Profits, Scale Economies, and the Gains from Trade and Industrial Policy

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Ahmad Lashkaripour   Volodymyr Lugovskyy

University of Michigan, March 2021

Indiana University

Industrial Policy is Back on the Scene[1]

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[1] See Aiginger and Rodrik (2020) for a detailed account.

Industrial Targeting via *Trade Restrictions* is Proliferating

Made in China 2025

- 2015 Initiative to promote Chinese manufacturing via trade barriers and subsidies.

National Trade Council

- Created in *Dec 2016* to promote US manufacturing (later became OTMP).
- Proposed tariffs on goods imported from China to counter *“Made in China 2025”*.

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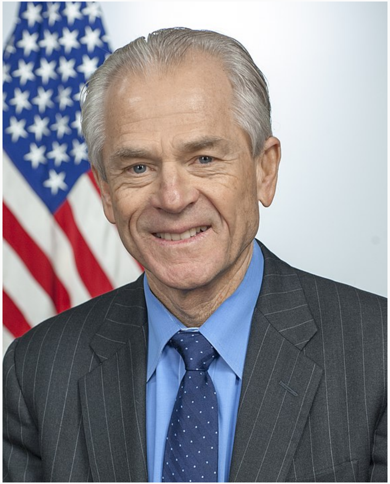

Old-but-Unresolved Policy Questions have Resurfaced

These developments have resurfaced some old-but-unresolved policy questions:

1. is trade policy an effective tool for correcting misallocation in domestic industries? (e.g., for correcting underproduction in manufacturing)
2. if not, should governments correct misallocation, *unilaterally*, with industrial subsidies to target industries?
3. or should they coordinate their industrial policies via deep trade agreements?

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Our Answers to these Questions Exhibit Important Gaps

Standard *theories* that speak to Question 1-3 overlook key policy considerations:

- typically based on partial equilibrium, *2-good×2-country* models.
- overlook multilateral considerations & key industry linkages.

The *quantitative* route has proven equally-elusive:

- quantitative trade model have advanced remarkably over the past two decades...
- ...but we lack credible estimates for parameters that govern the gains from trade and industrial policy. [exception: Bartelme *et. al.* (2019)]

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This Paper: *Roadmap*

Step #1. Derive analytic formulas for *1st-best* and *2nd-best* trade policies in an important class of *multi-industry—multi-country* quantitative trade models where misallocation occurs due to scale economies or markup distortions.

Step #2 Estimate the parameters that govern the gains from policy in theses frameworks using micro-level data.

Step #3 Plug the estimated parameters into the analytic optimal policy formulas to quantify the gains from trade and industrial policy under various scenarios.

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This Paper: *Main Findings*

1. Trade restrictions are an ineffective *second-best* measure for correcting misallocation in domestic industries.

2. Unilateral industrial policy is equally ineffective, as it triggers *immiserizing growth* in most countries.

3. What is the best remedy for misallocation in open economies? multilateral industrial policies that are coordinated via *deep* agreements.

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Conceptual Framework

Theoretical Model

We adopt a generalized *multi-country*, *multi-industry* Krugman model:

- general equilibrium + can tractably accommodate IO linkages
- accommodates the ToT-improving & misallocation-correcting cases for policy
- is isomorphic to a *Melitz-Pareto* model or an *Eaton-Kortum* model with Marshallian externalities (Kucheryavyy et. al., 2020).

The Economic Environment

- Many countries: $i, j, n = 1,...,\mathcal{N}$
  - Country i is populated by L_i workers who supply labor inelastically.
  - Labor is the only (primary) factor of production

- Many industries: $k, g = 1,..., \mathcal{K}$
  - Industries differ in terms of their trade elasticity, scale elasticity, *etc*.
  - Each industry is served by many firms (index ω)

Notation: Good's Indexes

- Goods are indexed by origin–destination–industry

$$\text{good } ij,k \;\sim\; \text{origin } i - \text{destination } j - \text{industry } k$$

- *Supply-side* variables are indexed by origin–industry

$$\text{subscript } i,k \;\sim\; \text{origin } i - \text{industry } k$$

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Preferences: Non-Parametric Across Industries

- Representative consumer's problem in country i

national income

$$\max_{\mathbf{Q}_i} \; U_i(\mathbf{Q}_i) \quad s.t. \; \sum_k \left(\tilde{P}_{i,k} Q_{i,k}\right) = Y_i$$

  - $\mathbf{Q}_i \equiv \{Q_{i,k}\}$ ~ composite industry-level consumption.
  - $\tilde{\mathbf{P}}_i \equiv \{\tilde{P}_{i,k}\}$ ~ "consumer" price index of industry-level composite.

- The Marshallian demand function for *industry k* goods in *market i*

$$Q_{i,k} = \mathcal{D}_{i,k}(\tilde{\mathbf{P}}_i, Y_i)$$

- The **Cobb-Douglas** case: $U_i\left(\mathbf{Q}_i\right) = \prod_{k=1}^{\mathcal{K}} Q_{i,k}^{e_{i,k}} \longrightarrow Q_{i,k} = e_{i,k} Y_i / \tilde{P}_{i,k}$

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- Cross-national aggregator: $Q_{i,k} = \left(\sum_{j\in\mathbb{C}} Q_{ji,k}^{\frac{\sigma_k-1}{\sigma_k}}\right)^{\frac{\sigma_k}{\sigma_k-1}}$
- Sub-national aggregator: $Q_{ji,k} = \left(\sum_{\omega\in\Omega_{j,k}} q_{ji,k}(\omega)^{\frac{\gamma_k-1}{\gamma_k}}\right)^{\frac{\gamma_k}{\gamma_k-1}}$
- The demand facing an firm-level variety ω (origin j–destination i–industry k):

$$q_{ji,k}(\omega) = \left(\frac{\tilde{p}_{ji,k}(\omega)}{\tilde{P}_{ji,k}}\right)^{-\gamma_k} \left(\frac{\tilde{P}_{ji,k}}{\tilde{P}_{i,k}}\right)^{-\sigma_k} \mathcal{D}_{i,k}\left(\tilde{\mathbf{P}}_i, Y_i\right)$$

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Production and Firms

- Firms compete under monopolistic competition.

- variety-specific marginal cost (*origin* i*–destination* j*–industry* k)

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- Entry is either free or restricted
  - **Free Entry:** endogenous number of firms + zero profits
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this presentation will focus on this case

Summarizing the Production Side

- The *producer* price of goods supplied by *origin i–industry k*:

$$P_{ij,k} = \text{constant} \times w_i \times L_{i,k}^{-\frac{1}{\gamma_k - 1}}$$

- The special case w/ constant-returns to scale: $\mu_k \longrightarrow 0$

- **Note:** firm-level *markup* in industry $k = 1 + \mu_k$

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The Rationale for Policy Intervention

From country i's standpoint, the market equilibrium exhibits 2 types of inefficiency:

1. Sectoral misallocation
   - There is sub-optimal output in high-returns-to-scale (high-μ) industries.
2. Unexploited ToT gains
   - **Export side:** the government can charge an additional markup on export goods.
   - **Import side:** the government can lower the price of imports via import restrictions.

- Governments have access to a complete set of tax instruments ⟶ they can target each inefficiency margin listed above and reach the *1st-best* outcome.

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Two Key Elasticities for Policy Analysis

$$\text{trade elasticity} \sim \sigma_k - 1 = \frac{\partial \ln \text{Trade value}}{\partial \ln \text{Trade cost}}$$

$$\text{scale elasticity} \sim \mu_k = \frac{\partial \ln \text{Variety-adjusted TFP}}{\partial \ln \text{Number of workers}}$$

- Lower σ_k ⟶ more scope for ToT manipulation in industry k
- Higher $\text{Var}_k(\mu_k)$ ⟶ greater degree of misallocation in the economy
- **Note:** non-nested CES preferences imply $\mu_k = \frac{1}{\sigma_k - 1}$ ⟶ impose an arbitrary link b/w the scale and trade elasticity (Benassy, 1996).

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Instruments of Policy

- Import tariffs, export subsidies, and industrial subsidies create a wedge b/w *producer prices* (P) and *consumer prices* ($\tilde{P}$):

$$\tilde{P}_{ij,k} = \frac{1 + t_{ij,k}}{(1 + x_{ij,k})(1 + s_{i,k})} P_{ij,k}$$

- Tax revenues are rebated to the consumers lump-sum.[2] Definition of equilibrium

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[2] **Note:** lump-sum transfers are isomorphic to uniform consumption subsidies in the present setup because the labor supply is inelastic—see Dixit, 1980 and Lashkaripour, 2020.

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First-Best Non-Cooperative Policy

Optimal Non-Cooperative Policy Problem

- Country i's unilaterally optimal policy problem

import tariff

vector of equilibrium outcome

$$\max_{\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i} \; W_i(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbb{X}) \qquad s.t. \; \text{Equilbrium conditons}$$

export subsidy

industrial subsidy

- **Note:** the solution to the above problem does *not* internalize country i's ToT externality on the rest of the world ⟶ it's sub-optimal from a global standpoint.

Dual approach for deriving 1st-best policies

Optimal Non-Cooperative Policy Problem

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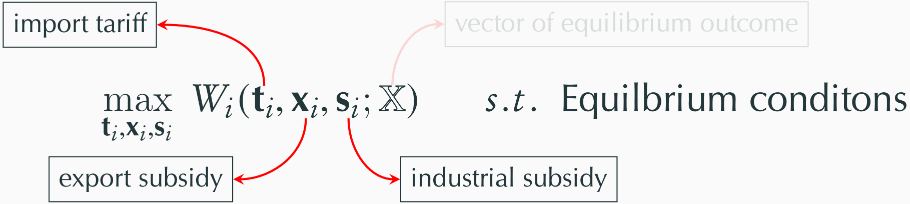

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Dual approach for deriving 1st-best policies

Optimal Non-Cooperative Policy Problem

- Country i's unilaterally optimal policy problem

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Dual approach for deriving 1st-best policies

Theorem 1: *Country i's (1st-Best) Optimal Policy*

[industrial subsidy] $$1 + s_{i,k}^{\star} = (1 + \mu_k)(1 + \bar{s}_i)$$

[import tariff] $$1 + t_{ji,k}^{\star} = (1 + \omega_{ji,k})(1 + \bar{t}_i)$$

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arbitrary tax shifters to account for multiplicity

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can be characterized in terms of σ_k, μ_k, and observable shares

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expenditure share on good ij, k

Special Case: *Multi-Industry Armington Model*

Perfectly competitive industries ($\mu_k = 0$) ⟶ our model reduces to a multi-industry ***Armington*** or ***Eaton-Kortum*** model and our optimal policy formulas reduce to:

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A Verbal Summary of Theorem 1

The unilaterally optimal (first-best) policy consists of

1. industrial subsidies ($\mathbf{s}_i$) that promote high-μ (*high-returns-to-scale*) industries.
2. import tariffs ($\mathbf{t}_i$) $+$ export subsidies ($\mathbf{x}_i$) that contract exports in low-σ industries.

Corollary: first-best optimal tariffs and export subsidies are *misallocation-blind*.

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Second-Best Trade Policy

Second-Best Optimal Trade Policy Problem

- Country i's 2nd-best optimal trade policy problem

import tariff

$$\max_{\mathbf{t}_i,\mathbf{x}_i,\mathbf{s}_i} W_i(\mathbf{t}_i, \mathbf{x}_i, \mathbf{s}_i; \mathbb{X}) \qquad s.t. \begin{cases} \text{Equilbrium conditons} \\ \mathbf{s}_i = \mathbf{0} \end{cases}$$

export subsidy

industrial subsidy

- **Note:** The restriction that $\mathbf{s}_i = \mathbf{0}$ may reflect institutional barriers or political economy pressures.

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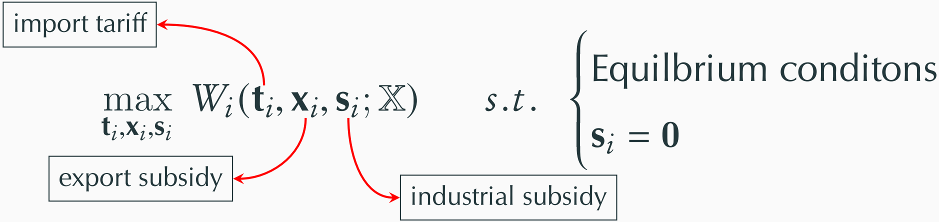

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Theorem 2: *Optimal 2nd-Best Trade Taxes/Subsides*

$$1 + t_{ji,k}^{\text{2nd-best}} = \frac{1 + (\sigma_k - 1)\lambda_{ii,k}}{1 + \frac{1+\overline{\mu}_i}{1+\mu_k}(\sigma_k - 1)\lambda_{ii,k}} \left(1 + t_{ji,k}^{\text{1st-best}}\right)$$

$$1 + x_{ij,k}^{\text{2nd-best}} = \frac{1 + \mu_k}{1 + \overline{\mu}_i} \left(1 + x_{ij,k}^{\text{1st-best}}\right)$$

Intuition

- 2nd-best trade taxes/subsidies mimic *1st-best* industrial subsidies...
- ... but—by the ***targeting principle***—they cannot replicate the *1st-best* outcome.

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average μ_k in economy i

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Tension between ToT and Misallocation-Correcting Objectives

- Correcting misallocation requires promoting high-μ industries.
- ToT improvement requires contracting export sales in low-σ industries.

Proposition

- If $\text{Cov}_k(\mu_k, \sigma_k) < 0 \implies$ correcting misallocation with trade policy worsens the terms-of-trade and *vice versa*.
- This tension makes trade policy an ineffective *misallocation-correcting* measure, beyond what is implied by the targeting principle.

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Avoiding Immiserizing Growth with Deep Agreements

- **Flip side:** If $\text{Cov}_k(\mu_k, \sigma_k) < 0 \implies$ using industrial subsidies, *unilaterally*, to correct misallocation causes *immiserizing growth*.
- **Why?** corrective industrial subsidies promote high-μ industries $\longrightarrow$ expand exports in low-σ industries by design $\longrightarrow$ worsen the ToT.
- The best remedy for misallocation in open economies:
  - Countries coordinate their industrial subsidies via *deep* trade agreements.
  - In this process, each country forgoes the (unilateral) ToT gains from policy but benefit for efficiency improvements in the RoW.

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Estimating the Key Policy Parameters

The Parameters that Govern the Gains from Policy

- The gains from optimal policy depend crucially on two sets of elasticities:[3]

  1. μ_k ~ industry-level scale elasticity
  2. $\sigma_k - 1$ ~ industry-level trade elasticity

- We posses plenty of estimates for trade elasticities, but μ_k is often normalized:
  - perfectly competitive models $\longrightarrow$ $\mu_k = 0$
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[3] **Note:** To account for firm-selection à la *Melitz-Chaney*, we need to estimate the shape of the Pareto distribution in addition to σ_k and $\mu_k = 1/(\gamma_k - 1)$.

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Estimation Strategy

- We propose a new methodology to jointly estimate μ_k and σ_k.
- We estimate a ***firm-level*** nest-CES import demand function with ***transaction-level*** trade data ($j, kt \sim$ origin j–product k–year t):

$$\underbrace{\ln X_{j,kt}(\omega)}_{\text{firm-leve sales}} = -(\sigma_k - 1) \ln \underbrace{\tilde{p}_{j,kt}(\omega)}_{\text{firm-level price}} + \left[1 - \frac{\sigma_k - 1}{\gamma_k - 1}\right] \ln \underbrace{\lambda_{j,kt}(\omega)}_{\text{within-national market share}} + \delta_{kt} + \varepsilon_{\omega jkt}$$

- Data Source: Universe of Colombian import transactions during 2007-2013, covering 226,288 exporting firms from 251 different countries. Estimation Details

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Quantifying the Gains from Policy

Sketch of Optimization-Free Quantitative Strategy

- Our goal is to simulate the counterfactual equilibrium under optimal policy.
- A bullet point summary of our quantitative strategy:
  1. Use exact hat-algebra ⟶ express optimal policy formulas in changes
  2. Use exact hat-algebra ⟶ express equilibrium conditions in changes
  3. Solve the system of equations derived under Steps (1) and (2)
- Step (3) determines the change in *real GDP* in response to optimal policy as a function of the following *sufficient statistics*:

$$\mathcal{B}_v \equiv \{\lambda_{ni,k}, e_{n,k}, r_{ni,k}, \rho_{i,k}, w_n \bar{L}_n, Y_n\}_{ni,k} \qquad \mathcal{B}_e = \{\sigma_k - 1, \mu_k\}_k$$

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national accounts data

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estimable parameters

Data Sources

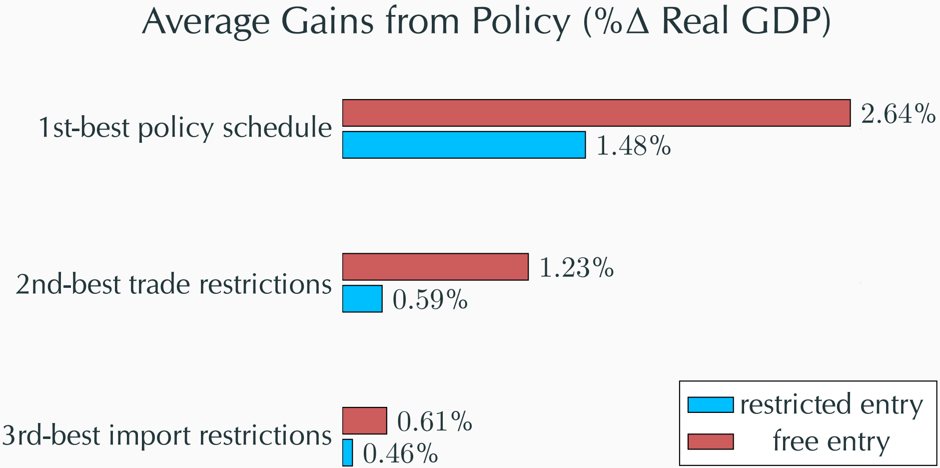
WORLD INPUT-OUTPUT DATABASE (2000-2014)

- production and expenditure by *origin×destination×industry*.
- 44 Countries + an aggregate of the rest of the world
- 56 Industries

UNCTAD-TRAINS Database:

- Average industry-level tariffs for all *44×43* country pairs.

Gains from Non-Cooperative Optimal Policies



Accounting for firm-selection

σ_k and μ_k estimated in levels

The Immiserizing Growth Effects of Industrial Policy

Welfare consequences of *corrective* industrial subsidies under **free entry**

- Unilateral adoption ⟶ **0.70%** decline in real GDP
- Coordinated via a deep agreement ⟶ **3.22%** rise in real GDP

Welfare consequences of *corrective* industrial subsidies under **restricted entry**

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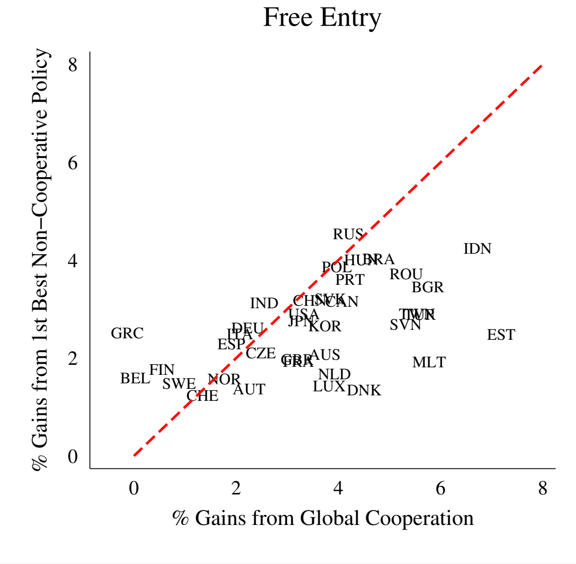
Deep Cooperation vs. Non-Cooperation



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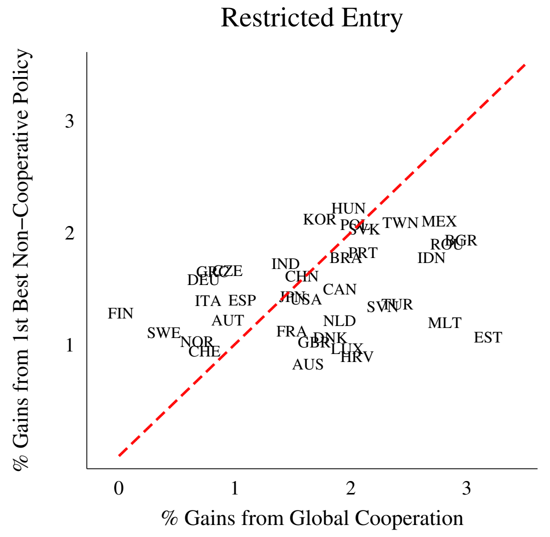

Concluding Remarks

- The gains from *terms-of-trade* manipulation are small!
- Trade restrictions are an ineffective *second-best* measure for correcting misallocation in domestic industries.
- Unilateral industrial policy is equally ineffective, as it triggers *immiserizing growth* in most countries.
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Thank you

References

Equilibrium for a given Vector of Taxes $(\mathbf{t}, \mathbf{x}, \mathbf{s}, \tau)$

1. Consumption choices are optimal: $\begin{cases} Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i) \\ \tilde{P}_{ji,k} = \frac{1+t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji,k} \end{cases}$

2. Production choices are optimal: $P_{ij,k} = C_{ij,k} \times w_i Q_{i,k}^{-\frac{\mu_k}{1+\mu_k}}$

3. Wage payments equal net sales: $w_i L_i = \sum_{j=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} \left[P_{ij,k} Q_{ij,k} \right]$

4. Income equals wage payments plus tax revenues: $Y_i = w_i L_i + \mathcal{R}_i(\mathbf{t}, \mathbf{x}, \mathbf{s})$

Return

Equilibrium for a given Vector of Taxes $(\mathrm{t}, \mathrm{x}, \mathrm{s}, \tau)$

1. Consumption choices are optimal: $\begin{cases} \mathrm{Q}_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i) \\ \tilde{P}_{ji,k} = \frac{1+t_{ji,k}}{(1+x_{ji,k})(1+s_{j,k})} P_{ji,k} \end{cases}$

2. Production choices are optimal: $P_{ij,k} = \mathrm{C}_{ij,k} \times w_i Q_{i,k}^{-\frac{\mu_k}{1+\mu_k}}$

3. Wage payments equal net sales: $w_i L_i = \sum_{j=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} \left[P_{ij,k} Q_{ij,k}\right]$

4. Income equals wage payments plus tax revenues: $Y_i = w_i L_i + \mathcal{R}_i(\mathbf{t}, \mathbf{x}, \mathbf{s})$

tax revenues

Return

Our Dual Approach to Characterizing $\mathbb{T}^\star$

Step 1–Reformulate the optimal policy problem

- The government in i chooses optimal consumer prices and abatement levels

$$\max_{\mathbb{T}_i} \; W_i(\mathbb{T}_i; \mathbb{X}_i) \quad \textbf{[P1]} \quad \xrightarrow{\text{reformulate}} \quad \max_{\mathbb{P}_i} \; W_i(\mathbb{P}_i; \mathbb{X}_i) \quad \textbf{[P1}'\textbf{]}$$

- Optimal taxes can be recovered from the optimal choice w.r.t. $\mathbb{P}_i$

$$1 + t^\star_{ji,k} = \frac{\tilde{P}^\star_{ji,k}}{P_{ji,k}}, \qquad 1 + x^\star_{ij,k} = \frac{P^\star_{ij,k}}{\tilde{P}_{ij,k}} \frac{P^\star_{ii,k}}{\tilde{P}_{ii,k}}, \qquad 1 + s^\star_{i,k} = \frac{P^\star_{ii,k}}{\tilde{P}_{ii,k}}$$

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Our Dual Approach to Characterizing $\mathbb{T}^\star$

Step 2–Derive F.O.C.s for the reformulated problem P1′

- This step is complicated by GE considerations ⟶ traditional theories bypass these complications by focusing on partial equilibrium 2-by-2 models.
- Intermediate Envelope Theorem: The first-order conditions associated with Problem P1′ can be derived ***as if***
    1. wages $\mathbf{w} = \{w_i\}$ are constant $\sim$ GE wage effects are *welfare-neutral*
    2. demand is income inelastic $\sim$ GE income effects are *welfare-neutral* at the optimum

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Our Dual Approach to Characterizing $\mathbb{T}^\star$

Step 3–Solve the system of F.O.C.s

- We use the primitive properties of Marshallian demand (i.e., *Cournot aggregation*, *homogeneity of degree zero*) to prove that the system of F.O.C.s admits a unique and trivial solution.

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Identification Strategy

Take first differences to eliminate the firm-product FE

$$\Delta \ln X_{j,kt}(\omega) = -(\sigma_k - 1)\Delta \ln \tilde{p}_{j,kt}(\omega) + (1 - \mu_k[\sigma_k - 1])\,\Delta \ln \lambda(\omega \mid j, kt) + \tilde{\delta}_{kt} + \Delta\varepsilon_{\omega jkt}$$

- **Identification Challenge:** $\Delta \ln p$ and $\Delta \ln \lambda$ maybe correlated with $\Delta\varepsilon$.
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Main Instrument

- Compile an external database on monthly exchange rates.
- Interact the change in monthly exchange rates w/ *prior* monthly export behavior to construct a *variety-specific* shift-share IV:

$$z_{j,kt}(\omega) = \sum_{m=1}^{12} \left([\text{share of month } m \text{ sales in } t-1] \times \Delta \ln \mathcal{E}_{j,t}(m) \right)$$

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Sector	ISIC4 codes	Estimated Parameter $\sigma_k - 1$	 $\frac{\sigma_k-1}{\gamma_k-1}$	 μ_k	Obs.	Weak Ident. Test
Agriculture & Mining	100-1499	6.212	0.875	0.141	11,962	2.51
		(2.112)	(0.142)	(0.167)		
Food	1500-1699	3.333	0.883	0.265	20.042	6.00
		(0.815)	(0.050)	(0.131)		
Textiles, Leather & Footwear	1700-1999	3.413	0.703	0.207	126,483	63.63
		(0.276)	(0.020)	(0.022)		
Wood	2000-2099	3.329	0.899	0.270	5,962	1.76
		(1.331)	(0.181)	(0.497)		
Paper	2100-2299	2.046	0.813	0.397	37,815	2.65
		(0.960)	(0.216)	(0.215)		
Petroleum	2300-2399	0.397	0.698	1.758	4,035	2.03
		(0.342)	(0.081)	(1.584)		
Chemicals	2400-2499	4.320	0.915	0.212	134,413	42.11
		(0.376)	(0.027)	(0.069)		

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Rubber & Plastic	2500-2599	3.599	0.582	0.162	107,713	7.22
		(0.802)	(0.041)	(0.039)		
Minerals	2600-2699	4.561	0.847	0.186	28,197	3.19
		(1.347)	(0.096)	(0.129)		
Basic & Fabricated Metals	2700-2899	2.959	0.559	0.189	155,032	16.35
		(0.468)	(0.024)	(0.032)		
Machinery	2900-3099	8.682	0.870	0.100	266,628	8.54
		(1.765)	(0.080)	(0.065)		
Electrical & Optical Equipment	3100-3399	1.392	0.631	0.453	260,207	17.98
		(0.300)	(0.015)	(0.099)		
Transport Equipment	3400-3599	2.173	0.289	0.133	86,853	5.09
		(0.589)	(0.028)	(0.036)		
N.E.C. & Recycling	3600-3800	6.704	0.951	0.142	70,974	8.51
		(1.133)	(0.100)	(0.289)		

Summary of Estimates

- High-μ sectors:
  1. Electrical & Optical Equipment
  2. Petroleum

- Low-μ sectors:
  1. Agriculture & Mining
  2. Wood

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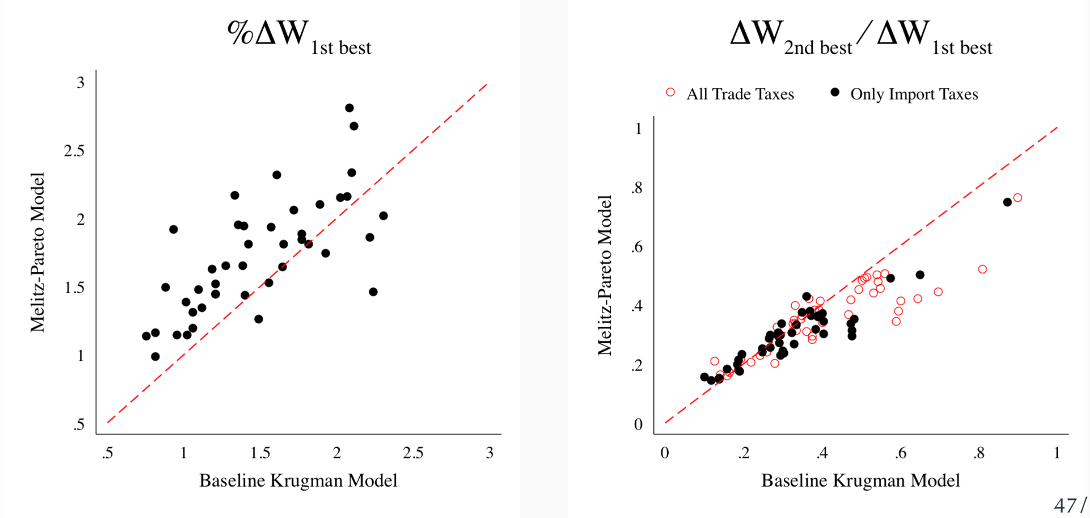
Accounting for Firm-Selection à la Melitz-Chaney

Return



Gains Implied by σ_k and μ_k Estimated in Levels

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