The Multi-Industry Trade Model with IO Linkages

International Trade (PhD), Fall 2019

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Overview

- In this lecture, we introduce inout-output (IO) linkages into the multi-industry trade model.
- Main implications
  - The gains from trade are larger once we account for IO linkages.
  - IO linkages aggravate distortions like markups, trade barriers, or tariffs.
- **References:**
  - *Costinot & Rodriguez-Clare (2014)*
  - *Caliendo & Parro (2014)*: application to NAFTA

Environment

- $j, \ i = 1, \ldots, N$ countries
- $k = 1, \ldots, K$ industries
- Labor is the only factor of production.
- Country i is endowed with L_i units of labor.
- Every good can be used as either a final consumption good or an intermediate input good.

Product Space

- All products are differentiated by country of origin *à la* Armington.
- ji, k indexes a good corresponding to *Exporter* $j \times$ *Importer* $i \times$ *industry* k.
- Every good ji, k can be used as
  1. a final consumption good
  2. an intermediate input good
- **Example:** milk can be used for final consumption or as an input for ice-cream production.

Demand for Final Goods: *Armington*

The preferences of the representative consumer are described by a *two-tier* Cobb-Douglas-CES utility function:

$$U_i(\mathbf{Q}_{1i}, \ldots, \mathbf{Q}_{Ni}) = \prod_{k=1}^{K} \left(Q_{1i,k}^{\rho_k} + \ldots + Q_{Ni,k}^{\rho_k} \right)^{\frac{\beta_{i,k}}{\rho_k}}$$

- Index ji, k corresponds to *Exporter* $j \times$ *Importer* $i \times$ *industry* k.
- $\sigma_k \equiv 1/(1 - \rho_k)$ is the elasticity of sub. between national varieties.
- $\beta_{i,k}$ is the *constant* share of spending on final goods from industry k.

Demand for Final Goods

- Consumer's problem (p is price, Y is income):

$$\max_{\mathbf{Q}_i} U_i(\mathbf{Q}_{1i}, ..., \mathbf{Q}_{Ni})$$

$$\text{s.t.} \sum_{k=1}^{K} \sum_{j=1}^{N} P_{ji,k} Q_{ji,k} \leqslant Y_i \qquad (\mathbf{CP})$$

- Demand function implied by CP:

$$P_{ji,k} Q_{ji,k} = \left(\frac{P_{ji,k}}{P_{i,k}} \right)^{-\epsilon_k} \beta_{i,k} Y_i$$

  - $\epsilon_k \equiv \rho_k / (1 - \rho_k)$
  - $P_{i,k} = \left[\sum_j P_{ji,k}^{-\epsilon_k} \right]^{-\frac{1}{\epsilon_k}}$ is the *industry-level* price index.

Supply

- Production in industry k employs (i) labor (L), and (ii) composite intermediate inputs for various industries (I_g).

- The production function for composite variety ji, k

$$Q_{ij,k} = \frac{1}{\tau_{ij,k} a_{j,k}} L_{ij,k}^{1-\alpha_{i,k}} \prod_{g=1}^{K} I_{i,g}^{\alpha_{i,gk}}$$

  - $\alpha_{i,kg}$ is the share of industry g inputs in production ($\alpha_{i,k} \equiv \sum_g \alpha_{i,gk}$)
  - The composite intermediate input from industry g is given by

$$I_{i,g} = \left[Q_{1i,g}^{\tilde{\rho}_g} + \ldots + Q_{Ni,g}^{\tilde{\rho}_g} \right]^{1/\tilde{\rho}_g}$$

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- **Key assumption:** $\tilde{\rho}_k = \rho_k \Longrightarrow P_{i,k}^{\mathcal{I}} = P_{i,k}$

Supply

- Perfect competition + cost minimization implies

$$P_{ij,k} = \tau_{ij,k} a_{i,k} \, w_i^{1-\alpha_{j,k}} \prod_{g=1}^{K} P_{i,g}^{\alpha_{i,gk}}$$

- Total spending on intermediate inputs from industry g

$$E_{i,g}^{\mathcal{I}} \equiv P_{i,g} I_{i,g} = \sum_{k=1}^{K} \alpha_{i,gk} R_{i,k}$$

where $R_{i,k}$ is total *gross* revenue collected by country i in industry k:

$$R_{i,k} = \sum_{j=1}^{N} P_{ij,k} Q_{ij,k}$$

A Summary of Aggregate Demand and Supply

– Share of spending on composite variety ji, k (*final* + *intermediate*)

$$\lambda_{ij,k} = \frac{P_{ij,k}^{-\epsilon_k}}{\sum_{\ell=1}^{N} P_{\ell j,k}^{-\epsilon_k}}$$

– Country i's total revenue from industry k sales:

$$R_{i,k} = \sum_{j=1}^{N} \lambda_{ij,k} E_{j,k}$$

– Country i's total expenditure on industry k goods

$$E_{i,k} = \underbrace{\sum_{j=1}^{N} (\lambda_{ji,k} \beta_{i,k} Y_i)}_{\text{final goods}} + E_{i,k}^{\mathcal{I}}$$

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$$E_{i,k} = \sum_{j=1}^{N} \left(\lambda_{ij,k} \beta_{j,k} w_j L_j\right) + \sum_{g=1}^{K} \left(\alpha_{i,gk} R_{i,g}\right)$$

Formal Definition of Equilibrium

Equilibrium consists of $N \times K$ price indexes, $\mathbf{P} \equiv \{P_{i,k}\}$, N wage rates, $\boldsymbol{w} \equiv \{w_i\}$, $N \times K$ industry-level expenditure levels, $\mathbf{E} \equiv \{E_{i,k}\}$, $N \times K$ industry-level revenue levels, $\mathbf{R} \equiv \{R_{i,k}\}$, such that

$$\begin{cases} P_{i,k}^{-\epsilon_k} = \sum_{j=1}^{N} \left[P_{ji,k}(w_j, \mathbf{P}_j)^{-\epsilon_k}\right] & \forall i,k \\ E_{i,k} = \sum_{j=1}^{N} \left(\lambda_{ij,k}(\boldsymbol{w}, \mathbf{P})\beta_{j,k} w_j L_j\right) + \sum_{g=1}^{K} \left(\alpha_{i,gk} R_{i,g}\right) & \forall i,k \\ R_{i,k} = \sum_{j=1}^{N} \lambda_{ij,k}(\boldsymbol{w}, \mathbf{P}) E_{j,k} & \forall i,k \\ w_i L_i = \sum_{k=1}^{K} (1-\alpha_{i,k}) R_{i,k} & \forall i \end{cases}$$

where

$$\begin{cases} P_{ij,k}(w_i, \mathbf{P}_i) = \tau_{ij,k} a_{i,k} w_i^{1-\alpha_{i,k}} \prod_{g=1}^{K} P_{i,g}^{\alpha_{i,gk}} & \forall i,k \\ \lambda_{ij,k}(\boldsymbol{w}, \mathbf{P}) = \frac{P_{ji,k}(w_j, \mathbf{P}_j)^{-\epsilon_k}}{\sum_{\ell=1}^{N} P_{\ell i,k}(w_\ell, \mathbf{P}_\ell)^{-\epsilon_k}} & \forall i,j,k \end{cases}$$

Gains From Trade

- Taking logs from $\lambda_{ii,k} = \left(\frac{P_{ii,k}}{P_{i,k}}\right)^{-\epsilon_k}$ yields the following:

$$\ln P_{i,k} = -\frac{1}{\epsilon_k} \ln \lambda_{ii,k} + \ln P_{ii,k}$$

- Plugging $P_{ii,k} = \tau_{ii,k} a_{i,k} w_i^{1-\alpha_{i,k}} \prod_g P_{i,g}^{\alpha_{i,gk}}$ into the above expression + assigning country i's labor as the numeraire ($w_i = 1$) implies

$$\ln P_{i,k} = \gamma_{i,k} - \frac{1}{\epsilon_k} \ln \lambda_{ii,k} + \sum_g \alpha_{i,gk} \ln P_{i,g}$$

where $\gamma_{i,k}$ is composed of structural parameters

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where $\gamma_{i,k}$ is composed of structural parameters $\Longrightarrow$ we can invert to above system to solve for $\{P_{i,k}\}$ in terms of $\{\lambda_{ii,k}\}$.

Gains From Trade

- Inverting the system presented above, delivers the following expression

$$P_{i,k} = \tilde{\gamma}_{i,k} \times \prod_{g=1}^{K} \lambda_{ii,g}^{\frac{\tilde{\alpha}_{i,kg}}{1-\sigma_g}}$$

  where $\tilde{\gamma}_{i,k}$ is composed of structural parameters and $\tilde{\alpha}$'s are elements of *Leontief inverse*

$$\tilde{\mathbf{A}}_i = (\mathbf{I}_K - \mathbf{A}_i)^{-1} = \mathbf{I} + \mathbf{A}_i + \mathbf{A}_i^2 + \ldots$$

  - $\tilde{\mathbf{A}}_i \equiv [\tilde{\alpha}_{i,kg}]_{k,g}$ is a $K \times K$ matrix
  - $\mathbf{A}_i \equiv [\alpha_{i,kg}]_{k,g}$is country i's $K \times K$ I-O matrix

Gains From Trade

- Welfare in country i is given by

$$W_i = \frac{w_i}{P_i}$$

Gains From Trade

– Welfare in country i is given by ($w_i = 1$)

$$W_i = \frac{1}{P_i} = \prod_{k=1}^{K} P_{i,k}^{-\beta_{i,k}}$$

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- Gains from trade

$$GT_i = 1 - \frac{W_i^A}{W_i} = 1 - \frac{P_i^A}{P_i}$$

Gains From Trade

- Welfare in country i is given by ($w_i = 1$)

$$W_i = \frac{1}{P_i} = \prod_{k=1}^{K} P_{i,k}^{-\beta_{i,k}}$$

- Gains from trade ($\lambda_{ii,k}^{A} = 1$)

$$GT_i = 1 - \prod_{k=1}^{K} \prod_{g=1}^{K} \lambda_{ii,g}^{\frac{\tilde{\alpha}_{i,kg}}{\epsilon_g}\beta_{i,k}}$$

Recap: *Evaluating GT in the Presence of IO Linkages*

- ***Step 1*:** compile industry-level data for domestic expenditure shares, $\{\lambda_{ii,k}\}$, and I-O matrix, $\mathbf{A}_i \equiv \{\alpha_{i,gk}\}_{k,g}$.[1]

- ***Step* 2:** compute the *Leontief inverse:* $\tilde{\mathbf{A}}_i = (\mathbf{I}_K - \mathbf{A}_i)^{-1}$.

- ***Step* 3:** plug data on $\lambda_{ii,k}$ and the elements of the *Leontief inverse* obtained in Step 2 into the gains from trade formula:

$$GT_i = 1 - \prod_{k=1}^{K} \prod_{g=1}^{K} \lambda_{ii,g}^{\frac{\tilde{\alpha}_{i,kg}}{\epsilon_g} \beta_{i,k}}$$

[1]The WIOD is the standard source for this type of data.

The Gains from Trade: *The Perfectly Comp. Case*

	% GT	
	w/o IO Linakges	w/ IO Linakges
Ireland	8%	37.1%
Belgium	7.8%	54.6%
Germany	4.5%	21.6%
China	2.6%	11.5%
U.S.	1.8%	8.3%

Source: *Costinot & Rodriguez-Clare (2014)* based on data from the 2008 WIOD, which cover 16 industries.

Some Intuition

- IO linkages multiply distortions (e.g., *trade costs*, *tariffs*, *markups*).
- The negative effect of trade barriers on welfare is multiplied by IO linkages $\Longrightarrow$ larger gains from removing trade barriers.
- Similarly, the gains/losses from tariff imposition also multiply through IO linkages (*Yi, 2003*; *Beshkar & Lashkaripour, 2019*).

Cross-Country Income Differences

- The IO model can partly explain cross-country income differences, which are puzzlingly large (*Jones, 2011*).
- To make this point, consider the case of autarky ($P_{i,k} = P_{ii,k}$ for all k).
- Perfectly competitive (efficient) price

$$P_{i,k} = a_{i,k} w_i^{1-\alpha_{i,k}} \prod_{g=1}^{K} \left(P_{i,g}\right)^{\alpha_{i,gk}}$$

- Price with industry-specific markup wedge

$$\tilde{P}_{i,k} = \underbrace{\mu_k}_{\text{markup}} a_{i,k} w_i^{1-\alpha_{i,k}} \prod_{g=1}^{K} \left(\tilde{P}_{i,g}\right)^{\alpha_{i,gk}}$$

Cross-Country Income Differences

- Combining the previous two expressions implies

$$\tilde{P}_{i,k} = \underbrace{\mu_k \left(\prod_g \mu_g^{\hat{\alpha}_{i,gk}} \right)}_{\text{amplified markup}} \underbrace{a_{i,k} w_i^{1-\alpha_{i,k}} \prod_{g=1}^{K} (P_{i,g})^{\alpha_{i,gk}}}_{P_{i,k}}$$

where $\hat{\mathbf{A}}_i = \mathbf{A}_i + \mathbf{A}_i^2 + \mathbf{A}_i^3 + \ldots \geqslant \mathbf{0}$.

- Hence, distance from the efficiency frontier is give by[2]

$$\frac{W_i^{\text{eff.}}}{W_i} = \prod_k \left(\mu_k^{\beta_{i,k}} \right) \underbrace{\prod_{k,g} \left(\mu_g^{\hat{\alpha}_{i,gk} \beta_{i,k}} \right)}_{\text{effect of IO linakges}}$$

[2] *Notation*: $W_i = w_i / P_i$ denotes real income p/c.

Two Important Takeaways

- Misallocation in less-developed economies can multiply through IO linkages, leading to a significant real income gap between more- and less-developed countries.

- Suppose the government has limited resources to tackle misallocation $\Longrightarrow$ it is optimal to target upstream industries first.

Performing Counterfactuals using Exact Hat-Algebra

The impact of a change in trade costs, $\{\hat{\tau}_{ji,k}\}$ can be determined using a system of $N + 3NK$ equations and unknowns (namely, $\{\hat{w}_i\}$, and $\{\hat{L}_{i,k}\}$):[3]

$$\begin{cases} \hat{P}_{i,k}^{-\epsilon_k} = \sum_{j=1}^{N} \left[\lambda_{ji,k} \hat{P}_{ji,k}^{-\epsilon_k}\right] & \forall i,k \\ \hat{E}_{i,k} E_{i,k} = \sum_{j=1}^{N} \left(\hat{\lambda}_{ij,k} \lambda_{ji,k} \beta_{j,k} \hat{w}_j w_j L_j\right) + \sum_{g=1}^{K} \left(\alpha_{i,gk} \hat{R}_{i,g} R_{i,g}\right) & \forall i,k \\ \hat{R}_{i,k} R_{i,k} = \sum_{j=1}^{N} \hat{\lambda}_{ij,k} \lambda_{ji,k} \hat{E}_{j,k} E_{j,k} & \forall i,k \\ \hat{w}_i w_i L_i = \sum_{k=1}^{K} (1 - \alpha_{i,k}) \hat{R}_{i,k} R_{i,k} & \forall i \end{cases}$$

where

$$\begin{cases} \hat{P}_{ij,k} = \hat{\tau}_{ji,k} \hat{w}_i^{1-\alpha_{i,k}} \prod_{g=1}^{K} \hat{P}_{i,g}^{\alpha_{i,gk}} & \forall i,k \\ \hat{\lambda}_{ij,k} = \frac{\hat{P}_{ji,k}^{-\epsilon_k}}{\sum_{\ell=1}^{N} \lambda_{\ell i,k} \hat{P}_{\ell i,k}^{-\epsilon_k}} & \forall i,j,k \end{cases}$$

[3] Highlighted variables are either observables or estimable parameters.

Application: *Caliendo & Parro (2014)*

- Infer $\{\hat{\tau}_{fh,k}\}$ from the change in tariff rates between 1993 and 2005 due to **NAFTA** $\Longrightarrow$ solve for $\hat{W}_i$ using the system of equations specified in the previous slide.

- Note that tariff changes raise/exhaust revenue $\Longrightarrow$ the previous system has to be amended to account for tariff revenues.

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- The welfare effects of NAFTA according to *Caliendo & Parro*'s analysis:
  - $\Delta W_{\text{MEX}} = 1.31\%$
  - $\Delta W_{\text{CAN}} = -0.06\%$
  - $\Delta W_{\text{USA}} = 0.08\%$