, p_{ji,k}(\omega), \tilde{p}_{i,k}; \varphi)^{\theta_k} \frac{P_{i,k}(\varphi, y)^{\theta_k - \gamma_k}}{\sum_{\varphi' \in \Phi_k} P_{i,k}(\varphi', y)^{-\gamma_k}},$$

where $P_{i,k}(\varphi, y) \equiv [\sum_{j \in C} \sum_{\omega \in \Omega_{i,k}(\varphi, y)} v_k(yw_i, p_{ji,k}(\omega), \tilde{p}_{i,k}; \varphi)^{\theta_k}]^{-1/\theta_k}$ is an income- and quality-specific demand shifter. Accordingly, the market-level Marshallian demand facing variety $\omega \in \Omega_{i,k}(\varphi, y)$ can be expressed as⁵⁰

$$q_k(w_i, p_{ji,k}(\omega), \varphi, \tilde{p}_{i,k}, P_i, G_i) = \left[\int_y D_k(yw_i, p_{ji,k}(\omega), \varphi, \tilde{p}_{i,k}) \lambda_k(y, p_{ji,k}(\omega), \varphi, \tilde{p}_{i,k}, P_i(y)) dG_i(y) \right] L_i$$

where $\mathcal{D}_k(\cdot)$ is the demand function associated with $V_k(\cdot)$.

In the above lemma, $\mathbf{P}_{i,k}(y) = \{P_{i,k}(\varphi, y)\}$ and $\mathbf{P}_{i,k} = \{\mathbf{P}_{i,k}(y)\}$ denote the vector of demand shifters for various quality levels and income groups. Likewise, I use $i_{i,k}(y) \equiv \{\Omega_{i,k}(\varphi, y)\}_{\varphi \in \Phi_k}$ to denote the complete set of varieties (from all quality levels) affordable to individuals with endowment y in market i .

At this point, it is useful to formally define homothetic preferences from the lens of the model. I say that individual-level preferences are homothetic iff

$$\partial \lambda_k(y, w_i, p_{ji,k}(\omega), \varphi, \tilde{p}_{i,k}, \mathbf{P}_i(y)) / \partial y = 0,$$

provided that $\partial i_{i,k}(y) / \partial y = 0$. The above definition outlines two sources of income-driven variation in individual-level demand. First, conditional on choosing from the same set of affordable varieties, $\lambda_k(\cdot)$ may vary with y due to the functional-form of the indirect utility function, $\mathcal{V}_k(\cdot)$. Second, $\lambda_k(\cdot)$ may vary with y due to the expansion of set $i_{i,k}(y)$ with y .⁵¹ In what follows, I apply the demand system characterized by Lemma 1 to two special cases: (i) continuous industries, and (ii) discrete industries.

B.1.1. Continuous industries

Continuous industries involve goods that are (i) infinitely divisible, and (ii) not directly substitutable with services. As a result, all continuous goods are affordable to all individuals by assumption, i.e., $\Omega_{i,k}(\varphi, y) = \Omega_{i,k}(\varphi)$ for all y . I implement these features in

⁵⁰ In order to make the notation more compact, I use $\mathbf{P}_i(y)$ instead $\mathbf{P}_i(yw_i)$.
⁵¹ Following the chain rule, if $\lambda_k(\cdot)$ is invariant to income, so is the individual-level demand elasticity, $\varepsilon_k(\cdot) \equiv \partial \ln \lambda_k(\cdot) / \partial \ln p_{ji,k}(\omega)$. Likewise, if preferences are homothetic (as defined above) the share of individuals that chose a variety with quality φ , namely, $\Lambda_{i,k}(y, \varphi) \equiv P_{i,k}(\varphi, y)^{-\gamma_k} / \sum_{\varphi' \in \Phi_k} P_{i,k}(\varphi', y)^{-\gamma_k}$, does not vary with y .
⁵² To be clear, the price of non-traded services does not enter the indirect utility function, because of the implicit assumption that continuous goods are not directly substitutable with services.
⁵³ See Bertoletti et al. (2018) for an elaborate review of indirectly additive preferences.

my general discrete choice model by assuming that individuals spend their entire budget on buying multiple units of their preferred variety: $q(\omega) = e_k y w_i / p_{j_i,k}(\omega)$. As we will see shortly, at the market-level, this choice structure is isomorphic to individuals having indirectly additive preferences and buying a fractional quantity of multiple varieties. The indirect utility, therefore, can be formulated as⁵²

$$v_k(\cdot) = v_k(e_k y w_i / p_{j_i,k}(\omega)) f_k(\varphi), \quad \omega \in \Omega_k(\varphi).$$

Plugging the above characterization of $v_k(\cdot)$ into Lemma 1, the probability that a consumer with income y in country i chooses variety $\omega \in \Omega_{j_i,k}(\varphi)$ is given by

$$\lambda_k(y, e_k w_i / p_{j_i,k}(\omega), \varphi, \mathbf{P}_{i,k}(y)) = v_k(y e_k w_i / p_{j_i,k}(\omega)) \frac{P_{i,k}(\varphi, y)^{\theta_k - \gamma_k}}{\sum_{\varphi' \in \Phi_k} P_{i,k}(\varphi', y)^{-\gamma_k}},$$

where $P_{i,k}(\varphi, y) = f_k(\varphi) [\sum_{j \in C} \sum_{\omega \in \Omega_{j,k}(\varphi)} v_k(y e_k w_i / p_{j_i,k}(\omega))^{\theta_k}]^{\frac{-1}{\theta_k}}$. Another characteristic of continuous industries is that all varieties are accessible to all consumers—i.e., $\Omega_{i,k}(y, \varphi) = \Omega_{i,k}(\varphi)$ for all y . Correspondingly, the market-level demand facing firm ω in market i can be calculated as

$$q_k(e_k w_i / p_{j_i,k}(\omega), \varphi, \mathbf{P}_{i,k}, G_i) = \left[\int_y (y e_k w_i / p_{j_i,k}(\omega)) \lambda_k(y, e_k w_i / p_{j_i,k}(\omega), \varphi, \mathbf{P}_{i,k}(y)) dG_i(y) \right] L_i, \tag{B.2}$$

where the above expression uses the fact that $D_k(\cdot) \equiv y e_k w_i / p_{j_i,k}(\omega)$. The above demand function is identical to that arising from indirectly additive preferences.⁵³ This class of preferences includes the CES demand system presented in Section 3 as a special case, where $v_k(x) = x$. The above demand function immediately characterizes the market-level price elasticity of demand, both in the general case and in the special case where preferences are homothetic (i.e., CES).

Lemma 2. [Demand Elasticity in Continuous Industries]

(a) The market-level Marshallian demand elasticity facing firm $\omega \in \Omega_{i,k}(\varphi)$ depends primarily on the average consumption level in market i (i.e., $e_k w_i / p_{j_i,k}(\omega)$):

$$\varepsilon_{j_i,k}(\omega) = \varepsilon_k(e_k w_i / p_{j_i,k}(\omega), \varphi; \mathbf{P}_{i,k}, G_i), \quad k \in \mathbb{K}_C.$$

(b) In the case where preferences are homothetic (i.e., $x v_k'(x) / v_k(x) = 1$), the market-level Marshallian demand elasticity is constant: $\varepsilon_{j_i,k}(\omega) = 1 + \theta_k$.⁵⁴

A formal proof for the above lemma is provided in Appendix C.2. But the basic intuition behind Lemma 2 can be provided by focusing on the case without income heterogeneity. In this special case, the market-level demand elasticity is given by

$$\varepsilon_k(\bar{q}_{j_i,k}(\omega), \varphi) = 1 + \theta_k \frac{\bar{q}_{j_i,k}(\omega) v_k'(\bar{q}_{j_i,k}(\omega))}{v_k(\bar{q}_{j_i,k}(\omega))}.$$

where $\bar{q}_{j_i,k}(\omega) \equiv e_k w_i / p_{j_i,k}(\omega)$. The above formula implies that the main source of variation in the market-level demand elasticity is the level of consumption. Moreover, $\varepsilon_k(\cdot)$ is strictly increasing in the average consumption level, \bar{q} , as long as individual-level demand satisfies the sub-convexity assumption, $\frac{\partial q v_k'(q)}{\partial q v_k(q)} < 0$, which is also known as Marshall's second law of demand (Mrázová and Peter Neary (2017)). The above property is at the core of many existing theories of PTM and IQS. That is, as individuals become richer and consume more, they become less price-sensitive. Correspondingly, firms charge higher markups in

⁵⁴ More generally, individual-level preferences are invariant to income if $x v_k'(x) / v_k(x) = \text{constant}_k$; but here to simplify the exposition focus on the case where $x v_k'(x) / v_k(x) = 1$.

⁵⁵ There is another well-known class of continuous demand functions, popularized by Arkolakis et al. (2019) (ACDR), where the market-level demand elasticity varies not directly as a function of individual consumption, but as function of price $p_{j_i,k}(\omega)$ relative to a market-level choke price. The ACDR class of preferences is closely related to the indirectly additive preferences nested by the present model. However, to formally reproduce the ACDR class of demand systems with a discrete choice framework, one has to impose stronger functional form assumptions on $v_k(\cdot)$ —see Thisse and Ushchev (2016) who reproduce the ACDR demand system using a multinomial logit framework.

⁵⁶ Theoretically, the choice of forgoing a discrete purchase can be thought of as follows. There is a choice category $\varphi \in \Phi_k$ (in addition to all existing categories), which consists of homogeneous varieties (i.e., $\theta \rightarrow \infty$ in this particular category). Each variety pertaining to this category is priced competitively at $\bar{p}_{i,k}$. Hence, choosing a variety from this category is akin to spending one's entire budget on services.

markets that are populated with high-income, less price-sensitive individuals. The special case where preferences are homothetic (i.e., $qv_k'(q)/v_k(q) = 1$), is isomorphic to the CES model. The fact that the homotheticity and constant elasticity of market-level demand coincide, is a basic corollary of Bergson's theorem. This result is significant, as it states that the market-level demand for continuous goods varies across low- and high-wage markets only and only if individual-level preferences are non-homothetic.⁵⁵

B.1.2. Discrete industries

Discrete industries involve goods that are (i) indivisible, and (ii) substitutable with services. I implement these features in my general discrete choice model by assuming that individuals acquire one unit of their preferred discrete variety, and spend the rest of their budget on corresponding services. They can also forgo paying an acquisition cost by spending their entire budget on services.⁵⁶ Considering this, an individual with endowment y who opts for variety, $\omega \in \Omega_k(\varphi, y)$, purchases one unit (i.e., $q(\omega) = 1$) and is left with $e_k y w_i - p_{ji,k}(\omega)$ to spare on services. The indirect utility associated with $\mathcal{U}_k(\cdot)$ can, therefore, be expressed as

$$v_k(\cdot) = v_k\left(\left[e_k w_i y - p_{ji,k}(\omega)\right] / \tilde{p}_{i,k}\right) f_k(\varphi), \quad \omega \in \Omega_{i,k}(y, \varphi)$$

Plugging the above formulation into Lemma 1, implies that the individual-level probability of choosing variety $\omega \in \Omega_{i,k}(y, \varphi)$ is given by

$$\lambda_k\left(y, p_{ji,k}(\omega) / \tilde{p}_{i,k}, \varphi, \mathbf{P}_{i,k}(y)\right) = v_k\left(\tilde{e}_{i,k} y - \frac{p_{ji,k}(\omega)}{\tilde{p}_{i,k}}\right)^{\theta_k} \frac{P_{i,k}(\varphi, y)^{\theta_k - \gamma_k}}{\sum_{\varphi' \in \Phi_k} P_{i,k}(\varphi', y)^{-\gamma_k}}, \tag{B.3}$$

where $\tilde{e}_{i,k} \equiv e_k / \tilde{a}_{i,k}$, and $P_i(\varphi, y) = f_k(\varphi) [\sum_{j \in C} \sum_{\omega \in \Omega_{j,k}(y, \varphi)} v_k(\tilde{e}_{i,k} y - p_{ji,k}(\omega) / \tilde{p}_{i,k})^{\theta_k}]^{\frac{1}{\theta_k}}$. Noting that $\mathcal{D}_k(\cdot) = 1$, the market-level demand facing variety ω can, therefore, be expressed as

$$q_k\left(p_{ji,k}(\omega) / e_k w_i, p_{ji,k}(\omega) / \tilde{p}_{i,k}, \varphi, \mathbf{P}_{i,k}, G_i\right) = \left[\int \frac{p_{ji,k}(\omega)^\infty}{e_k w_i} \lambda_k\left(y, p_{ji,k}(\omega) / \tilde{p}_{i,k}, \varphi, \mathbf{P}_{i,k}(y)\right) dG_i(y) \right] L_i. \tag{B.4}$$

where, unlike the continuous case, the above aggregation takes into account that variety ω is affordable only to consumers with an endowment $y \geq p_{ji,k}(\omega) / e_k w_i$.

Before moving forward, it should be noted that the above framework differs from existing discrete choice trade models (e.g., Verhoogen (2008) and Fajgelbaum et al. (2011)) in two key aspects. First, the present setup allows for discrete goods to be combined with "non-traded" services, whereas prior discrete choice models do not feature a non-traded sector.⁵⁷ Second, the present model allows for affordability constraints to bind. Using Eq. (B.4), I can derive the market-level demand elasticity both in the general case and in the homothetic case.

Lemma 3. [Demand Elasticity in Discrete Industries]

(a) The market-level Marshallian demand elasticity facing firm $\omega \in \Omega_{i,k}(\varphi)$ is primarily a function of its overall affordability (i.e., $p_{ji,k}(\omega) / e_k w_i$) and its price relative to non-traded services (i.e., $p_{ji,k}(\omega) / \tilde{p}_{i,k}$):

$$\varepsilon_{ji,k}(\omega) = \varepsilon_k\left(p_{ji,k}(\omega) / e_k w_i, p_{ji,k}(\omega) / \tilde{p}_{i,k}, \varphi; \mathbf{P}_i, G_i\right), \quad k \in \mathbb{K}_{\mathcal{D}}.$$

(b) In the case where preferences are homothetic (i.e., $v_k'(x) / v_k(x) = 1$), the market-level demand elasticity is given by

$$\varepsilon_{ji,k}(\omega) = \theta_k p_{ji,k}(\omega) / \tilde{p}_{i,k} + \rho_{ji,k}(\omega) \mathcal{H}_i\left(p_{ji,k}(\omega) / e_k w_i\right) p_{ji,k}(\omega) / e_k w_i, \quad k \in \mathbb{K}_{\mathcal{D}}.$$

⁵⁷ To be specific, the present framework can replicate Verhoogen, 2008 under the following assumptions: (i) $v(x) = e^{\alpha x}$ and $f(x) = e^{\beta x}$; (ii) all varieties are affordable; (iii) services are costlessly traded across markets, i.e., $\tilde{p}_i = 1$ for all i (Verhoogen (2008) though allows $f(\cdot)$ to be market-specific). Likewise, the present framework can replicate the framework in Fajgelbaum et al. (2011) under the following assumptions: (i) $V(y, \tilde{p}_i, p_{oi}, \varphi) = \exp(\varphi(y - p_{oi}) / \tilde{p}_i)$, (ii) all varieties are affordable, and (iii) services are costlessly traded across markets, i.e., $\tilde{p}_i = 1$ for all i . Note, however, that the framework in Fajgelbaum et al. (2011) does not satisfy Assumption A1, which as stated earlier is not consequential to any of our claims that follow.

⁵⁸ IGFR stands for Increasing Generalized Failure Rate, which is satisfied by nearly all empirically-relevant income distributions (see Lariviere and Porteus (2001) and Lariviere (2006)).

⁵⁹ In the discrete case the existence of a solution is implied by the fact that $e_k(\cdot)$ is increasing in $v_{ji,k}(\varphi)$.

where $\rho_{ji,k}(\omega)$ reflects the importance of the marginal consumers and $\mathcal{H}_i(\cdot)$ denotes the hazard rate of the income distribution in market i .

The above lemma, which is proven in [Appendix C.3](#), indicates that one source of cross-market variation in $\varepsilon_k(\cdot)$ is the functional-form of $v_k(\cdot)$. Specifically, if $v_k'(x)/v_k(x) \neq 1$, preferences will be non-homothetic and will naturally produce cross-market heterogeneity in demand. The demand elasticity corresponding to the homothetic case differs from the formula presented in [Section 3](#), because it is derived under an arbitrary endowment distribution, $G_i(y)$. The empirically-relevant case, though, is one where the income distribution satisfies the IGFR property.⁵⁸ In that case, $\mathcal{H}_i(x)x$ is increasing and $\varepsilon_{ji,k}(\omega)$ is lower the more the affordable variety ω . The term $\rho_{ji,k}(\omega)$, is the purchase probability of variety ω 's marginal consumers relative to its average consumer. Theoretically, $\rho_{ji,k}(\omega)$ may be greater than 1, even when preferences are homothetic. That is because the marginal consumer buying ω may be choosing from a narrower set of affordable varieties. However, for any variety ω , we can set $\rho_{ji,k}(\omega) \approx 1$ to a first-order approximation if (i) $v_k'(\cdot)/v_k(\cdot) = 1$, and (ii) $\Omega_{i,k}(\varphi)$ is composed of a discrete set of varieties.

B.2. Equilibrium

Given [Lemmas 2 and 3](#), the demand elasticity facing variety ω in market i is uniquely determined by $p_{ji,k}(\omega)/w_i$ plus market-level demand shifters, $\mathbf{P}_{i,k}$. The symmetry of firms within economy j entails that they all charge the same price $p_{ji,k}(\varphi)$ for industry k goods as a function of quality φ . Defining $v_{ji,k}(\varphi) \equiv p_{ji,k}(\varphi)/w_i$, we can express the optimal monopoly pricing equation as follows⁵⁹:

$$v_{ji,k}(\varphi) = \frac{\varepsilon_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) \tau_{ji} a_{j,k} c(\varphi) w_j}{\varepsilon_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) - 1} \frac{1}{w_i}$$

where $\mathbf{P}_{i,k} = \mathbf{P}_{i,k}(\mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k})$ is uniquely determined by $\mathbf{v}_{i,k} \equiv \{v_{ji,k}(\varphi)\}_{\omega \in \Omega_k(\varphi), \varphi \in \Phi_k}$, the vector of aggregate wage rates, $\mathbf{w} = \{w_i\}_i$, and number of firms $\mathbf{N}_k = \{N_{i,k}(\varphi)\}_{i \in \mathcal{I}, \varphi \in \Phi_k}$. Similarly, given $\mathbf{v}_{i,k}$, \mathbf{w} , and \mathbf{N}_k we can uniquely pin down the matrix of industry-level bilateral trade values as

$$X_{j,i,k}(\varphi, \mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k}, G_i) = v_{ji,k}(\varphi) N_{j,k}(\varphi) \bar{\lambda}_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) Y_i \tag{B.5}$$

where $\bar{\lambda}_k(\cdot) \equiv q_k(\cdot)/L$ denotes the market-level probability of choosing an imported variety. Subsequently, given the matrix $\mathbf{X}_k \equiv \{X_{ji,k}(\varphi, \mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k}, G_i)\}_{j, i \in \mathcal{I}, \varphi \in \Phi_k}$, we can calculate the variable profits in each country and industry.

So, altogether, we can define the equilibrium as a vector of normalized prices $\mathbf{v}_{i,k}$, wage rates \mathbf{w} , number of firms \mathbf{N}_k , and market-specific demand shifters, $\mathbf{P}_{i,k} = \mathbf{P}_{i,k}(\mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k})$, that satisfy (i) the optimal pricing equation, (ii) the balanced trade condition (BT), and (iii) the zero-profit condition (FE):

$$\begin{cases} v_{ji,k}(\varphi) = \frac{\varepsilon_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) \tau_{ji} a_{j,k} c(\varphi) w_j}{\varepsilon_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) - 1} \frac{1}{w_i} & \forall \varphi \in \Phi_k, k \in \mathcal{K} \\ \sum_j \sum_k \sum_{\varphi \in \Phi_k} X_{ji,k}(\varphi, \mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k}, G_i) = \sum_{i \in \mathcal{I}} \sum_k \sum_{\varphi \in \Phi_k} X_{i\ell,k}(\varphi, \mathbf{w}, \mathbf{N}_k, \mathbf{v}_{i,k}, G_i) & \forall k \in \mathcal{K} \quad (\mathbf{BT}), \\ N_{j,k}(\varphi) = \left\{ \sum_i \frac{v_{ji,k}(\varphi) \bar{\lambda}_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i) Y_i}{\varepsilon_k(v_{ji,k}(\varphi), \varphi, \mathbf{P}_{i,k}, G_i)} \frac{L_j}{Y_j} \right\} \frac{1}{f_k^e(\varphi)} & \forall k \in \mathcal{K} \quad (\mathbf{FE}) \end{cases}$$

By Walras' law, the satisfaction of the BT and FE conditions ensures that labor markets also clear in each country.

B.3. IQS and PTM: discrete vs. continuous industries

Recall that, when preferences are homothetic, the demand elasticity is (a) constant and uniform across markets in continuous industries, but (ii) variable and heterogeneous across markets in discrete industries. This outcome immediately implies that we can produce the following analog of [Proposition 1](#), using the general non-parametric model.

Proposition 4. (a) *Non-homothetic preferences and Ricardian technology differences are the only drivers of international quality specialization and pricing-to-market in continuous industries; but (b) in discrete industries, international quality specialization and pricing-to-market may be alternatively driven by affordability constraints and cross-national heterogeneity in the price of non-traded services.*

The proof of the above theorem follows trivially from [Lemmas 2 and 3](#), but needs a basic qualification. There exist a class of continuous preferences that are *not* indirectly additive, but are homothetic and imply a non-constant demand elasticity, e.g., Kimball preferences. In that case, the above proposition should be restated as follows: PTM may occur under homothetic preferences, but not PTM as a function of income per capita, which is the focus of this paper.

Appendix C. Proofs and derivations

C.1. Demand shares under general discrete choice preferences

Suppose the utility of the individual ι with income Y can be expressed as

$$U_\iota(\omega) = \ln V_k(Y, p_{j,i,k}(\omega), \tilde{p}_{i,k}) + \xi_\iota(\omega), \quad \omega \in \Omega_{j,k}(\varphi)$$

In this appendix, I invoke the theorem of “general extreme value” to characterize the share of consumers who choose variety ω as their preferred variety. To this end, define $H_k(\mathbf{V})$ as follows

$$H_k(\mathbf{V}) = \sum_{\varphi \in \Phi_k} \left(\sum_{j \in \mathbb{C}} \sum_{\omega \in \Omega_{j,k}(\varphi, Y)} A(\varphi) V(\omega)^{\theta_k} \right)^{\gamma_k / \theta_k}$$

where $\mathbf{V} \equiv \{V(\omega)\}$, with $V(\omega) \equiv V_k(Y, p_{j,i,k}(\omega), \tilde{p}_{i,k})$. Note that $H_k(\cdot)$ is a continuous and differentiable function of vector \mathbf{V} and has the following properties:

$$H_k(\cdot) \geq 0;$$

- (i) $H_k(\cdot)$ is a homogeneous function of rank θ_k ; $H_k(\rho \mathbf{V}) = \rho^{\gamma_k} H_k(\mathbf{V})$; $\lim_{V(\omega) \rightarrow \infty} H_k(\mathbf{V}) = \infty, \forall \omega \in \cup_{j \in \mathbb{C}} \Omega_{j,k}$;
- (ii) the m 'th partial derivative of $H_k(\cdot)$ with respect to a generic combination of m variables $V(\omega)$, is non-negative if m is odd and non-positive if m is even.

Following Ben-Akiva et al. (1985), if $\xi_\iota(\omega)$'s (the idiosyncratic taste parameters) are drawn from the following distribution

$$F_k(\xi) = \exp(-H_k(\exp(\xi))) = \exp\left(-\sum_{\varphi \in \Phi_k} \left(\sum_{j \in \mathbb{C}} \sum_{\omega \in \Omega_{j,k}(\varphi, Y)} A(\varphi) \exp(\xi(\omega))^{\theta_k} \right)^{\gamma_k / \theta_k}\right),$$

then the probability of choosing variety $\omega \in \Omega_{i,k}(\varphi, Y)$, namely, $\lambda(\omega) \equiv \lambda(Y, \tilde{p}_{i,k}, p_{\omega i}, \varphi)$ is given by

$$\begin{aligned} \lambda(\omega) &= \frac{V(\omega) \frac{\partial H_k(\mathbf{V})}{\partial V(\omega)}}{\gamma_k H_k(\mathbf{V})} = \frac{A(\varphi) V(\omega)^{\theta_k} \left(\sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{j,k}(\varphi, Y)} A(\varphi) V(\omega')^{\theta_k} \right)^{\frac{\gamma_k}{\theta_k} - 1}}{\sum_{\varphi' \in \Phi_k} \left(\sum_{\omega' \in \Omega_{i,k}(\varphi', Y)} A(\varphi') V(\omega')^{\theta_k} \right)^{\frac{\gamma_k}{\theta_k}}} \\ &= \frac{V(\omega)^{\theta_k}}{\sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{j,k}(\varphi, Y)} V(\omega')^{\theta_k}} \cdot \frac{\left(A(\varphi) \sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{j,k}(\varphi, Y)} V(\omega')^{\theta_k} \right)^{\frac{\gamma_k}{\theta_k}}}{\sum_{\varphi' \in \Phi_k} \left(A(\varphi') \sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{j,k}(\varphi', Y)} V(\omega')^{\theta_k} \right)^{\frac{\gamma_k}{\theta_k}}} \end{aligned}$$

The expression for $g_k(\cdot)$ follows trivially from the above expression by integrating over all consumers in market i .

C.2. Demand elasticity in continuous industries

This appendix derives the demand elasticity in continuous industries, which amounts to proving Lemma 2 from Appendix B. Claim (a) in Lemma 2 can be proven by taking the derivative of $q_k(\cdot)$ w.r.t. $p_{ji,k}(\omega)$:

$$\begin{aligned} \frac{\partial q_k(p_{ji,k}(\omega), \dots)}{\partial p_{ji,k}(\omega)} &= \frac{\partial}{\partial p_{ji,k}(\omega)} \left[\int_y \mathcal{D}_k(e_k y w_i, p_{ji,k}(\omega)) \lambda_k(y, e_k w_i / p_{ji,k}(\omega), \varphi, \mathbf{P}_{i,k}(y)) dG_i(y) \right] L_i \\ &= \left[\int_y \left(\frac{\partial \mathcal{D}_k(\cdot)}{\partial p_{ji,k}(\omega)} \lambda_k(y, e_k w_i / p_{ji,k}(\omega), \varphi; \mathbf{P}_{i,k}(y)) + \mathcal{D}_k(e_k y w_i, p_{ji,k}(\omega)) \frac{\partial \lambda_k(\cdot)}{\partial p_{ji,k}(\omega)} \right) dG_i(y) \right] L_i \\ &= - \left[\int_y \frac{1}{p_{ji,k}(\omega)} \left(1 + \theta_k \frac{e_k y w_i v'_k(e_k y w_i / p_{ji,k}(\omega))}{p_{ji,k}(\omega) v_k(e_k y w_i / p_{ji,k}(\omega))} \right) \mathcal{D}_k(e_k y w_i, p_{ji,k}(\omega)) \lambda_k(\cdot) dG_i(y) \right] L_i \\ &= - \frac{q_k(\cdot)}{p_{ji,k}(\omega)} \left[1 + \theta_k \int_y \frac{e_k y w_i v'_k(y w_i / p_{ji,k}(\omega))}{p_{ji,k}(\omega) v_k(y w_i / p_{ji,k}(\omega))} dG_{i,k}(y, w_i / p_{ji,k}(\omega), \varphi; \mathbf{P}_i) \right], \end{aligned}$$

where $G_{i,k}$ denotes the demand-weighted income distribution, which augments the actual income distribution, $G_i(\cdot)$, by weighing each income group by their demand:

$$dG_{i,k}(y, w_i / p_{ji,k}(\omega), \varphi; \mathbf{P}_i) \equiv \frac{\lambda_k(y, e_k w_i / p_{ji,k}(\omega), \varphi, \mathbf{P}_i(y)) dG_i(y)}{\int_{y'} \lambda_k(y', e_k w_i / p_{ji,k}(\omega), \varphi, \mathbf{P}_i(y')) \cdot dG_i(y')}$$

Using the above expressions, we can immediately calculate the demand elasticity, $\varepsilon_k(\cdot) = 1 + \partial \ln q_k(\cdot) / \partial \ln p_{ji,k}(\omega)$, as follows:

$$\varepsilon_k(e_k w_i / p_{ji,k}(\omega), \varphi; \mathbf{P}_i, G_i) = 1 + \theta_k \left[\int_y \frac{v'_k(y e_k w_i / p_{ji,k}(\omega))}{y v_k(y e_k w_i / p_{ji,k}(\omega))} y dG_{i,k}(y; e_k w_i / p_{ji,k}(\omega), \varphi) \right] \frac{e_k w_i}{p_{ji,k}(\omega)}.$$

The above equation clearly states that apart from the aggregate demand shifters, \mathbf{P}_i , the income distribution, $G_i(\cdot)$, and the quality level, φ , which applies to all firms in set $\Omega_{i,k}(\varphi)$, $\varepsilon_k(\cdot)$ only depends on the average demand level, $e_k w_i / p_{ji,k}(\omega)$.

To prove Claim (b), I first need to characterize the conditions under which $\lambda_k(\cdot)$ is invariant to income. To this end, defining $x_{ji,k}(\omega) \equiv e_k y / p_{ji,k}(\omega)$, we can take the derivative of $\lambda_k(\cdot)$ as follows:

$$\begin{aligned} \frac{\partial \ln \lambda_k(x_\omega, \varphi, \mathbf{P}_i)}{\partial \ln y} &= \theta_k \left(\frac{\partial \ln v_k(x_{ji,k}(\omega))}{\partial \ln y} - \sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{jk}(y, \varphi)} \frac{\partial \ln v_k(x_{ji,k}(\omega')) \lambda_k(x_{ji,k}(\omega'), \varphi, \mathbf{P}_i)}{\partial \ln y \Lambda_k(\varphi, y)} \right) \\ &\quad - \gamma_k \left[\sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{jk}(y, \varphi)} \frac{\partial \ln v_k(x_{ji,k}(\omega')) \lambda_k(x_{ji,k}(\omega'), \varphi, \mathbf{P}_i)}{\partial \ln y \Lambda_k(\varphi, y)} \right. \\ &\quad \left. - \sum_{\varphi' \in \Phi_k} \sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{jk}(y, \varphi')} x_{ji,k}(\omega') \frac{\partial \ln v_k(x_{ji,k}(\omega'))}{\partial \ln y} \lambda_k(x_{ji,k}(\omega'), \varphi', \mathbf{P}_i) \right], \end{aligned} \tag{C.1}$$

where $\Lambda_k(\varphi, y) \equiv P_{i,k}(\varphi, y)^{-\gamma_k} / \sum_{\varphi' \in \Phi_k} [P_{i,k}(\varphi', y)^{-\gamma_k}]$ denotes the share of consumers choosing quality φ . Given that the probability shares add up to one, i.e.,

$$\sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{jk}(y, \varphi)} \lambda_k(x_{ji,k}(\omega'), \varphi, \mathbf{P}_i) / \Lambda_k(\varphi, y) = \sum_{\varphi' \in \Phi_k} \sum_{j \in \mathbb{C}} \sum_{\omega' \in \Omega_{jk}(y, \varphi')} \lambda_k(x_{ji,k}(\omega'), \varphi', \mathbf{P}_i) = 1,$$

a sufficient condition for $\partial \ln \lambda_k(\cdot) / \partial y = 0$ is that $\partial \ln v_k(x) / \partial \ln y = \text{constant}$. Given that $x \equiv ey/p$, this condition can be otherwise stated as

$$x v'_k(x) / v_k(x) = c_k,$$

where c_k is an industry-level constant. It follows trivially from Eq. (C.1) that the above condition is also necessary for the invariance of $\lambda_k(\cdot)$ w.r.t. y . In particular, if $\partial \ln \lambda_k(x_{ji,k}(\omega), \varphi, \mathbf{P}_i) / \partial \ln y = 0$ is to be satisfied simultaneously for all ω , then it should be the

case that $xv_k'(x)/v_k(x) = c_k$. Plugging this condition into the general demand elasticity formula, yields the constant elasticity expression:

$$\varepsilon_k(e_k w_i / p_{j_i,k}(\omega), \varphi; \mathbf{P}_i, G_i) = 1 + \theta_k c_k,$$

where the formula stated under Lemma 2 concerns the special case, where $c_k = 1$.

C.3. Demand elasticity in discrete industries

Claim (a) can be proven by applying the Leibniz integral rule as follows:

$$\frac{\partial q_k(p_{j_i,k}(\omega), \dots)}{\partial p_{j_i,k}(\omega)} = -\lambda_k\left(\frac{p_{j_i,k}(\omega)}{e_k w_i}, p_{j_i,k}(\omega), \tilde{p}_{i,k}, \varphi, \mathbf{P}_i(y)\right) g_i\left(p_{j_i,k}(\omega)/e_k w_i\right) L_i / e_k w_i +$$

$$-\frac{\theta_k}{\tilde{p}_{i,k}} \left(\int \frac{p_{j_i,k}(\omega)}{e_k w_i} \frac{v_k\left(y \frac{e_k w_i - p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}\right)}{v_k\left(\frac{e_k w_i - p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}\right)} \lambda_k\left(y, p_{j_i,k}(\omega), \tilde{p}_{i,k}, \varphi, \mathbf{P}_i(y)\right) dG_i(y) \right) L_i.$$

where, as defined in the main text, $\bar{\lambda}_k(p_{j_i,k}(\omega), \tilde{p}_{i,k}, \varphi, \mathbf{P}_i(y)) \equiv q_k(\cdot) / [1 - G_i(p_{j_i,k}(\omega)/e_k w_i)] L_i$ denotes the purchase probability of the average consumer who can afford ω . Recalling the definition of $G_i(\cdot)$ from Appendix C.2, the above equation immediately implies the following expression for the demand elasticity, $\varepsilon_k(\cdot) = \partial \ln q_k(\cdot) / \partial \ln p_{j_i,k}(\omega)$:

$$\varepsilon_k\left(\frac{p_{j_i,k}(\omega)}{e_k w_i}, \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}, \varphi, \mathbf{P}_i, G_i\right) = \left[\frac{\lambda_k\left(\frac{p_{j_i,k}(\omega)}{e_k w_i}, \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}, \varphi, \mathbf{P}_i(y)\right) g_i\left(p_{j_i,k}(\omega)/e_k w_i\right)}{\bar{\lambda}_k\left(p_{j_i,k}(\omega), \varphi, \tilde{p}_{i,k}, \mathbf{P}_i(y)\right) [1 - G_i(p_{j_i,k}(\omega)/e_k w_i)]} \right] \frac{p_{j_i,k}(\omega)}{e_k w_i}$$

$$+ \theta_k \left[\int \frac{p_{j_i,k}(\omega)}{e_k w_i} \frac{v_k\left(\tilde{e}_{i,k} y - \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}\right)}{v_k\left(\tilde{e}_{i,k} y - \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}\right)} dG_{i,k}\left(y; \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}}, \varphi, \mathbf{P}_i\right) \right] \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}},$$

where $\tilde{e}_k \equiv e_k / \tilde{a}_{i,k}$. The above equation clearly states that apart from the aggregate demand shifters, \mathbf{P}_i , the income distribution, $G_i(\cdot)$, and the quality level, φ , which applies to all firms in set $\Omega_{j_i,k}(\varphi, y)$, $\varepsilon_k(\cdot)$ only depends on (i) the degree of affordability $p_{j_i,k}(\omega)/e_k w_i$, and (ii) the price relative to services, $p_{j_i,k}(\omega)/\tilde{p}_{i,k}$.

To prove Claim (b), I first need to characterize the conditions under which $\lambda_k(\cdot)$ is invariant to income. To this end, we can take the same exact steps outlined earlier in the proof of Lemma 2, which imply that individual demand probabilities are (conditionally) invariant to y iff $\partial \ln v_k(x) / \partial \ln y = \text{constant}$. In the case of discrete demand industries, where $x = (y - p_{j_i,k}(\omega)) / \tilde{p}_{i,k}$, this condition simply reduces to

$$v_k'(x)/v_k(x) = c_k,$$

where c_k is an industry-level constant. Plugging this condition into the general demand elasticity formula, yields the following formulation:

$$\varepsilon_k\left(\rho_{oi}/e_k w_i, p_{j_i,k}(\omega)/\tilde{p}_{i,k}, \varphi, \mathbf{P}_i, G_i\right) = c_k \theta_k \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}} + \rho_{j_i,k}(\omega) \mathcal{H}_i\left(\frac{p_{j_i,k}(\omega)}{e_k w_i}\right) \frac{p_{j_i,k}(\omega)}{e_k w_i},$$

where $\mathcal{H}_i(y) = g_i(y)/(1 - G_i(y))$ the hazard rate of the income distribution in market i . Also, note that the formula stated under Lemma 3 concerns the special case, where $c_k = 1$. The demand elasticity in benchmark model presented in 3 can be attained as a special case of the above formula, where (i) $c_k = 1$, and (ii) the endowment distribution is exponential, with a constant hazard rate $\mathcal{H}_i(\cdot) = \zeta_i$. So, given that $\rho_{j_i,k}(\omega) = \rho_{j_i,k}(p_{j_i,k}(\omega)) \approx 1$ to a first-order approximation, the above elasticity formula in this special case reduces to Eq. (8) from Section 3:

$$\varepsilon_{j_i,k}(\varphi) = \theta_k \frac{p_{j_i,k}(\omega)}{\tilde{p}_{i,k}} + \zeta_i \frac{p_{j_i,k}(\omega)}{e_k w_i}.$$

C.4. Deriving Eq. (10)

Continuous industries ($k \in \mathbb{K}_C$): In the case of continuous industries, the CES demand implies that

$$\frac{Q_{j,i,k}(\varphi_H)/Q_{j,i,k}(\varphi_L)}{Q_{j,i,k}(\varphi_H)/Q_{j,i,k}(\varphi_L)} = 1 \Leftrightarrow \frac{p_{j,i,k}(\varphi_H)/p_{j,i,k}(\varphi_L)}{p_{j,i,k}(\varphi_H)/p_{j,i,k}(\varphi_L)} = 1, \quad k \in \mathbb{K}_C.$$

The right-hand side of the above condition holds due to the assumption price exhibits a constant markup and the country-specific cost-shifter is quality-neutral: $p_{j,i,k} = \mu_k \tau_{j,i,k} a_{j,k} c(\varphi) w_i$.

Discrete industries ($k \in \mathbb{K}_D$): The demand facing a firm supplying discrete varieties from country j to market i is given by:

$$q_{j,i,k}(\varphi) = \left[\int \frac{p_{j,i,k}(\varphi)^\infty}{e_k w_i} \exp(-p_{j,i,k}(\varphi)/\theta_k) \Psi_{i,k}(\varphi, y) e^{-\zeta_i(y-\bar{y}_i)} dy \right] \varphi \in \Phi_k, \tag{C.2}$$

In the above equation, $\Psi_{i,k}(\varphi, y)$ is a quality \times income-specific demand shifters, which can be expressed as follows:

$$\Psi_{i,k}(\varphi, y) = \frac{\left[\sum_{\ell \in \mathbb{C}} N_{\ell,k}(\varphi) \exp(-p_{j,i,k}(\varphi)/\theta_k) \right]^{\frac{\gamma_k-1}{\theta_k}}}{\sum_{\varphi \in \Phi_k} \left[\sum_{\ell \in \mathbb{C}} N_{\ell,k}(\varphi) \exp(-p_{j,i,k}(\varphi)/\theta_k) \right]^{\frac{\gamma_k}{\theta_k}}} L_i.$$

Since, $\Psi_{i,k}(\varphi, y)$ is piecewise differentiable with $\partial \Psi_{i,k}(\varphi, y)/\partial y = 0$, we can perform integration by parts (piece-by-piece) to rewrite Eq. (C.2) as follows:

$$\begin{aligned} q_{j,i,k}(\varphi) &= \exp(-p_{j,i,k}(\varphi)/\bar{p}_i \theta_k) \left[\frac{1}{\zeta_i} e^{-\zeta_i \left(\frac{p_{j,i,k}(\varphi)}{e_k w_i} - \bar{y}_i \right)} \right] \Psi_{i,k}(\varphi, p_{j,i,k}(\varphi)/e_k w_i) \\ &= A_i \exp(p_{j,i,k}(\varphi)/w_i)^{-\vartheta_{i,k}} \Psi_{i,k}(\varphi, p_{j,i,k}(\varphi)/e_k w_i), \end{aligned}$$

where $1/\vartheta_{i,k} \equiv \theta_k/\bar{a}_i + \zeta_i/e_k$ and $A_i \equiv e^{\zeta_i \bar{y}_i}/\zeta_i$. Noting that (i) $Q_{j,i,k}(\varphi) = N_{j,k}(\varphi) q_{j,i,k}(\varphi)$ and (ii) $p_{j,i,k}(\varphi) = \tau_{j,i,k} a_{j,k} w_j c_k(\varphi) + \vartheta_{i,k} w_i$, the above equation yields the following expression:

$$\begin{aligned} \frac{Q_{j,i,k}(\varphi_H)}{Q_{j,i,k}(\varphi_L)} &= \frac{N_{j,k}(\varphi_H)}{N_{j,k}(\varphi_L)} \exp\left([p_{j,i,k}(\varphi_L) - p_{j,i,k}(\varphi_H)] \frac{\vartheta_{i,k}}{w_i} \frac{\Psi_{i,k}(\varphi_H, p_{j,i,k}(\varphi_H)/e_k w_i)}{\Psi_{i,k}(\varphi_L, p_{j,i,k}(\varphi_L)/e_k w_i)} \right) \\ &= \frac{N_{j,k}(\varphi_H)}{N_{j,k}(\varphi_L)} \exp([c_k(\varphi_L) - c_k(\varphi_H)] \tau_{j,i,k} a_{j,k} w_j) \frac{\vartheta_{i,k}}{w_i} \frac{\Psi_{i,k}(\varphi_H, p_{j,i,k}(\varphi_H)/e_k w_i)}{\Psi_{i,k}(\varphi_L, p_{j,i,k}(\varphi_L)/e_k w_i)}. \end{aligned}$$

Finally, given that to a first-order approximation $\Psi_{i,k}(\varphi, y) = \Psi_{i,k}(\varphi, \bar{y}_i)$ for all y , then for any two suppliers j and ℓ ,

$$\frac{\Psi_{i,k}(\varphi_H, p_{j,i,k}(\varphi_H)/e_k w_i)/\Psi_{i,k}(\varphi_L, p_{j,i,k}(\varphi_L)/e_k w_i)}{\Psi_{i,k}(\varphi_H, p_{\ell,i,k}(\varphi_H)/e_k w_i)/\Psi_{i,k}(\varphi_L, p_{\ell,i,k}(\varphi_L)/e_k w_i)} \approx 1.$$

That being the case, Eq. (C.3) immediately implies Expression 10 for $k \in \mathbb{K}_D$.

C.5. Proof of Proposition 2

Let $r_{j,i,k}(\varphi) = Q_{j,i,k}(\varphi)/\sum_{\varphi' \in \Phi_k} Q_{j,i,k}(\varphi')$ and let us only focus on a discrete industry $k \in \mathbb{K}_D$. An immediate corollary of Eq. (10) (derived in the previous appendix) is that for any φ_H and $\varphi_L \in \Phi_k$ such that $\varphi_H > \varphi_L$:

$$\frac{\partial r_{j,i,k}(\varphi_H)/r_{j,i,k}(\varphi_L)}{\partial \bar{p}_i}, \frac{\partial r_{j,i,k}(\varphi_H)/r_{j,i,k}(\varphi_L)}{\partial \bar{p}_j}, \frac{\partial r_{j,i,k}(\varphi_H)/r_{j,i,k}(\varphi_L)}{\partial a_{j,k}} > 0.$$

Combining the above relationship with the fact that $\sum_{\varphi \in \Phi_k} r_{j,i,k}(\varphi) = 1$, the above expression simply implies that

$$\frac{\partial^2 r_{j,i,k}(\varphi)}{\partial \varphi \partial \tilde{p}_i}, \frac{\partial^2 r_{j,i,k}(\varphi)}{\partial \varphi \partial \tilde{p}_j}, \frac{\partial^2 r_{j,i,k}(\varphi)}{\partial \varphi \partial a_{j,k}} > 0.$$

Now let us focus on the effect of \tilde{p}_i on the aggregate export unit price, $\bar{p}_{j,i,k}$. Given that (a) $\frac{\partial p_{j,i,k}(\varphi)}{\partial \tilde{p}_i}$, (b) $\frac{\partial p_{j,i,k}(\varphi)}{\partial \varphi} > 0$, and (c) $\frac{\partial^2 r_{j,i,k}(\varphi)}{\partial \varphi \partial \tilde{p}_i} > 0$, it immediately follows that

$$\frac{\partial \bar{p}_{j,i,k}}{\partial \tilde{p}_i} = \sum_{\varphi \in \Phi_k} \left(p_{j,i,k}(\varphi) \frac{\partial r_{j,i,k}(\varphi)}{\partial \tilde{p}_i} + \frac{\partial p_{j,i,k}(\varphi)}{\partial \tilde{p}_i} r_{j,i,k}(\varphi) \right) > 0, \quad k \in \mathbb{K}_D.$$

Along the same line arguments we can establish that $\frac{\partial \bar{p}_{j,i,k}}{\partial \tilde{p}_j} > 0$ and $\frac{\partial \bar{p}_{j,i,k}}{\partial a_{j,k}} > 0$ if $k \in \mathbb{K}_D$. Meanwhile, the fact $\frac{\partial \bar{p}_{j,i,k}}{\partial \tilde{p}_i} = \frac{\partial \bar{p}_{j,i,k}}{\partial \tilde{p}_j} = 0$ if $k \in \mathbb{K}_C$ follows trivially from all components of $r_{j,i,k}(\varphi)$ and $p_{j,i,k}(\varphi)$ being invariant to \tilde{p}_j, \tilde{p}_i . Relatedly, $\frac{\partial \bar{p}_{j,i,k}}{\partial a_{j,k}} = 0$ follows from $r_{j,i,k}(\varphi)$ being invariant to $a_{j,k}$.

C.6. Gains from trade formulas

This appendix derives the gains from trade formula presented in Section 6 under Eq. (16). Recall that this equation is derived under the assumption that (a) the traded sector is entirely discrete (labeled \mathcal{D}), and (b) consumers spend $e_{i,\mathcal{D}}$ of their total budget on consumption categories where traded discrete goods are combined with non-traded services. Based on these assumptions, Eq. (13) implies that

$$GT_i = 1 - \left(\lambda_{i,\mathcal{D}} / \hat{N}_{i,\mathcal{D}} \right)^{\frac{e_{i,\mathcal{D}}}{\theta}}.$$

To arrive at Eq. (16) from the above expression, we need to characterize $\hat{N}_{i,\mathcal{D}}$. To this end, we can appeal to the free-entry condition:

$$N_{i,\mathcal{D}} w_i f^e = \sum_{i \in \mathcal{C}} (p_{i,\mathcal{D}} - c_{i,\mathcal{D}}) Q_{i,\mathcal{D}}.$$

where $Q_{i,\mathcal{D}} = \lambda_{i,\mathcal{D}} L \ell$. In the absence of affordability constraints, $p_{i,\mathcal{D}} = c_{i,\mathcal{D}} + w \ell / \tilde{a} \ell \theta$. Assuming $\tilde{a} \ell = 1$ for all ℓ , and substituting for $p_{i,\mathcal{D}} - c_{i,\mathcal{D}}$ in the above expression yields

$$N_{i,\mathcal{D}} w_i f^e = \sum_{i \in \mathcal{C}} \frac{w \ell}{\theta} \lambda_{i,\mathcal{D}} L \ell = \sum_{i \in \mathcal{C}} \left(\frac{1}{\theta} \lambda_{i,\mathcal{D}} \frac{w \ell L \ell}{w_i L_i} \right) w_i L_i.$$

Rearranging the above equation implies, $N_{i,\mathcal{D}} = \sum_{i \in \mathcal{C}} (\lambda_{i,\mathcal{D}} \frac{w \ell L \ell}{w_i L_i}) \frac{L_i}{\theta f^e}$. Accordingly, under autarky, $N_{i,\mathcal{D}}^A = \frac{L_i}{\theta f^e}$, which yields

$$\hat{N}_{i,\mathcal{D}} = N_{i,\mathcal{D}}^A / N_{i,\mathcal{D}} = \sum_{i \in \mathcal{C}} \left(\lambda_{i,\mathcal{D}} \frac{w \ell L \ell}{w_i L_i} \right).$$

Plugging the above expression back in to expression for GT_i , yields 16.

C.7. Derivation of Eq. (17)

As shown in the previous appendix, the number of firms located in country 1 (i.e., the Free Entry equation) can be stated as

$$N_{1,D} = \left(\lambda_{12,D} \frac{w_2 L_2}{w_1 L_1} + \lambda_{11,D} \frac{w_1 L_1}{w_1 L_1} \right) \frac{L_1}{\theta f^e} = \left(\lambda_{12,D} \frac{w_2 L_2}{w_1 L_1} + \lambda_{11,D} \right) \frac{L_1}{\theta f^e}. \tag{C.4}$$

Rearranging the *Balanced Trade* equation, the term $\lambda_{12,D} \frac{w_2 L_2}{w_1 L_1}$ (in Eq. (C.4)) can be alternatively expressed as

$$\begin{aligned} \lambda_{12,D} \frac{w_2 L_2}{w_1 L_1} &= \left(\frac{\tau a \frac{w_2}{w_1} + \frac{1}{\theta}}{\tau a \frac{w_1}{w_2} + \frac{1}{\theta}} \right) \lambda_{21,D} \\ &= \lambda_{21,D} \left[1 + \frac{\tau a \frac{w_2}{w_1} - \tau a \frac{w_1}{w_2}}{\tau a \frac{w_1}{w_2} + \frac{1}{\theta}} \right] = \lambda_{21,D} \left[1 + \frac{\frac{w_2}{w_1} - \frac{w_1}{w_2}}{\frac{w_1}{w_2} + \frac{1}{a\tau\theta}} \right]. \end{aligned}$$

Plugging the above expression for $\lambda_{12,D} \frac{w_2 L_2}{w_1 L_1}$ back into Eq. (C.4) yields the following expression for $N_{1,D}$:

$$N_{1,D} = \left(\lambda_{21,D} \left[1 + \frac{\frac{w_2}{w_1} - \frac{w_1}{w_2}}{\frac{w_1}{w_2} + \frac{1}{a\tau\theta}} \right] + \lambda_{11,D} \right) \frac{L_1}{\theta f^e} = \left(1 + \frac{\frac{w_2}{w_1} - \frac{w_1}{w_2}}{\frac{w_1}{w_2} + \frac{1}{a\tau\theta}} \lambda_{21,D} \right) \frac{L_1}{\theta f^e}, \tag{C.5}$$

where the second line follows from $\lambda_{21,D} + \lambda_{11,D} = 1$. Following the same exact steps, we can derive the following expression for $N_{2,D}$:

$$N_{2,D} = \left(1 + \frac{\frac{w_1}{w_2} - \frac{w_2}{w_1} \lambda_{21,D}}{\frac{w_2}{w_1} + \frac{1}{a\tau\theta}} \right) \frac{L_2}{\theta f^e}. \tag{C.6}$$

Eqs. (C.5) and (C.6) can be alternatively formulated as

$$N_{i,D} = \left(1 + \frac{\frac{w_{-i}}{w_i} - \frac{w_i}{w_{-i}} \lambda_{-ii,D}}{\frac{w_i}{w_{-i}} + \frac{1}{a\tau\theta}} \right) \frac{L_i}{\theta f^e}.$$

Appendix D. Discrete trade with firm-level heterogeneity

Below, I outline the implications of firm-heterogeneity for the model. Before doing so, note that the firm-level predictions of the model, with regards to PTM, are not sensitive to firm-level heterogeneity by construction. The macro-level predictions, regarding IQS, however, can change with the introduction of firm heterogeneity.

In the baseline model, $c_k(\varphi, z)$ denotes the marginal labor requirement for producing quality φ in industry k . In the baseline case, z differs across countries but is the same across all firms in a given country. So, to model firm heterogeneity I make the following amendments to the baseline model:

- (i) Firms are ex-ante identical in industry k . After paying the entry cost, firms are assigned a productivity, z , which is the realization of a random variable drawn independently across firms from a Pareto distribution:

$$F_{j,k}(z) = 1 - A_{j,k} z^{-\eta_k},$$

where $A_{j,k}$ is a country×industry-specific productivity shifter.

- (ii) Firms have to incur a fixed cost, $f_{i,k}(\varphi)$, to serve market i in quality level $\varphi \in \Phi_k$.

Extending the discussion in Appendix B, the profits facing each firm with productivity z can be expressed as $\pi_k(\nu_{ji,k}(\varphi, z), \varphi, \mathbf{P}_i, k)$, where $\nu_{ji,k}(\varphi, z) \equiv p_{ji,k}(\varphi, z)/w_i$. The zero-profit cutoff is, therefore, determined by the following equation:

$$\pi_k(\nu_{ji,k}^*(\varphi, z), \varphi, \mathbf{P}_{i,k}) = w_i f_{i,k}(\varphi).$$

Importantly, the above equation implies that $\nu_{ji,k}^*(\varphi, z)$ should be uniform across all exporting countries. Namely,

$$\nu_{ji,k}(\varphi, z_{ji,k}^*(\varphi)) \equiv \nu_{i,k}^*(\varphi).$$

Correspondingly, the total value of exports from country j to market i in quality-level $\varphi \in \Phi_k$ is given by

$$X_{ji,k}(\varphi) = N_{j,k}(\varphi) \left(\int_{z_{ji}^*(\varphi)}^{\infty} \nu_{ji,k}(\varphi, z) \lambda_k(\nu_{ji,k}(\varphi, z), \varphi, \mathbf{P}_{i,k}) w_i L_i dF_{j,k}(z) \right) w_i L_i. \quad (\text{D.1})$$

To present the next step transparently, let $m_k(\nu_{ji,k}(\varphi, z), \varphi, \mathbf{P}_{i,k}) \equiv \varepsilon/\varepsilon - 1$ denote the markup charged by a firm with characteristics (φ, z) , exporting industry k goods from country j to market i . Then, by definition, $m_k(\nu_{ji,k}(\varphi, z), \varphi, \mathbf{P}_{i,k}) w_j \tau_{ji} c_k(\varphi) / w_i z = \nu_{ji,k}(\varphi, z)$; and we can write $d \ln z$ in terms of $d \ln \nu$ as follows

$$d \ln z = \left(1 + \frac{\partial \ln m_k}{\partial \ln \nu} \right) d \ln \nu.$$

The above expression, in turn, implies that

$$dz = \frac{w_j \tau_{ji} c_k(\varphi) m_k(\nu, \varphi, \mathbf{P}_{i,k}) d\nu}{w_i \nu \left(1 + \frac{\partial \ln m_k}{\partial \ln \nu} \right)^{-1} \nu}.$$

Given the Pareto productivity distribution, the above equation yields the following formulation for $dF_{j,k}(z)$:

$$\begin{aligned} dF_{j,k}(z) &= \eta_k A_{j,k} z^{-\eta_k - 1} dz \\ &= \eta_k A_{j,k} \left(\frac{w_j \tau_{ji} c_k(\varphi) m_k(\nu, \varphi, \mathbf{P}_{i,k})}{w_i \nu \left(1 + \frac{\partial \ln m_k}{\partial \ln \nu} \right)^{-1}} \right)^{-\eta_k} \frac{d\nu}{\nu}. \end{aligned}$$

Plugging the above equation back into Eq. (D.1), yields the following expression for $X_{ji}(\varphi)$:

$$X_{ji}(\varphi) = \psi_k A_{j,k} N_j(\varphi) \left(\frac{\tau_{ji,k} w_j}{w_i} \right)^{-\eta_k} w_i L_i \mathcal{Q}_{i,k}(\varphi),$$

where

$$\mathcal{Q}_{i,k}(\varphi) \equiv \int_0^{\nu_{i,k}^*(\varphi)} \lambda_k(\nu, \varphi, \mathbf{P}_{i,k}) \left[\frac{c_k(\varphi) m_k(\nu, \varphi, \mathbf{P}_{i,k})}{\nu \left(1 + \frac{\partial \ln m_k}{\partial \ln \nu} \right)^{-1}} \right]^{-\eta_k} d\nu,$$

depends only on the characteristics of the importing market, i . Letting $E_i(\varphi)$ denote the total expenditure on quality $\varphi \in \Phi_k$, the budget constraint of consumers in market i entails that $\sum_{j \in C} X_{ji}(\varphi) = E_i(\varphi)$. This condition plus the above equation, imply the

Table E1Grouping products based on Broda-Weinstein elasticities (Dependent: $\ln \bar{p}_{i,k}$).

Regressor (log)	$\sigma_{BW} \leq 3.6$			$\sigma_{BW} > 3.6$		
	(1)	(2)	(3)	(1)	(2)	(3)
\bar{p}_i		-0.051 (0.045)			0.021 (0.025)	
$\bar{p}_i \times \text{Discrete}$		0.129*** (0.019)	0.134*** (0.017)		0.165*** (0.026)	0.158*** (0.016)
\bar{p}_j	0.158*** (0.002)			0.054*** (0.001)		
$\bar{p}_j \times \text{Discrete}$	0.095*** (0.003)		0.186*** (0.022)	0.022*** (0.002)		0.047** (0.021)
$RCA_{j,k}$	0.026*** (0.000)		0.032*** (0.005)	0.021*** (0.000)		0.031*** (0.004)
$RCA_{j,k} \times \text{Discrete}$	0.035*** (0.001)		0.036*** (0.011)	0.061*** (0.001)		0.082*** (0.010)
Controls for GDP & GDP p/c	Y	Y	...	Y	Y	...
Controls for Distance	Y	Y	...	Y	Y	...
HS6 FE	Y	Y
Exporter×HS6 FE	N	Y	N	N	Y	N
Importer×HS6 FE	Y	N	N	Y	N	N
Exporter×Importer FE	Y	Y
Observations	2,204,000			2,492,000		

Note: This table corresponds to estimating Eq. (12) on more- and less differentiated goods as implied by Broda-Weinstein elasticity estimates, σ_{BW} . Robust standard errors are reported in parentheses. * indicates statistically significant at 10% level, ** indicates statistically significant at 5% level, and *** indicates statistically significant at 1% level. Unit prices are constructed using the BACI database in 2011; the price of non-traded services are constructed using the ICP database in the same year; and each observation in the sample is weighted by trade quantity.

following gravity equation describing macro-level export values:

$$X_{ji}(\varphi) = \frac{A_{j,k} N_j(\varphi) (\tau_{j,i,k} w_j)^{-\eta_k}}{\sum_{i \in C} A_{i,k} N_i(\varphi) (\tau_{i,i,k} w_i)^{-\eta_k}} E_i(\varphi).$$

Considering the above expression, the prediction of model regarding the *home-market driven* quality specialization remains intact. That is, high-wage countries, which have a greater home-market demand for quality, would host a greater number of high-quality producing firms. Accordingly, due to their higher $N_j(\varphi)$ in high-quality (high- φ) categories, they would export relatively more high-quality goods. However, based on the above equation, the prediction that affordability constraints induce IQS no longer holds. One should nonetheless keep in mind that the break-down of affordability-driven IQS is a mere artifact of the Pareto assumption, rather than firm heterogeneity per se.

Appendix E. Additional test the model

Section 5 tested Proposition 2 while restricting the BACI dataset to sub-samples of (i) only *final goods* and (ii) only *differentiated goods*. In this appendix I present an alternative test that groups HS6 product by their underlying demand elasticity. To this end, I use the estimated elasticities by Broda and Weinstein (2006). I classify HS6 products as less- and more- differentiated based on the following criteria:

- (i) *More-differentiated* HS6 products exhibit a lower-or-equal-to-median elasticity, i.e., $\sigma_{BW} \leq 3.6$.
- (ii) *Less-differentiated* HS6 products exhibit an above-median demand elasticity, i.e., $\sigma_{BW} > 3.6$.

I then re-estimate Eq. (12) separately on these two sub-groups of products. The estimation results are reported in Table E, and corroborate the model's prediction that export price levels –in discrete product categories– vary systematically with the price of non-traded services in the origin and destination economy as well the origin economy's degree of comparative advantage. At the same time, these same patterns are either non-existent or weaker in continuous product categories—refer to Section 5 for a more elaborate discussion of these predictions.

Appendix F. Estimation of θ

This appendix describes how the triple difference procedure developed by Caliendo and Parro (2015) can be employed to estimate θ . This procedure relies on applied tariff data. So, before outlining the procedure, I describe how the data on applied tariffs is compiled.

The UNCTAD-TRAINS Data The United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAINS) reports data on applied tariff for 2008, spanning 31 two-digit (in ISIC rev.3) sectors, 181 importers, and 245 export partners. In

line with [Caliendo and Parro \(2015\)](#), I use the *simple tariff line average* of the *effectively applied tariff* (AHS) denoting it by $t_{j,i,k}$. In instances where tariff data are missing for 2008, I use tariff data for the nearest available year, giving priority to earlier years. To aggregate the UNCTAD-TRAINS data into individual WIOD industries, I closely follow the methodology outlined in [Kucheryavy et al. \(2016\)](#). Importantly, individual European Union (EU) member countries are not represented in the UNCTAD-TRAINS data during the 2000–2014 period. So, I infer their applied tariff rates from applied EU tariffs, given that intra-EU trade is subject to zero tariffs while all EU members impose a common external tariff on non-members.

Continuous Model. Eq. (7), bilateral trade shares in continuous traded industries can be characterized as

$$\lambda_{j,i,k} = N_{j,k} \Omega_{i,k} [c_{j,i,k} (1 + t_{j,i,k})]^{-\theta} \quad k \in \mathbb{K}_C,$$

where $\Omega_{i,k} \equiv \sum_n (N_{n,k} [c_{n,i,k} (1 + t_{n,i,k})]^{-\theta}) e_{i,k}$ can be treated as an importer \times industry fixed effect, and $c_{j,i,k}$ denotes the marginal cost of producing and transporting industry k goods from origin j to destination i . Suppose $\ln c_{j,i,k} = \ln \nu_{j,k} + \ln d_{j,i,k} + \varepsilon_{j,i,k}$, where $d_{j,i,k} = d_{i,j,k}$ is a systematic and symmetric component that accounts for the cost-equivalent effect of distance, common language, and common border, and $\varepsilon_{j,i,k}$ is a random disturbance term that represents deviation from symmetry. Using this decomposition, we can produce the following estimating equation for any triplet (j, i, n) :

$$\ln \frac{\lambda_{j,i,k} \lambda_{i,n,k} \lambda_{n,j,k}}{\lambda_{i,j,k} \lambda_{n,i,k} \lambda_{j,n,k}} = -\theta \ln \frac{(1 + t_{j,i,k})(1 + t_{i,n,k})(1 + t_{n,j,k})}{(1 + t_{i,j,k})(1 + t_{n,i,k})(1 + t_{j,n,k})} + \tilde{\varepsilon}_{j,i,n,k},$$

where $\tilde{\varepsilon}_{j,i,n,k} \equiv \theta_k (\varepsilon_{j,i,k} - \varepsilon_{j,n,k} + \varepsilon_{i,n,k} - \varepsilon_{n,i,k} + \varepsilon_{n,j,k} - \varepsilon_{j,n,k})$. The above equation can be used to attain unbiased and consistent estimates for θ under the identifying assumption that $\text{Cov}(t_{j,i,k}, \varepsilon_{j,i,k}) = 0$. I estimate the above equation using data on trade shares, $\lambda_{j,i,k}$, from the WIOD and matching data on applied tariffs, $t_{j,i,k}$, from the UNCTAD-TRAINS. As noted earlier, the original WIOD data features 16 traded industries. Since I am interested in attaining an economy-wide estimate for θ , I perform the estimation while pooling data across all 16 industries, treating them as continuous.

Discrete Model. Eq. (7), trade shares in discrete traded industries can be characterized as:

$$\lambda_{j,i,k} = N_{j,k} \Omega_{i,k} \exp(c_{j,i,k} + \bar{t}_{j,i,k})^{-\theta/\bar{p}_j}, \quad k \in \mathbb{K}_D.$$

where $\Omega_{i,k} \equiv \sum_n (N_{n,k} \exp(c_{n,i,k} + \bar{t}_{n,i,k})^{-\theta/\bar{p}_i}) e_{i,k}$ can be once again treated as an importer \times industry fixed effect, and $\bar{t}_{j,i,k}$ is the per-unit equivalent of the ad-valorem tariff rate, $t_{j,i,k}$. Suppose $c_{j,i,k}/\bar{p}_j = \nu_{j,k} + \delta_{i,k} + d_{j,i,k} + \varepsilon_{j,i,k}$, where $d_{j,i,k} = d_{i,j,k}$ is a systematic and symmetric component that accounts for the cost-equivalent of distance, common language, and common border, and $\varepsilon_{j,i,k}$ is a random disturbance term that represents deviation from symmetry. This decomposition produces the following estimating equation for any triplet (j, i, n) :

$$\ln \frac{\lambda_{j,i,k} \lambda_{i,n,k} \lambda_{n,j,k}}{\lambda_{i,j,k} \lambda_{n,i,k} \lambda_{j,n,k}} = -\theta \left[\frac{\bar{t}_{j,i,k}}{\bar{p}_i} + \frac{\bar{t}_{i,n,k}}{\bar{p}_n} + \frac{\bar{t}_{n,j,k}}{\bar{p}_j} - \frac{\bar{t}_{i,j,k}}{\bar{p}_j} - \frac{\bar{t}_{n,i,k}}{\bar{p}_i} - \frac{\bar{t}_{j,n,k}}{\bar{p}_n} \right] + \tilde{\varepsilon}_{j,i,n,k},$$

where $\tilde{\varepsilon}_{j,i,n,k} \equiv \theta_k (\varepsilon_{j,i,k} - \varepsilon_{j,n,k} + \varepsilon_{i,n,k} - \varepsilon_{n,i,k} + \varepsilon_{n,j,k} - \varepsilon_{j,n,k})$. The above equation can be used to attain unbiased and consistent estimates for θ under the identifying assumption that $\text{cov}(t_{j,i,k}, \varepsilon_{j,i,k}) = 0$. I estimate the above equation using data on trade shares and applied tariffs from the the WIOD and UNCTAD-TRAINS. Since I am interested in attaining an economy-wide estimate for θ , I perform the estimation while pooling data across the 16 traded WIOD industries, treating them all as discrete.

Appendix G. Out-of-sample performance of the discrete model

This appendix investigates the out-of-sample predictive power of the discrete model that is calibrated to data in [Section 6](#). The data moment of interest is the share of expenditure on traded versus non-trade sectors. Recall that the model is calibrated to fit only the total share of expenditure on categories where traded and non-traded services are bundled in the consumption basket.

In line with my analysis in [Section 6](#), suppose the traded sector consists of only discrete industries. In that case, the L_i consumers in economy i purchase in total $Q_i = K \times L_i$ units of traded goods across all K industries. Hence, letting \bar{p}_i denote the average price of tradable consumption in economy i , the total expenditure on non-traded services can be calculated as $\bar{E}_i = w_i L_i - \bar{p}_i Q_i$. Correspondingly, the *share* of service expenditure is given by

$$\tilde{e}_i \equiv \frac{\bar{E}_i}{Y_i} = \frac{w_i L_i - \bar{p}_i Q_i}{w_i L_i} = 1 - \kappa \frac{\bar{p}_i}{\bar{p}_i},$$

where $\kappa \equiv K/\bar{a}$. Given the above equation and data on \bar{p}_i/\bar{p}_i , we can calculate the model's prediction with regards to the share of service expenditure as $\ln(1 - \tilde{e}_i) = \ln \kappa + \ln \bar{p}_i/\bar{p}_i$. The data on \bar{p}_i/\bar{p}_i is readily accessible for 182 countries from the 2011 ICP data, which was described in [Section 5](#). The same data also reports the factual share of non-traded service expenditure, \tilde{e}_i , for each economy. Using this information, I contrast the predicted values for \tilde{e}_i with factual values in [Fig. G1](#). Encouragingly, the discrete

model performs remarkably well in predicting the cross-national variation in service expenditure. Finally, note that by allowing for \tilde{a}_i to be a free-moving parameter, I can perfectly match \tilde{e}_i in all countries. The gains from trade implied in that case, though, closely resemble those implied by my benchmark analysis.

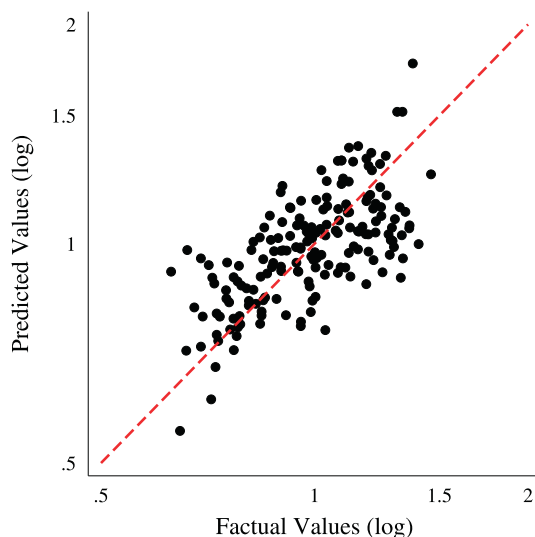
Fig. G1. Discrete model's predictive power w.r.t. relative service expenditure.

Note: This graph compares data and predicted values for (relative) service expenditure across 182 countries, including the 31 countries in the WIOD sample. The predicted values are obtained from a model that (i) treats the entire traded sector as discrete, and (ii) the entire service sector as one that complements discrete consumption. See Appendix G for more details about the construction of each data point and predicted value.

Table H1

Classification of Industries in the WIOD.

WIOD sector	Sector's description	Category	
1	Agriculture, Hunting, Forestry and Fishing	Traded	
2	Mining and Quarrying		
3	Food, Beverages and Tobacco		
4	Textiles and Textile Products		
5	Leather and Footwear		
6	Wood and Products of Wood and Cork		
7	Pulp, Paper, Paper, Printing and Publishing		
8	Coke, Refined Petroleum and Nuclear Fuel		
9	Chemicals and Chemical Products		
10	Rubber and Plastics		
11	Other Non-Metallic Mineral		
12	Basic Metals and Fabricated Metal		
13	Machinery, Nec		
14	Electrical and Optical Equipment		
15	Transport Equipment		
16	Manufacturing, Nec; Recycling		Non-Traded Service
17	Electricity, Gas and Water Supply		
18	Construction		
19	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel		
20	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles		
21	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods		
22	Hotels and Restaurants		
23	Inland Transport		
24	Water Transport		
25	Air Transport		
26	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies	Non-Traded Service, bundled w/Tradables	
27	Post and Telecommunications		
28	Financial Intermediation		
29	Real Estate Activities		
30	Renting of M&Eq and Other Business Activities		
31	Education		
32	Health and Social Work		
33	Public Admin and Defence; Compulsory Social Security		
34	Other Community, Social and Personal Services		
35	Private Households with Employed Persons		



Appendix H. Classification of the WIOD industries

This appendix describes my classification of the 35 industries in the WIOD sample, which is used to compute $e_{i,C}$ and $e_{i,D}$. My classification groups industries into three different categories:

- (i) Traded industries (as classified by Costinot and Rodriguez-Clare (2014));
- (ii) Non-traded industries whose services are bundled with traded discrete goods in the consumption basket; and
- (iii) Other non-traded service industries.

The assignment of the WIOD industries into these categories is reported in Table H1. To briefly explain my classification, consider the industries supplying transportation services (i.e., industries numbers 23–24 in Table H1). A consumer may purchase a personal car, jet, or yacht, and spend the rest of their transportation budget on other types of ground, air, or sea transportation services that are infeasible to perform with their personally-acquired transportation device. Similarly, a consumer may purchase a personal grill or an RV (Recreational Vehicle) and spend the rest of their food and travel budget on hotel and restaurant services (i.e., industry number 22 in Table H1). Similar examples can be constructed for household- and health-related services (i.e., industries numbers 32–35 in Table H1). But the same argument cannot be readily extended to real estate services or services supplying residential water, electricity, and gas. Relatedly, financial services are classified as solely non-traded, because they concern a non-repetitive service that has to be frequently customized to match the consumer's needs on a case-to-case basis.

References

- Alcalá, Francisco, 2016. Specialization across goods and export quality. *J. Int. Econ.* 98, 216–232.
- Alessandria, George, Kaboski, Joseph P., 2011. Pricing-to-market and the failure of absolute PPP. *Am. Econ. J. Macroecon.* 3 (1), 91–127.
- Antoniades, Alexis, 2015. Heterogeneous firms, quality, and trade. *J. Int. Econ.* 95 (2), 263–273.
- Arkolakis, C., Costinot, A., Rodriguez-Clare, A., 2012. Clare, 2012, new trade models, same old gains. *Am. Econ. Rev.* 102 (1), 94.
- Arkolakis, Costas, Costinot, Arnaud, Donaldson, Dave, Rodriguez-Clare, Andrés, January 2019. The elusive pro-competitive effects of trade. *Rev. Econ. Stud.* 86 (1), 46–80.
- Atkeson, Andrew, Burstein, Ariel, 2008. Pricing-to-market, trade costs, and international relative prices. *Am. Econ. Rev.* 98 (5), 1998–2031.
- Auer, Raphael A., Chaney, Thomas, Sauré, Philip, 2018. Quality pricing-to-market. *J. Int. Econ.* 110, 87–102.
- Baldwin, Richard, Harrigan, James, 2011. Zeros, quality, and space: trade theory and trade evidence. *Am. Econ. J. Microecon.* 3 (2), 60–88.
- Banerjee, Abhijit V., 2003. Contracting constraints, credit markets, and economic development Abhijit V. Banerjee. *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress.* vol. 3. Cambridge University Press 1.
- Bekkers, Eddy, Simonovska, Ina, 2015. The Balassa-Samuels effect and pricing-to-market: the role of strategic complementarity. *Econ. Lett.* 126, 156–158.
- Ben-Akiva, Moshe E., Lerman, Steven R., Lerman, Steven R., 1985. *Discrete Choice Analysis: Theory and Application to Travel Demand.* vol. 9. MIT press.
- Benhabib, Jess, Rogerson, Richard, Wright, Randall, 1991. Homework in macroeconomics: household production and aggregate fluctuations. *J. Polit. Econ.* 99 (6), 1166–1187.
- Bertoletti, Paolo, Etro, Federico, Simonovska, Ina, 2018. International trade with indirect additivity. *Am. Econ. J. Microecon.* 10 (2), 1–57.
- Bhagwati, Jagdish N., 1984. Why are services cheaper in the poor countries? *Econ. J.* 94 (374), 279–286.
- Broda, Christian, Weinstein, David E., 2006. Globalization and the gains from variety. *Q. J. Econ.* 121 (2), 541–585 URL: <http://qje.oxfordjournals.org/content/121/2/541.short>.
- Caliendo, Lorenzo, Parro, Fernando, January 2015. Estimates of the trade and welfare effects of NAFTA. *Rev. Econ. Stud.* 82 (1), 1–44 rdu035.
- Chen, Natalie, Juvenal, Luciana, 2014. Quality, Trade, and Exchange Rate Pass-Through. International Monetary Fund, pp. 14–42.
- Choi, Yo Chul, Hummels, David, Xiang, Chong, 2009. Explaining import quality: the role of the income distribution. *J. Int. Econ.* 77 (2), 265–275.
- Cooley, Thomas F., Prescott, Edward C., 1995. Economic growth and business cycles. *Front. Bus. Cycle Res.* 1.
- Costinot, Arnaud, Rodriguez-Clare, Andrés, 2014. Trade theory with numbers: quantifying the consequences of globalization. *Handb. Int. Econ.* 4, 197.

- Costinot, Arnaud, Donaldson, Dave, Kyle, Margaret, Williams, Heidi, 2016. The More we Die, the More we Sell? A Simple Test of the Home-Market Effect. Tech. rep. National Bureau of Economic Research.
- De Loecker, Jan, Eeckhout, Jan, 2017. The Rise of Market Power and the Macroeconomic Implications. Tech. rep. National Bureau of Economic Research.
- Dingel, Jonathan I., October 2017. The determinants of quality specialization. *Rev. Econ. Stud.* 84 (4), 1551–1582 [rdw054](#).
- Drăgulescu, Adrian, Yakovenko, Victor M., 2001. Evidence for the exponential distribution of income in the USA. *Eur. Phys. J. B-Condens. Matter Complex Syst.* 20 (4), 585–589.
- Fajgelbaum, Pablo D., Khandelwal, Amit K., 2014. Measuring the Unequal Gains from Trade. Tech. rep. National Bureau of Economic Research.
- Fajgelbaum, Pablo, Grossman, Gene M., Helpman, Elhanan, 2011. Income distribution. *Prod. Qual. Int. Trade, IJ. Polit. Econ.* 118 (4) 721.
- Feenstra, Robert C., Romalis, John, 2014. International prices and endogenous quality. *Q. J. Econ.* 129 (2), 477–527.
- Feenstra, Robert C., Romalis, John, Schott, Peter K., 2002. US imports, Exports, and Tariff Data, 1989–2001. Tech. rep. National Bureau of Economic Research URL <http://www.nber.org/papers/w9387>.
- Fernandez-Villaverde, Jesus, Krueger, Dirk, 2011. Consumption and saving over the life cycle: how important are consumer durables? *Macroecon. Dyn.* 15 (5), 725–770.
- Flam, Harry, Helpman, Elhanan, 1987. Vertical product differentiation and North-South trade. *Am. Econ. Rev.* 810–822.
- Foellmi, Reto, Hepenstrick, Christian, Josef, Zweimüller, 2017. International arbitrage and the extensive margin of trade between rich and poor countries. *Rev. Econ. Stud.* 85 (1), 475–510.
- Gabaix, Xavier, 2009. Power laws in economics and finance. *Annu. Rev. Econ.* 1 (1), 255–294.
- Gaulier, Guillaume, Zignago, Soledad, 2010. Baci: International Trade Database at the Product-Level (the 1994–2007 Version).
- Hallak, Juan Carlos, 2006. Product quality and the direction of trade. *J. Int. Econ.* 68 (1), 238–265 URL <http://www.sciencedirect.com/science/article/pii/S0022199605000516>.
- Hallak, Juan Carlos, Schott, Peter K., 2011. Estimating cross-country differences in product quality. *Q. J. Econ.* 126 (1), 417–474 URL <http://qje.oxfordjournals.org/content/126/1/417.short>.
- Harrigan, James, Ma, Xiangjun, Shlychkov, Victor, 2015. Export prices of US firms. *J. Int. Econ.* 97 (1), 100–111.
- Head, Keith, Mayer, Thierry, Ries, John, 2010. The erosion of colonial trade linkages after independence. *J. Int. Econ.* 81 (1), 1–14.
- Hummels, David, Klenow, Peter J., 2005. The variety and quality of a nation's Exports. *Am. Econ. Rev.* 95 (3).
- Hummels, David, Lugovskyy, Volodymyr, 2009. International pricing in a generalized model of ideal variety. *J. Money Credit Bank.* 41 (s1), 3–33.
- Jaimovich, Esteban, Merella, Vincenzo, 2015. Love for quality, comparative advantage, and trade. *J. Int. Econ.* 97 (2), 376–391.
- Khandelwal, Amit, 2010. The long and short (of) quality ladders. *Rev. Econ. Stud.* 77 (4), 1450–1476 URL <http://restud.oxfordjournals.org/content/77/4/1450.short>.
- Kmenta, Jan, 1997. *Elements of Econometrics*. University of Michigan Press.
- Krugman, Paul, 1980. Scale economies, product differentiation, and the pattern of trade. *Am. Econ. Rev.* 70 (5), 950–959 URL <https://doi.org/10.2307/1805774>.
- Kucheryavyi, Konstantin, Lyn, Gary, Rodriguez-Clare, Andrés, 2016. Grounded by Gravity: A Well-Behaved Trade Model with Industry-Level Economies of Scale. Tech. rep. National Bureau of Economic Research.
- Lariviere, Martin A., 2006. A note on probability distributions with increasing generalized failure rates. *Oper. Res.* 54 (3), 602–604.
- Lariviere, Martin A., Porteus, Evan L., 2001. Selling to the newsvendor: an analysis of price-only contracts. *Manuf. Serv. Oper. Manag.* 3 (4), 293–305.
- Lewis, W. Arthur, 1954. Economic development with unlimited supplies of labour. *Manch. Sch.* 22 (2), 139–191.
- Lugovskyy, Volodymyr, Skiba, Alexandre, 2015. How geography affects quality. *J. Dev. Econ.* 115, 156–180.
- Manova, Kalina, Zhang, Zhiwei, 2012. Export prices across firms and destinations. *Q. J. Econ.* 127 (1), 379–436.
- Matsuyama, Kiminori, 2000. A Ricardian model with a continuum of goods under nonhomothetic preferences: demand complementarities, income distribution, and north-south trade. *J. Polit. Econ.* 108 (6), 1093–1120.
- Matsuyama, Kiminori, 2015. The Home Market Effect and Patterns of Trade between Rich and Poor Countries.
- McFadden, Daniel, et al., 1978. *Modelling the Choice of Residential Location*. Institute of Transportation Studies, University of California.
- Mrázová, Monika, Peter Neary, J., 2017. Not so demanding: demand structure and firm behavior. *Am. Econ. Rev.* 107 (12), 3835–3874.
- Murphy, Kevin M., Shleifer, Andrei, 1997. Quality and trade. *J. Dev. Econ.* 53 (1), 1–15.
- Murphy, Kevin M., Shleifer, Andrei, Vishny, Robert W., 1989. Industrialization and the big push. *J. Polit. Econ.* 97 (5), 1003–1026.
- Pierce, Justin R and Peter K Schott. 2012. "A concordance between ten-digit US Harmonized System Codes and SIC/NAICS product classes and industries." *Journal of Economic and Social Measurement* 37 (1,2):61–96.
- Proudman, James, Redding, Stephen, 2000. Evolving patterns of international trade. *Rev. Int. Econ.* 8 (3), 373–396.
- Rauch, James E., 1999. Networks versus markets in international trade. *J. Int. Econ.* 48 (1), 7–35 URL <http://www.sciencedirect.com/science/article/pii/S0022199698000099>.
- Reiss, Peter C., Wolak, Frank A., 2007. Structural econometric modeling: rationales and examples from industrial organization. *Handb. Econ.* 6, 4277–4415.
- Rosenstein-Rodan, Paul N., 1943. Problems of industrialisation of eastern and South-Eastern Europe. *Econ. J.* 53 (210/211), 202–211.
- Samuelson, Paul A., 1994. Facets of Balassa-Samuelson thirty years later. *Rev. Int. Econ.* 2 (3), 201–226.
- Schott, Peter K., 2004. Across-product versus within-product specialization in international trade. *Q. J. Econ.* 119 (2), 647–678 URL <http://qje.oxfordjournals.org/content/119/2/647.short>.
- Simonovska, Ina, 2015. Income differences and prices of tradables: insights from an online retailer. *Rev. Econ. Stud.* 82 (4), 1612–1656.
- Solon, Gary, Haider, Steven J., Wooldridge, Jeffrey M., 2015. What are we weighting for? *J. Hum. Resour.* 50 (2), 301–316.
- Sposi, Michael, 2015. Trade barriers and the relative price of tradables. *J. Int. Econ.* 96 (2), 398–411.
- Thisse, Jacques-Francois, Ushchev, Philip, 2016. When Can a Demand System Be Described by a Multinomial Logit with Income Effect?
- Timmer, Marcel, Erumban, Abdul A., Gouma, Reitze, Los, Bart, Temurshoev, Umed, de Vries, Gaaitzen J., Arto, I-aki, Genty, Valeria Andreoni AurŽlien, Neuwahl, Frederik, Francois, Joseph, et al., 2012. *The World Input-Output Database (WIOD): Contents, Sources and Methods*. Tech. rep. Institute for International and Development Economics.
- Verhoogen, Eric A., 2008. Trade, quality upgrading, and wage inequality in the Mexican manufacturing sector. *Q. J. Econ.* 123 (2), 489–530.
- Wooldridge, Jeffrey M., 1995. Selection corrections for panel data models under conditional mean independence assumptions. *J. Econ.* 68 (1), 115–132.