Introduction to Quantitative Trade Models

International Trade (PhD), Spring 2023

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Background

- The class of trade models covered in this class (*e.g.*, Armington, Krugman, Eaton-Kortum, Melitz-Pareto) deliver a common macro-level representation for general equilibrium.

- These models have two appealing features:
  1. They predict trade values consistent with a gravity equation:

$$\text{Trade Value}_{in} \propto \frac{\text{GDP}_i \times \text{GDP}_n}{\text{Distance}_{in}^{\beta}} \qquad (\text{origin } i, \ \text{destination } n)$$

     which amounts to good in-sample predictive power *w.r.t.* trade flows.

  2. They can be used to perform counterfactual analyses based on easy-to-obtain sufficient statistics: (1) trade shares, (2) national accounts data, and (3) trade elasticities.

Road Map of Today's Lecture

- *First*, we present the common representation of general equilibrium implied by quantitative trade models (*e.g.*, Armington, Krugman, Eaton-Kortum, Melitz-Pareto).

- *Second*, we overview the *ex-post* and *ex-ante* applications of these models, highlighting their merits relative to alternative research designs (*e.g.*, diff-in-diff, shift-share).

- *Third*, we discuss the structural estimation of these models and the exact hat-algebra technique for obtaining counterfactual (or out-of-sample) predictions.

Environment

- The global economy consist of $N > 1$ countries.
- We use $i, j, n \in \{1, .., N\}$ to index countries
- Labor is the only factor of production
- Country i is endowed with L_i units of labor

[1] See Adao, Costinot, Donaldson (2017, AER) and Costinot & Rodriguez-Clare (2018, JEP).

Environment

- The global economy consist of $N > 1$ countries.
- We use $i, j, n \in \{1, .., N\}$ to index countries
- Labor is the only factor of production
- Country i is endowed with L_i units of labor
- ***Note***: The class of trade models we study can be alternatively cast as a fictitious endowment economy in which trade values reflect the international demand for each country's labor services.[1]

[1] See Adao, Costinot, Donaldson (2017, AER) and Costinot & Rodriguez-Clare (2018, JEP).

Exogenous Parameters or Variables

- L_i is country i 's labor endowment
- χ_i encompasses information on country i's technological endowment
- τ_{in} is the iceberg trade cost associated with origin i's sales to destination n
- ϵ is the elasticity of trade values w.r.t. trade costs (i.e., the trade elasticity)
- D_i is country i's trade deficit vis-à-vis the rest of the world ($\sum D_i = 0$).

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Note: only L_i and D_i are directly observable, the remaining parameters must be estimated.

Endogenous Equilibrium Outcomes

Main independent outcome

- the vector of national-level wages $\{w_1, ..., w_N\}$

Outcomes determined by wages exogenous parameters

- λ_{in} ~ the share of country *n*'s expenditure on goods originating from country *i*
- E_n ~ country *n*'s total expenditure (GDP + deficit)

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Note that λ_{in} and E_n are readily observable, whereas w_i is difficult to measure as it represents a national-level index of factor prices.

The General Equilibrium

For a given vector of parameters $\{\epsilon, \chi_i, L_i, D_i, \tau_{in}\}_{i,n}$ equilibrium is a vector of wages, $\{w_i\}_i$, such that labor markets clear in each country:

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}\left(w_1, ..., w_N\right) E_n\left(w_n\right)}_{\text{country } i\text{'s sales to country } n} = w_i L_i \quad , \forall i$$

with bilateral expenditure shares (λ_{in}) and national expenditure (E_n) given by

$$\begin{cases} \lambda_{in}\left(w_1, ..., w_N\right) = \frac{\chi_i \left(\tau_{in} w_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \chi_j \left(\tau_{jn} w_j\right)^{-\epsilon}} & \forall i, j \\ E_n\left(w_n\right) = w_n L_n + D_n & \forall n \end{cases}$$

The General Equilibrium

- Given $\{\epsilon, L_i, D_i, \chi_i, \tau_{ij}\}_{i,j}$, the vector of wages $\{w_1, ..., w_N\}$ can be computed by solving a non-linear system of *N-equations* and *N-unknowns*[2]

$$\underbrace{\frac{1}{w_i}\sum_{n=1}^{N}\left[\frac{\chi_i\left(\tau_{in}w_i\right)^{-\epsilon}}{\sum_{j=1}^{N}\chi_j\left(\tau_{jn}w_j\right)^{-\epsilon}}\left(w_nL_n+D_n\right)\right]}_{\text{demand for country } i\text{'s labor}} = \overbrace{L_i}^{\text{labor supply}}$$

- Workhorse trade models can be cast as a fictitious endowment economy in which countries directly exchange labor services.

- The main equilibrium outcome is a vector of wages that equalizes the supply and demand for each country's labor.

[2] Link to Matlab routine that solves the above system

The General Equilibrium

- When mapping trade models to data is useful to specify equilibrium in terms of national income or GDP ($Y_i = w_i L_i$) rather than wages.

- Given $\{\epsilon, L_i, D_i, \tilde{\chi}_i, \tau_{ij}\}_{i,j}$, equilibrium can be alternatively defined as a vector $\{Y_1, ..., Y_N\}$ that solve the following system of equations

$$\sum_{n=1}^{N} \left[\frac{\tilde{\chi}_i \left(\tau_{in} Y_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau_{jn} Y_j\right)^{-\epsilon}} \left(Y_n + D_n\right) \right] = Y_i, \quad \text{where} \quad \tilde{\chi}_i \equiv \chi_i L_i^{\epsilon}$$

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- The above formulation is also useful for deriving the gravity equation.

The Gravity Equation

– Let $X_{in} = \lambda_{in} \times E_n$ denotes trade flows from origin i to destination n

$$X_{in} = \frac{\tilde{\chi}_i \left(\tau_{in} Y_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau_{jn} Y_j\right)^{-\epsilon}} E_n$$

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$$X_{in} = \tau_{in}^{-\epsilon} \quad \underbrace{\tilde{\chi}_i \left(Y_i\right)^{-\epsilon}}_{\Phi_i} \quad \underbrace{\frac{E_n}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau_{jn} Y_j\right)^{-\epsilon}}}_{\Omega_n}$$

The Gravity Equation

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$$X_{in} = \tau_{in}^{-\epsilon} \, \Phi_i \, \Omega_n$$

- $\tau_{in}^{-\epsilon}$ represents trade frictions relating to taste differences, transport costs, or policy.
- Φ_i is the *exporter fixed effect*, summarizing all relevant information on origin i
- Ω_n is the *importer fixed effect*, summarizing all relevant information on destination n

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The Gravity Equation

- The Labor Market Clearing condition specifies Φ_i in terms of Y_i

$$\sum_{n=1}^{N} X_{in} = \Phi_i \sum_{n=1}^{N} \left[\tau_{in}^{-\epsilon}\Omega_n\right] = Y_i \quad \Longrightarrow \quad \Phi_i = \frac{Y_i}{\sum_n \Omega_n \tau_{in}^{-\epsilon}} \qquad (*)$$

- The national-level budget constraint specifies Ω_i in terms of E_i

$$\sum_{n=1}^{N} X_{ni} = \sum_{n=1}^{N} \left[\Phi_n \tau_{ni}^{-\epsilon}\right] \Omega_i = E_i \quad \Longrightarrow \quad \Omega_i = \frac{E_i}{\sum_n \Phi_n \tau_{ni}^{-\epsilon}} \qquad (**)$$

- Combining equation $(*)$ and $(**)$ and noting that $\tau_{in}^{-\epsilon} \sim \text{Dist}_{in}^{-\beta}$, yields

$$X_{in} = \frac{Y_i}{\sum_n \Omega_n \text{Dist}_{in}^{-\beta}} \times \frac{E_n}{\sum_n \Phi_n \text{Dist}_{ni}^{-\beta}} \times \text{Dist}_{in}^{-\beta}$$

An Implicit Property of Quantitative Trade Models

Proposition. If trade trade costs are symmetric and there are no *aggregate* trade imbalances, then trade values are bilaterally balanced

$$\begin{cases} \tau_{ji} = \tau_{ij} & \forall i, j \\ D_i = 0 & \forall i \end{cases} \implies X_{ij} = X_{ji} \quad (\forall i, j)$$

- The above proposition can be proven by appealing to Equations $(*)$ and $(**)$, and showing that $\Phi_i = \Omega_i$ *if* $\tau_{ji} = \tau_{ij}$ and $D_i = 0$.
- **Implication:** bilateral trade imbalances may be a mere reflection of aggregate trade imbalances rather than asymmetric trade barriers.

Applications of Quantitative Trade Models

- Quantitative trade models can be used to examine the *ex-ante* or *ex-post* impacts of shocks to the global economy.

Example of ex-ante application

- What is the impact of a eliminating aggregate trade imbalances?
- The shock we seek to examine ($D_i \to 0$) has not materialized yet, so non-structural research designs such as *diff-in-diff* or *shift-share* are not applicable.

Example of ex-post application

- What was the impact of NAFTA on the US economy?
- The NAFTA shock ($\Delta\tau^{\text{NAFTA}} < 0$) has already materialized, but non-structural research designs (if applicable) may fail to identify the GE effects of NAFTA.

Two Approaches to Performing Counterfactual Analyses

- The noted applications require that we simulate the counterfactual equilibrium that emerges after say the NAFTA shock. This task can be accomplished in two ways.

First Approach

- Estimate the full parameters of the model
- shock the parameters and re-solve the model to obtain counterfactual outcome

Second Approach

- Apply the exact the hat-algebra technique
- Under this approach we no longer need to estimate τ_{ni} or $\tilde{\chi}_i$, since the information on these parameters if fully embedded in expenditure shares and income levels.

Class Assignment

- Quantitative trade models predict trade flows are given by

$$X_{in} = \frac{\tilde{\chi}_i \left(\tau_{in} Y_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau_{jn} Y_j\right)^{-\epsilon}} E_n$$

  and satisfy the adding up constraint $\sum_n X_{in} = Y_i$ for all i.

- X_{ni}, Y_i, and E_i are observable in the data.
- **How would you estimate $\tilde{\chi}_i$, τ_{in}, and ϵ?**

Class Assignment

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- X_{ni}, Y_i, and E_i are observable in the data.
- **How would you estimate $\tilde{\chi}_i$, τ_{in}, and ϵ?**
- I will create an "Announcement" on Canvas. Submit your answer as a comment underneath the announcement before Tuesday, next week.

Estimation of Quantitative Trade Models

Estimation Setup

- Data points: $\mathbb{D} = \left\{X_{ni}^{data}, Y_i^{data}, E_i^{data}\right\}_{i,n}$
- Unobserved parameters: $\Theta = \{\tau_{in}, \tilde{\chi}_i, \epsilon\}_{i,n}$
- Model's prediction *w.r.t.* trade flows, given $\left\{Y_i^{data}\right\}_i$ and $\left\{E_i^{data}\right\}_i$

$$X_{in}(\Theta; \mathbb{D}) = \frac{\tilde{\chi}_i \left(\tau_{in} Y_i^{data}\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau_{jn} Y_j^{data}\right)^{-\epsilon}} E_n^{data}$$

Note: ϵ cannot be separately identified from τ_{in} with information on $\mathbb{D}$

- Parameter combinations $\{\tilde{\chi}_i, \tau_{in}, \epsilon\}_{i,n}$ and $\{\tilde{\chi}_i, \tau'_{in}, \epsilon'\}_{i,n}$ are observationally equivalent in terms of their prediction vis-à-vis $\mathbb{D}$ *iff* $\tau_{in}^{-\epsilon} = (\tau'_{in})^{-\epsilon'}$.

Generic Estimation Strategy

- We can normalize ϵ and estimate the remaining elements of Θ by minimizing the distance between the model's predictions and data *subject to* equilibrium constraints:

$$\min_{\Theta} \sum_{n,i} \left(\log X_{in} \left(\Theta ; \mathbb{D} \right) - \log X_{in}^{data} \right)^2 \qquad s.t. \qquad \sum_{n} X_{in} \left(\Theta ; \mathbb{D} \right) = Y_i^{data} \quad (\forall i)$$

- The above problem is exactly identified, *i.e.*, there exists a Θ^* such that

$$X_{in} \left(\Theta^* ; \mathbb{D} \right) = X_{in}^{data} \quad (\forall i, n)$$

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- We can use Θ^* to perform counterfactuals (*e.g.*, eliminating trade imbalances), *but* this task can be performed more efficiently with *exact hat algebra*.

Estimating the Determinants of Trade Costs

- We can use a similar strategy to estimate the determinants of τ_{in}.
- Suppose we have data on bilateral distance, FTAs, common language, common border, and conflict for many country pairs.
- We can parameterize bilateral trade costs as

$$\tau_{in} = \bar{\tau} \left(\mathrm{Dist}_{in}\right)^{\beta_d} \cdot \beta_f^{\mathsf{FTA}_{in}} \cdot \beta_l^{\mathsf{Lang}_{in}} \cdot \beta_b^{\mathsf{Border}_{in}} \cdot \beta_c^{\mathrm{Conflict}_{in}}$$

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Interpretation of Parameters

- $\beta_f = 0.75$ implies that a typical FTA reduces trade costs by 25%
- $\beta_c = 1.5$ implies that conflict increases trade costs by 50%; *etc.*

Estimation

- Reduced set of parameters: $\tilde{\Theta} = \{\tilde{\chi}_i, \beta_d, \beta_f, \beta_l, \beta_b, \beta_c, \epsilon\}$
- We can normalize ϵ and estimate the remaining elements of $\tilde{\Theta}$ as

$$\min_{\tilde{\Theta}} \sum_{n,i} \left(\log X_{in} \left(\tilde{\Theta}; \mathbb{D} \right) - \log X_{in}^{data} \right)^2 \qquad s.t. \qquad \sum_{n} X_{in} \left(\tilde{\Theta}; \mathbb{D} \right) = Y_i^{data} \quad (\forall i)$$

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- The estimation of β's unveils policy-relevant shocks for counterfactual analysis—*e.g.*,

$$\text{aboloshing FTAs} \qquad \sim \qquad \Delta \ln \tau'_{in} \approx \begin{cases} \beta_f - 1 & \text{if } \ \text{FTA}_{in} = 1 \\ 0 & \text{if } \ \text{FTA}_{in} = 0 \end{cases}$$

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- The estimation of β's unveils policy-relevant shocks for counterfactual analysis—*e.g.*,

$$\text{global conflict} \qquad \sim \qquad \Delta \ln \tau'_{in} \approx \begin{cases} 0 & \text{if Conflict}_{in} = 1 \\ \beta_c - 1 & \text{if Conflict}_{in} = 0 \end{cases}$$

The Exact Hat-Algebra Approach

Definition of Equilibrium

- For any set of exogenous parameters and variables $\{\tau_{in}, \tilde{\chi}_i, D_i, \epsilon\}$, equilibrium is a vector of national GDP levels, $\mathbf{Y} = \{Y_1, ..., Y_N\}$, that satisfy

$$Y_i = \sum_{n=1}^{N} \left[\lambda_{in}(\mathbf{Y}) \times \overbrace{(Y_n + D_n)}^{E_n} \right], \qquad (\forall i)$$

where the expenditure share $\lambda_{in}(\mathbf{Y})$ is given by

$$\lambda_{in}(\mathbf{Y}) = \frac{\tilde{\chi}_i (\tau_{in} Y_i)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j (\tau_{jn} Y_j)^{-\epsilon}}, \qquad (\forall i, n)$$

Hat-Algebra Notation

For a generic variable (x)

- x ~ baseline value under the status quo
- x' ~ counterfactual value after some external shock
- $\widehat{x} \equiv \frac{x'}{x}$

Example: suppose countries i and n sign an FTA that lowers their bilateral trade cost by 25% and increases their bilateral trade value by 15%:

$$\hat{\tau}_{in} = \hat{\tau}_{ni} = 0.75; \qquad \widehat{X}_{in} = \widehat{X}_{ni} = 1.15$$

Counterfactual Expenditure Shares

- Consider an external shock to trade costs: $\left\{\hat{\tau}_{in}\right\}_{i,n}$
- Considering that exogenous parameters ($\tilde{\chi}_i$ and ϵ) are unaffected by the shock, counterfactual expenditure shares are

$$\lambda'_{in} = \frac{\tilde{\chi}_i \left(\tau'_{in} Y'_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\tau'_{jn} Y'_j\right)^{-\epsilon}}$$

- Noting that $\tau'_{in} = \hat{\tau}_{in}\tau_{in}$ and $Y'_i = \widehat{Y}_i Y_i$ we can rewrite this equation as

$$\lambda'_{in} = \frac{\tilde{\chi}_i \left(\hat{\tau}_{in}\tau_{in} \widehat{Y}_i Y_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \tilde{\chi}_j \left(\hat{\tau}_{jn}\tau_{jn} \widehat{Y}_j Y_j\right)^{-\epsilon}} = \frac{\lambda_{in} \left(\hat{\tau}_{in} \widehat{Y}_i\right)^{-\epsilon}}{\sum_{j=1}^{N} \lambda_{jn} \left(\hat{\tau}_{jn} \widehat{Y}_j\right)^{-\epsilon}}$$

Counterfactual Equilibrium

- Labor-market clearing condition in the counterfactual equilibrium:

$$Y_i' = \sum_{n=1}^{N} [\lambda_{in}' \times (Y_n' + D_n)]$$

Counterfactual Equilibrium

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- The above system determines $\left\{ \widehat{Y}_1, ..., \widehat{Y}_N \right\}$ with information on observables $\mathbb{D} = \{Y_i, D_i, \lambda_{in}\}_{i,n}$ and the trade elasticity, ϵ

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- Given $\widehat{Y}_i$, we can calculate the change in trade values in response to $\{\hat{\tau}_{in}\}_{i,n}$ as

$$\widehat{X}_{in} = \widehat{\lambda}_{in} \times \underbrace{\frac{Y_n \widehat{Y}_n + D_n}{Y_n + D_n}}_{\widehat{E}_n}, \qquad \text{where} \qquad \widehat{\lambda}_{in} = \frac{\left(\hat{\tau}_{in} \widehat{Y}_i \right)^{-\epsilon}}{\sum_{j=1}^{N} \lambda_{jn} \left(\hat{\tau}_{jn} \widehat{Y}_j \right)^{-\epsilon}}$$

Example: *the US and the Rest of the World*

- *Two countries*: US ($i = 1$) and ROW ($i = 2$)

$$\boldsymbol{\lambda} = \begin{bmatrix} 0.88 & 0.02 \\ 0.12 & 0.98 \end{bmatrix}; \qquad \mathbf{Y} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}; \qquad \mathbf{D} = \begin{bmatrix} 0.04 \\ -0.04 \end{bmatrix}$$

- Suppose international trade costs fall by 20%:

$$\hat{\boldsymbol{\tau}} = \begin{bmatrix} 1 & 0.80 \\ 0.80 & 1 \end{bmatrix}$$

Example: *the US and the Rest of the World*

- System of equations specifying labor-market clearing conditions:

$$Y_1 \widehat{Y}_1 = \frac{\lambda_{11}\left(\hat{\tau}_{11}\widehat{Y}_1\right)^{-\epsilon} \times \left(Y_1\widehat{Y}_1 + D_1\right)}{\lambda_{11}\left(\hat{\tau}_{11}\widehat{Y}_1\right)^{-\epsilon} + \lambda_{21}\left(\hat{\tau}_{21}\widehat{Y}_2\right)^{-\epsilon}} + \frac{\lambda_{12}\left(\hat{\tau}_{12}\widehat{Y}_1\right)^{-\epsilon} \times \left(Y_2\widehat{Y}_2 + D_2\right)}{\lambda_{12}\left(\hat{\tau}_{12}\widehat{Y}_1\right)^{-\epsilon} + \lambda_{22}\left(\hat{\tau}_{22}\widehat{Y}_2\right)^{-\epsilon}}$$

$$Y_2 \widehat{Y}_2 = \frac{\lambda_{21}\left(\hat{\tau}_{21}\widehat{Y}_2\right)^{-\epsilon} \times \left(Y_1\widehat{Y}_1 + D_1\right)}{\lambda_{11}\left(\hat{\tau}_{11}\widehat{Y}_1\right)^{-\epsilon} + \lambda_{21}\left(\hat{\tau}_{21}\widehat{Y}_2\right)^{-\epsilon}} + \frac{\lambda_{22}\left(\hat{\tau}_{22}\widehat{Y}_2\right)^{-\epsilon} \times \left(Y_2\widehat{Y}_2 + D_2\right)}{\lambda_{12}\left(\hat{\tau}_{12}\widehat{Y}_1\right)^{-\epsilon} + \lambda_{22}\left(\hat{\tau}_{22}\widehat{Y}_2\right)^{-\epsilon}}$$

- Assuming $\epsilon = 5$, solving the system implies[3]

$$\widehat{\mathbf{Y}} = \begin{bmatrix} 1.025 \\ 1.062 \end{bmatrix} \implies \widehat{\mathbf{X}} = \begin{bmatrix} 0.86 & 3.66 \\ 2.22 & 1.01 \end{bmatrix}$$

[3] See Canvas for the Matlab code that generates these numbers.

Example: *the US and the Rest of the World*

- System of equations specifying labor-market clearing conditions:

$$\widehat{Y}_1 = \frac{0.88\left(\widehat{Y}_1\right)^{-\epsilon} \times \left(\widehat{Y}_1 + 0.04\right)}{0.88\left(\widehat{Y}_1\right)^{-\epsilon} + 0.12\left(0.80\,\hat{Y}_2\right)^{-\epsilon}} + \frac{0.02\left(0.80\,\widehat{Y}_1\right)^{-\epsilon} \times \left(4\,\widehat{Y}_2 - 0.04\right)}{0.02\left(0.80\,\widehat{Y}_1\right)^{-\epsilon} + 0.98\left(\widehat{Y}_2\right)^{-\epsilon}}$$

$$4\,\widehat{Y}_2 = \frac{0.12\left(0.80\,\widehat{Y}_2\right)^{-\epsilon} \times \left(\widehat{Y}_1 + 0.04\right)}{0.88\left(\widehat{Y}_1\right)^{-\epsilon} + 0.12\left(0.80\,\widehat{Y}_2\right)^{-\epsilon}} + \frac{0.98\left(\widehat{Y}_2\right)^{-\epsilon} \times \left(4\,\widehat{Y}_2 - 0.04\right)}{0.02\left(0.80\,\widehat{Y}_1\right)^{-\epsilon} + 0.98\left(\widehat{Y}_2\right)^{-\epsilon}}$$

- Assuming $\epsilon = 5$, solving the system implies[3]

$$\widehat{\mathbf{Y}} = \begin{bmatrix} 1.025 \\ 1.062 \end{bmatrix} \implies \widehat{\mathbf{X}} = \begin{bmatrix} 0.86 & 3.66 \\ 2.22 & 1.01 \end{bmatrix}$$

[3] See Canvas for the Matlab code that generates these numbers.

Taking Stock

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- The exact hat-algebra approach enables us to perform counterfactuals without estimating the trade cost or technology parameters (τ and $\tilde{\chi}$).
- Performing counterfactuals requires two sets of sufficient statistics:
  1. Observable statistics: λ_{in}, Y_i, and E_i.
  2. Trade Elasticity: ϵ

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- Performing counterfactuals requires two sets of sufficient statistics:
  1. Observable statistics: λ_{in}, Y_i, and E_i.
  2. Trade Elasticity: ϵ
- In the class of models we study, the change in welfare in response to an external shock, $\{\hat{\tau}_{in}, \widehat{\chi}_i\}_{i,n}$, can be also calculated using *exact hat-algebra* as

$$\hat{W}_i = \hat{\tau}_{ii}^{-1} \times \widehat{\chi}_i^{\frac{1}{\epsilon}} \times \widehat{\lambda}_{ii}^{-\frac{1}{\epsilon}}$$

- In the earlier example: $\hat{\tau}_{ii} = \hat{\chi}_i = 1 \longrightarrow \widehat{W}_i = \widehat{\lambda}_{ii}^{-\frac{1}{\epsilon}}$.

Accompanying Code

‘

- Link to the code and data accompanying this lecture, which includes
  1. Matlab code for MPEC estimation
  2. Matlab code for nested fixed point estimation
  3. Matlab code corresponding to the exact hat-algebra example

Accompanying Code

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- Link to the code and data accompanying this lecture, which includes
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- Class assignment: modify ‘‘`HAT_ALGEBRA_EXAMPLE.m`’’ to calculate the effect of eliminating aggregate trade imbalances ($D_i \rightarrow D_i' = 0$) on US’s exports & imports.