Can Trade Policy Mitigate Climate Change?

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February 2024

Abstract

Trade policy is often cast as a solution to the free-riding problem in international climate agreements. This paper examines the extent to which trade policy can deliver on this promise. We incorporate global supply chains of carbon and climate externalities into a multi-country, multi-industry general equilibrium model of trade. By deriving theoretical formulas for optimal carbon and border taxes, we quantify the maximum efficacy of two trade policy solutions to the free-riding problem. First, we show that border taxes, when used as non-contingent, indirect mechanisms for carbon taxation, have limited potential to mitigate global emissions even under optimal design. However, Nordhaus's (2015) *climate club* framework, in which border taxes are used as contingent penalties to deter free-riding, is highly effective. The climate club can achieve up to 69% of the emissions reduction under globally optimal carbon pricing, while ensuring global participation and maintaining free trade. This success depends on major economic powers like the U.S., E.U., and China forming an initial alliance of core members and leveraging their collective trade penalties to compel participation by reluctant governments.

1 Introduction

Climate change is accelerating at an alarming rate, yet governments have been unsuccessful in forging an agreement to effectively tackle this pressing issue. Major climate agreements, like the 1997 Kyoto Protocol and the 2015 Paris Climate Accord, have failed to deliver a meaningful reduction in global carbon emissions. This failure is often attributed to the *free-riding* problem: Countries have an incentive to free-ride on the rest of the world's reduction in carbon emissions without undertaking proportionate abatement themselves.

*This paper has previously circulated as "Trade, Firm Delocation, and Optimal Climate Policy." We are grateful to Jonathan Eaton, Vernon Henderson, Sam Kortum, Volodymyr Lugovskyy, Ishan Nath, William Nordhaus, Ralph Ossa, Heitor Pellegrina, Robert Staiger, and John Sturm Becko for their valuable comments, and participants in many seminars and conferences for their helpful discussions and feedback. Email: ffarrokh@purdue.edu and alashkar@indiana.edu.

The shortcoming of existing climate agreements has led experts to propose alternative solutions that are resistant to free-riding. Two canonical trade policy proposals have emerged:

Proposal 1: Climate-conscious governments use *carbon border taxes* as a second-best policy to curb untaxed carbon emissions beyond their jurisdiction.

Proposal 2: Climate-conscious governments form a *climate club*, using collective and contingent trade penalties to incentivize climate cooperation by reluctant governments (Nordhaus, 2015).

While both proposals combine carbon pricing with trade policy, they differ starkly in their approach. Proposal 1 is grounded in unilateralism. It presumes that global climate cooperation is improbable, but unilateral policies can serve as a viable second best solution. Proposal 2 relies on the premise that unilateral action is insufficient and that the failure of past multilateral agreements could be reversed through better institutional design.

The maximal efficacy of these proposals remains unclear due the challenges in characterizing their *optimal* design within *quantitative* frameworks. Traditional theories of optimal trade and environmental policy are limited to stylized models that preclude quantitatively important considerations. Existing quantitative studies examine simplified variants of these proposals that are not optimal, sidestepping the computational challenges associated with optimal policy analysis. Thus, they reveal only a fraction of what these proposals could potentially achieve.

We overcome these challenges by combining optimal policy analysis with quantitative general equilibrium modeling. First, we incorporate global carbon supply chains and climate externalities into a multi-country, multi-industry general equilibrium trade model. Second, we derive theoretical formulas for optimal carbon border taxes and climate club penalties that internalize climate damage from carbon emissions and terms-of-trade effects under rich general equilibrium considerations. Third, we map our model and optimal policy formulas to data on trade, production, and emissions to evaluate the maximal effectiveness of carbon border taxes and climate clubs.

Section 2 presents our theoretical framework, that is a general equilibrium semi-parametric model of international trade with many countries and industries. Our framework incorporates production, distribution, and utilization of fossil fuel energy which gives rise to international climate externalities. The resulting framework is particularly attractive as it combines the carbon externality and terms-of-trade rationales for policy intervention in a tractable fashion. Section 3 derives theoretical formulas for optimal carbon and border taxes in our general equilibrium framework. Our optimal policy formulas represent a notable advance over traditional theories.

In addition to internalizing multilateral leakage and ripple effects through carbon supply chains, our formulas pave the way for an in depth quantitative analysis of the above canonical climate policy proposals.

Our derivation of optimal policy is grounded in an envelope result that transforms our general equilibrium optimal policy problem into a more manageable pseudo-partial equilibrium problem. To elaborate, the first-order conditions associated with our optimal policy problem feature complex terms representing the general equilibrium elasticities of domestic wage and demand quantities with respect to policy. Our envelope result shows that these general equilibrium elasticities are redundant at the optimum, provided that the government is afforded sufficient policy instruments. In other words, we can fully characterize the optimal policy formulas without characterizing these complex general equilibrium elasticities. This result circumvents one of the main complications impeding optimal policy derivation in general equilibrium settings.

Our analytical formulas indicate that the *unilaterally* optimal domestic carbon tax equals the disutility from carbon emissions for domestic households. This policy choice is inefficient from a global standpoint as it does not internalize the home country's carbon externality on foreign residents. Unilaterally optimal import tariffs and export subsidies are composed of two components: a conventional terms-of-trade-driven component and carbon border adjustments. Relevant to Proposal 1, these carbon adjustments impose a tax on imported goods based on the carbon content per dollar value and provide a subsidy to exported goods based on the carbon intensity of competing foreign varieties. Relevant to Proposal 2, the unilaterally optimal border taxes represent the trade penalties that maximize welfare transfers from free-riders to climate club members.

To better understand these non-cooperative policy choices and elucidate the free-riding problem, we compare them with optimal policy under global cooperation. The first-best policy from a global standpoint features zero border taxes/subsidies and a globally optimal carbon tax that equals the *global* disutility from carbon emissions. Importantly, the globally optimal carbon tax rate greatly exceeds the unilaterally optimal rate as it penalizes a country's carbon externality on not only its own residents but also foreign households. Governments acting in their own self-interest, therefore, have incentives to deviate from the globally optimal rate, thus perpetuating the free-riding problem in climate action.

Sections 4 and 5 leverage our optimal tax formulas and the sufficient statistics for counterfactual analysis to determine the maximal efficacy of carbon border taxes and the climate club proposal in reducing carbon emissions. The sufficient statistics for counterfactual policy analysis are obtained as follows: First, observable shares are constructed from national and environmental accounts data. Second, the governments' perceived disutility from climate change is inferred from their applied environmentally-related taxes. Third, structural parameters including the industry-level trade elasticities and the energy demand elasticity are estimated using cross-sectional tax and expenditure data, utilizing conventional identification strategies. The required data on trade, production, carbon emissions, and taxes are primarily taken from the GTAP Database for 2014 augmented by several auxiliary data sources. Our final database covers 18 broadly defined industries, including energy, representing the entire vector of production across 13 major countries, the European Union, and five aggregate regions containing neighboring blocs of countries.

Our analysis reveals that carbon border taxes have limited efficacy in reducing carbon emissions, even when designed optimally. The adoption of unilaterally optimal border taxes by all countries cuts global carbon emissions by a mere 1.2%. This modest reduction amounts to just 3.2% of the reduction attainable under the globally-first-best carbon taxes. The inefficacy of carbon border taxes stems from three factors. First, carbon border taxes are not granular enough to incentivize firm-level abatement abroad. They impose penalties based on the *average* carbon intensity of all firms within a given country and industry. Since individual firms take these averages as given, border taxes have limited ability to spur abatement among foreign firms. Second, carbon border taxes cannot target emissions from non-traded goods, which account for a significant share of global emissions—in fact, most emissions originate from less-traded industries. Third, carbon border taxes cannot prevent carbon leakage through general equilibrium price changes. As pre-tax energy prices fall in response to border taxes, energy use and carbon emissions tend to increase in countries without a domestic carbon tax.

To examine the climate club, we solve a sequential game where *core* members move first, followed by other countries. Core members and non-core countries that join the club abide by the rules of membership: they impose unilaterally optimal trade penalties against non-members and commit to free trade among members. Furthermore, they raise domestic carbon prices to meet a specified *carbon tax target*. Non-members can use their trade taxes to retaliate against club members but keep their other taxes unchanged. When considering joining the club, countries weigh the cost of higher domestic carbon taxes against the benefits of evading the climate club's collective trade penalties.[1]

[1] Analyzing the climate club proposal quantitatively poses two major challenges. First, computing optimal trade penalties in a strategic game involving many players is practically infeasible with numerical optimization methods. We circumvent this issue by leveraging our theoretical formulas for optimal trade penalties. Second, solving the climate club game suffers from the curse of dimensionality, requiring that one searches over an excessively large number of possible outcomes. To overcome this challenge, we shrink the space of possible outcomes using a procedure that closely mimics the iterative elimination of dominated strategies.

In setting the climate club's carbon tax target, we balance two considerations. The first is a trade-off reminiscent of the Laffer curve. Higher taxes encourage greater emission cuts per member, but also discourage participation—yielding an inverted U-shape relationship between global carbon reduction and the carbon tax target. The second consideration is upholding free trade. Trade penalties against non-members are intended as deterrent threats, so the ideal target must be set at a level that elicits universal participation, rendering the imposition of such penalties unnecessary. Considering these dual objectives, our analysis sets the carbon tax target at the maximal rate that results in an inclusive club of all nations.

We find that the climate club framework can effectively reduce global carbon emissions, but its success hinges critically on the makeup of core members. If the EU and US initiate a climate club as core members, universal participation will be attained at a maximal carbon tax target of 53 ($/tCO2), yielding a 18.3% reduction in global carbon emissions. Though substantial, the EU-US alliance lacks the necessary market power to elicit a higher tax target. However, by incorporating China as a core member, the maximal carbon tax target can be raised to 90 ($/tCO2) leading to a 28.2% reduction in global carbon emissions. This figure represents 69% of the emissions reduction achievable under globally first-best carbon taxes, evaluated at the social cost of carbon equal to 156 ($/tCO2). Overall, the climate club's efficacy in mitigating climate change relies on assembling an influential group of core members and setting an appropriate carbon tax target. Moreover, comparing the efficacy of the climate club to carbon border taxes reveals that trade policy is more effective at deterring free-riding than an indirect mechanism for carbon taxation.

Lastly, Section 6 demonstrates the robustness of our quantitative results, showing they remain similar across several alternative model specifications and extensions.

Related Literature

Our work contributes to several areas of literature. First, we contribute to *theoretical* analyses of trade and environmental policy. Early works such as Markusen (1975); Copeland (1996); Hoel (1996), use partial equilibrium or two-country models to study how unilaterally-applied trade taxes can mitigate transboundary environmental damages. More recent research by Kortum and Weisbach (2020) and Weisbach et al. (2023) characterizes unilaterally-optimal carbon policy in a two-country Dornbusch et al. (1977) model, emphasizing the effectiveness of combining supply and demand-side carbon taxes. Another body of literature examines multilateral policies, looking at issue linkages between trade and climate policy (Barrett, 1997; Nordhaus, 2015; Maggi, 2016; Nordhaus, 2021; Harstad, Forthcoming). Our work advances this literature by characterizing optimal policy in a multi-country and industry general equilibrium model amenable to rich quantitative analysis.

Second, our analysis is related to *quantitative* examinations of environmental and energy-related policies in open economies, e.g., Babiker (2005); Elliott et al. (2010); Taheripour et al. (2019); Farrokhi (2020). Our paper is especially relevant to studies analyzing the efficacy of carbon border adjustment policies, including Böhringer et al. (2016); Larch and Wanner (2017); Shapiro (2021). Although these studies feature rich specifications of the global economy, they lack a concept of optimal policy design. Consequently, they do not reveal the full potential of trade policy for reducing carbon emissions. We complement this literature by utilizing optimal policy formulas to uncover the frontier of trade and climate policy outcomes.

Third, our work relates to an emerging literature characterizing optimal policy in modern quantitative trade models, e.g., Costinot et al. (2015); Bartelme et al. (2021); Beshkar and Lashkaripour (2020); Lashkaripour (2021); Caliendo and Parro (2022); Lashkaripour and Lugovskyy (2023). These studies have bridged a longstanding divide between classic partial equilibrium trade policy frameworks and modern general equilibrium trade theories. Our envelope result advances this effort towards closing the gap. It shows that optimal policy formulas can be derived without characterizing complex general equilibrium elasticities, removing a primary impediment to general equilibrium optimal policy analysis. This particular result sharpens and extends the result in Lashkaripour and Lugovskyy (2023) to settings with global carbon supply chains and international consumption externalities, an example of which is climate change damage.

Lastly, we contribute to the growing research on trade and the environment. This literature has made significant advances in bringing a spatial dimension to integrated assessment models, as reviewed by Desmet and Rossi-Hansberg (2023). It embeds environmental issues ranging from local pollution to global deforestation into trade models, e.g., Shapiro and Walker (2018) and Farrokhi et al. (2023). See Copeland and Taylor (2004) and Copeland et al. (2021) for reviews of the literature on trade and the environment and Staiger (2021) for how the existing world trade system can handle climate and environmental issues.

2 Theoretical Framework

The global economy consists of multiple countries indexed by $i, j, n \in \mathbb{C} \equiv \{1, ..., N\}$ and multiple industries indexed by $k, g \in \mathbb{K} \equiv \{0, 1, ..., K\}$. Each country i is endowed by $\bar{L}_i$ workers and $\bar{R}_i$

carbon reserves. Workers are perfectly mobile across industries but immobile across countries, and each worker supplies one unit of labor inelastically. Production in the global economy can be thought of as a two stage process. First, each country's energy industry (indexed by $k = 0$) employs labor and carbon reserves—as a specific input—to produce energy. Second, other industries (indexed by $k = 1, ..., K$) employ labor and energy to produce final goods. Markets are perfectly competitive[2] and goods in all industries are internationally traded.

We denote quantities of energy in terms of their CO_2 emission content. Along the carbon supply chain, we count CO_2 emissions when energy is used by final good producers and households. Since every individual producer or consumer is infinitesimally small, they do not internalize the impact of their production or consumption choices on CO_2 emissions.[3]

2.1 Prices and Tax Instruments

Subscript (ji, k) indexes a variety corresponding to *origin* j*—destination* i*—industry* k—i.e., a variety of industry k that is produced in origin j and shipped to destination i. Country i's government has access to the following tax instruments:

1. Import tax, $t_{ji,k}$, applied to imported variety ji, k ($t_{ii,k} = 0$ by design);
2. Export subsidy, $x_{ij,k}$, applied to exported variety ij, k ($x_{ii,k} = 0$ by design);
3. Carbon tax, $\tau_{i,k}$, applied to the carbon content of energy use;

Border taxes/subsidies create a wedge between the after-tax *consumer price*, $\tilde{P}_{ji,k}$, and the before-tax *producer price*, $P_{ji,k}$, of each variety (ji, k),

$$\tilde{P}_{ji,k} = \frac{(1 + t_{ji,k})}{(1 + x_{ji,k})} \times P_{ji,k}, \qquad k = 0, 1, ..., K. \tag{1}$$

A representative "energy distributer" in country i purchases varieties of energy from international suppliers $j = 1, ..., N$, at prices $\left\{\tilde{P}_{ji,0}\right\}_j$, and aggregates them into a composite energy bundle with price $\tilde{P}_{i,0} = \tilde{\mathrm{P}}_{i,0}\left(\tilde{P}_{1i,0}, ..., \tilde{P}_{Ni,0}\right)$. This bundle is sold to domestic producers after the inclusion of an end-use-specific carbon tax, which creates a wedge between $\tilde{P}_{i,0}$ and the final price paid by producers for use in industry k:

$$\tilde{P}_{i,0k} = \tilde{P}_{i,0} + \tau_{i,k}, \qquad k = 1, ..., K. \tag{2}$$

[2] In Section 6.3, we consider a more general case with monopolistic competition and firm entry.

[3] Throughout the paper, we use "energy" as a shorthand for "fossil fuel energy" and we use "carbon emissions" interchangeably with "CO_2 emissions".

where $\tilde{P}_{i,0k}$ denotes the price of energy input for use in industry $k = 1, ..., K$ (after the inclusion of all taxes) and $\tau_{i,k}$ is the carbon tax. The above-listed tax instruments are sufficient for obtaining the first-best policy outcome under cooperative and non-cooperative scenarios. Additional tax instruments (e.g., production or consumption taxes) are redundant as their effects can be perfectly mimicked with the appropriate choice of existing instruments.

2.2 Consumption

The representative household in country i maximizes a non-parametric utility function $U_i(\mathbf{C}_i)$ by choosing the vector of consumption quantities, $\mathbf{C}_i = \{C_{ji,k}\}_{j,k\geq 1}$ subject to the budget constraint,

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{K} \tilde{P}_{ji,k} C_{ji,k}, \tag{3}$$

where E_i denotes national household expenditure, and $\tilde{P}_{ji,k}$ is the consumer price index of variety ji, k (Equation 1). Let $\tilde{\mathbf{P}}_i = \{\tilde{P}_{ji,k}\}_{j\in \mathbb{C}, k\geq 1}$ denote the entire vector of consumer prices in country i. The household's utility maximization implies an indirect utility function, $V_i\left(E_i, \tilde{\mathbf{P}}_i\right)$, and a Marshallian demand function for each variety ji, k,

$$C_{ji,k} = \mathcal{D}_{ji,k}\left(E_i, \tilde{\mathbf{P}}_i\right), \qquad k = 1, ..., K. \tag{4}$$

We denote the elasticity of demand for variety (ji, k) with respect to the price of variety (ni, g) by:

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(E_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,g}}, \qquad \varepsilon_{ji,k} \sim \varepsilon_{ji,k}^{(ji,k)}; \tag{5}$$

with the own price elasticity of demand defined as $\varepsilon_{ji,k} \sim \varepsilon_{ji,k}^{(ji,k)} \leq -1$ (we use "$\sim$" as a shorthand for *defined as*).

We use β and λ to denote household expenditure shares. The within-industry expenditure share on variety ji, k (origin j–destination i–industry k) is denoted by $\lambda_{ji,k}$, and the overall expenditure share of country i on industry $k \neq 0$ is denoted by $\beta_{i,k}$,

$$\lambda_{ji,k} \equiv \frac{\tilde{P}_{ji,k} C_{ji,k}}{\sum_{n=1}^{N} \tilde{P}_{ni,k} C_{ni,k}}, \qquad \beta_{i,k} \equiv \frac{\sum_{n=1}^{N} \tilde{P}_{ni,k} C_{ni,k}}{\sum_{n=1}^{N} \sum_{g=1}^{K} \tilde{P}_{ni,g} C_{ni,g}} = \frac{\sum_{n=1}^{N} \tilde{P}_{ni,k} C_{ni,k}}{E_i}. \tag{6}$$

A familiar special case is the Cobb-Douglas-CES form, where a constant fraction of expenditure, $\beta_{i,k}$, is spent on industry k whose varieties are differentiated by source countries under a constant elasticity of substitution, σ_k. The demand function in this special case is:

$$\text{[special case: Cobb-Douglas-CES]} \qquad \mathcal{D}_{ji,k}\left(E_i, \tilde{\mathbf{P}}_i\right) = \frac{b_{ji,k}\tilde{P}_{ji,k}^{-\sigma_k}}{\sum_n b_{ni,k}\tilde{P}_{ni,k}^{1-\sigma_k}}\beta_{i,k}E_i,$$

with demand elasticities given by $\varepsilon_{ji,k}^{(ni,k)} = -\sigma_k \mathbb{1}_{n=j} + (\sigma_k - 1)\lambda_{ji,k}$ and $\varepsilon_{ji,k}^{(ni,g)} = 0$ if $g \neq k$.

2.3 Production

Energy Extraction. The extraction industry ($k = 0$) in each country j produces energy by employing exogenously-given carbon reserves, $\bar{R}_j$, as specific input and labor, $L_{j,0}$, as variable input under a Cobb-Douglas technology:

$$Q_{j,0} = \bar{\varphi}_{j,0}\left(\frac{L_{j,0}}{1-\phi_j}\right)^{1-\phi_j}\left(\frac{\bar{R}_j}{\phi_j}\right)^{\phi_j}. \tag{7}$$

Here, $\bar{\varphi}_{j,0}$ is an exogenous productivity parameter and $Q_{j,0}$ is the output quantity of energy which can be thought of as *carbon supply* from each economy j. Production flows of energy are freely and internationally traded. Hence, the producer price of energy in origin country j shipped to destination i, $P_{ji,0}$, equals across destinations:

$$P_{ji,0} = P_{jj,0} = \frac{1}{\bar{\varphi}_{j,0}} w_j^{1-\phi_j} r_j^{\phi_j}, \tag{8}$$

where w_j is the wage rate in country j, and r_j is the rental rate of carbon reserves there.[4]

Energy Distribution. A representative energy distributer in each country i purchases varieties of energy $\{C_{ji,0}\}_i$ from international suppliers $j = 1, .., N$, aggregates them into a bundle of energy, $Z_i = Z_i\left(C_{1i,0}, ..., C_{Ni,0}\right)$, and sells this energy bundle to domestic final-good producers. The price of the energy bundle, $\tilde{P}_{i,0}$, is determined by a homogeneous-of-degree-one aggregator:

$$\tilde{P}_{i,0} = \tilde{\mathcal{P}}_{i,0}\left(\tilde{P}_{1i,0}, ..., \tilde{P}_{Ni,0}\right). \tag{9}$$

The energy price aggregator, $\tilde{P}_{i,0}$, is implied by a homothetic system of demand for international sources of energy. The distributor's demand for variety $(ji, 0)$ is, accordingly, a function of total expenditure on energy varieties, $E_{i,0} = \sum_j \tilde{P}_{ji,0}C_{ji,0}$, and the vector of (pre-carbon-tax) energy

[4] This specification implies an energy supply curve, $P_{jj,0} = \bar{p}_{j,0} \times w_j \times Q_{j,0}^{\tilde{\phi}_j}$, where $\bar{p}_{j,0} = \left(\phi_j^{\phi_j}\bar{\varphi}_{j,0}\right)^{1/\left(1-\phi_j\right)}$ is an exogenous shifter and $\tilde{\phi}_j \equiv \phi_j / \left(1-\phi_j\right) > 0$ is the inverse energy supply elasticity.

prices, $\tilde{\mathbf{P}}_{i,0} = \left\{\tilde{P}_{1i,0}, ..., \tilde{P}_{Ni,0}\right\}$. Namely,

$$C_{ji,0} = \widetilde{\mathcal{D}}_{ji,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right). \tag{10}$$

As earlier, we use $\varepsilon_{ji,0}^{(ni,0)} = \partial \ln \widetilde{\mathcal{D}}_{ji,0}\left(.\right) / \tilde{P}_{ni,0}$ as the price elasticity of demand for energy varieties. A special case of the above specification is the CES aggregator, which implies the following price and quantity equations:[5]

$$\text{[special case: CES]} \qquad \tilde{\mathcal{P}}_{i,0}\left(\tilde{\mathbf{P}}_{i,0}\right) = \left[\sum_j b_{ji,0}\tilde{P}_{ji,0}^{1-\sigma_0}\right]^{\frac{1}{1-\sigma_0}}; \quad \widetilde{\mathcal{D}}_{ji,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right) = \frac{b_{ji,0}\tilde{P}_{ji,0}^{-\sigma_0}E_{i,0}}{\sum_n b_{ni,0}\tilde{P}_{ni,k}^{1-\sigma_0}}.$$

Note that $\tilde{P}_{ni,0}$ includes border taxes on energy but not the carbon tax. The latter is applied after bundling of energy varieties, so that the final price of the energy bundle paid by final-good producers k is $\tilde{P}_{i,0k} = \tilde{P}_{i,0} + \tau_{i,k}$.

Household Energy Consumption. Our setup accommodates energy use by households, which we model by making use of a fictitious industry that helps us maintain a compact notation. The fictitious industry $k_0 \in \{1, ..., K\}$ purchases the energy bundle, at price $\tilde{P}_{i,0} + \tau_{i,k_0}$, and converts it without generating any value added into a final good of the same price. This fictitious industry is nontradeable and sells exclusively to domestic households. Therefore, households' consumption of final good k_0 corresponds to their energy consumption and their associated CO_2 emission.

Production of Final Goods. Production of final good $k = 1, ..., K$ in country i is conducted by a representative producer that combines labor and the energy input using a constant-reruns-to-scale production function,

$$Q_{i,k} = \bar{\varphi}_{i,k} F_{i,k}\left(L_{i,k}, Z_{i,k}\right). \tag{11}$$

The arguments $L_{i,k}$ and $Z_{i,k}$ denote the quantity of labor and energy inputs, and $\bar{\varphi}_{i,k} > 0$ is a Hick-neutral productivity shifter. International trade in final goods is subject to iceberg trade costs, $\bar{d}_{in,k} \geq 1$, with $\bar{d}_{ii,k} = 1$. Consequently, per cost minimization, the competitive producer price of variety in, k equals:

$$P_{in,k} = \frac{\bar{d}_{in,k}}{\bar{\varphi}_{i,k}} \times \mathscr{C}_{i,k}\left(w_i, \tilde{P}_{i,0k}\right), \tag{12}$$

where $\mathscr{C}_{i,k}$ is a homogeneous-of-degree-one input price aggregator. We denote the cost share of energy by $\alpha_{i,k}$. Assuming that the demand for inputs is separable and homothetic, we can specify

[5] The finite elasticity of substitution between energy sources, as shown in Farrokhi (2020), can be micro-founded via aggregation over sourcing choices of input-users who face variability in transport costs vis-a-vis exporters.

the cost share of energy, $\alpha_{i,k}$, and energy use, $Z_{i,k}$, as:

$$\alpha_{i,k} \equiv \frac{\tilde{P}_{i,0k} Z_{i,k}}{Y_{i,k}} = \alpha_{i,k}\left(\tilde{P}_{i,0k}/\mathscr{C}_{i,k}\right), \tag{13}$$

where $\alpha_{i,k}(.)$ is a function of the relative price of of energy bundle inclusive of the carbon tax, $\tilde{P}_{i,0k} = \tilde{P}_{i,0} + \tau_{i,k}$, and $Y_{i,k} = P_{ii,k}Q_{i,k}$ denotes the total value of sales by origin i–industry k.

A canonical special case of our setup is the case of CES production function, with

$$F_{i,k}\left(L_{i,k}, Z_{i,k}\right) = \left[(1-\bar{\kappa}_{i,k})^{\frac{1}{\varsigma}} L_{i,k}^{\frac{\varsigma-1}{\varsigma}} + (\bar{\kappa}_{i,k})^{\frac{1}{\varsigma}} Z_{i,k}^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}},$$

where $\bar{\kappa}_{i,k} \in [0,1]$ represents exogenous energy intensity, and $\varsigma > 0$ is the elasticity of substitution between labor and energy inputs. In this special case, the input cost aggregator and the energy cost share take the following familiar forms:

$$\text{[special case: CES]} \qquad \mathscr{C}_{i,k} = \left[(1-\bar{\kappa}_{i,k})w_i^{1-\varsigma} + \bar{\kappa}_{i,k}\tilde{P}_{i,0k}^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}}, \qquad \alpha_{j,k} = \bar{\kappa}_{i,k}\left(\frac{\tilde{P}_{i,0k}}{\mathscr{C}_{i,k}}\right)^{1-\varsigma};$$

where ς regulates the "energy demand elasticity."

Crucially, a carbon tax, $\tau_{i,k}$, raises the consumer price of energy, $\tilde{P}_{i,0k}$, resulting in a lower energy use $Z_{i,k}$ per unit of output in the production of final goods. Under the CES special case, a higher energy demand elasticity, ς, implies a greater reduction in energy use in response to a carbon tax.

2.4 CO_2 Emissions

Aggregate CO_2 emission from each industry $k = 1, ..., K$ equals:

$$Z_{i,k} = \mathrm{z}_{i,k}\left(\alpha_{i,k}\right) \times Q_{i,k}, \tag{14}$$

where $\mathrm{z}_{i,k}(.)$ represents the production "technique" as a function of the energy cost share and $Q_{i,k}$ is total industry-level quantity of output.[6] Country i's total CO_2 emissions, Z_i, and (before carbon tax) total expenditure on energy, $E_{i,0}$, aggregate energy uses and expenditures across industries:

$$Z_i = \sum_{k=1}^{K} Z_{i,k}, \qquad E_{i,0} = \tilde{P}_{i,0} Z_i. \tag{15}$$

[6] In relation to the decomposition of emissions a la Copeland and Taylor (2004), $\mathrm{z}_{i,k}\left(\alpha_{i,k}\right)$ represents the "technique" effect, $Q_{i,k}$ the "scale" effect, and the vector of $\left\{Z_{i,k}\right\}_k$ the "composition" effect.

Lastly, note that under the special case with CES production, the emission level described non-parametrically by Equation (14) becomes

$$\text{[special case: CES]} \qquad\qquad Z_{i,k} = \bar{z}_{i,k}\alpha_{i,k}^{\frac{\varsigma}{\varsigma-1}} \times Q_{i,k},$$

where $\bar{z}_{i,k} \equiv \bar{\kappa}_{i,k}^{\frac{1}{1-\varsigma}} / \bar{\varphi}_{i,k}$ is a constant shifter. Lastly, global CO_2 emission can be calculated by summing over national CO_2 emissions:

$$Z^{(global)} \equiv \sum_i Z_i \tag{16}$$

2.5 General Equilibrium

Tax Revenues and National Income. We denote by T_i the tax revenues collected by country i's government from imports, exports, and carbon taxes and rebated to consumers in that country,

$$T_i = \underbrace{\sum_{k=1}^{K} \left[\tau_{i,k} Z_{i,k}\right]}_{\text{carbon tax}} + \underbrace{\sum_{k=0}^{K} \sum_{n \neq i} \left[\frac{t_{ni,k}}{1+t_{ni,k}} \tilde{P}_{ni,k} C_{ni,k}\right]}_{\text{import taxes}} - \underbrace{\sum_{k=0}^{K} \sum_{n \neq i} \left[\frac{x_{in,k}}{1+t_{in,k}} \tilde{P}_{in,k} C_{in,k}\right]}_{\text{export subsidies}} \tag{17}$$

Let $Y_{i,k} = P_{ii,k} Q_{i,k}$ denote sales of country $i-$industry k,

$$Y_{i,k} = P_{ii,k} Q_{i,k}, \tag{18}$$

Industry sales, on aggregate, generate an income level of $\sum_{k=0}^{K} Y_{i,k} = w_i \bar{L}_i + r_i \bar{R}_i$ in each country i. We assume trade is balanced, so that national income is the sum of the wage bill, rental payments to carbon reserves, and tax revenues:

$$Y_i = w_i \bar{L}_i + \Pi_i + T_i, \qquad \text{where} \qquad \Pi_i = r_i \bar{R}_i. \tag{19}$$

Definition of General Equilibrium. For a given set of taxes $\{t_{ji,k}, x_{ij,k}, \tau_{i,k}\}$, a *general equilibrium* is a vector of consumption, production and input use, $\{C_{ji,k}, Q_{i,k}, L_{i,k}, Z_{i,k}\}$, final goods and energy input prices, $\{P_{ji,k}, \tilde{P}_{ji,k}, \tilde{P}_{i,0}, \tilde{P}_{i,0k}\}$, wage and rental rates, $\{w_i, r_i\}$, and income, sales and expenditure levels, $\{Y_{i,k}, Y_i, E_i, E_{i,0}\}$, such that equations (1)-(19) hold; goods market clear, whereby national consumption expenditure equals national income, $E_i = Y_i$, and total output in each industry equals demand,

$$Q_{i,k} = \sum_{n=1}^{N} \bar{d}_{in,k} C_{in,k} \tag{20}$$

and the factor markets clear according to:

$$w_i \bar{L}_i = \sum_{k=1}^{K} \left[\left(1 - \alpha_{i,k}\right) Y_{i,k} \right] + \left(1 - \phi_i\right) Y_{i,0}, \qquad \Pi_i \equiv r_i \bar{R}_i = \phi_i Y_{i,0}; \tag{21}$$

where in each country the wage rate clears the labor market and the rental rate of carbon reserves clears the market for energy extraction.[7]

3 Optimal Policy and the Free-Riding Problem

Our analysis builds on the realization that the globally optimal climate outcome is politically infeasible due to free-riding incentives, but climate-conscious countries can use trade policies to target global emissions. In this section, we first characterize the unilaterally optimal carbon and border taxes, elucidating the two rationales for policy intervention from a unilateral standpoint. Next, we characterize the globally optimal policy to highlight the free-riding problem in climate agreements. Finally, we discuss two trade policy remedies for the free-riding problem: carbon border taxes and the climate club. We explain how our theoretical optimal policy results provide the groundwork for quantitatively evaluating these policies. To set the context, we begin with a formal definition of policy objectives.

Social Welfare with Climate Damage. The welfare of the representative consumer in country i is the utility from consumption net of the disutility from CO_2 emissions.[8] Namely,

$$W_i = V_i \left(E_i, \tilde{\mathbf{P}}_i\right) - \delta_i \times Z^{(global)}. \tag{22}$$

The first term on the right-hand side represents the indirect utility from consumption and the second term is the disutility from global CO_2 emissions. δ_i is a parameter that (in principle) represents the disutility per unit of CO_2 emissions for country i's residents. However, since individual producers or consumers, take $Z^{(global)}$ as given, they do not internalize the externality from their energy consumption on the disutility from CO_2 emissions. Governments, meanwhile, can influ-

[7] The definition of general equilibrium ensures the *balance of trade.* Specifically, national exports equals national imports, $D_i \equiv \sum_k \sum_n X_{ni,k} - \sum_k \sum_j X_{ij,k} = 0$, where $X_{ij,k} \equiv \left(1 + x_{ij,k}\right) P_{ij,k} C_{ij,k}$ denotes each variety's trade flow outside the border of the exporting country and before the application of taxes by importing country.

[8] We exclude political economy factors for two reasons. First, they predominantly influence within-country distributional outcomes, which our analysis does not focus on. Within a similar framework, Ossa (2016) finds that "*optimal tariffs and their average welfare effects are quite similar with and without political economy pressures. This is because political economy pressures are more about the intra-national rather than the international redistribution of rents.*" Second, quantifying political economy weights is infeasible due to over-identification issues. For any hypothetical tax schedule, there exists a set of political weights that would rationalize it as optimal.

ence CO_2 emissions and internalize them in their policy choice. So, for all practical purposes, δ_i hereafter represents the disutility from CO_2 emissions as *perceived* by governments—meaning that our analysis does not rule out that δ_i may be disconnected from the actual climate cost facing country i's residents.[9] With this in mind, we turn to characterizing optimal policy under various scenarios.

3.1 Unilaterally Optimal Policy Problem

Unilaterally optimal policies apply to non-cooperative settings, where governments choose policies to maximize national welfare as specified by Equation (22) without considering effects on foreign households. The government in country i can utilize a comprehensive set of tax instruments denoted by $\mathbb{I}_i \equiv \left\{t_{ji,k}, x_{ij,k}, \tau_{i,k}\right\}_{j,k}$. The unilateral optimal policy choice is formally defined below, with an expansive formulation of the optimal policy problem provided in Appendix A.1.

Definition. The *Unilaterally Optimal Policy* for country i consists of taxes, $\mathbb{I}_i^* \equiv \{t^*_{ji,k}, x^*_{ij,k}, \tau^*_{i,k}\}_{j,k}$, that maximize country i's welfare in general equilibrium:

$$\mathbb{I}_i^* = \arg\max \;\; W_i\left(\mathbb{I}_i, \bar{\mathbb{I}}_{-i}\right) \quad \text{subject to general equilibrium Equations (1)} - \text{(21)};$$

where W_i is described by Equation (22) and $\bar{\mathbb{I}}_{-i}$ denotes policy choices in the rest of the world, which country i takes as given.

The unilaterally optimal policy seeks to correct the two sources of inefficiencies in the decentralized equilibrium from country i's *unilateral* standpoint: First, private energy production and consumption decisions fail to internalize the associated climate externality on country i's residents (as measured by δ_i). Second, country i's producers fail to internalize their collective market power when pricing the goods, so there is unexploited market power which country i's government can exploit to improve its terms of trade vis-a-vs the rest of the world.[10]

The *targeting principle* provides some guidance on the unilaterally optimal policy choices. Domestic carbon taxes are the first-best remedy for correcting carbon emissions from domestic economic activity. Border taxes (based on the carbon content of goods) are the unilaterally optimal instrument for correcting foreign emissions. And border taxes (based on national-level market power) are the first-best instrument for manipulating the terms-of-trade. However, precisely

[9] We also examine an alternative specification where δ_i maps to estimates of country-level climate change damage.

[10] Similarly, country i's consumers fail to internalize their collective monopsony power when purchasing foreign varieties, justifying import tariffs to exploit national-level import market power.

characterizing the optimal policy is complicated within a multi-country, multi-industry general equilibrium model. Below, we introduce a method to bypass certain complexities that come with deriving optimal policies in general equilibrium.

3.2 Dual Technique for Deriving Optimal Policy Formulas

We first convert the optimal policy problem that involves selecting taxes, $\mathbb{I}_i \equiv \left\{t_{ji,k}, x_{ij,k}, \tau_{i,k}\right\}_{j,k}$, into an equivalent problem where the government selects after-tax prices and energy cost shares, $\mathbb{P}_i = \left\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \alpha_{i,k}\right\}_{j,k}$. The optimal taxes from the original optimal policy problem, $\mathbb{I}_i^*$, can be recovered from the solution to the *reformulated* problem, $\mathbb{P}_i^*$, as:

$$1 + t_{ji,k}^* = \frac{\tilde{P}_{ji,k}^*}{P_{ji,k}}, \qquad \left(1 + x_{ij,k}^*\right)^{-1} = \frac{\tilde{P}_{ij,k}^*}{P_{ij,k}}, \qquad \tau_{i,k}^* = \frac{\alpha_{i,k}^* Y_{i,k}}{Z_{i,k}} - \tilde{P}_{i,0}.$$

Solving the first-order conditions (henceforth, F.O.C.s) with respect to policy $\mathbb{P}_i$ is difficult, as it involves characterizing complex general equilibrium (henceforth, GE) derivatives. We introduce a method to bypass this difficulty. To communicate our method succinctly, specify country i's welfare as an explicit function of policy ($\mathbb{P}_i$), wages ($\mathbf{w} \equiv \{w_n\}_n$), and domestic demand quantities ($\mathbf{C}_i \equiv \{C_{ni,k}\}_{n,k}$). Note that wages and domestic demand quantities are implicit functions of policy that satisfy labor market clearing and the interdependent system of demand schedules. The F.O.C.s under this representation can be expressed as

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln \mathbb{P}_i} = \frac{\partial W_i\left(\mathbb{P}_i; \mathbf{w}, \mathbf{C}_i\right)}{\partial \ln \mathbb{P}_i} + \frac{\partial W_i\left(\mathbb{P}_i; \mathbf{w}, \mathbf{C}_i\right)}{\partial \ln \mathbf{C}_i} \underbrace{\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \mathbb{P}_i}}_{\text{GE elasticity}} + \frac{\partial W_i\left(\mathbb{P}_i; \mathbf{w}, \mathbf{C}_i\right)}{\partial \ln \mathbf{w}} \underbrace{\frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln \mathbb{P}_i}}_{\text{GE elasticity}} = 0, \tag{23}$$

where $\frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln \mathbb{P}_i}$ and $\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \mathbb{P}_i}$ are "GE elasticities of wages and domestic demand quantities" with respect to policy. Characterizing these GE elasticities requires implicit differentiation of equilibrium conditions, which is analytically difficult. Traditional derivations of optimal policy circumvent this difficulty using two assumptions: (1) preferences are quasi-linear and separable, which reduces $\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \mathbb{P}_i}$ into a structural parameter;[11] (2) The economy contains a large, traded, and homogenous sector that sets $\frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln \mathbb{P}_i} = 0$. While these assumptions facilitate solving Equation 23, they limit the model's suitability for quantitative applications. Our method, however, allows analytical progress under the considerably milder assumptions that country i's policy does not alter *(A1)*

[11] For instance, consider the utility function, $U = C^{(\text{numeraire})} + \Sigma_k u(C_k)$, where $u(C_k) = \frac{\eta}{\eta-1}(C_k^{\frac{\eta-1}{\eta}} - 1)$ for each good k. With this parameterization, C_k depends only on $\tilde{P}_k$ and does not depend on income or prices of other goods. Therefore, the GE elasticity $(\mathrm{d}\ln C_k / \mathrm{d}\ln \tilde{P}_g)$ equals $-\eta$ if $g = k$ and equals zero if $k \neq g$.

relative wages among foreign countries, and *(A2)* aggregate factor income shares abroad.[12]

Our first result states that the GE elasticity of the domestic wage is redundant for solving Equation 23, since $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial \ln w_i} = 0$.[13] The foreign wage elasticities are also redundant per *A1* and Walras' law. Hence, the last term in Equation 23 can be disregarded altogether. Our second result asserts that a necessary condition for optimality is $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial \ln \mathbf{C}_i} = 0$, meaning that the optimal policy can be obtained without characterizing $\frac{\mathrm{d} \ln \mathbf{C}_i}{\mathrm{d} \ln \mathbb{P}_i}$. These two results, proven in Appendix A.4, reduce the optimality conditions from Equation 23 containing complex GE elasticities to a simpler set of equations:[14]

$$\frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i\right)}{\partial \mathbb{P}_i} = 0, \qquad\qquad \frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i\right)}{\partial \mathbf{C}_i} = 0. \tag{24}$$

The first equation, $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial \mathbb{P}_i} = 0$ is automatically satisfied *w.r.t.* the price of domestically-consumed goods per Roy's identity. Solving it *w.r.t.* other policy instruments determines the optimal carbon and export taxes. The second equation, $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial \mathbf{C}_i} = 0$, determines the optimal taxes on goods consumed domestically. The following proposition summarizes our results.

Proposition 1. *The GE elasticities of wages and domestic demand quantities w.r.t. policy,* $\left\{\frac{d \ln \mathbf{w}}{d \ln \mathbb{P}_i}, \frac{d \ln \mathbf{C}_i}{d \ln \mathbb{P}_i}\right\}$ *are redundant for the optimal design of policy. The optimal policy,* $\mathbb{P}_i^*$*, can be derived by solving the system of equations (24) rather than (23), bypassing the need to characterize these GE elasticities.*

This proposition can be framed as an *envelope* result: If the government has adequate tax instruments to control the price of all goods consumed and produced domestically, its optimal policy will internalize any welfare gains from perturbing wages or domestic demand quantitates. Hence, the F.O.C.s and implied optimal policy formulas could be derived *as if* $\mathbf{w}$ was constant and $\mathbf{C}_i$ resulted from a partial equilibrium demand system.[15] Deriving optimal policy formulas still requires solving the system of equations under 24. This feat can be accom-

[12] Neither *A1* nor *A2* is required in a two-country model with labor as the only factor of production. Beyond two countries, *A1* abstracts from relative wage effects among foreign countries $n, j \neq i$, i.e., $\mathrm{d}\left(w_n/w_j\right)/\mathrm{d}\mathbb{P}_i \approx 0$; and *A2* assumes that $\Pi_n/w_n L_n$ (for $n \neq i$) is unaffected by country i's policy. These assumptions preclude the need to trace income effects abroad, without restricting cross-price effects. Moreover, they impose no restrictions on how domestic variables respond to policy.

[13] This result is a reflection of the Lerner symmetry. It holds insofar as the government has access to sufficient tax instruments but the optimality of the policy choice is not required.

[14] We are not the first to make analytical progress on this front. We build on the optimal policy framework in Lashkaripour and Lugovskyy (2023), and advance it in two aspects. First, we incorporate international consumption externalities, broadening the applicability of our results to issues like climate policy. Second, we sharpen their envelope result. Specifically, Lashkaripour and Lugovskyy (2023) show that demand functions can be treated as income inelastic to simplify $(\mathrm{d}\ln \mathbf{C}_i/\mathrm{d}\ln \mathbb{P}_i)$ in the F.O.C.s. We demonstrate that this term is entirely redundant for characterizing optimal policy, so its simplification is unnecessary.

[15] Proposition 1 transforms a complex general equilibrium problem into a simpler pseudo-partial equilibrium variant.

plished non-parametrically, drawing on well-known micro envelope conditions like Shephard's and Hotelling's lemmas. The non-parametric F.O.C.s can then be simplified using standard identities like the Slutsky theorem and Cournot aggregation. Details of the derivations are provided in Appendix A.

3.3 Unilaterally Optimal Policy Formulas

We build on Proposition 1 to derive the unilaterally optimal policy formulas. These formulas are characterized by a set of sufficient statistics, making them suitable for quantitative analysis using data. To present our formulas, we define some auxiliary variables: We denote by $v_{n,k}$ the CO_2 emission per unit value of output in country $n-$industry k, and let $\rho_{ni,k}$ denote market i's share from that industry's total sales, $Y_{n,k}$. More formally,

$$v_{n,k} = \frac{Z_{n,k}}{Y_{n,k}}, \qquad \rho_{ni,k} = \frac{P_{ni,k}C_{ni,k}}{Y_{n,k}} \tag{25}$$

Additionally, we denote the elasticities of demand for the composite energy input (equivalently, CO_2 emissions) with respect to the energy input price at the industry and national levels as

$$\zeta_{n,k} \equiv \frac{\partial \ln Z_{n,k}}{\partial \ln \tilde{P}_{n,0}}, \qquad \zeta_n \equiv \frac{\partial \ln Z_n}{\partial \ln \tilde{P}_{n,0}} = \sum_{k\neq 0} \left(\frac{Z_{n,k}}{Z_n}\right) \zeta_{n,k}. \tag{26}$$

In the special case with CES production functions, $\zeta_{n,k} = -\varsigma\,(1-\alpha_{n,k})$, with (ς) as the elasticity of substitution between energy and labor inputs. Below, we present the unilaterally optimal policy formulas, noting that, by Lerner symmetry, the optimal border tax-cum-subsidies are unique only up to a uniform and arbitrary tax shifter, $\bar{t}_i \geq 0$.[16]

Proposition 2. *Country i's unilaterally optimal policy consists of (i) uniform carbon taxes ($\tau^*_{i,k} = \tau^*_i$), given by*

$$\tau^*_i = \tilde{\delta}_i \equiv \delta_i \tilde{P}_i,$$

(ii) import tariffs and export subsidies on final goods ($k \geq 1$) that are unique up to a uniform and arbitrary tax-shifter, $\bar{t}_i \geq 0$, augmented by a carbon border adjustment based on the CO_2 content per unit value of

This method has similarities with Costinot et al. (2015) who divide their general equilibrium optimal policy problem into *inner* and *outer* problems. Their inner problem holds wages fixed and leverages additive separability to break down a high-dimensional, multi-good problem into independent cell problems. The outer problem then solves for the optimal wage. Proposition 1 complements the primal approach of Costinot et al. (2015) as it does not impose separability restrictions, making it applicable to a wider range of demand and supply structures.

[16] For a clearer presentation, the export subsidy formulas are reported for additively separable preferences across industries and generalized separability within industries. General formulas are provided in Appendix A.10.

imported goods $v_{n,k}$ (Eq. 25),

$$1 + t^*_{ni,k} = (1 + \bar{t}_i) + \tau^*_i v_{n,k}$$
$$1 + x^*_{in,k} = \frac{1 + \varepsilon_{in,k}}{\varepsilon_{in,k}} \sum_{j \neq i} \left[\left(1 + t^*_{ji,k}\right) \frac{\lambda_{jn,k}}{1 - \lambda_{in,k}} \right]$$

(iii) import tariffs and export subsidies on energy,

$$1 + t^*_{ni,0} = (1 + \bar{t}_i)(1 + \omega_{ni,0}) + \sum_{\ell \neq i} \sum_{j \neq i} \left[\tilde{\psi}^{(i,0)}_{jn} \rho_{j\ell,0} \zeta_\ell \frac{\tau^*_i}{\tilde{P}_{\ell,0}} \right]$$
$$1 + x^*_{in,0} = \frac{1 + \varepsilon_{in,0}}{\varepsilon_{in,0}} \sum_{j \neq i} \left[\left(1 + t^*_{ji,0}\right) \frac{\lambda_{jn,0}}{1 - \lambda_{in,0}} \right] - \left(\Lambda_{in,0} + \zeta_n \frac{\tau^*_i}{\tilde{P}_{n,0}} \right) \frac{(1 + \bar{t}_i)}{\varepsilon_{in,0}},$$

where $\Lambda_{in,0} = \sum_{g \neq 0} \left[\alpha_{n,g} Y_{n,g}\right] / \sum_j \left[\tilde{P}_{jn,0} Q_{jn,0}\right]$ is the fraction of energy exports re-imported via the carbon supply chain; $\omega_{ni,0} = \frac{\phi_n}{1-\phi_n} \sum_{\ell \neq i} \left[\rho_{\ell i,0} \tilde{\psi}^{(i,0)}_{\ell n}\right]$ is the inverse export supply elasticity of energy (for flows from n to i), where $\tilde{\psi}^{(i,0)}_{\ell n} = \psi^{(i,0)}_{\ell n} Y_{\ell,0} / Y_{n,0}$ represents backward linkages in the energy sector;[17] λ and ρ represent international expenditure and sales shares (Eqs. 6 and 25); ζ is the demand elasticity of composite energy input (Eq. 26), and ε denotes the Marshallian demand elasticities (Eq. 5).

The unilaterally optimal carbon tax, τ^*_i, corrects only the carbon externality imposed on households in country i.[18] Specifically, it equals the welfare cost per unit of CO_2 emissions to residents of country i adjusted for the consumer price index, i.e., $\tilde{\delta}_i \sim \delta_i \tilde{P}_{i,0}$. The unilaterally optimal border taxes, however, pursue two objectives. First, they seek to manipulate the terms of trade in country i's favor. Second, they include a carbon border tax component that indirectly taxes the carbon externality of foreign production and consumption.

To better understand carbon border taxes, it is helpful to examine a small open economy under Cobb-Douglass CES preferences. Under the CES assumption, the import demand elasticity takes the form $\varepsilon_{in,k} = -\sigma_k + (\sigma_k - 1)\lambda_{in,k}$. The small open economy assumption sets $\lambda_{ij,k} \approx \rho_{ji,k} \approx 0$.

[17] Specifically, $\psi^{(i,0)}_{\ell n}$ is entry (ℓ, n) of matrix $\mathbf{\Psi}^{(i,0)} \equiv \text{inv}\left(\mathbf{I}_N + \frac{\phi_i}{1-\phi_i}\left[\mathbb{1}_{n \neq i} \sum_{m \neq i} \rho_{nm,0} \varepsilon^{(\ell m,0)}_{nm,0}\right]_{n,\ell}\right)$, measuring the exposure of country ℓ's energy output to demand for country n's energy, as detailed in Appendix A.2.

[18] Alternatively, the carbon tax could be applied at the point of energy extraction with appropriate adjustments to energy border taxes. See Appendix C.3 for optimal policy formulas featuring an explicit extraction tax.

Plugging these into our general optimal policy formulas yields a simplified representation:

$$\begin{cases} \tau_i^* = \tilde{\delta}_i \sim \delta_i \tilde{P}_i & \text{[carbon tax]} \\ t_{ni,k}^* = \bar{t}_i + \tau_i^* v_{n,k} \qquad t_{ni,0}^* = \bar{t}_i & \text{[import tax]} \\ 1 + x_{in,k}^* = (1+\bar{t}_i)\,\frac{\sigma_k - 1}{\sigma_k} + \tau_i^* \sum_{j \neq i} \left[\lambda_{jn,k} v_{j,k}\right] \frac{\sigma_k - 1}{\sigma_k} & \text{[export subsidy (non-energy)]} \\ 1 + x_{in,0}^* = (1+\bar{t}_i)\,\frac{\sigma_0 - 1}{\sigma_0} + \tau_i^* \frac{1}{\sigma_0} \frac{\zeta_n}{\tilde{P}_{n,0}} & \text{[export subsidy (energy)]} \end{cases}$$

The optimal import tax on final-good variety ni, k, which is *unaffected* by the CES and small open economy simplification, can be decomposed as:

$$t_{ni,k}^* = \bar{t}_i + \underbrace{\tau_i^* \times v_{n,k}}_{\text{Carbon Border Tax}} . \tag{27}$$

The uniform tariff component $\bar{t}_i$ reflects the standard terms-of-trade rationale for import taxation.[19] The carbon border tax component mimics the unilaterally-optimal domestic carbon tax. It taxes the carbon content per dollar value of imports, $v_{n,k}$, at the unilaterally optimal rate, τ_i^*. Remarkably, the unilaterally optimal border tax rate coincides with the accounting border adjustment that neutralizes the domestic cost disadvantage caused by carbon-pricing. Our formula presents a welfare rationale for these widely-used border adjustment schemes.[20]

The unilaterally optimal export subsidy on final-good variety in, k can be similarly decomposed as

$$1 + x_{in,k}^* = (1+\bar{t}_i)\,\frac{\sigma_k - 1}{\sigma_k} + \underbrace{\tau_i^* \times \sum_{j \neq i} \left[\lambda_{jn,k} v_{j,k}\right] \frac{\sigma_k - 1}{\sigma_k}}_{\text{Carbon Border Subsidy}}, \tag{28}$$

where the first component corresponds to the optimal markup on exports from the terms-of-trade standpoint. The carbon border subsidy depends on the average carbon intensity of competing foreign varieties in market n, namely, $\sum_{j \neq i} \left[\lambda_{jn,k} v_{j,k}\right]$. This differs from accounting border adjustment schemes that simply rebate the carbon taxes toward exports. The optimal carbon border subsidy seeks to mimic a carbon tax, τ_i^*, on foreign varieties sold to market $n \neq i$. It accomplishes this

[19] This element of our formula echoes the familiar result that, absent climate externalities, optimal tariffs are uniform across differentiated constant-returns-to scale industries. The logic follows the Lerner symmetry: a uniform tariff is akin to uniform export tax, allowing governments to elicit a markup on the nationally-differentiated labor content of exports, thereby improving their terms of trade.

[20] The above carbon border tax configuration does not account for origin country carbon tax rates, therefore risking double taxation. This is due to the non-cooperative nature of these taxes since governments may doubly tax the carbon externality to generate revenue. As shown in Appendix B, double taxation is avoided in a cooperative setting. The optimally cooperative carbon border tax is $\left(\tau^\star - \tau_n\right) \times v_{n,k}$, taxing the difference between the globally optimal rate $\tau^\star$ and the origin country rate, thus preventing double taxation.

by subsidizing the price of domestically produced exports varieties. Since domestically produced and foreign varieties are substitutable, the subsidy lowers demand for foreign goods in market $n \neq i$, imitating the demand drop if those goods were taxed directly.

Turning to border taxes on energy varieties, the uniform tariff, $\bar{t}_i$, on energy imports is motivated by terms-of-trade considerations.[21] Since imported energy varieties, after bundling and distribution, are subjected to a domestic carbon tax τ_i^*, no additional import duty on energy is needed. The optimal policy, however, includes a carbon-based tax on energy exports equal to $\tau_i^* \times \frac{1}{\sigma_0}\left(\zeta_n / \tilde{P}_{n,0}\right)$. The rationale is that country i would ideally levy a tax on country n's composite energy input at an ad valorem rate of $\tau_i^* / \tilde{P}_{n,0}$. This policy is infeasible, but the energy export tax is passed on to foreign's energy price, imitating this intended tax. Echoing this logic, the optimal export tax rate depends on the magnitude of tax passthrough, which is determined by the foreign countries's energy input demand elasticity, $\zeta_n \equiv \frac{\partial \ln Z_n}{\partial \ln \tilde{P}_{n,0}} < 0$, and the elasticity of substitution between international energy varieties, σ_0.

Having covered the basic intuition from the small open economy case, let us revisit the general formulas presented under Proposition 2. The optimal export subsidy for non-energy goods depends on the import demand elasticity ε, which is itself determined by structural parameters (like σ in the case of CES) and endogenous expenditure shares. The optimal border taxes on energy, meanwhile, account for general equilibrium linkages, which are non-trivial for large economies. Import taxes on energy constricts export supply and increase the marginal cost of energy extraction abroad. This triggers price changes that alters global energy demand, prompting further energy price shifts worldwide. These general equilibrium ripple effects are captured by the backward linkage matrix, $\mathbf{\Psi}^{(i,0)}$, whose elements determine the optimal import tax rate. Energy export subsidies, meanwhile, influence the cost of foreign goods using those inputs. Some of these goods are imported by country i and face a carbon border tax upon importation. The optimal energy export subsidy is, therefore, adjusted to prevent double marginalization. The optimal adjustment depends on $\Lambda_{in,0}$, which is the fraction of energy exports re-imported via the energy supply chain.

3.4 Globally Optimal Carbon-Pricing and Free-Riding

This section characterizes the optimal carbon policy from a global standpoint. Comparing the globally optimal policy with the unilaterally optimal policy, derived earlier, elucidates the free-

[21] A small open economy's optimal energy import tax has no climate-driven element, since imported energy varieties face a carbon tax after bundling and distribution. However, for a large economy, the optimal energy import tax internalizes climate impacts arising from general equilibrium linkages, as Proposition 2 indicates. We elaborate on these general equilibrium linkages in the next paragraph.

riding problem that impedes cooperation on climate action. We obtain the globally optimal policy by solving a *global* planning problem, where the planner selects tax instruments $\mathbb{I} \equiv \{\mathbb{I}_i\}_{i \in \mathbb{C}}$ and lump-sum international transfers, $\boldsymbol{\Delta} \equiv \{\Delta_i\}_i$, to maximize an internationally representative social welfare function. Letting $\tilde{\mathbb{I}} \equiv \{\mathbb{I}, \boldsymbol{\Delta}\}$ denote the policy set, the planing problem can be formulated compactly as

$$\max_{\tilde{\mathbb{I}}} \sum \vartheta_i \log W_i \left(\tilde{\mathbb{I}}\right) \quad \text{subject to General Equilibrium Equations (1)} - (21),$$

where $W_i \sim V_i \left(E_i + \Delta_i, \tilde{\mathbf{P}}_i\right) - \delta_i \times Z^{(global)}$ is country i's climate-adjusted welfare under policy, with $\sum_i \Delta_i = 0$. The inclusion of income transfers in the problem is essential, as it separates redistribution, addressed via transfers, from climate-related externalities, addressed via taxes.

Capitalizing on a variation of Proposition 2, we derive the globally optimal policy in Appendix B. The optimal policy from a global perspective involves carbon taxes that correct the worldwide externality of carbon emissions, along with zero trade taxes[22]:

$$\tau^{\star}_{i,k} = \sum_i \tilde{\delta}_i \sim \tau^{\star}, \qquad\qquad t^{\star}_{i,k} = x^{\star}_{i,k} = 0 \qquad (\forall i, k)\,. \tag{29}$$

The finding that globally optimal border taxes are zero (and carbon-blind) resonates with the targeting principle. Border taxes are an inefficient policy for reducing carbon emissions compared to directly targeted carbon taxes. In the unilateral case, carbon border taxes were justified since country i's government could not directly tax foreign carbon inputs. This missing policy limitation no longer applies in the globally optimal context.

The *free-riding problem* stems from the gap between the unilaterally optimal and globally optimal carbon tax rates. Specifically,

$$\tau^*_i = \tilde{\delta}_i \, < \, \tau^{\star} = \sum_n \tilde{\delta}_n.$$

This means that if all other countries commit to $\tau^{\star}$, country i's welfare-maximizing government will be inclined to lower its carbon tax rate from $\tau^{\star}$ to τ^*_i. Strategic behavior by all governments in this manner triggers a race to the bottom in climate action, similar to what we are witnessing today. In the next section, we discuss two potential solutions to the free-riding problem.

3.5 Two Remedies for the Free-Riding Problem

Two types of policies can mitigate the free-riding problem, both involving border tax measures:

[22] Transfers, $\Delta_i = (\pi_i \times \sum_i E_i) - E_i$, are pinned down by the optimal income shares: $\pi^{\star}_i = \left(\vartheta_i \frac{V_i}{W_i}\right) / \left(\sum_n \left[\vartheta_n \frac{V_n}{W_n}\right]\right)$.

Proposal 1. Governments use border taxes as a *second-best* policy to correct the climate externality of foreign emissions on their citizens. The maximal efficacy of this proposal will be realized if carbon border tax rates are set to the optimal rate specified by Proposition 2.

Proposal 2. Climate-conscious governments forge a climate club and leverage *contingent* trade penalties to deter free-riding. The maximal efficacy of this proposal will be realized if the trade penalties are applied based on the unilaterally optimal import and export tax rates (t^* and x^*) specified by Proposition 2.

A key difference is that Proposal 1 is rooted in unilateral action, while Proposal 2 seeks to revive multilateral climate efforts through better policy design. In theory, Proposal 2 could achieve first-best carbon pricing together with free trade. However, poorly-designed trade penalties and carbon tax targets for club members could also decouple the climate club from the rest of the world. We must clarify that our notion of optimal trade penalties refers to penalties that maximize welfare transfers from free-riders to climate club members. Accordingly, the optimal trade penalties coincide with the unilaterally optimal trade tax/subsides specified by Proposition 2—that is, they elevate the climate club's terms of trade with non-members to its maximal level while also taxing out-of-club carbon emissions. In Section 6.2, we discuss policy designs when free-riding is not a concern or trade penalties are chosen differently.

4 Mapping Theory to Data

This section describes how our general equilibrium model is mapped to data to simulate counterfactual policy outcomes. First, we describe our quantitative strategy for determining counterfactual optimal policy outcomes, identifying the sufficient statistics required for implementation. We then detail the data sources from which the noted sufficient statistics are obtained. For our quantitative analyses, we assume that the production function of final goods and the energy distributor has a CES form and the households' demand function has a Cobb-Douglas-CES functional form. The *baseline equilibrium*, to which we introduce the optimal policy interventions, corresponds to the status quo in 2014 (see Section 4.2). We are interested in counterfactual outcomes when taxes are revised from their applied levels to their optimal rates under the non-cooperative and climate club scenarios.

The baseline equilibrium under the status quo is characterized by the following statistics:

1. expenditure shares $\{\lambda_{ji,k}, \beta_{i,k}\}$ and employment shares $\{\ell_{i,k}\}$, where $\ell_{i,k} \equiv L_{i,k}/\bar{L}_i$ is country

i's share of employment in industry k,

2. CO_2 emissions, energy input cost shares, and CO_2 intensity values, $\{Z_{i,k}, \alpha_{i,k}, v_{i,k}\}$,

3. pre-carbon-tax price of energy $\{\tilde{P}_{i,0}\}$, and national accounting of income $\{w_i\bar{L}_i, r_i\bar{R}_i, Y_i\}$.

Let $\mathscr{B}^V$ stack the above-mentioned *baseline* variables, and let $\mathscr{B}^T \equiv \{x_{ij,k}, t_{ji,k}, \tau_{i,k}\}$ contain the applied policy variables—both of which are observable. Also, let $\mathscr{B}^\Theta = \{\tilde{\delta}_i, \phi_i, \varsigma, \sigma_k\}$ denote the set of structural parameters of the model consisting of carbon disutility parameters, cost share of carbon reserves, energy input demand elasticity, and trade elasticities; with $\mathscr{B} \equiv \{\mathscr{B}^V, \mathscr{B}^T, \mathscr{B}^\Theta\}$ denoting the set of sufficient statistics for conducting counterfactual policy analyses.

Let z' denote the value of a generic variable z in the counterfactual equilibrium, with $\widehat{z} \equiv z'/z$ denoting the corresponding change using the exact hat-algebra notation. To determine counterfactual outcomes under each of the policy scenarios, we solve a system of equations consisting of equilibrium conditions and optimal tax formulas. The solution to this system determines the optimal tax and subsidy rates, $\mathscr{R}^T = \left\{x'_{ij,k}, t'_{ji,k}, \tau'_{i,k}\right\}$, as well as *changes* to all general equilibrium variables, $\mathscr{R}^V = \left\{\hat{\lambda}_{ji,k}, \hat{\ell}_{i,k}, \hat{Z}_{i,k}, \hat{\alpha}_{i,k}, \hat{v}_{i,k}, \hat{w}_i, \hat{r}_i, \hat{Y}_i\right\}$, with $\mathscr{R} \equiv \{\mathscr{R}^T, \mathscr{R}^V\}$ denoting the full set of optimal policy outcomes solved given the sufficient statistics in $\mathscr{B}$.

4.1 Quantitative Strategy

Baseline Policies. Our analysis sets the baseline import tariffs, $\{t_{ji,k}\}$, export subsidies, $\{x_{ij,k}\}$, and carbon taxes, $\{\tau_{i,k}\}$, to the applied rates observed in data. These rates reflect the current-but-evolving sentiments of governments regarding trade and climate issues. In particular, governments are generally cooperative on trade policy and comply with WTO rules, but non-cooperative on climate policy. Moreover, government policies are currently undergoing a major shift, with governments beginning to adjust their unilateral climate policies to match growing public concern about climate change. Under this sentiment, our optimal policy framework predicts that governments aim to implement the following policies in the *long-run*: (i) import tariffs and export subsidies necessitated by carbon border taxation; and (ii) unilaterally optimal carbon taxes. The data is broadly consistent with these long-run objectives. Applied tariffs are near zero, reflecting the early stages of carbon border taxes after multiple (though incomplete) rounds of trade liberalization under the WTO/GATT; export subsidies are minimal due to the WTO's prohibition; and carbon taxes are below optimal but increasing worldwide to match the updated $\tilde{\delta}_i$'s.[23]

[23] Testing our optimal policy framework with a snapshot of contemporary policy data is challenging for two rea-

Counterfactual Policy Scenarios. We evaluate Proposal 1 by simulating the *non-cooperative* equilibrium in which each country adopts its unilaterally-optimal policy. Under this situation, country i's policy, $\mathscr{R}_i^T \equiv \left\{x'_{ij,k}, t'_{ji,k}, \tau'_{i,k}\right\}_{j,k}$ is determined by the optimal policy formulas presented under Proposition 2 as a function of $\mathscr{B}$ and $\mathscr{R}^V$.[24] The change in non-policy variables is, similarly, described by general equilibrium conditions as a function of $\mathscr{R}^T$ and $\mathscr{B}$. Appendix D outlines the equations describing the change in non-policy variables as a function of policy change $\mathscr{R}^T$ and the sufficient statistics $\mathscr{B}$. Jointly solving $\mathscr{R}^T = f\left(\mathscr{R}^V; \mathscr{B}\right)$ and $\mathscr{R}^V = g\left(\mathscr{R}^T; \mathscr{B}\right)$ determines optimal policy and counterfactual equilibrium outcomes as a function of the observable and estimable sufficient statistics in $\mathscr{B}$. Likewise, our analysis of the climate club uses the unilaterally optimal trade taxes described by Proposition 2 as contingent trade penalties, and simulates counterfactual policy outcomes using the same logic.

Interpreting Counterfactual Policy Outcomes. Before discussing the data and results, two clarifications are in order. First, our counterfactual analyses measure long-run outcomes depending on whether governments maintain their current policy stance or form a climate club. Results relating to Proposal 1 measure outcomes if heightened climate concerns prompt governments to abandon shallow trade cooperation while continuing to raise domestic carbon taxes until they reach the unilaterally optimal rate. Proposal 2 results measure outcomes if climate considerations are integrated into existing international trade agreements. Second, the primary goal of our optimal policy framework is to trace out the frontier of policy outcomes, not to necessarily explain government behavior. Actual policies often fall short of this frontier due to various obstacles. But the policy frontier remains an effective tool for gauging long-term policy efficacy—a point we come back in Section 6.2 when discussing the EU's unilateral policy frontier.

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sons. First, policies transition gradually rather than shift instantly to desired levels. Trade liberalization under the GATT/WTO exemplifies this gradual process, playing out over many decades and rounds of negotiations. Second, we are in a transitional policy period, undergoing a major shift in governmental and public attitudes toward trade and climate policy. This development has sparked recent reforms aimed at new policy objectives. Given the static nature of our model, it is difficult to test it against these dynamic, transitional policies. Yet our optimal policy formulas align with broader evidence on government behavior, beyond those highlighted above. Empirical evidence shows governments account for terms of trade effects in policymaking (Broda et al., 2008). Additionally, the EU CBAM design echoes the unilaterally optimal policy predicted by our theory.

[24] The resulting equilibrium constitutes the Nash equilibrium of a one-shot game, wherein every country selects their best policy response given applied policies in the rest of world. Lashkaripour (2021) and Lashkaripour and Lugovskyy (2023) use a similar logic to quantify the counterfactual impact of non-cooperative trade policies.

4.2 Data and Parameters: Sufficient Statistics

In this section, we describe the sufficient statistics for conducting counterfactual policy analysis, which include data on trade, production, and CO_2 emissions (labeled as $\mathscr{B}^V$), applied taxes ($\mathscr{B}^T$), and the structural parameters of our model ($\mathscr{B}^\Theta$).

Trade, Production, and Expenditure. We take data on international trade and production from the Global Trade Analysis Project (GTAP), which reports the matrix of international trade flows from each country-industry origin to each country-industry destination for the year 2014. We consolidate our sample into ($K+1=18$) "industries," comprising $K=17$ non-energy ISIC-level industries and one composite energy industry, and ($N=19$) "countries," consisting of the 13 countries with the largest GDP plus 6 aggregate regions. Tables 1 and 2 list the industries and countries in our sample, along with their key characteristics. Our final sample manifests as a $19\times 19\times 18$ matrix of free-on-board flows, with element $X_{ij,k}^{(fob)} = \tilde{P}_{ji,k}C_{ji,k}/\left(1+t_{ij,k}\right)$ corresponding to origin j–destination$i$–industry k.[25]

CO_2 Emissions and Carbon Accounting. We obtain information on CO_2 emissions from the GTAP database.[26] We count CO_2 emissions at the location of energy use by end-users (i.e., non-energy industries and households). We consolidate all energy types into one composite energy industry, denoted as industry "0," calculating the CO_2 emissions associated with direct and indirect energy usage. For example, consider the steel industry. It *directly* generates emissions, e.g., from burning coal at the location of steel production. Moreover, steel production *indirectly* generates emissions by using electricity, the production of which involves burning coal. We observe direct emissions in the data and calculate the indirect emissions, as elaborated below.[27]

Initially, consider a closed economy, denoting energy types by $e \in \{1, ..., E\}$.[28] Let $Z_e^{(direct)}$ denote the direct CO_2 emissions from production of energy type e and Y_e as its gross output. By accounting, Y_e comprises total usage for both energy generation and non-energy production, with $X_{ee'}$ representing the amount of type e energy used for type e' energy generation and C_e representing energy usage for non-energy production. To generate one dollar of type e' energy, $a_{ee'}$ dollars

[25] To be consistent with our framework, we purge the data from trade imbalances following Ossa (2016).

[26] In our analysis, CO_2 emissions are exclusively associated with the use of fossil fuels. Therefore, we exclude emissions from *(i)* non-CO_2 greenhouse gas emissions such as methane, *(ii)* CO_2 emissions that are associated with the production process such as those in the cement industry.

[27] We track the indirect emissions associated with energy purchases, and do not account for the energy embedded in other intermediate inputs.

[28] The data differentiates between the following energy types: coal, crude oil, natural gas, refined oil products and electricity & gas manufacture.

of type e energy inputs are required, leading to $X_{ee'} = a_{ee'} Y_{e'}$. Input-Output accounting entails that $Y_{(E\times 1)} = A_{(E\times E)} Y_{(E\times 1)} + C_{(E\times 1)}$, from which we derive $Y = (I-A)^{-1} C$, where $(I-A)^{-1} \equiv B$ is the Leontief inverse describing energy input-output flows. The effective carbon intensity for each energy type (i.e., emissions per dollar of output) is then given by $\tilde{v}_{e'} = \sum_{e=1}^{E} \left[b_{ee'} \left(Z_e^{(direct)} / Y_e \right) \right]$, where $b_{ee'}$ is the entry (e, e') of the Leontief inverse.

The emissions per dollar of output in non-energy sectors ($k = 1, ..., K$) encompass direct emissions, denoted by $Z_k^{(direct)}$, arising from combustion of fossil fuels during production, as well as indirect emissions tied to energy generation. The latter can be computed as $Z_k^{(indirect)} = \sum_e \tilde{v}_e X_{ek}$, where $\tilde{v}_e$ was defined above, and X_{ek} denotes the value from type e energy inputs used by industry k. The total emissions for industry k are thus represented by $Z_k = Z_k^{(direct)} + Z_k^{(indirect)}$.

The above procedure can be extrapolated to open economies as follows. Let vector $Y_{(NE\times 1)} = [Y_{ne}]$ represent gross energy output by type for each country n; let $A = [a_{ne,ie'}]_{(NE\times NE)}$ denote the global energy input-output matrix; and let $C = [C_{ne}]_{(NE\times 1)}$ represent total energy sales to non-energy sectors by type and country. The accounting equation for energy flows can be expressed as $Y = AY + C$, implying an $NE \times NE$ global Leontief Inverse matrix $B = (I-A)^{-1} = [b_{ne,ie'}]$. The effective emission per dollar of output generated by energy type e' in country i equals $\tilde{v}_{i,e'} = \sum_{n,e} b_{ne,ie'} (Z_{n,e}/Y_{n,e})$ and the indirect emissions associated with country $i-$industry k are represented by $Z_{i,k}^{(indirect)} = \sum_{n,e'} \tilde{v}_{n,e'} X_{ne',ik}$. The total emissions per industry is the sum of direct and indirect emissions, $Z_{i,k} = Z_{i,k}^{(direct)} + Z_{i,k}^{(indirect)}$. A similar procedure yields the total emissions associated with household consumption, $Z_{i,\text{hhd}}$.

Table A.2 reports total emissions (as the sum of direct and indirect emissions) by industries and households, which we use throughout our analysis.

Stylized Facts about Global CO_2 Emissions. We highlight key statistics that will aid in interpreting the findings from our quantitative analysis in Section 5. First, emissions from production constitute three-fourths of global CO_2 emissions, with the remaining one-fourth being generated by households (Appendix Table A.2). Second, more-tradeable industries exhibit lower CO_2 emission shares (Appendix Figure A.1). For example, Electronics & Machinery, Textiles & Leather, and Motor Vehicles industries, that are highly tradeable, as indicated by their high trade-to-GDP ratio, collectively account for only six percent of the global CO_2 emissions from production (Table 1). Lastly, low and middle-income countries are major contributors to global CO_2 emissions, with China alone accounting for over a quarter of these emissions. This proportion reaches 60% when considering all non-OECD countries (Table 2).

Energy Input Cost Shares. We construct energy input cost shares using data on sales and energy input expenditures. Our assumption that energy is freely traded implies a uniform (pre-carbon-tax) energy price across countries, denoted as P^Z. For the year 2014, our data sets P^Z at \$122 per tonne of CO_2, a figure that closely aligns with independent data on production quantities and primary energy prices for that year.[29] The energy input cost share can be calculated as $\alpha_{n,k} = \left(P^Z Z_{n,k}\right) / \left(P_{nn,k} Q_{n,k}\right)$, where $Z_{n,k}$ and $P_{nn,k}Q_{n,k}$ represent total CO_2 emissions and gross output in country $n-$industry k. Global average values of $\alpha_{n,k}$ for each industry are reported in Table 1.

Cost Share of Carbon Reserves. The GTAP database reports the value added associated with each factor of production, including natural resources. We set the cost share of carbon reserves in the energy extraction industry, ϕ_i, based on the value added share of natural resources in each country's primary energy sector, which consists of coal, crude oil and natural gas. The calibrated values of ϕ_i range between 0.31 and 0.49 across countries, with an average value of 0.37.

Baseline Taxes. We acquire data on applied tariffs from the GTAP database via the Market Access Map of International Trade Centre that reports tariffs at the level of 6-digit HS products in 2014. For each *origin–destination–industry* triplet, we use the simple average of the tariffs across HS products, except when the tax-imposing country is a member of the European Union (EU). In such cases, we assign applied tariffs based on the fact that intra-EU trade is subject to no tariffs and EU members apply a common tariff on non-members. In accordance with the World Trade Organization rules, we assume that applied export subsidies are negligible and set $x_{ij,k} = 0$ in our baseline equilibrium. We infer carbon taxes in 2014 from the the World Bank's Carbon Pricing Dashboard. To attain a harmonized measure of carbon taxes across countries, we calculate the ratio of taxes raised from climate policies to the aggregate CO_2 emissions within each country, which we designate as the national carbon tax for each respective country.[30]

Perceived Disutility from Carbon Emissions. We recover the perceived disutility from CO_2 emissions through governments' revealed preferences for tackling environmental issues. Specifically, we postulate that the perceived national disutility from carbon is proportional to applied environmentally-related taxes per unit of CO_2 emissions, adjusted for the respective country's

[29] Specifically, dividing the global sales of primary energy—consisting of coal, crude oil, and natural gas—by the global output quantity of primary energy which maps to the global CO_2 emission, delivers the pre-tax global carbon price.

[30] In 2014, carbon taxes were zero in most countries and substantially lower than their unilaterally-optimal levels in other countries. Consolidating the impact of climate policies into a single carbon tax metric is difficult, especially with limited data on sectoral variations. This consideration, however, remains inconsequential for our 2014 baseline because: *(i)* most countries lacked climate policies, and *(ii)* carbon taxes, whether directly applied or indirectly implied, were still very low, even in the EU.

Table 1: Industry-Level Statistics

Industry	CO_2 Emission (% of Total)	Trade-to-GDP Ratio	Carbon Intensity (v)	Energy Cost Share (α)	Trade Elasticity ($\sigma - 1$)
Agriculture	4.2%	8.9%	100.0	0.030	3.80
Other Mining	1.9%	28.9%	183.0	0.055	10.16
Food	3.3%	8.0%	45.9	0.015	3.80
Textile	1.9%	22.8%	59.7	0.021	4.25
Wood	0.5%	8.4%	61.0	0.026	6.50
Paper	2.1%	8.9%	125.9	0.061	6.55
Chemicals	9.5%	21.9%	179.5	0.062	8.60
Plastics	1.8%	13.5%	89.1	0.056	8.60
Nonmetallic Minerals	8.6%	6.0%	458.4	0.121	5.27
Metals	14.7%	14.6%	205.2	0.066	5.99
Electronics and Machinery	3.0%	30.0%	42.0	0.022	3.98
Motor Vehicles	1.2%	23.3%	34.0	0.014	4.88
Other Manufacturing	0.6%	21.5%	42.0	0.032	4.80
Construction	1.5%	0.6%	59.2	0.025	5.94
Wholesale and Retail	3.6%	2.4%	34.7	0.017	5.94
Transportation	27.3%	10.5%	498.3	0.171	5.94
Other Services	14.5%	3.1%	26.7	0.012	5.94

Note: This table shows for every of the 17 non-energy industries the share from world industrial CO_2 emission (not including households' emission), world-level trade-to-GDP ratio, global average carbon intensity (CO_2 emissions per dollar of output) normalized by that of agriculture, energy cost shares reported as unweighted mean across countries within each industry, and estimated trade elasticities.

size. If perceptions of climate damage were symmetric across governments, the disutility from CO_2 emissions would merely scale with country size. To account for the size effect, we impose that $\left(\tilde{\delta}_i/\tilde{\delta}_j\right) \propto \left(L_i/L_j\right)$, where L_i denotes country i's population. However, governments' attitudes towards climate change are markedly diverse—even after accounting for size effects. We do not intend to explain these differences, but posit that governmental concern for climate damage can be inferred from policy stance toward environmental issues. Under this assumption, we assert that $\left(\tilde{\delta}_i/\tilde{\delta}_j\right) \propto \left(T_i^{(env)}/Z_i\right) / \left(T_j^{(env)}/Z_j\right)$, where $T_i^{(env)}$ denotes the environmentally-related taxes collected by country i, sourced from OECD-PINE. These considerations lead to the following proportionality condition:

$$\text{(a)} \qquad \frac{\tilde{\delta}_i/L_i}{\tilde{\delta}_j/L_j} = \frac{T_i^{(env)}/Z_i}{T_j^{(env)}/Z_j}.$$

Table 2: Country-Level Statistics

Country	Share from World			Carbon	
	Output	CO_2 Emission	Population	Intensity (v)	Disutility ($\tilde{\delta}$)
Australia	1.8%	1.2%	0.3%	147.5	1.5
EU	25.9%	12.3%	7.5%	100.0	53.2
Brazil	2.8%	1.6%	2.8%	135.3	6.0
Canada	1.9%	1.6%	0.5%	176.1	1.2
China	17.8%	26.7%	18.9%	378.1	20.9
Indonesia	1.0%	1.5%	3.5%	302.0	0.5
India	2.4%	6.4%	17.9%	620.4	12.5
Japan	6.2%	3.6%	1.8%	127.6	6.0
Korea	2.2%	1.7%	0.7%	188.7	3.2
Mexico	1.4%	1.4%	1.7%	218.8	0.3
Russia	1.9%	4.4%	2.0%	436.5	0.2
Saudi Arabia	0.4%	1.5%	0.4%	752.4	0.0
Turkey	1.0%	1.1%	1.1%	245.5	4.9
USA	20.4%	17.2%	4.4%	162.0	6.8
Africa	2.6%	3.6%	15.9%	285.3	22.2
RO Americas	3.0%	2.7%	4.1%	194.7	9.8
RO Asia and Oceania	5.1%	5.5%	11.8%	253.0	6.6
RO Eurasia	0.7%	2.2%	1.9%	671.6	0.1
RO Middle East	1.6%	3.9%	2.8%	494.9	0.3

Note: This table shows for every of the 19 regions (13 countries + the EU + Africa + 4 "Rest Of" regions as collection of neighboring countries), their share from world output, CO_2 emission, and population, and carbon intensity (CO_2 emissions per dollar of output) normalized by that of the EU, and CPI-adjusted disutility from one tonne of CO_2 emission, which sum to the social cost of carbon.

Moreover, we impose an adding up constraint, wherein the global sum of perceived disutility from CO_2 emissions equates the global Social Cost of CO_2. In particular,

$$\text{(b)} \qquad \sum_i \tilde{\delta}_i = \text{SC-CO}_2, .$$

We calibrate SC-CO_2 based on the latest release of the United States Environmental Protection Agency (EPA)'s Final Report on the Social Cost of Greenhouse Gases. From this report, we adopt the middle scenario discount rate, yielding a SC-CO_2 of \$156.2 per tonne of CO_2 in 2014.[31] By consolidating conditions, (a) and (b), we recover the CPI-adjusted disutility from carbon emissions, $\tilde{\delta}_i$. Table 2 reports our calibrated values of $\tilde{\delta}_i$ for each country in the sample.[32]

[31] Specifically, Table A.5 of the EPA's publication reports 193 (\$/tCO_2) for 2020 and 230 (\$/tCO_2) for 2030, both in dollars of 2020, based on a 2% annual discount rate (as the middle scenario between 1.5% and 2.5%). Using a linear projection to the year 2014 (which is the year in our baseline data) and adjusting for the inflation, we obtain a SC-CO_2 of 156.2 (\$/tCO_2) for the year 2014 in terms of dollars of 2014.

[32] We also experiment with an alternative calibration of $\tilde{\delta}_i$ based on country-level social cost of carbon. See Figure A.4

Trade Elasticities. We estimate the industry-level trade elasticities, $(\sigma_k - 1)$, using an identification strategy resembling that of Caliendo and Parro (2015). Under Cobb-Douglas-CES demand, the free-on-board value corresponding to origin i–destination j–industry k, denoted by $X_{ij,k}^{(fob)} \equiv P_{ij,k} C_{ij,k}$, is given by

$$X_{ij,k}^{(fob)} = (1 + t_{ij,k})^{-\sigma_k} \left(\bar{d}_{ij,k} P_{ii,k}\right)^{1-\sigma_k} P_{j,k}^{\sigma_k - 1} \beta_{j,k} E_j,$$

where $(1 + t_{ij,k})$ is the ad valorem tariff rate, $P_{ii,k}$ is the producer price in the origin country, and $\tilde{P}_{j,k}$ and $\beta_{j,k} E_j$ are the industry-level consumer price index and expenditure in the importing country. We specify the bilateral trade cost in industry k as $\bar{d}_{ij,k} = \bar{d}_{i,k} \times \bar{d}_{j,k} \times \bar{d}_{i \leftrightarrow j,k} \times \exp(\epsilon_{ij,k})$, dissecting it into origin and destination fixed effects alongside a symmetric dyad fixed effect, encapsulating the effect of gravity-related variables such as distance, common currency, or common border. From the above relationships we obtain the following estimating equation:

$$\ln X_{ij,k}^{(fob)} = -\sigma_k \ln(1 + t_{ij,k}) + D_{i,k} + D_{j,k} + D_{i \leftrightarrow j,k} + \epsilon_{ij,k}, \tag{30}$$

where $D_{i \leftrightarrow j,k} = (1 - \sigma_k) \ln \bar{d}_{i \leftrightarrow j,k}$ is a symmetric dyad fixed effect, while $D_{i,k} = (1 - \sigma_k) \ln \left(\bar{d}_{i,k} P_{ii,k}\right)$ and $D_{j,k} = \ln \left[\left(P_{j,k} / d_{j,k}\right)^{\sigma_k - 1} \beta_{j,k} E_j \right]$ represent importer and exporter fixed effects. Utilizing data on trade values and applied tariffs, we estimate σ_k under the identifying assumption that applied tariffs are uncorrelated with idiosyncratic variations in bilateral trade costs, $\epsilon_{ij,k}$. Detailed estimation results are reported in Appendix Table A.3, with point estimates replicated in Table 1.[33]

Energy Demand Elasticity. According to cost minimization, the following equation describes the quantity of energy inputs relative to total input cost, $Z_{i,k} / TC_{i,k}$, in country i and industry k:

$$\ln \left(\frac{Z_{i,k}}{TC_{i,k}} \right) = -\varsigma \ln \tilde{P}_{i,0k} + \underbrace{(1 - \varsigma) \ln \mathscr{C}_{i,k} + \ln \bar{\kappa}_{i,k}}_{= \Phi_i^{(energy)} + \Phi_k^{(energy)} + \epsilon_{i,k}^{(energy)}}. \tag{31}$$

and Section 6.1. In both cases we recover CPI-adjusted disutility parameters, $\tilde{\delta}_i = \tilde{P}_i \delta_i$ which is sufficient for our counterfactual equilibrium analyses.

[33] Two points warrant mention. First, lacking information on service trade tariffs, we set the trade elasticity of services to the average from non-service industries. Second, the table does not list the energy industry since, by accounting of carbon flows, we assign CO_2 emissions to consumption (and not production) of energy. For completeness, we note that global energy trade-to-GDP ratio is 24.6%, with energy trade elasticity estimated at $(\sigma_0 - 1) = 10.16$, derived from pooling observations on energy flows with Other Mining. The reason is that otherwise we would lose too many observations as energy is traded typically in one direction between many country pairs whereas our specification requires a symmetric origin-destination fixed effect.

The right-hand side variables include the after-tax price of energy, $\tilde{P}_{i,0k}$,[34] the marginal cost, $\mathscr{C}_{i,k}$, and the constant input demand parameter, $\bar{\kappa}_{i,k}$. We allow the combined effect of the latter two terms to systematically vary by country and industry through the fixed effects, $\Phi_i^{(energy)}$ and $\Phi_k^{(energy)}$, with $\epsilon_{i,k}^{(energy)}$ denoting the unobserved energy demand residual.

Our identification strategy relies on two assumptions. First, an individual industry's energy demand residual (in a given country) has a negligible impact on the global pre-tax energy prices. This assumption entails that each industry is small relative to the global energy market where pre-tax energy prices are set. Second, we assume that the unobserved energy demand residual is uncorrelated with energy tax rates after controlling for country and industry fixed effects. Table 3 reports our estimation results. Our preferred specification corresponds to Column (3) which corresponds to an energy demand elasticity of 0.645.[35]

Table 3: Estimation: Energy Demand Elasticity

	(1)	(2)	(3)
Estimate	-0.928	-0.626	-0.645
S.E.	(0.140)	(0.143)	(0.170)
Industry FE	Y	Y	Y
Country FE	N	N	Y
Additional Controls	N	Y	N
R-squared	0.456	0.516	0.629
Observations	2,040	2,006	2,040

Note: This table reports OLS estimates of the energy demand elasticity $(-\varsigma)$, based on Equation (31). Standard errors are clustered at the country level. Each observation is a pair of country-industry across 120 countries and 17 non-energy industries. Energy use for each industry aggregates purchases of refined oil products, electricity and manufactured gas in oil equivalent units. Total cost for each industry equals payments to factors of production and intermediate inputs. All columns include an industry FE and Column (3) additionally includes a country FE. Column (2) controls for country-level effects using variables that approximate country-wide unobserved production costs and energy demand, consisting of industrial expenditure per worker, national energy reserves, and domestic expenditure share in the energy sector.

Magnitudes of Optimal Border Taxes. To lay the groundwork for our assessment of Proposals 1 and 2, we discuss the magnitude of unilaterally optimal border taxes as implied by our calibrated model. Recall that unilaterally optimal border policies involve both import tariffs and export subsidies. Consider first the case where governments exhibit no concern for climate change. In this

[34] The after-tax price of energy which we use here includes fuel taxes that are not related to climate change. In our quantitative analysis, these non-climate-related fuel taxes are captured by exogenous energy demand shifters.

[35] Our elasticity parameters align with the long-run estimates in the literature, reflecting our focus on long-term outcomes. In their meta-analysis, Labandeira et al. (2017) report an average long-run energy demand elasticity of -0.596 (ours is -0.645). Our calibration implies a greater-than-one energy supply elasticity which is closer to an elasticity one would obtain from a long-run history of oil field extractions or explorations, e.g., see Appendix E.2 of Kortum and Weisbach (2021) and Dahl and Duggan (1998). Lastly, our trade elasticity, ranging between 3.8 and 8.6 across manufacturing industries, is in line with larger and long-run estimates in the literature (Alessandria et al., 2021).

case, optimal border policies solely include terms-of-trade driven components. Per the Lerner symmetry, only the ratio of the optimal tariff (t^*) to export subsidy (x^*) is determined, and exhibits a median of 17% across non-energy product varieties, i.e., $(1+t^*) \,/\, (1+x^*) \simeq 17\%$. The corresponding 10th and 90th percentiles of optimal tariff-to-subsidy ratios stand at 12% and 26%, respectively. These ratios tend to be larger in industries with a lower trade elasticity and exhibit modest variation across countries.[36]

Carbon border taxes/subsidies constitute a modest fraction of the optimal border tax/subsidy rate. They vary noticeably across countries imposing the taxes, as they align with each country's unilaterally optimal carbon tax, $\tau_i^* = \tilde{\delta}_i$. They also vary considerably across industries and are more punitive in industries with higher carbon intensities. Figure A.2 in the appendix elucidates this point by showcasing EU's unilaterally optimal carbon import taxes, evaluated at $\tau_{EU}^* = 53$ ($/t$CO_2$). "Nonmetallic Minerals," "Metals" and "Chemicals" in the manufacturing sector and "Transportation" in services have the highest unilaterally-optimal carbon import tax, with median rates ranging from 2-7% and reaching 10% for exporters at the 90th percentile of carbon intensity.

5 Quantitative Assessment of Climate Proposals 1 and 2

In this section, we provide a quantitative assessment of two prominent climate proposals that combine carbon taxes with border measures to address the free-riding problem. We examine the efficacy of each proposal by reporting the changes in carbon emissions and welfare resulting from these policies, compared to the status quo.

5.1 Proposal 1: Non-Cooperative Carbon Border Taxes

Under Proposal 1, border taxes are employed as a second-best policy to cut (under-taxed) carbon emissions by non-cooperative trading partners. To gauge maximal efficacy, we simulate a non-cooperative Nash equilibrium in which each government enacts its best policy response consisting of unilaterally optimal border and carbon taxes. The resulting change in CO_2 emissions (Z), real consumption (V), and climate-adjusted welfare (W) are reported in Table 4.

The first panel (titled "Noncooperative: Carbon + Border Tax") reports changes in economic

[36] Our optimal border taxes are broadly consistent with, but on the lower side of existing estimates obtained from models without carbon externalities, e.g., Ossa (2014); Lashkaripour (2021). The terms-of-trade component of optimal border taxes largely depend on the industry-level trade elasticity, $(\sigma_k - 1)$, with a higher trade elasticity implying a lower degree of national-level market power. Our estimates of trade elasticity are on average 5.91, which is higher than the estimates of trade elasticity in Ossa (2014); Lashkaripour (2021).

outcomes relative to the status quo when all governments adopt their unilaterally optimal carbon and border tax measures non-cooperatively. To understand these results, note that domestic carbon taxes are virtually zero under the status quo. Therefore, the carbon reduction reported in this panel represents the combined reduction from both elevating the domestic carbon tax and border tax rates to their unilaterally optimal rates.

To isolate the net contribution of border taxes to carbon reduction, the middle panel in Table 4 (titled "Noncooperative: Carbon Tax") reports outcomes under unilaterally optimal domestic carbon taxes that are not supplemented with any border measures. The difference between the numbers presented in the first and middle panels represents the net contribution of non-cooperative border taxes to economic outcomes, including carbon emissions. To put the non-cooperative outcomes in perspective, the panel "Globally Cooperative" presents the effects of globally optimal (first-best) carbon taxes.[37]

The results in Table 4 suggest that optimally-designed non-cooperative border taxes deliver a 1.2% reduction in global CO_2 emissions (i.e., $^{(1-0.064)}/_{(1-0.054)} - 1 = 1.2\%$), complementing the 5.4% reduction attained through unilaterally optimal domestic carbon taxes.[38] This stands in contrast to the additional 37.6% reduction in global CO_2 emissions when domestic carbon prices are elevated to their first-best level (i.e., $^{(1-0.410)}/_{(1-0.054)} - 1 = 37.6\%$). To rephrase, non-cooperative border taxes replicate only 3.2% of the potential CO_2 reduction under global cooperation (i.e., $^{1.2}/_{37.6} = 3.2\%$)—highlighting the limited effectiveness of non-contingent carbon border taxation in addressing transboundary CO_2 emissions.

The limited efficacy of carbon border taxes stems from three main factors. First, carbon border taxes applied to non-energy goods are not granular enough to incentivize carbon abatement by foreign firms. Carbon border taxes penalize CO_2 emissions based on the *origin country–industry*'s average carbon intensity—a metric individual firms cannot directly influence. Consequently, carbon border taxes fail to encourage individual foreign firms to abate, by substituting energy with cleaner inputs.

Second, border taxes cannot cut the CO_2 emissions from non-traded goods, which constitute a significant portion of worldwide emissions. The "Transportation" sector, for example, is responsible for over 25% of global industrial CO_2 emissions, yet it has a trade-to-GDP ratio of just 0.10 (see Table 1). Appendix Figure (A.1) compares the tradeability of industries to their global emis-

[37] The outcomes presented in the last panel exclude the lump-sum inter-country transfers necessary for ensuring Pareto improvements. The country weights in the global planner's problem could be chosen to ensure such Pareto improvements. Here, we simply set these weights based on GDP share of countries in status quo.

[38] Let $\Delta x_A = (\widehat{x}_A - 1)$ be the percentage change in a generic variable x under counterfactual policy "A" with $\widehat{x}_A \equiv x_A/x$ as the corresponding "hat" value. Then, the percentage change for a move from counterfactual A to B equals: $(\widehat{x}_B/\widehat{x}_A - 1) = ([1 + \Delta x_A]/[1 + \Delta x_B] - 1)$.

Table 4: The Impact of Non-cooperative and Cooperative Tax Policies

Country	Non-Cooperative						Globally Cooperative		
	Carbon + Border Tax			Carbon Tax					
	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW
Australia	0.6%	-0.6%	-0.5%	1.9%	-0.0%	0.1%	-39.6%	-1.2%	-0.4%
EU	-22.2%	-0.3%	-0.0%	-21.2%	-0.0%	0.2%	-38.5%	-0.4%	1.7%
Brazil	-1.5%	-0.1%	0.3%	-1.0%	0.0%	0.3%	-39.4%	0.3%	2.6%
Canada	8.3%	-1.6%	-1.5%	3.5%	-0.1%	0.0%	-42.6%	-1.2%	-0.6%
China	-9.7%	-0.1%	0.1%	-8.3%	0.0%	0.1%	-39.0%	-1.7%	-0.6%
Indonesia	1.7%	-0.2%	-0.1%	2.4%	-0.0%	0.1%	-42.9%	-3.1%	-2.7%
India	-6.7%	-0.4%	0.5%	-5.3%	0.0%	0.7%	-44.0%	4.5%	10.8%
Japan	-2.2%	-0.3%	-0.1%	-0.6%	0.0%	0.1%	-39.1%	-1.5%	-0.5%
Korea	0.4%	0.3%	0.6%	0.9%	0.0%	0.2%	-39.9%	1.6%	3.2%
Mexico	4.6%	-1.3%	-1.2%	3.0%	-0.0%	0.0%	-41.5%	-1.3%	-1.1%
Russia	7.3%	-1.3%	-1.3%	3.5%	-0.2%	-0.2%	-43.8%	-0.0%	0.1%
Saudi Arabia	12.2%	-3.9%	-3.9%	4.8%	-0.6%	-0.6%	-45.8%	-0.6%	-0.5%
Turkey	-6.0%	-0.6%	0.2%	-0.0%	0.1%	0.8%	-39.1%	1.9%	7.6%
USA	-3.8%	-0.3%	-0.3%	-1.9%	0.0%	0.0%	-43.0%	-1.7%	-1.3%
Africa	-14.1%	-1.2%	0.1%	-10.2%	-0.1%	1.1%	-41.7%	8.4%	20.6%
RO Americas	-6.2%	-0.7%	-0.2%	-3.4%	-0.0%	0.4%	-41.5%	2.0%	5.6%
RO Asia	-5.8%	-1.0%	-0.9%	-0.9%	0.0%	0.2%	-40.6%	-0.9%	0.4%
RO Eurasia	2.5%	-1.1%	-1.1%	3.6%	-0.1%	-0.1%	-44.2%	-2.2%	-2.1%
RO Middle East	5.9%	-2.5%	-2.5%	3.9%	-0.3%	-0.3%	-43.3%	0.0%	0.2%
Global	-6.5%	-0.5%	-0.2%	-5.4%	-0.0%	0.2%	-41.0%	-0.6%	1.1%

Note: This table shows for every country the change to CO_2 emission, real consumption, and welfare from the baseline to noncooperative and cooperative equilibrium. In the baseline, each country's tariffs and carbon taxes are set at their applied rates in 2014 and export subsidies are zero. Optimal policy formulas for the noncooperative and cooperative outcomes are detailed in Sections 3.1and 3.4 and our quantitative implementation is described in Sections 4.1 and 4.2.

sions share. Notably, the industries that have a trade-to-GDP ratio below 0.15 are responsible for over 80% of global CO_2 emissions from production.

Third, border taxes cause leakage via general equilibrium price adjustments. In particular, they reduce global demand for energy, leading to lower pre-tax energy prices worldwide. This in turn causes a drop in the after-tax price of energy in major energy exporting countries like Russia and Saudi Arabia, which have lesser care for climate change. As a result, their CO_2 emissions rise with carbon border taxes, dampening the overall reduction in global emissions.[39]

The modest CO_2 reduction achieved with non-cooperative border taxes is offset by sizable consumption losses in certain countries. On aggregate, the global real consumption decreases

[39] Our carbon border tax specification exhibits similarities and differences with the carbon border adjustment mechanism (CBAM) in the European Green Deal. Both unilaterally levy duties on the carbon content of imports. However, the CBAM aims to target firm-level emissions when possible, exempting exporters who demonstrate abatement through monitoring. Thus, while the CBAM faces the second and third limitations highlighted above, the extent to which the first limitation applies is unclear. Additionally, the CBAM allows deduction of carbon taxes already paid in the origin country. This bears similarity to the globally-optimal carbon border taxes analyzed in Appendix C.1.

by 0.5% under these taxes, with only a negligible benefit from reductions in CO_2 emissions. By comparison, globally optimal carbon taxes deliver a 41.0% reduction in global CO_2 emissions, paired with mere 0.6% loss to real consumption, which translates to a 1.1% increase in *climate-adjusted* welfare.

5.2 Proposal 2: Climate Club with Contingent Trade Penalties

Under Nordhaus's (2015) climate club proposal, border taxes are used as a *contingent* penalty device to deter free-riding. We begin by specifying the climate club as a sequential game. A group of "core" countries move first and all countries simultaneously play afterwards. The game is characterized by a given set of core countries, denoted by $\mathbb{C}^{(\text{core})}$, and a "carbon tax target," denoted by $\tau^{(\text{target})}$. Given $\left(\mathbb{C}^{(\text{core})}, \tau^{(\text{target})}\right)$, governments play according to the following rules:

Members. A member country must raise its domestic carbon tax to $\tau^{(\text{target})}$, set zero border taxes against other members, and impose unilaterally optimal trade taxes, as penalty, against non-members. By design, core countries adhere to these rules, while others conform only if they opt to join the club.

Non-members. Non-member nations retaliate by imposing their unilaterally optimal trade taxes against member countries. Other than this, non-member countries retain their status quo tax policies—preserving existing tariffs against other non-member countries and maintaining their zero or near-zero carbon taxes.

For a game $\left(\mathbb{C}^{(\text{core})}, \tau^{(\text{target})}\right)$, a partitioning of countries into non-core club members $\mathbb{C}^{(\text{member})}$, and non-members $\mathbb{C}^{(\text{non-member})}$ constitutes a Nash equilibrium if *(i)* No non-core country has an incentive to deviate from the partition to which it belongs. *(ii)* Each core country's welfare improves (compared to the baseline) under this partition.

Quantitative Challenges.— Analyzing the climate club game in-depth poses significant challenges for two main reasons. First, iteratively determining optimal trade penalties for various countries across all conceivable partitions is practically infeasible with brute-force numerical optimization techniques. Our formulas for optimal border taxes, however, offer an analytical representation of these penalties, effectively circumventing this issue. Second, identifying all possible equilibria of the climate club game is complicated by the curse of dimensionality. Without a technique to shrink the outcome space, our analysis would involve examining $2^{N-N^{(\text{core})}}$ combinations of national strategies.[40] We address this challenge by noting that the severity of climate club penalties increases with the club's size. Consequently, the pay-off from joining the club rises with size, al-

[40] With nineteen countries in our sample and supposing one core member, we would be required to solve for 4.7 million general equilibrium outcomes. Each partitioning of non-core countries, $\left(\mathbb{C}^{(\text{member})}, \mathbb{C}^{(\text{non-member})}\right)$, maps to a different general equilibrium outcome, amounting to 2^{18} cases. Additionally, for a given partitioning, checking whether any of the eighteen non-core countries has an incentive to unilaterally deviate corresponds to a new general equilibrium outcome. Therefore, in total, there are $18 \times 2^{18} \approx 4.7$ million general equilibrium outcomes to check.

lowing us to shrink the outcome space via iterative elimination of dominated strategies.[41]

Carbon Tax Target.— The selection of the carbon tax target $\tau^{(target)}$ is based on two key considerations. First, there is an inverted U-shaped relationship between the club's emission reduction and the carbon tax target, akin to the *Laffer curve*. When weighing club membership, non-core countries compare the costs of raising their carbon tax against trade penalties from club members. While a higher carbon tax target prompts more emission reduction per member, it also deters participation due to higher costs. This creates a trade-off: an excessively high tax target reduces membership limiting global emission cuts, while an overly low target delivers a limited reduction in global emissions despite maximal participation. Second, the climate club's aim is to cut emissions without triggering decoupling between club members and the rest of the world. That is, trade penalties are meant to deter free-riding without being exercised in equilibrium. To achieve this, $\tau^{(\text{target})}$ must be ideally set to the maximal carbon tax target that supports an inclusive club of all nations as the Nash equilibrium.[42]

Solution Algorithm.— We employ the following two-tier procedure to identify the maximal carbon tax target. In the inner tier, we use the following iterative procedure for a given carbon tax target: In the first iteration, climate club penalties are applied only by core members. We identify non-core countries that would benefit from unilaterally joining the club, adding them to the club in the subsequent rounds. In the second iteration, climate club penalties are applied by core members plus those added from the previous round. We re-evaluate the gains from unilateral club membership under the new penalties and update the club accordingly. We repeat this procedure until we achieve convergence. The resulting outcome is an equilibrium club of all nations if: *(i)* the converged set corresponds to the set of all non-core countries who have no incentive to unilaterally withdraw, *(ii)* the welfare of core countries has increased relative to the status quo.[43] In the outer tier, we incrementally increase the carbon tax target from a small initial value until we identify the maximum target at which the club of all nations is formed.

While we specify the climate club as a static one-shot game, our procedure offers a glimpse into the club's potential growth trajectory. For example, consider a club with the EU and US

[41] This procedure requires that the benefits of membership increase as the climate club grows larger. This typically holds since a bigger club can impose harsher trade penalties on non-members. However, the relationship may not hold universally due to a caveat: As the climate club expands, global energy demand decreases, lowering the pre-tax price of energy worldwide. These general equilibrium price effects can diminish the desirability of club membership by raising the opportunity cost of carbon pricing. We cannot theoretically preclude scenarios where these general equilibrium forces undermine the link between club size and membership benefits. Instead, in the spirit of irreversible actions in theories of gradual coalition formation (Seidmann and Winter, 1998), we assume that exiting the climate club damages reputation and carries a non-pecuniary cost that intensifies with the club's size. Therefore, even if escalating trade sanctions prove insufficient, the non-pecuniary cost of existing ensures that the benefits of maintaining membership rise as the club grows.

[42] In our quantitative exercises, this maximal target typically aligns with the peak of the emission reduction along the Laffer curve. The maximum emission reduction can be attained only when large developing countries such as India or Indonesia are in the club. At the same time, these countries are nearly marginal in joining the club or staying out. As a result, although that is not theoretically the case, in practice aiming for an inclusive club of all nations typically aligns with achieving the maximum emission reduction.

[43] The second stage among non-core countries constitutes a *coalition-proof* equilibrium under the assumption referenced

Table 5: Climate Club Game with the EU & US as Core and Carbon Tax Target of 53 ($/tCO2)

Round 1	Brazil, Canada, Korea, Turkey, RO Eurasia
Round 2	Russia, RO Americas
Round 3	Africa
Round 4	Mexico
Round 5	Saudi Arabia
Round 6	Japan
Round 7	China, Indonesia, RO Asia, RO Middle East
Round 8	Australia, India

Note: This table shows the convergence of our solution method via successive rounds to a club of all nations, for the case in which the EU and US are core members and the carbon tax target is at its maximal value of 53 \$/tCO_2. A country unilaterally evaluates to join or leave at each round given the club configuration at its previous round.

as core members and a carbon tax target of 53 ($/tCO2), as detailed in Table 5. In Round 1, five countries with stronger trade ties to the EU and US find it beneficial to join. Given this outcome, two additional countries opt to join in Round 2 to evade penalties by the EU, US, and the five other members that joined in Round 1. Following this iterative process, the club eventually includes all non-core countries after eight rounds. At this point, we assess the benefits for the first movers—the EU and US—and find that their core membership has improved their national welfare compared to the status quo. It is worth noting this example uses the maximal carbon tax target of 53 ($/tCO2), as a higher target fails to deliver global participation.

The progression of country memberships in the above example reflect the *gravity* structure of trade relations. Nations like Turkey and Canada join early given their robust trade ties with the EU and US. As the club expands, it attracts more distant countries with strong trade connections with the evolving club's collective body. Accordingly, the club's expansion occurs by the membership from the West toward the East.

Outcomes Under Various Makeup of Core Members.— We analyze three climate club scenarios, each with a distinct composition of core countries. Initially, we consider the European Union (EU) as the sole core member, recognizing its status as a leader in environmental commitment. Subsequently, we explore a scenario where the United States joins the EU, forming a larger core. Our final scenario includes the EU, US, and China as the core members of the club.

For each scenario, Table 6 reports the maximal carbon tax target and the resulting global CO2 reduction. With the EU as the only core member, the maximal carbon price target is 37 ($/tCO2), leading to a 13.7% decrease in global emissions. When the EU and US unite as core members, the maximal target rises to 53 ($/tCO2), delivering a 18.3% reduction in global emissions. The addition of China as a core member further amplifies the club's impact: it allows for a maximal

in Footnote 41. Specifically, suppose that once a country joins the club, the increasing costs of exiting prevent it from leaving in subsequent rounds. Under this assumption as a universal feature, our procedure coincides with the *iterative elimination of dominated strategies*, allowing us to narrow the set of potential outcomes. Moreover, the resulting outcome is a coalition-proof equilibrium provided that the iterative elimination of dominated strategies converges to a unique outcome, which is the case in our analysis (Moreno and Wooders, 1996). Lastly, we highlight that, for completeness, we always verify that the resulting outcome constitutes a Nash equilibrium even without the assumption in Footnote 41.

carbon price target of 90 ($/tCO2), culminating in a 28.2% reduction in global emissions. This is substantial when contrasted with the 41.0%, obtainable under first-best carbon pricing.[44]

Table 6: Climate Club Outcomes

Core	Max Carbon Price Target ($/tCO2)	Reduction in Global CO_2
EU	37	13.7%
EU+USA	53	18.3%
EU+USA+CHN	90	28.2%

Note: This table shows the climate club outcomes of the maximal carbon price target and the corresponding reduction in global CO_2 emissions for each scenario of the core countries

These findings suggest that a well-structured climate club could achieve more than two-thirds of the first-best reduction in global carbon emissions (28.2/41.0≈0.69). The extent of this success, however, hinges critically on the initial composition of core members and the appropriate selection of the carbon tax target.

6 Discussions

In this section, we examine the robustness of our results to alternative parameterizations, characterize alternative policy designs, and discuss extensions to our framework.

6.1 Sensitivity Analysis and External Validity

We redo our analysis under five alternative specifications, with results reported in Tables A.6 and A.7 . First, we set the social cost of carbon at 92 ($/tCO2) compared to 156 ($/tCO2) in our main analysis. This choice is consistent with the EPA's estimate under a 2.5% (instead of 2%) annual discount rate. Second, we leverage *country-level* estimates for the social cost of carbon from Ricke et al. (2018) to re-calibrate the carbon disutility parameters, $\tilde{\delta}_i$.[45] Third, we assign a uniform trade elasticity, $\sigma_k \equiv \sigma = 6.7$, to all industries, by estimating Equation (30) on a pooled sample. With a uniform σ_k, export market power is determined solely by size rather than comparative advantage across industries. Fourth, we consider a Cobb-Douglas production function for final goods, corresponding to a unitary substitution elasticity between energy and labor inputs (i.e., $\varsigma \to 1$), compared to $\varsigma = 0.65$ in our main specification. Lastly, we consider an alternative (inverse) energy supply elasticity by setting $\phi_i/(1 - \phi_i)$ to 2.0 for all countries, compared to an average of 0.6 in our main specification. Following Kortum and Weisbach (2020), this choice aligns with data

[44] Tables A.4 and A.5 in the appendix show the rounds of succession when the core consists of the EU or EU+US+China. In addition, Figure A.3 in the appendix shows, for each of the three scenarios of core countries, the welfare gains of staying *vs.* withdrawing for each of non-core countries, and it reports the welfare improvement for each core country *vs.* the status quo.

[45] Specifically, we calibrate the disutility parameters by assuming that the relative disutility is proportional the country-level cost of carbon and that the disutility parameters add up to SC-CO_2 = 156.

on the distribution of extraction costs among oil fields.[46] For each specification, Table A.6 reports the effects of non-cooperative border taxes, while Table A.7 reports outcomes associated with the climate club. Across all scenarios tested, the qualitative results remain identical and quantitative results are similar to our main specification.

We additionally conduct two external validation checks on our model. First, we conduct an IV-based test in the spirit of Adao et al. (2023). To this end, we use our model to simulate countries' emission response to *observed* changes in carbon taxes from 2014 to 2022. We then check whether the difference between the vector of model-implied and observed emission changes is uncorrelated with the vector of carbon tax changes. Following Adao et al. (2023), a significant non-zero correlation would suggest that our model is misspecified. Encouragingly, as Figure A.7 shows, the noted correlation is statistically indistinguishable from zero in our case.[47] Second, we compare our model's predicted emissions reductions from globally-applied carbon taxes to estimates from other modeling approaches, including integrated assessment models, computable general equilibrium models, and ex-post empirical studies. Figure A.8 plots our model's projected global emission reductions against the global carbon tax rate, benchmarked against projections from leading studies in the literature. Despite differences in underlying assumptions, our results fall within the range reported across these previous analyses, providing additional support for the credibility of our model.

6.2 Alternative Policy Designs

Our main analysis focused on policy proposals that can address the *free-riding* problem. Our analysis of Proposal 1 considered the most effective carbon border tax design that is resilient to free-riding, as characterized by Proposition 2. For Proposal 2, we focused on optimal penalties that maximize welfare transfers from non-members to members of the climate club. But it is important to note that border taxes can deliver even greater emission reductions when free-riding is *not* the central concern. These taxes can also be more punitive than the unilaterally-optimal taxes used in our climate club analysis. Below, we explore alternative border tax designs, which are relevant when free-riding is less of a concern or when countries are willing to exert harsher sanctions on free-riders.

First, consider a global economy where governments are willing to cooperate on climate is-

[46] Our specification of energy production, Equation (7), is isomorphic to the one in Kortum and Weisbach (2020). The latter assumes a continuum of fields that are heterogenous in their extraction costs, as captured by the unit labor requirement, a. Let $Q_0 = E(\bar{a}) = \text{constant} \times \bar{a}^{\epsilon}$ represent the amount of energy that can be extracted with a unit labor requirement $a < \bar{a}$. This formulation is equivalent to the production function, $Q_0 = \text{constant} \times L_0^{1-\phi}$, assumed in this paper by setting $(1-\phi) = \epsilon/(\epsilon+1)$. The choice of $\epsilon = 0.5$ implied by the empirical distribution of extraction costs, yields $\phi = 2/3$, which corresponds to an inverse energy supply elasticity of $\phi/(1-\phi) = 2$.

[47] We view the test outcome as merely *suggestive*, since data limitations prevent us from running a more formal version of Adao et al.'s (2023) test. For that, we need data on CO2 emissions and carbon taxes at the level of industries (and at the household level). Data on CO2 emissions at the level of individual industries for recent years are currently unavailable. In addition, the task of collapsing all climate policy measures of 2022 into a carbon-tax-equivalent at the level of individual industries (and as faced by households) is beyond the scope of this paper.

sues but face political pressures that prevent them from implementing *first-best* carbon taxes. In this scenario, border taxes could serve as a *second-best* cooperative solution. We characterize the globally optimal border taxes under this scenario in Appendix B. Quantitatively, we find that this policy reduces global carbon emissions by only 0.9%, which is comparable to the non-cooperative border taxes examined earlier. The main takeaway here is that carbon border taxes have limited efficacy in reducing global emissions regardless of whether they address international free-riding or domestic political constraints. Instead, their ineffectiveness stems from the three structural limitations discussed in Section 5.1.

Second, envision a scenario where the home country's government assigns a non-zero weight to foreign welfare when designing its policy. The resulting optimal policy choices in that case trace out the home country's *unilateral policy frontier*. Each point on this frontier corresponds to a specific set of weights assigned to foreign countries' welfare. As detailed in Appendix B, placing more importance on foreign welfare dilutes the terms-of-trade component of the optimal border taxes. And when the weights on foreign welfare are sufficiently large, the home country's optimal policy has no negative externality on other countries—aligning with the optimal policy framework in Kortum and Weisbach (2020).

Figure A.5 in the appendix illustrates the EU's unilateral policy frontier. As the EU assigns a greater weight to non-EU countries, its optimal policy moves along the frontier to a point where it preserves non-EU's welfare. This policy, labeled "Externality-Free", elevates the EU's welfare by 0.19% compared to 0.32% under our baseline Unilaterally-Optimal policy. Moreover, global emissions drop by 3.4% under the Externality-Free policy compared to 1.9% under the Unilaterally-Optimal policy. The Externality-Free policy, however, is difficult to implement due to free-riding incentives. It effectively raises non-EU welfare to the detriment of EU countries compared to other policies on the frontier (top panel of Figure A.6). Additionally, policies that assign a greater weight to non-EU welfare trigger more carbon leakage, as displayed in the bottom panel of Figure A.6. The reason is that a higher non-EU weight prompts the EU to raise its domestic carbon tax, bringing it closer to the social cost of carbon. This increase in tax reduces the EU's energy demand and consequently lowers global pre-tax energy prices, prompting higher energy use and carbon emissions in non-EU regions.

The unilateral policy frontier, moreover, identifies the range of penalties that a country could impose on its trade partners. Most notably, it covers cases where the home country assigns a negative weight to foreign welfare, as studied in Sturm Becko (Forthcoming). Under one such weighting scheme, a country could achieve the maximal reduction in foreign welfare without reducing its own welfare. The noted policy lies on the westernmost point of the frontier, labeled as "Maximal Sanction" in Figure A.5. In the context of a climate club, applying these extreme trade sanctions would have ambiguous effects on the club's efficacy. On one hand, the sanctions would make non-membership more costly. On the other hand, they would dilute the benefits of membership by prioritizing harm to foreign countries over domestic welfare. Nevertheless, incorporating sanctions (implied by negative weights on foreign welfare) into the climate club analysis and identifying optimal weights presents a worthwhile avenue for future research.

Lastly, the unilateral policy frontier shows the limitations of unilateral policy, regardless of whether governments implement optimal policies or not. The frontier shown in Figure A.5 determines the range of potential welfare outcomes possible under unilateral policy. Suboptimal policy decisions would result in outcomes inside the frontier. The maximum welfare increase realizable for the EU under unilateral policy is less than 0.4%, contrasting sharply with the 1.1% increase feasible under the climate club. Similarly, emissions reductions are capped at around 5% under the EU's unilateral policy, compared to around 14% with the climate club initiated by the EU (Figure A.6 and Table 6). Essentially, even if governments do not optimize policies, the climate club's frontier remains far more promising.

6.3 Extensions to Richer Settings

(a) **Increasing-returns to scale.** We introduce increasing-returns to scale in final-good industries as in Krugman (1980), with details provided in Appendix E.1. In this setting, scale economies arise from love-for-variety, governed by the elasticity of substitution, γ_k, between firm varieties.[48] Firms' entry decisions do not internalize the full benefits of introducing new varieties, leading to inefficient entry and output across industries. Consequently, optimal policy aims to address inter-industry scale distortions while also managing the terms of trade and reducing emissions. For a small open economy under Cobb-Douglas-CES preferences, the unilaterally optimal policy formulas become:

$$\begin{cases} \tau_i^* = \tilde{\delta}_i \sim \delta_i \tilde{P}_i, \qquad s_{i,k}^* = \frac{1}{\gamma_k - 1} & \text{[carbon tax \& domestic subsidy]} \\ t_{ni,k}^* = \bar{t}_i + \frac{\gamma_k - 1}{\gamma_k} \tau_i^* v_{n,k} \qquad t_{ni,0}^* = \bar{t}_i & \text{[import tax (energy and non-energy)]} \\ 1 + x_{in,k}^* = (1 + \bar{t}_i) \frac{\sigma_k - 1}{\sigma_k} + \frac{\gamma_k - 1}{\gamma_k} \tau_i^* \sum_{j \neq i} \left[\lambda_{jn,k} v_{j,k} \right] \frac{\sigma_k - 1}{\sigma_k} & \text{[export subsidy (non-energy)]} \\ 1 + x_{in,0}^* = (1 + \bar{t}_i) \frac{\sigma_0 - 1}{\sigma_0} + \tau_i^* \frac{1}{\sigma_0} \frac{\zeta_n}{\tilde{P}_{n,0}} & \text{[export subsidy (energy)]} \end{cases} \tag{32}$$

The above policy differs from the constant-returns to scale variant in two ways. First, it includes production subsidies, denoted by $s_{i,k}$. The optimal production subsidy is carbon-blind and corrects scale distortions by favoring high-returns-to-scale (low-γ) industries. Second, carbon border taxes are adjusted to account for scale economies, as they exert two countervailing effects on foreign emissions: they lower emissions by reducing output (Q), but concurrently, raise the per-unit emissions (Z/Q). The latter effect occurs because per unit emissions decline with output scale at an elasticity, $(\gamma_k - 1)^{-1}$. To balance this trade-off, the optimal carbon border tax is revised downwards by a factor of $\frac{\gamma_k - 1}{\gamma_k}$. In the limit where $\gamma_k \to \infty$, industry k operates under constant-returns to scale and the above formulas reduce to the baseline formulas presented earlier.

Tables A.8 and A.9 in the appendix show the impacts of Proposals 1 (non-cooperative carbon border taxes) and 2 (climate club) under increasing-returns to scale. The analysis uses scale

[48] As shown in Appendix E.1, this extended model is isomorphic to a setting with *external* economies of scale.

elasticities derived from the estimates in Lashkaripour and Lugovskyy (2023).[49] The results indicate that carbon pricing policies deliver smaller reductions in *global* emissions due to the same trade-off highlighted earlier. Specifically, under increasing-returns to scale (IRS), contractions in output, Q, coincide with an increase in per-unit emissions (Z/Q). While this trade-off moderates the overall impact of policy on emissions, the relative efficacy of Proposals 1 and 2 is virtually unchanged compared to the baseline constant-returns-to-scale (CRS) scenario.[50]

(b) **Firm heterogeneity.** We consider two sources of firm heterogeneity: differences in (1) total factor productivity and (2) carbon intensity across firms. The Krugman extension of our framework can readily handle the former, but modeling heterogeneity in carbon intensity is more challenging due to data limitations. Specifically, the formulas described in Equation 32 remain valid if there is firm heterogeneity only in total factor productivity. The formulas apply without qualification if serving new markets does not require paying a fixed overhead cost. In the presence of fixed costs, however, the optimal policy must account for self-selection of the most productive firms into export markets, à la Melitz (2003). As shown in Appendix E.2, the Krugman extension of our model is isomorphic to the Melitz model if firm-level productivity follows a Pareto distribution. Therefore, Equation 32 describes optimal policy in the Melitz-Pareto case, albeit with a reinterpretation of parameters—indicating our quantitative results would be unchanged.

A richer extension could incorporate firm heterogeneity in both productivity and carbon intensity (see Cherniwchan et al. (2017)). Here, border taxes could alter the average carbon intensity of exporting firms. This consideration lends itself to policy designs that target firm-level abatement, such as the Carbon Border Adjustment Mechanism (CBAM) referenced in footnote 39. But how this consideration affects optimal policy design depends on information asymmetry between governments and foreign firms. For instance, if governments know that more productive firms tend to be less carbon intensive, they could set a higher carbon border tax to deter entry by small, carbon-intensive firms. Yet with only industry-level data on carbon intensities, governments may implement voluntary certification schemes that incentivize low-emissions firms to disclose their output and emission levels (Cicala et al., 2022). Quantifying the global impacts of border taxes in either case requires international firm-level emissions data, which is presently unavailable.[51]

[49] In this extension of our model, a necessary condition for uniqueness is $\mu_k \equiv (\sigma_k - 1) / (\gamma_k - 1) \leq 1$. Therefore, we use the estimates of μ_k from Lashkaripour and Lugovskyy (2023), which guarantee $\mu_k \leq 1$, together with our estimates of trade elasticity σ_k to recover the love-of-variety parameters, γ_k.

[50] Despite similar aggregate results, some differences are noticeable at the level of individual countries. For example, Figure A.9 compares the change in CO_2 emissions under Proposal 1 between our main model (CRS) and extended model (IRS). Under the IRS model, when firms are subjected to border tax hikes, they tend to relocate to larger markets to evade such taxes. These delocation effects can raise the scale of production and CO_2 emissions even in climate-conscious regions like the EU that charge a relatively high carbon tax.

[51] Specifically, one requires firm-level data, with information on energy use, sales and exports, from different sectors and in multiple countries. For instance, Shapiro and Walker (2018) document that in the US manufacturing, more productive firms are cleaner in terms of non-CO_2 local air pollution. However, little is known about the extent to which this relationship holds for CO_2 emissions particularly in large carbon-emitting developing countries, and in non-manufacturing particularly in agriculture, energy, and transportation sectors.

7 Conclusion

We examined two prominent climate policy proposals that leverage trade policy to address the free-riding problem in climate action. One proposal calls for carbon border taxes as a second-best device to curb transboundary emissions, while the other, the climate club, advocates for border taxes as a penalty device to encourage cooperation by reluctant governments. We characterized optimal policies to evaluate these proposals in a general equilibrium trade model with global carbon supply chains and climate externalities. Our findings indicated that carbon border taxes can achieve only a modest reduction in global emissions even when designed optimally, whereas the climate club can be remarkably successful. This success, however, hinges on the makeup of the club's core members and its carbon tax target.

Our analysis puts forth methods that have implications beyond the scope of this paper. First, carbon border taxes could be targeted towards individual firms given appropriate monitoring regimes. Collecting firm-level emissions data to quantitatively assess such targeted policies represents a promising direction for future research. Incorporating distributional considerations into the optimal climate policy calculus is another potential avenue – for instance, through the inclusion of an international climate fund, technology transfers to developing nations, or supply-side carbon policies. Furthermore, our analysis excludes dynamic considerations such as potential climate tipping points or technological innovation. These factors provide justifications for dynamic policies or industrial policies that subsidize green technologies. They also rationalize policy packages that concurrently promote both climate change mitigation and adaptation.

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Appendices for “Can Trade Policy Mitigate Climate Change?” (for Online Publication)

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A Unilaterally Optimal Policy

A.1 Expansive Statement of the Unilaterally Optimal Policy Problem

Theoretical Setting. This section outlines the unilaterally optimal policy problem for the government of country i as the tax-imposing country which we also refer to as *home*. The home's government has access to a full set of policy choices in the reformulated problem, namely the consumer prices of all goods that country i produces, the consumer prices of all goods that country i consumes, and energy cost shares in country i , $\mathbb{P}_i \equiv \left\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \alpha_{i,k}\right\}_{j,k}$.[52] For a clearer exposition, we focus on the case in which tax rates in foreign countries are set to zero.

To clarify the notation which we adopt below, $\tilde{\mathbf{P}}_i \equiv \left\{\tilde{P}_{ji,k}\right\}_{j,k\neq 0} \in \mathbb{P}_i$ denotes the vector of final-good consumer prices in country i, $\tilde{\mathbf{P}}_{i,0} \equiv \left\{\tilde{P}_{ji,0}\right\}_j \in \mathbb{P}_i$ contains the (pre-carbon-tax) consumer prices of energy varieties in country i, and $\mathbf{P}_{-in} \equiv \left\{P_{jn,k}\right\}_{j\neq i,k}$ contains the producer price of all goods sold to country n excluding those sourced from origin i. Furthermore, we make use of a notation in which all the general equilibrium relationships, with the exception of the labor market cleaning condition, are specified as an explicit function of policy instruments $\mathbb{P}_i$ and wage rates $\mathbf{w} \equiv [w_i]_i$, $\mathbb{X} = (\mathbb{P}_i; \mathbf{w})$, as detailed in *Panel I* of Table A.1. We call a policy-wage combination *feasible*, if the labor market clearing conditions, as specified in *Panel II* of Table A.1, hold. The *general equilibrium* is a feasible policy-wage combination that satisfies the equations in *Panel I*.

Formal Statement of the Unilaterally Optimal Policy Problem. The government of country i chooses $\mathbb{P}_i$ to maximize its welfare:

$$W_i\left(\mathbb{X}\right) = V_i\left(E_i\left(\mathbb{X}\right), \tilde{\mathbf{P}}_i\right) - \delta_i \sum_n \sum_k Z_{n,k}\left(\mathbb{X}\right)$$

subject to the system of general equilibrium as detailed in Table A.1.

[52] Given that $\alpha_{i,k} = \tilde{P}_{i,k0} Z_{i,k} / (P_{ii,k} Q_{i,k})$, we could equivalently make the after-carbon-tax price of energy ($\tilde{P}_{i,k0} \equiv \tilde{P}_{i,0} + \tau_{i,k}$) a policy choice. However, our derivations are more straightforward when $\alpha_{i,k}$ is a policy choice.

Table A.1: System of General Equilibrium

Panel I. Equilibrium Constraints as an Explicit Function of Policy and Wages, $\mathbb{X} \equiv (\mathbb{P}_i; w)$

Demand Quantities

(D.1) Final Goods $$C_{n\ell,k}(\mathbb{X}) = \begin{cases} \mathcal{D}_{ni,k}\left(E_i(\mathbb{X}), \tilde{\mathbf{P}}_i\right) & \text{if } \ell = i, \quad k = 1,..,K \\ \mathcal{D}_{n\ell,k}\left(E_\ell(\mathbb{X}), \tilde{\mathbf{P}}_{i\ell}, \mathbf{P}_{-i\ell}(\mathbb{X})\right) & \text{if } \ell \neq i, \quad k = 1,..,K \end{cases}$$

(D.2) Energy $$C_{n\ell,0}(\mathbb{X}) = \begin{cases} \widetilde{\mathcal{D}}_{ni,0}\left(E_{i,0}(\mathbb{X}), \tilde{\mathbf{P}}_{i,0}\right) & \text{if } \ell = i, \quad k = 0 \\ \widetilde{\mathcal{D}}_{n\ell,0}\left(E_{\ell,0}(\mathbb{X}), \tilde{P}_{i\ell,0}, \mathbf{P}_{-i\ell,0}(\mathbb{X})\right) & \text{if } \ell \neq i, \quad k = 0 \end{cases}$$

Expenditure/Income

(E.1) Aggregate $$E_n(\mathbb{X}) = Y_n(\mathbb{X}) \equiv \begin{cases} w_i L_i + \Pi_i(\mathbb{X}) + T_i(\mathbb{X}) & \text{if } n = i \\ w_n L_n + \Pi_n(\mathbb{X}) & \text{if } n \neq i \end{cases}$$

(E.2) Profits $$\Pi_n = \phi_n P_{nn,0}(\mathbb{X})\, Q_{n,0}(\mathbb{X}) \qquad \text{for all } n$$

(E.3) Tax Revenue $$T_i(\mathbb{X}) = \textstyle\sum_{n\neq i}\sum_k \left[\left(\tilde{P}_{ni,k} - P_{ni,k}(\mathbb{X})\right) C_{ni,k}(\mathbb{X})\right] \\ \textstyle\sum_n \sum_k \left[\left(\tilde{P}_{in,k} - \left[1-\alpha_{i,k}\right] P_{in,k}(\mathbb{X})\right) C_{in,k}(\mathbb{X})\right] - \tilde{P}_{i,0}(\mathbb{X})\, Z_i(\mathbb{X})$$

(E.4) Energy Exp. $$E_{n,0}(\mathbb{X}) = \begin{cases} \sum_k \alpha_{i,k} P_{ii,k}(\mathbb{X})\, Q_{i,k}(\mathbb{X}) & \text{if } n = i \\ \sum_k \alpha_{n,k}\left(w_n, \tilde{P}_{n,0}(\mathbb{X})\right) P_{nn,k}(\mathbb{X})\, Q_{n,k}(\mathbb{X}) & \text{if } n \neq i \end{cases}$$

Producer Prices

(P.1) Final goods $$P_{\ell n,k}(\mathbb{X}) = \begin{cases} \bar{d}_{in,k}\bar{p}_{i,k} C_{i,k}\left(w_i, \alpha_{i,k}\right) & \text{if } \ell = i,\ k = 1,...,K \\ \bar{d}_{\ell n,k}\bar{p}_{\ell,k}\mathscr{C}_{\ell,k}\left(w_\ell, \tilde{P}_{\ell,0}(\mathbb{X})\right) & \text{if } \ell \neq i,\ k = 1,...,K \end{cases}$$

(P.2) Energy $$P_{\ell n,0} = \bar{d}_{\ell n,0}\bar{p}_{\ell,0} Q_{\ell,0}^{\frac{\phi_\ell}{1-\phi_\ell}} \bar{w}_\ell \qquad \text{for all } \ell, \text{if } k = 0$$

Distribution-level Energy Price

(P.3) distr. energy $$\tilde{P}_{n,0}(\mathbb{X}) = \begin{cases} \tilde{\mathcal{P}}_{i,0}\left(\tilde{\mathbf{P}}_{i,0}\right) & \text{if } n = i \\ \tilde{\mathcal{P}}_{n,0}\left(\tilde{P}_{in,0}, \mathbf{P}_{-in,0}(\mathbb{X})\right) & \text{if } n \neq i \end{cases}$$

Output & Emission

(Q) Output $$Q_{n,k}(\mathbb{X}) = \textstyle\sum_\ell \bar{d}_{n\ell,k} C_{n\ell,k}(\mathbb{X}) \qquad \text{for all n, for } k = 0, 1, ..., K$$

(Z) Emission $$Z_n(\mathbb{X}) = \textstyle\sum_k \mathcal{Z}_{n,k}(\mathbb{X}), \quad \mathcal{Z}_{n,k}(\mathbb{X}) = \begin{cases} \mathrm{z}_{i,k}\left(\alpha_{i,k}\right) Q_{i,k}(\mathbb{X}) & \text{if } n = i \\ \mathrm{z}_{n,k}\left(\alpha_{n,k}\left(w_n, \tilde{P}_{n,0}(\mathbb{X})\right)\right) Q_{n,k}(\mathbb{X}) & \text{if } n \neq i \end{cases}$$

Panel II. Feasibility of Policy-Wages

$$(\mathbb{P}_i, \mathbf{w}) \in \mathcal{E} \quad : \quad w_\ell \bar{L}_\ell = \begin{cases} \sum_{k\neq 0}\left[\left(1-\alpha_{i,k}\right) P_{ii,k}(\mathbb{X})\, Q_{i,k}(\mathbb{X})\right] \\ \qquad + \left(1-\phi_i\right) P_{ii,0}(\mathbb{X})\, Q_{i,0}(\mathbb{X}) & \text{if } \ell = i \\ \sum_{k\neq 0}\left[\left(1-\alpha_{\ell,k}\left(w_\ell, \tilde{P}_{\ell,0}(\mathbb{X})\right)\right) P_{\ell\ell,k}(\mathbb{X})\, Q_{\ell,k}(\mathbb{X})\right] \\ \qquad + \left(1-\phi_\ell\right) P_{\ell\ell,0}(\mathbb{X})\, Q_{\ell,0}(\mathbb{X}) & \text{if } \ell \neq i \end{cases}$$

Notes: This table shows the system of general equilibrium relationships. Panel I presents the equations that describe demand, expenditures and the balance of budget, producer prices, energy price at the level of distribution (before application of carbon taxes), output quantities and emission levels. Panel II describes the feasible policy-wage outcome as the one that satisfies labor market conditions.

Non-parametric Functions. Our dual approach in deriving the optimal policy formulas does not impose any functional form on demand or production functions of final goods. Specifically, we can accommodate the following functions non-parametrically:

1. Indirect utility, $V_i\left(E_i, \tilde{\mathbf{P}}_i\right)$, and its corresponding Marshallian demand, $\mathcal{D}_{ni,k}\left(E_i, \tilde{\mathbf{P}}_i\right)$, are characterized by elasticities of price and income:

$$\varepsilon_{ni,k}^{(ji,g)} \equiv \frac{\partial \ln \mathcal{D}_{ni,k}\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln \tilde{P}_{ji,g}}, \qquad \eta_{ni,k} \equiv \frac{\partial \ln \mathcal{D}_{ni,k}\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln E_i}$$

2. Energy distributor's demand for international energy varieties, $\widetilde{\mathcal{D}}_{ni,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right)$, is homothetic and characterized by price elasticities of demand:

$$\varepsilon_{ni,0}^{(ji,0)} \equiv \frac{\partial \ln \widetilde{\mathcal{D}}_{ni,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right)}{\partial \ln \tilde{P}_{ji,0}}$$

   and its corresponding distribution-level price, $\tilde{\mathcal{P}}_{i,0}\left(\tilde{\mathbf{P}}_{i,0}\right)$.

3. Cost minimization given the constant-returns-to-scale production function of final goods ($k > 0$), $Q_{n,k} = \bar{\varphi}_{n,k} F_{n,k}\left(L_{n,k}, Z_{n,k}\right)$, implies:

   (a) Input price aggregator, as a homogeneous-of-degree-one function $\mathcal{C}_{i,k}\left(w_i, \alpha_{i,k}\right)$ for home country i and $\mathscr{C}_{n,k}\left(w_n, \tilde{P}_{n,0}\right)$ for foreign $n \neq i$;

   (b) Energy cost share function $\alpha_{n,k}\left(w_n, \tilde{P}_{n,0}\right)$ for foreign $n \neq i$ (with $\alpha_{i,k}$ for home i regarded as a policy choice);

   (c) Energy input use functions (equivalently, carbon emission functions) $\mathcal{Z}_{i,k}\left(\alpha_{i,k}, Q_{i,k}\right) = \mathrm{z}_{i,k}\left(\alpha_{i,k}\right) Q_{i,k}$ for home i and $\mathcal{Z}_{n,k}\left(\alpha_{n,k}, \tilde{P}_{n,0}, Q_{n,k}\right) = \mathrm{z}_{n,k}\left(\alpha_{n,k}\left(\bar{w}_n, \tilde{P}_{n,0}\right)\right) Q_{n,k}$ for foreign $n \neq i$ (with aggregate emission levels $Z_n = \sum_k \mathcal{Z}_{n,k}(.)$ for home $n = i$ or foreign $n \neq i$) that are characterized by energy input demand elasticities:

$$\zeta_{n,k} \equiv \frac{\partial \ln \mathcal{Z}_{n,k}(.)}{\partial \ln \tilde{P}_{n,0}}, \qquad \zeta_n \equiv \frac{\partial \ln Z_n(.)}{\partial \ln \tilde{P}_{n,0}}$$

Functional Forms for *Quantitative* Analysis. In our derivations of the optimal policy, we make no parametric assumptions about the above generic functions. However, for our quantitative analysis, we adopt the following functional forms:

1. *Cobb-Douglas-CES preferences for final goods* deliver the following indirect utility:

$$V_i\left(E_i, \tilde{\mathbf{P}}_i\right) = E_i / \tilde{P}_i, \qquad \text{where} \qquad \tilde{P}_i = \prod_{k=1}^{K} \left(\sum_{j=1}^{N} b_{ji,k} \tilde{P}_{ji,k}^{1-\sigma_k}\right)^{\frac{\beta_{i,k}}{1-\sigma_k}};$$

   and, Marshallian demand functions, for home i and foreign $\ell \neq i$:

$$\mathcal{D}_{ni,k}\left(E_i, \tilde{\mathbf{P}}_i\right) = \frac{b_{ni,k} \tilde{P}_{ni,k}^{1-\sigma_k}}{\sum_j b_{ji,k} \tilde{P}_{ji,k}^{1-\sigma_k}} \beta_{i,k} E_i, \qquad \mathcal{D}_{n\ell,k}\left(E_\ell, \tilde{\mathbf{P}}_{i\ell}, \mathbf{P}_{-i\ell}\right) = \frac{b_{n\ell,k} P_{n\ell,k}^{1-\sigma_k}}{b_{i\ell,k} \tilde{P}_{i\ell,k} + \sum_{j \neq i} b_{j\ell,k} P_{j\ell,k}^{1-\sigma_k}} \beta_{\ell,k} E_\ell$$

and, price elasticities of demand:

$$\varepsilon_{ji,k}^{(ni,g)} = \begin{cases} -\left[1 + (\sigma_k - 1)(1 - \lambda_{ni,k})\right] & j = n,\ g = k \\ (\sigma_k - 1)\lambda_{ni,k} & j \neq n,\ g = k \\ 0 & g \neq k \end{cases}$$

(*Note:* In the above specification, income elasticities of demand equal unity. In the general case, however, they do not appear in the optimal policy formulas.)

2. *CES aggregator over international energy varieties* delivers the following demand function (for the energy distributor) for home i and foreign $\ell \neq i$

$$\widetilde{\mathcal{D}}_{ni,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right) = \frac{b_{ni,0}\tilde{P}_{ni,0}^{1-\sigma_0}}{\sum_j b_{ji,0}\tilde{P}_{ji,0}^{1-\sigma_0}} E_{i,0}, \qquad \widetilde{\mathcal{D}}_{n\ell,0}\left(E_{\ell,0}, \tilde{P}_{i\ell,0}, \mathbf{P}_{-i\ell,0}\right) = \frac{b_{n\ell,0}P_{n\ell,0}^{1-\sigma_0}}{b_{i\ell,0}\tilde{P}_{i\ell,0}^{1-\sigma_0} + \sum_{j\neq i} b_{ji,0}P_{ji,0}^{1-\sigma_0}} E_{\ell,0}$$

and, price elasticities of demand:

$$\varepsilon_{ji,0}^{(ni,0)} = \begin{cases} -\left[1 + (\sigma_k - 1)(1 - \lambda_{ni,0})\right] & j = n \\ (\sigma_k - 1)\lambda_{ni,0} & j \neq n \end{cases}$$

and, the aggregate price of the energy composite for home i and foreign $\ell \neq i$,

$$\tilde{\mathcal{P}}_{i,0}\left(\tilde{\mathbf{P}}_{i,0}\right) = \left[\sum_{j=1}^{N} b_{ji,0}\tilde{P}_{ji,0}^{1-\sigma_0}\right]^{\frac{1}{1-\sigma_0}}, \quad \tilde{\mathcal{P}}_{\ell,0}\left(\tilde{P}_{i\ell,0}, \mathbf{P}_{-i\ell,0}\right) = \left[b_{i\ell,0}\tilde{P}_{i\ell,0}^{1-\sigma_0} + \sum_{j\neq i} b_{j\ell,0}P_{j\ell,0}^{1-\sigma_0}\right]^{\frac{1}{1-\sigma_0}}$$

3. *CES production function of final goods*, is described by:

$$F_{i,k}\left(L_{i,k}, Z_{i,k}\right) = \left[(1 - \bar{\kappa}_{i,k})^{\frac{1}{\varsigma}} L_{i,k}^{\frac{\varsigma-1}{\varsigma}} + (\bar{\kappa}_{i,k})^{\frac{1}{\varsigma}} Z_{i,k}^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}$$

where ς is the elasticity of substitution between labor and energy input, implying:

(a) Input price aggregator (equivalently, marginal cost net of total factor productivity) for home i and foreign $\ell \neq i$,[53]

$$\mathcal{C}_{i,k}\left(w_i, \alpha_{i,k}\right) = (1 - \kappa_{i,k})\, w_i \left(1 - \alpha_{i,k}\right)^{\frac{1}{\varsigma-1}}, \qquad \mathscr{C}_{n,k}\left(w_n, \tilde{P}_{n,0}\right) = \left((1 - \kappa_{n,k})\, w_n^{1-\varsigma} + \kappa_{n,k}\tilde{P}_{n,0}^{1-\varsigma}\right)^{\frac{1}{1-\varsigma}}.$$

(b) Energy input cost share function for foreign country $n \neq i$,

$$\alpha_{n,k}\left(w_n, \tilde{P}_{n,0}\right) = \left(\frac{\kappa_{n,k}}{1 - \kappa_{n,k}} \frac{\tilde{P}_{n,0}}{w_n}\right)^{1-\varsigma},$$

(c) Energy input use (corresponding to carbon emission) function:

$$\mathcal{Z}_{i,k}(.) = \mathrm{z}_{i,k}\left(\alpha_{i,k}\right) Q_{i,k} \qquad \text{where} \quad \mathrm{z}_{i,k}\left(\alpha_{i,k}\right) = \bar{\mathrm{z}}_{i,k}\alpha_{i,k}^{\frac{\varsigma}{\varsigma-1}};$$

[53] For foreign countries, the marginal cost $\mathscr{C}_{n,k}\left(w_n, \tilde{P}_{n,0}\right)$ simply falls from the CES structure. For the home country, cost minimization implies the labor cost share as $1 - \alpha_{i,k} = \left((1 - \kappa_{i,k})\, w_i / \mathcal{C}_{i,k}\right)^{1-\varsigma}$, which delivers: $\mathcal{C}_{i,k}\left(w_i, \alpha_{i,k}\right) = (1 - \kappa_{i,k})\, w_i \left(1 - \alpha_{i,k}\right)^{\frac{1}{\varsigma-1}}$.

and, the elasticities of:

$$\zeta_{n,k} = -\varsigma\left(1-\alpha_{n,k}\right), \qquad \zeta_n = -\varsigma\left(1-\alpha_n\right)$$

where $\alpha_n \equiv \sum_{g\neq 0}\alpha_{n,g}\frac{Z_{n,g}}{Z_n}$ is the average energy input cost share in country n.

Auxiliary Variables. For completeness, we also reproduce the following auxiliary variables, which we make use of in our derivations:

$$\text{[industry-level total sales]} \qquad Y_{n,k} = P_{nn,k}Q_{n,k}$$

$$\text{[industry-level carbon intensity]} \qquad v_{n,k} = \frac{Z_{n,k}}{Y_{n,k}}$$

$$\text{[within-industry international sales shares]} \qquad \rho_{ni,k} = \frac{P_{ni,k}C_{ni,k}}{\sum_\ell P_{n\ell,k}C_{n\ell,k}} = \frac{P_{ni,k}C_{ni,k}}{Y_{n,k}}$$

$$\text{[within-industry international expenditure shares]} \qquad \lambda_{ni,k} = \frac{\tilde{P}_{ni,k}C_{ni,k}}{\sum_m \tilde{P}_{mi,k}C_{mi,k}}$$

$$\text{[cross-industry expenditure shares]} \qquad \beta_{i,k} = \frac{\sum_m \tilde{P}_{mi,k}C_{mi,k}}{\sum_k\sum_m \tilde{P}_{mi,k}C_{mi,k}} = \frac{\sum_m \tilde{P}_{mi,k}C_{mi,k}}{E_i}$$

$$\text{[CPI-adjusted climate damage cost]} \qquad \tilde{\delta}_i = \tilde{P}_i \times \delta_i; \quad \text{where} \quad \tilde{P}_i \equiv \left(\frac{\partial \ln V_i}{\partial \ln E_i}\right)^{-1}$$

Details of Tax Revenues and Nominal Income. As shown by Equation (E.1), national expenditure in country i equals total income as the sum of wage payments (w_iL_i), the surplus paid to energy reserves (Π_i), and tax revenues (T_i). Namely,

$$Y_i = w_iL_i + \Pi_i + T_i,$$

where the surplus paid to energy reserves, $\Pi_i \sim \Pi_i\left(P_{ii,0}, w_i\right)$, equals total sales minus labor input costs: $\Pi_i\left(P_{ii,0}, w_i\right) = \max_l P_{ii,0}Q_{i,0}\left(l\right) - w_il$. Country i's tax revenues, $T_i = T_i^z + T_i^q$, are composed of carbon tax revenues (T_i^z) and trade tax revenues (T_i^q). Carbon tax revenues are determined by the wedge between the final price of energy inputs for producers in each industry k ($\tilde{P}_{i,0k}$) and the price of the composite energy input, $\tilde{P}_{i,0} = \tilde{\mathcal{P}}_{i,0}\left(\tilde{P}_{1i,0}, ..., \tilde{P}_{Ni,0}\right)$. Specifically,

$$T_i^z = \sum_k\left(\tilde{P}_{i,0k} - \tilde{P}_{i,0}\right)Z_{i,k}.$$

Considering that $\tilde{P}_{i,0k}Z_{i,k} = \alpha_{i,k}P_{ii,k}Q_{i,k}$, where $\tilde{P}_{i,k0} = \tilde{P}_{i,0} + \tau_{i,k}$ is the after-carbon-tax consumer price of energy, $\alpha_{i,k}$ is the energy cost share, and $Q_{i,k} = \sum_n \bar{d}_{in,k}C_{in,k}$ is the output quantity of origin i–industry k, we can rewrite carbon tax revenues as

$$T_i^z = \sum_{k\neq 0}\left[\alpha_{i,k}P_{ii,k}Q_{i,k}\right] - \tilde{P}_{i,0}\sum_{k\neq 0}Z_{i,k}.$$

The pre-carbon-tax price index of the composite energy input, $\tilde{P}_{i,0} = \tilde{\mathcal{P}}_{i,0}\left(\tilde{P}_{1i,0}, ..., \tilde{P}_{Ni,0}\right)$, aggregates over the *after-trade tax* price of internationally-sourced energy varieties, where function $\tilde{\mathrm{P}}_{i,0}\left(.\right)$ is homogeneous of degree one. Trade tax revenues, T_i^q, include taxes on energy sales at the border and domestically (although the latter is redundant given carbon taxation) as well as taxes on all other goods sold to or from country i. Namely,

$$T_i^q = \sum_n \sum_k \left[\left(\tilde{P}_{ni,k} - P_{ni,k} \right) C_{ni,k} \right] + \sum_{n \neq i} \sum_k \left[\left(\tilde{P}_{in,k} - P_{in,k} \right) C_{in,k} \right].$$

Plugging the expressions for T_i^z and T_i^q back into our original expression for Y_i and noting that $P_{ni,k} = d_{ni,k} P_{ii,k}$ and $\alpha_{i,0} = 0$ (by definition, the energy extraction does not use the energy input), yields

$$\begin{aligned} Y_i = & w_i L_i + \Pi_i \left(P_{ii,0}; w_i \right) + \sum_{n \neq i} \sum_k \left[\left(\tilde{P}_{ni,k} - P_{ni,k} \right) C_{ni,k} \right] \\ & + \sum_n \sum_k \left[\left(\tilde{P}_{in,k} - \left[1 - \alpha_{i,k} \right] P_{in,k} \right) C_{in,k} \right] - \tilde{P}_{i,0} \sum_{k \neq 0} Z_{i,k}, \end{aligned} \tag{A.1}$$

Assumptions about Foreign Income. We impose two assumptions: Country i' policy choice $\mathscr{P} \in \mathbb{P}_i$ does not alter **(A1)** the relative wage rates among foreign countries, i.e., $\mathrm{d} \left(w_n / w_{n_0} \right) / \mathrm{d}\mathscr{P} = 0$ for all $n, n_0 \neq i$; and **(A2)** aggregate factor income ratios abroad, i.e., $\mathrm{d} \left(\Pi_n / w_n L_n \right) / \mathrm{d}\mathscr{P} = 0$ for all $n \neq i$. Assumption *A1* in combination with Walras' law ensure that $\mathbf{w}_{-i}$ is invariant to $\mathscr{P} \in \mathbb{P}_i$. Though, home's policy still affects its own wage w_i relative to foreign wages. Assumption *A2* asserts that income in a foreign country ($n \neq i$) is pinned down by its wage rate. Namely, $E_n = Y_n = \left(1 + \overline{\pi}_n \right) w_n L_n$, where $\overline{\pi}_n = \Pi_n / w_n L_n$ is invariant to $\mathscr{P} \in \mathbb{P}_i$. Together, assumptions *A1* and *A2* imply that nominal income in foreign countries is unaffected by home's policy up to a choice of numeraire. Note that both assumptions would be redundant when considering a two-country model with labor as the sole factor of production.

A.2 Characterizing General Equilibrium Effects of Policy on Output Quantities

As an intermediate step in our derivations of the optimal policy formulas, we will need to trace out the impact of policy $\mathscr{P} \in \mathbb{P}_i$ on output quantities, $\left\{ Q_{n,k} \right\}_{n,k}$. These derivatives emerge nontrivially in the system of general equilibrium particularly when production functions are decreasing or increasing returns to scale. This section characterizes these general equilibrium effects on output quantities, first for the energy industry and then for final-good industries. Later, when we expand the F.O.C.s of the optimal policy problem, we make use of the intermediate results which we derive here.

Energy Industry. The production technology in the energy sector features decreasing returns to scale, as energy extraction uses carbon reserves as industry-specific inputs. Accordingly, the producer price of energy depends on each country's energy output along its upward-sloping supply curve. So, when deriving the F.O.C.s, we must track the interdependence between energy output quantities and prices. These are general equilibrium relations by nature, wherein a policy change in country i alters the scale of production and energy prices globally, creating ripple effects. To characterize these ripple effects, we note that the origin ℓ's energy output is the implicit solution to Equations (D.2) and (Q.1), which we reproduce as the following system of equations:

$$Q_{\ell,0} \equiv \mathcal{Q}_{\ell,0} \left(C_{\ell 1,0}, ..., C_{\ell N,0} \right) = \sum_{n=1}^{N} \bar{d}_{\ell n,0} C_{\ell n,0}, \tag{A.2}$$

$$\text{where} \quad C_{\ell n,0} = \begin{cases} \widetilde{\mathcal{D}}_{\ell n,0} \left(E_{n,0}, \tilde{P}_{in,0}, \mathbf{P}_{-in,0} \right) & n \neq i \\ \widetilde{\mathcal{D}}_{\ell i,0} \left(E_{i,0}, \tilde{\mathbf{P}}_{i,0} \right) & n = i \end{cases}$$

Here, $\mathcal{Q}_{\ell,0}(.)$ denotes the industry-level energy output as a function of all destination-specific quantities of demand for *origin* ℓ's energy, with $\widetilde{\mathcal{D}}_{\ell n,0}(.)$ denoting the Marshallian demand function that takes as inputs total energy expenditure, $E_{n,0}$, and the entire vector of after-border-tax (but before-carbon-tax) energy prices $(\tilde{\mathbf{P}}_{n,0})$. In the home country $(n = i)$, all after-border-tax prices are policy choices i.e., $\tilde{\mathbf{P}}_{n,0} \sim \tilde{\mathbf{P}}_{i,0} \equiv \{\tilde{P}_{ji,0}\}_j \subset \mathbb{P}_i$. In foreign countries $(n \neq i)$, only the after-tax price of goods sourced from country i is determined by policy, $\{\tilde{P}_{in,0}\}_{n\neq i} \subset \mathbb{P}_i$. The remaining prices are equal to producer prices (without the tilde notation), which are determined endogenously in equilibrium. More specifically, the producer price of energy, $P_{\ell n,0} \in \mathbf{P}_{-in,0}$, is an explicit function of the origin country's energy output, $Q_{\ell,0}$, reflecting decreasing-returns to scale in production. As indicated by Equation (P.2), we reproduce the noted function as:

$$P_{\ell n,0} \equiv \mathcal{P}_{\ell n,0}\left(Q_{\ell,0}, \bar{w}_\ell\right) = d_{\ell n,0}\bar{p}_{\ell,0} \times \left(\mathcal{Q}_{\ell,0}(.)\right)^{\frac{\phi_\ell}{1-\phi_\ell}} \times \bar{w}_\ell, \qquad (\forall \ell). \tag{A.3}$$

We can apply the Implicit Function Theorem to the system of interdependent Equations (A.2) and (A.3) to specify the scale effects associated with policy $\mathscr{P} \in \mathbb{P}_i$—i.e., $\frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathscr{P}}$. For this purpose, let $Q_{n,0}\left(\tilde{\mathbb{P}}_i; \bar{\mathbf{w}}\right)$ be the implicit solution to the system specified by Equations (A.2) and (A.3). Next, we characterize the elasticity of $Q_{n,0}$ w.r.t. each policy choice $\mathscr{P} \in \mathbb{P}_i$, organizing our derivations for all policy choices except export prices and separately for export prices.

Policy Choices other than Export Prices. We first characterize the elasticity of $Q_{n,0}$ w.r.t. a non-export-price policy choice $\mathscr{P} \in \mathbb{P}_i - \left\{\tilde{\mathbf{P}}_{in}\right\}_{n\neq i}$, holding all other elements of $\tilde{\mathbb{P}}_i$ and wages $\mathbf{w} = \bar{\mathbf{w}}$ fixed. Holding wages constant, reflects our forthcoming result about the neutrality of GE wage effects and assumption A1. Note that per assumption A2, the constancy of wages ensures that $\mathbf{E}_{-i,0}$ is invariant to $\tilde{\mathbb{P}}_i$ as well; through $E_{i,0}$ varies with policy due to revenue effects. Considering these points, we can now specify the change in $Q_{n,0}$ in response to $\mathscr{P}$ as the sum of direct demand effects in market i and indirect demand effects in other markets where prices are not pinned to the choice of $\mathbb{P}_i$. In particular,

$$\begin{aligned}\frac{\partial \ln Q_{n,0}\left(\mathbb{P}_i; \mathbf{w}\right)}{\partial \ln \mathscr{P}} =& \frac{\partial \ln \mathcal{Q}_{n,0}(.)}{\partial \ln C_{ni,0}} \overbrace{\left(\mathbb{1}_{\mathscr{P}\in\tilde{\mathbf{P}}_{i,0}} \frac{\partial \ln \widetilde{\mathcal{D}}_{ni,0}(.)}{\partial \ln \mathscr{P}} + \frac{\partial \ln \widetilde{\mathcal{D}}_{ni,0}(.)}{\partial \ln E_{i,0}} \frac{\mathrm{d} \ln E_{i,0}}{\mathrm{d} \ln \mathscr{P}}\right)}^{\sim \mathrm{d}\ln C_{ni,0}/\mathrm{d}\ln\mathscr{P}} \\ &+ \sum_{\ell\neq i}\sum_{m\neq i}\left[\frac{\partial \ln \mathcal{Q}_{n,0}(.)}{\partial \ln C_{nm,0}} \frac{\partial \ln \widetilde{\mathcal{D}}_{nm,0}(..., P_{\ell m,0})}{\partial \ln P_{\ell m,0}} \frac{\partial \ln \mathcal{P}_{\ell m,0}(.)}{\partial \ln Q_{\ell,k}} \frac{\partial \ln Q_{\ell,0}\left(\mathbb{P}_i; \mathbf{w}\right)}{\partial \ln \mathscr{P}}\right],\end{aligned} \tag{A.4}$$

Here, our application of the chain rule uses the functional relationship, $C_{ni,0} = \widetilde{\mathcal{D}}_{ni,0}\left(E_{i,0}, \tilde{\mathbf{P}}_{i,0}\right)$. To clarify, the sum in the second line of the above equation collects the (first-order) general equilibrium linkages relates to (decreasing-returns-to) scale effects in the rest of the world $(\ell, m \neq i)$.[54] Our derivation, notice, imposes assumption *A1* that $\mathbf{w}_{-i}$ is invariant to $\mathbb{P}_i$ up to normalization of one wage element per Walras's law. The reason the sum in the second line excludes country i is that all prices associated with country i are pinned down by country i's policy, $\mathbb{P}_i$. Hence, for goods sold to or supplied by country i, the scale-driven changes to producer prices do not impact the associated consumer prices and, correspondingly, demand. Appealing to Equation (**??**) and the functional form of (A.3) and invoking our choice of notation for Marshallian demand elasticities, we can specify the

[54] We omit second-order terms. For instance, the output-scale-driven change to producer prices changes energy rents, Π_i, which in turn alters net expenditure and, correspondingly, final demand for all goods including energy. We omit these second-order effects in Equation A.4.

partial derivatives in Equation (A.4) as

$$\frac{\partial \ln \mathcal{Q}_{n,0}(.)}{\partial \ln C_{ni,0}} = \frac{C_{ni,0}}{Q_{i,0}} \sim \rho_{ni,0}, \qquad \frac{\partial \ln \widetilde{\mathcal{D}}_{nm,0}(.)}{\partial \ln P_{\ell m,0}} \sim \varepsilon_{nm,0}^{(\ell m,0)}, \qquad \frac{\partial \ln \mathcal{P}_{\ell m,0}(.)}{\partial \ln Q_{\ell,0}} = \frac{\phi_\ell}{1-\phi_\ell}.$$

Plugging these expressions back into Equation A.4, we can express the system of equations specified by A.4 in matrix notation as:

$$\left[\frac{\partial \ln Q_{n,0}(\tilde{\mathbb{P}}_i;\mathbf{w})}{\partial \ln \mathscr{P}}\right]_n = \mathbf{\Psi}^{(i,0)} \left[\rho_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln \mathscr{P}}\right]_n.$$

where:

$$\mathbf{\Psi}^{(i,0)} \equiv \left[\psi_{\ell n}^{(i,0)}\right]_{\ell,n} = \mathrm{inv}\left(\mathbf{I}_N + \frac{\phi_i}{1-\phi_i}\left[\mathbb{1}_{n\neq i}\sum_{m\neq i}\rho_{nm,0}\varepsilon_{nm,0}^{(\ell m,0)}\right]_{n,\ell}\right) \tag{A.5}$$

We hereafter use $\psi_{\ell n}^{(i,0)}$ to denote the (ℓ, n) entry of matrix $\mathbf{\Psi}^{(i,0)}$. Since the consumer price of energy goods supplied by country i is determined by $\mathbb{P}_i$, we can verify that the i'th column of $\mathbf{\Psi}^{(i,0)}$ is equal to that of the identity matrix, $\mathbf{I}_N$. The following lemma formalizes this result, which is used later to simplify the F.O.C.s.

Remark 1. Matrix $\mathbf{\Psi}^{(i,0)} \equiv \left[\psi_{\ell n}^{(i,0)}\right]_{\ell,n}$, given by Equation (A.5), satisfies: $\psi_{ii}^{(i,0)} = 1$ for home country i and $\psi_{ni}^{(i,0)} = 0$ for all foreign countries $n \neq i$.

It follows from Remark 1 that for the the price of domestically consumed goods, $\mathscr{P} \sim \tilde{P}_{ji,k}$, we have

$$\frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} = \mathbb{1}_{n=i} \times \rho_{ii,0}\frac{\mathrm{d}\ln C_{ii,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}} + \sum_{\ell\neq i}\left[\psi_{n\ell}^{(i,0)}\rho_{\ell i,0}\frac{\mathrm{d}\ln C_{\ell i,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right], \tag{A.6}$$

where the terms $\left\{\frac{\mathrm{d}\ln C_{\ell i,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right\}_\ell$ encompass demand adjustments that channel through price or income effects. As we show later, one does not need to unpack these terms as they are redundant at the optimum policy point.

Export Prices. The same logic applies when considering the change in output levels in response to an export price instrument, $\tilde{P}_{in,k} \in \mathbb{P}_i$. In particular, extrapolating from the logic outlined above, we get:

$$\frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ij,k}} = \mathbb{1}_{n=i} \times \rho_{nj,0}\frac{\mathrm{d}\ln C_{nj,0}}{\mathrm{d}\ln \tilde{P}_{ij,k}} + \sum_{\ell\neq i}\left[\psi_{n\ell}^{(i,0)}\rho_{\ell j,0}\frac{\mathrm{d}\ln C_{\ell j,0}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right], \tag{A.7}$$

where, if we abstract from income effects in the rest of the world, can simplified by setting $\frac{\mathrm{d}\ln C_{nj,0}}{\mathrm{d}\ln \tilde{P}_{ij,0}} \approx \varepsilon_{nj,0}^{(ij,0)}$ in case of energy prices ($k = 0$) and $\frac{\mathrm{d}\ln C_{nj,0}}{\mathrm{d}\ln \tilde{P}_{ij,k}} \approx 0$ if $k \neq 0$.

Notational Convention. When specifying the F.O.C.s, we break down and formulate our expression for $\frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}$ in Equation A.6 as follows

$$\frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} = \mathbb{1}_{n=i} \times \rho_{ii,0}\frac{\mathrm{d}\ln C_{ii,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}} + \sum_{\ell\neq i}\left[\psi_{n\ell}^{(i,0)}\rho_{\ell i,0}\frac{\mathrm{d}\ln C_{\ell i,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right] \sim \frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \mathbf{C}_{i,0}}\frac{\mathrm{d}\ln \mathbf{C}_{i,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}}$$

The last representation is useful for presenting our envelop result as it decomposes $\frac{\partial \ln Q_{n,0}(\mathbb{P}_i;\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}$ into a partial derivative w.r.t. domestic demand quantities and the policy-induced change in theses quantities.

Final-good Industries. In our main specification, the production technology of final goods are constant returns to scale. Hence, we can fully track producer prices, given by Equation (P.1), through cost functions:

$$P_{\ell n,k} = \begin{cases} \bar{d}_{in,k}\bar{p}_{i,k}\mathcal{C}_{i,k}\left(w_i, \alpha_{i,k}\right) & \ell = i,\ k \geq 1 \\ \bar{d}_{\ell n,k}\bar{p}_{\ell,k}\mathscr{C}_{\ell,k}\left(w_\ell, \tilde{P}_{\ell,0}\right) & \ell \neq i,\ k \geq 1 \end{cases}$$

That is, producer prices of final goods ($k \geq 1$) do not depend on production quantities; that is, $\partial \ln P_{\ell n,k}/\partial Q_{\ell,k} = 0$ for $k = 1, ..., K$. Note that we will revisit the scale effects in final-good industries for the extension of our model in which we allow for increasing returns to scale.

The supply curve in final good industries is flat meaning that the impact of policy on output quantities is equal to demand adjustments. Namely, for $k = 1, ..., K$,

$$\frac{\partial \ln Q_{n,k}\left(\mathbb{P}; \mathbf{w}\right)}{\partial \ln \mathscr{P}} = \rho_{ni,k}\frac{\mathrm{d} \ln C_{ni,k}}{\mathrm{d} \ln \mathscr{P}} + \sum_{j \neq i}\left[\mathbb{1}_{\mathscr{P} \in \tilde{\mathbf{P}}_j} \times \rho_{nj,k}\frac{\mathrm{d} \ln C_{nj,k}}{\mathrm{d} \ln \mathscr{P}}\right] \tag{A.8}$$

where we $\frac{\mathrm{d} \ln C_{ni,k}}{\mathrm{d} \ln \mathscr{P}}$ encompasses demand adjustments in the domestic economy that channel through income and/or price effects. The second term is non-zero if $\mathscr{P}$ is an export price instrument that directly modifies demand in any foreign market j. Throughout the analysis we make use of assumption *A2* that the aggregate factor income ratios in the rest of the world are invariant to country i's policy—i.e., $\bar{\pi}_n = \Pi_n/w_n L_n$ remains constant for $n \neq i$.

A.3 Generic Statement of First-Order Conditions *w.r.t.* $\mathscr{P} \in \mathbb{P}_i$

Our goal is to solve the unilateral policy problem of country i as detailed in Appendix A.1 under our formulation of the general equilibrium detailed in Table A.1. The F.O.C. with respect to a generic policy instrument $\mathscr{P} \in \mathbb{P}_i = \left\{\alpha_{i,k}, \tilde{P}_{ji,k}, \tilde{P}_{ii,k}, \tilde{P}_{ij,k}\right\}_{j,k}$, can be decomposed into four terms:

$$\begin{aligned} \frac{\mathrm{d}W_i\left(\mathbb{P}_i, \mathbf{w}\right)}{\mathrm{d} \ln \mathscr{P}} = & \underbrace{\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln \mathscr{P}} \times \mathbb{1}_{\mathscr{P} \in \tilde{\mathbf{P}}_i}}_{\text{consumer price effect}} + \underbrace{\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial E_i}\frac{\mathrm{d}\left(Y_i\left(\mathbb{P}_i\right)\right)}{\mathrm{d} \ln \mathscr{P}}}_{\text{income effect}} \\ & - \underbrace{\delta_i \sum_n \frac{\mathrm{d}Z_n\left(\mathbb{P}_i\right)}{\mathrm{d} \ln \mathscr{P}}}_{\text{emission effect}} + \underbrace{\frac{\partial W_i\left(\mathbb{P}_i, \mathbf{w}\right)}{\partial \ln \mathbf{w}} \times \frac{\mathrm{d} \ln \mathbf{w}}{\mathrm{d} \ln \mathscr{P}}}_{\text{wage effect}} \quad = 0 \end{aligned} \tag{A.9}$$

The first term on the right-hand side represents the direct welfare impact of perturbing consumer prices via policy. The second term represents the welfare gains from raising income through tax revenues or payments to energy reserves (holding wages constant). The third term represents the impact of policy on global carbon emissions. The fourth term represents the indirect impact on welfare through adjustments in wage rates.

We take two steps in our way of showing that it is sufficient to expand only some, and not all, the components of the above F.O.C.s. First, we show that the fourth term, namely the wage effect, is redundant (Lemma 1). Second, we show that the second and third components, namely the income and emission effects, can be greatly simplified according to an envelope-type result (Lemma 2). Proposition 1 in the main text combines the results from these two lemmas.

A.4 Neutrality of General Equilibrium Wage Effects

This section demonstrates the neutrality of wage effects in the reformulated unilateral policy problem. To do so, it is useful to rewrite a more compact version of Equation (A.9) in which the F.O.C.s associated with each policy instrument, $\mathscr{P} \in \mathbb{P}_i$, is expressed as the sum of direct effects of policy (holding wages constant) and indirect GE effects through wage adjustments:

$$\frac{\mathrm{d}W_i\left(\mathbb{P}_i,\mathbf{w}\right)}{\mathrm{d}\ln\mathscr{P}} = \frac{\partial W_i\left(\mathbb{P}_i,\mathbf{w}\right)}{\partial\ln\mathscr{P}} + \frac{\partial W_i\left(\mathbb{P}_i,\mathbf{w}\right)}{\partial\ln\mathbf{w}} \times \underbrace{\frac{\mathrm{d}\ln\mathbf{w}}{\mathrm{d}\ln\mathscr{P}}}_{\text{GE wage elasticities}} = 0.$$

Considering this expression, the following lemma notes the neutrality of wage effects:

Lemma 1. *Under assumption A1, the GE wage elasticities are redundant for the design of any policy in which the government of country i chooses all policy instruments in $\mathbb{P}_i$ as $\frac{\partial W_i(\mathbb{P}_i,\mathbf{w})}{\partial\ln\mathbf{w}} = 0$.*

Proof. Consider, first, country i's wage rate, $w_i \in \mathbf{w}$. Noting that all elements of $\tilde{\mathbf{P}}_i$ are policy choices included in $\mathbb{P}_i$, we can write:

$$\frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i} = \frac{\partial V_i\left(E_i,\tilde{\mathbf{P}}_i\right)}{\partial E_i}\frac{\partial Y_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i} - \delta_i\sum_n\frac{\partial Z_n\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i},$$

where the first term on the right-hand side imposes the equilibrium constraint, $E_i\left(\mathbb{X}\right) = Y_i\left(\mathbb{X}\right)$, according to Equation (E.1). Note that $\frac{\partial Z_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i} \sim \frac{\partial\mathcal{Z}_i(\boldsymbol{\alpha}_i,\mathbf{Q}_i)}{\partial\ln\mathbf{Q}_i}\cdot\frac{\partial\ln\mathbf{Q}_i}{\partial\ln E_i}\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i}$, since $\boldsymbol{\alpha}_i \in \mathbb{P}_i$, where $\mathcal{Z}_i\left(\boldsymbol{\alpha}_i,\mathbf{Q}_i\right)$ is a vector whose elements are $\mathcal{Z}_i\left(\alpha_i,Q_i\right) \equiv \sum_k\left[z_{i,k}\left(\alpha_{i,k}\right)Q_{i,k}\right]$ given by Equation (Z). Meanwhile, $\frac{\partial Z_n(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i} = 0$ for $n \neq i$, since foreign emissions are fully determined by $\mathbb{P}_i$ and $\mathbf{w}_{-i}$. Accordingly, holding $\mathbf{w}_{-i}$ and all elements of $\mathbb{P}_i$ fixed, country i's wage does not influence energy demand in foreign countries. Taking note of these relationships delivers:

$$\frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i} = \left[\frac{\partial V_i\left(E_i,\tilde{\mathbf{P}}_i\right)}{\partial E_i} - \delta_i\frac{\partial\mathcal{Z}_i\left(\boldsymbol{\alpha}_i,\mathbf{Q}_i\right)}{\partial\ln\mathbf{Q}_i}\cdot\frac{\partial\ln\mathbf{Q}_i}{\partial\ln E_i}\right]\frac{\partial Y_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}. \tag{A.10}$$

Next, we characterize $\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i}$, demonstrating that it is zero at the optimum ($\mathbb{P}_i = \mathbb{P}_i^*$). Taking derivatives from Equation (E.1) and invoking Hotelling's lemma, the impact of wages on national income can be specified as:[55]

$$\frac{\partial Y_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}=w_iL_i+\Pi_i-\sum_k\left[\left(1-\alpha_{i,k}\right)\frac{\partial\ln P_{ii,k}\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}P_{ii,k}Q_{i,k}\right]+\left(\frac{\partial\left(T_i+\Pi_i\right)}{\partial\ln\mathbf{Q}_i}\cdot\frac{\partial\ln\mathbf{Q}_i}{\partial\ln E_i}\right)\frac{\partial Y_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i},$$

where $\frac{\partial(T_i+\Pi_i)}{\partial\ln\mathbf{Q}_i}\cdot\frac{\partial\ln\mathbf{Q}_i}{\partial\ln E_i}$ collects the sum of income effects, whereby a wage-driven change in total income (and thus expenditure) modifies country i's entire demand schedule, $\mathbf{Q}_i\equiv\left\{Q_{ni,k}\right\}_{n,i}$, and the resulting revenues, $T_i+\Pi_i$. With a slight abuse of notation, we are setting $\alpha_{i,0}=0$ in the above equation (Note that the energy extraction industry does not use the energy input). Considering that $\left(\frac{\partial\ln P_{ii,k}(.)}{\partial\ln w_i}\right)_{\mathbf{Q}_i}=1$ and rearranging the above equation, yields

$$\frac{\partial Y_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}=\underbrace{\left(w_i\bar{L}_i+\Pi_i-\sum_k\left[\left(1-\alpha_{i,k}\right)P_{ii,k}Q_{i,k}\right]\right)}_{=0}\left(1-\frac{\partial\left(T_i+\Pi_i\right)}{\partial\ln\mathbf{Q}_i}\cdot\frac{\partial\ln\mathbf{Q}_i}{\partial\ln E_i}\right)^{-1}=0,$$

where the last line derives from the factor market clearing condition in home, whereby $w_iL_i+\Pi_i-\sum_k\left[\left(1-\alpha_{i,k}\right)P_{ii,k}Q_{i,k}\right]=0$. Plugging $\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i}=0$ into Equation A.10, establishes our claim regrading the neutrality of welfare with respect to w_i, given the choice of $\mathbb{P}_i$. Namely,

$$\frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}=0.$$

The neutrality of wage effects in foreign countries follows from Walras's law and assumption *A1* under which home's policy does not alter the relative wage rates among foreign countries, i.e., $\mathrm{d}\left(w_n/w_j\right)/\mathrm{d}\mathscr{P}=0$ for all $n,j\neq i$. In particular, setting $w_j\in\mathbf{w}_{-i}$ as the numeraire by Walras' law, this assumption entails that $\frac{\mathrm{d}\ln\mathbf{w}_{-i}}{\mathrm{d}\ln\mathscr{P}}=0$, implying that $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln\mathbf{w}_{-i}}=\mathbf{0}$. Notice that *A1* is redundant in a two-country setting. $\square$

In order for Lemma 1 to hold, the government of country i must have access to the prices of all goods that country i supplies, but it is important to note that the optimality of the policy choice is *not* required. In particular, the intuition behind the welfare neutrality of the domestic wage rate, i.e., $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial\ln w_i}=0$, can be put as follows: home's wage rate affects home's welfare exclusively through its impact on home's income; meanwhile, an increase in home's wage has two opposing effect on home's income: *(i)* it increases wage bills, *(ii)* it decreases tax revenues because it raises home's producer

[55] Recall that $\Pi_i=\mathit{\Pi}_i\left(P_{ii,0},w_i\right)$ where $\mathit{\Pi}_i\left(P_{ii,0},w_i\right)=\max_l P_{ii,0}F_{i,0}\left(l\right)-w_il$. The Hotelling's lemma, then, implies:

$$\frac{\partial\mathit{\Pi}_i\left(P_{ii,0},w_i\right)}{\partial\ln P_{ii,0}}=P_{ii,0}Q_{i,0},\qquad\qquad\frac{\partial\mathit{\Pi}_i\left(P_{ii,0},w_i\right)}{\partial\ln w_i}=-w_iL_{i,0}.$$

Given $\mathbf{Q}_i$, $P_{ii,0}$ is fully determined by the domestic wage rate, w_i, i.e., $P_{ii,0}=\bar{p}_{i,0}\times Q_{i,0}^{\frac{\phi_i}{1-\phi_i}}\times w_i$. Recalling that $w_iL_{i,0}=\left(1-\phi_i\right)P_{ii,0}Q_{i,0}$ and $\Pi_i=\phi_iP_{ii,0}Q_{i,0}$, it follows that:

$$\left(\frac{\partial\Pi_i}{\partial\ln w_i}\right)_{\mathbf{Q}_i}=\frac{\partial\mathit{\Pi}_i\left(P_{ii,0},w_i\right)}{\partial\ln P_{ii,0}}\left(\frac{\partial\ln P_{ii,0}}{\partial\ln w_i}\right)_{\mathbf{Q}_i}+\frac{\partial\mathit{\Pi}_i\left(P_{ii,0},w_i\right)}{\partial\ln w_i}=P_{ii,0}Q_{i,0}-w_iL_{i,0}=\Pi_i$$

Therefore,

$$\begin{aligned}\frac{\partial\Pi_i}{\partial\ln w_i}&=\left(\frac{\partial\Pi_i}{\partial\ln w_i}\right)_{\mathbf{Q}_i}+\frac{\partial\Pi_i}{\partial\ln\mathbf{Q}_i}\frac{\partial\ln\mathbf{Q}_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial\ln w_i}\\&=\Pi_i+\frac{\partial\Pi_i}{\partial\ln\mathbf{Q}_i}\frac{\partial\ln\mathbf{Q}_i\left(\mathbb{P}_i;\mathbf{w}\right)}{\partial E_i}\frac{\partial Y_i}{\partial\ln w_i}\end{aligned}$$

prices under the dual problem in which the consumer prices of made-in-home goods are held fixed. These two forces exactly offset each other insofar as the domestic labor market condition holds.

Lemma 1 implies that we can hold the wage rates fixed in the system of F.O.C.s. of the optimal policy problem. This is a very useful result because it allows us to disregard difficult-to-characterize but redundant general equilibrium wage elasticities, as we articulate below.

A.5 Generic Statement of the First-order Conditions

Following Lemma 1—the welfare neutrality of wage effects—we can henceforth hold wages fixed, $\mathbf{w} = \bar{\mathbf{w}}$ in the system of F.O.C.s and express country i's income as $E_i = \bar{w}_i \bar{L}_i + T_i + \Pi_i$. Consequently, the unilaterally optimal policy problem facing the government in country i can be further reformulated as the maximization of $W_i\left(\mathbb{P}_i; \bar{\mathbf{w}}\right)$ subject to the equilibrium equations in *Panel I* of Table A.1, with no need to track wage adjustments through invoking the labor market clearing conditions. Accordingly, the generic statement of the F.O.C., described by Equation (A.9), simplifies to:

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln\mathscr{P}} = \underbrace{\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln\mathscr{P}} \times \mathbb{1}_{\mathscr{P}\in\tilde{\mathbf{P}}_i}}_{\text{consumer price effect}} + \underbrace{\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial E_i}\frac{\mathrm{d}\left(T_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right) + \Pi_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)\right)}{\mathrm{d}\ln\mathscr{P}}}_{\text{income effect}} - \underbrace{\delta_i \sum_n \frac{\mathrm{d}Z_n\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)}{\mathrm{d}\ln\mathscr{P}}}_{\text{emission effect}} = 0 \tag{A.11}$$

Anticipating our envelope result, it will prove useful to rearrange the F.O.C.s, specified in Equation (A.11), into: *(i)* terms that represent the effect of policy holding domestic demand quantities, $\mathbf{C}_i \equiv \{C_{ni,k}\}_{n,k}$, fixed; and *(ii)* terms that account for the GE changes in domestic demand quantities. We also drop

$$\begin{aligned}\frac{\mathrm{d}W_i}{\mathrm{d}\ln\mathscr{P}} =& \frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln\mathscr{P}} \times \mathbb{1}_{\mathscr{P}\in\tilde{\mathbf{P}}_i} + \left(\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial E_i}\frac{\partial\left(T_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right) + \Pi_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)\right)}{\partial\ln\mathscr{P}} - \delta_i\frac{\partial\sum_n Z_n\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)}{\partial\ln\mathscr{P}}\right)_{\mathbf{C}_i} \\ &+ \underbrace{\left[\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial E_i}\frac{\partial\left(T_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right) + \Pi_i\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)\right)}{\partial\ln\mathbf{C}_i} - \delta_i\frac{\partial\sum_n Z_n\left(\mathbb{P}_i;\bar{\mathbf{w}}\right)}{\partial\ln\mathbf{C}_i}\right]}_{\text{domestic quantity effect}} \times \underbrace{\frac{\mathrm{d}\ln\mathbf{C}_i}{\mathrm{d}\ln\mathscr{P}}}_{\text{GE domestic quantitiy elasticity}} = 0\end{aligned} \tag{A.12}$$

Equation (A.12) is equivalent to the F.O.C. in the main text, i.e., Equation (23). To see this correspondence, let us reproduce Equation (23),

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln\mathscr{P}} = \frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i\right)}{\partial\ln\mathscr{P}} + \frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i\right)}{\partial\ln\mathbf{C}_i}\underbrace{\frac{\mathrm{d}\ln\mathbf{C}_i}{\mathrm{d}\ln\mathscr{P}}}_{\text{GE elasticity}} + \frac{\partial W_i\left(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i\right)}{\partial\ln\mathbf{w}}\underbrace{\frac{\mathrm{d}\ln\mathbf{w}}{\mathrm{d}\ln\mathscr{P}}}_{\text{GE elasticity}} = 0$$

The first term on the right-hand side, $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial\ln\mathscr{P}}$, corresponds to the first line of Equation (A.12). The second term, $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial\ln\mathbf{C}_i}\frac{\mathrm{d}\ln\mathbf{C}_i}{\mathrm{d}\ln\mathscr{P}}$, corresponds to the second line of Equation (A.12). The third (and last) term can be set to zero by noting that, $\frac{\partial W_i(\mathbb{P}_i;\mathbf{w},\mathbf{C}_i)}{\partial\ln\mathbf{w}} = 0$ per Lemma 1.

Our goal is now to unpack the F.O.C.s, as specified in Equation (A.12), separately for each of the policy instruments. Namely,

1. Imported and domestic consumer prices ($\mathscr{P} = \tilde{P}_{ji,k}$ for all j including $j = i$, and all k);
2. Energy cost shares ($\mathscr{P} = \alpha_{i,k}$ for all $k \neq 0$); and,

3. Exported consumer prices ($\mathscr{P} = \tilde{P}_{ij,k}$ for all $j \neq i$ and all k).

We begin our F.O.C.s with respect to imported and domestic consumer prices, $\mathscr{P} = \left\{\tilde{P}_{ji,k}\right\}_{j,k}$. The reason that we begin with $\mathscr{P} = \left\{\tilde{P}_{ji,k}\right\}_{j,k}$ is that we will use the F.O.C.s *w.r.t.* $\mathscr{P} = \left\{\tilde{P}_{ji,k}\right\}_{j,k}$ to produce our *envelope* result. Specifically, in the F.O.C.s *w.r.t.* ($\mathscr{P} = \tilde{P}_{ji,k}$), we show that the first line of Equation (A.12) equals zero; and so, the solution is obtained by setting the "domestic quantity effect," as specified in the second line of Equation (A.12), to zero. This, in turn, means that the "GE domestic quantity elasticities" need not be characterized. Having established this result, we proceed to unpack the F.O.C.s *w.r.t.* energy cost shares ($\mathscr{P} = \alpha_{i,k}$) and exported consumer prices ($\mathscr{P} = \tilde{P}_{ij,k}$), where the entire second line of Equation (A.12) will be redundant.

A.6 The Envelope Result & Proof of Proposition 1

A.6.1 The Envelope Result

We follow Equation (A.12) to inspect the F.O.C. *w.r.t.* domestic or imported consumer prices, $\mathscr{P} = \tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$. To establish the envelope result, it is sufficient to unpack the components that emerge conditional on holding domestic quantities fixed—i.e., the terms in the first line of Equation (A.12) for $\mathscr{P} = \tilde{P}_{ji,k}$. We will inspect the terms in the second line of Equation (A.12) in Section A.7 where we complete our derivations for the F.O.C. *w.r.t.* $\mathscr{P} = \tilde{P}_{ji,k}$.

Consumer Price Effects. The indirect utility from consumption, $V_i\left(E_i, \tilde{\mathbf{P}}_i\right)$, is an explicit function of non-energy consumer prices, since $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$ if $k \neq 0$. The welfare effects associated with the noted price can be obtained by appealing to Roy's identify. Namely,

$$\frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial \ln \tilde{P}_{ji,k}} = -\mathbb{1}_{k\neq 0} \times \frac{\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)}{\partial E_i} \tilde{P}_{ji,k} C_{ji,k} \qquad \text{[Roy's identity]}. \tag{A.13}$$

The above relationship, notice, only applies to non-energy prices. Energy goods, under our notational choice, are not directly consumed and have no *CPI effect* associated with them—i.e., $\partial V_i\left(E_i, \tilde{\mathbf{P}}_i\right)/\partial \ln \tilde{P}_{ji,0} = 0$.

Income Effects. Recall that country i's income is: $Y_i = \bar{w}_i \bar{L}_i + T_i + \Pi_i$, as described by Equation (A.1). Country i's tax revenues, in turn, can be rewritten equivalently under a slightly different format:

$$\begin{aligned} T_i = & \sum_{n\neq i}\sum_{k}\left[\left(\tilde{P}_{ni,k} - P_{ni,k}\right) C_{ni,k}\right] + \sum_{n}\sum_{k}\left[\tilde{P}_{in,k} C_{in,k}\right] \\ & - \left[P_{ii,0} Q_{i,0}\right] + \sum_{k\neq 0}\left[\left(1-\alpha_{i,k}\right) P_{ii,k} Q_{i,k}\right] - \tilde{P}_{i,0}\sum_{k\neq 0} Z_{i,k} \end{aligned}$$

Using the above expression, the impact of policy on tax revenues, holding domestic demand fixed, equals:

$$\left(\frac{\partial T_i\left(\mathbb{P}_i; \bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} = \tilde{P}_{ji,k} C_{ji,k} + P_{ii,0} Q_{i,0} \underbrace{\frac{\partial \ln \mathcal{P}_{ii,0}\left(.\right)}{\partial \ln Q_{i,0}}}_{\frac{\phi_i}{1-\phi_i}} \left(\frac{\partial \ln Q_{i,0}\left(\mathbb{P}_i; \bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} - \mathbb{1}_{k=0} \times \sum_{g\neq 0} \tilde{P}_{i,0} Z_{i,g} \frac{\partial \ln \tilde{\mathcal{P}}_{i,0}\left(\tilde{P}_{1i,0}, \ldots, \tilde{P}_{Ni,0}\right)}{\partial \ln \tilde{P}_{ji,0}}$$

The first term on the right-hand side shows that an increase in (imported) consumer price $\tilde{P}_{ji,k}$ raises the tax revenue mechanically in proportion to demand quantity. The second term represents the fact that an increase in $\tilde{P}_{ji,k}$ changes the domestic energy output via general equilibrium effects—as discussed in Section A.2—altering energy export tax revenues. Note that, as specified earlier, the inverse supply elasticity in the energy sector is given by $\frac{\partial \ln \mathcal{P}_{ii,0}(w_i,Q_{i,0})}{\partial \ln Q_{i,0}} = \frac{\phi_i}{1-\phi_i}$. The last term, represents the change in the composite energy price index, $\tilde{P}_{i,0}$. This term is relevant only when the taxed good (ji,k) pertains to the energy sector ($k = 0$), in which case it can characterized by appealing to Shepherd's lemma:

$$\sum_{g\neq 0}\left[\tilde{P}_{i,0}Z_{i,g}\frac{\partial \ln \tilde{\mathcal{P}}_{i,0}(.)}{\partial \ln \tilde{P}_{ji,0}}\right] = \tilde{P}_{ji,0}C_{ji,0}. \qquad \text{(Shepherd's Lemma)}$$

Turning to the surplus from carbon reserves, we obtain:

$$\left(\frac{\partial \Pi_i(\mathbb{P}_i;\bar{\mathbf{w}})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} = \frac{\partial \mathit{\Pi}_i(.)}{\partial \ln P_{ii,0}}\frac{\partial \ln \mathcal{P}_{ii,0}(.)}{\partial \ln Q_{i,0}}\left(\frac{\partial \ln Q_{i,0}(\mathbb{P}_i;\bar{\mathbf{w}})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i}$$

The term on the right-hand side represents the change in the surplus paid to energy reserves, which recall is fully determined by the output unit prices, $P_{ii,0}$, and the input price, w_i—i.e., $\Pi_i = \mathit{\Pi}_i(P_{ii,0}, w_i)$. Per Hotelling's lemma, moreover,

$$\frac{\partial \mathit{\Pi}_i(P_{ii,0},w_i)}{\partial \ln P_{ii,0}}\frac{\partial \ln P_{ii,0}}{\partial \ln Q_{i,0}} = P_{ii,0}Q_{i,0}\frac{\phi_i}{1-\phi_i} \qquad \text{(Hotelling's Lemma)}$$

Putting together the policy impact on T_i and Π_i, we obtain:

$$\left(\frac{\partial\left[T_i(\mathbb{P}_i;\bar{\mathbf{w}})+\Pi_i(\mathbb{P}_i;\bar{\mathbf{w}})\right]}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} = \mathbb{1}_{k\neq 0}\times \tilde{P}_{ji,k}C_{ji,k} \tag{A.14}$$

Carbon Emission Effects. As shown later in Section **??**, the effect of $\tilde{P}_{ji,k}$ on carbon emissions channels exclusively through changes in domestic consumption quantities, holding other policy variables (including energy input shares $\boldsymbol{\alpha}_i \in \mathbb{P}_i$) fixed. This is because carbon emissions depend on energy input shares and output quantities. The former is a policy choice. The latter react to policy via general equilibrium changes in domestic demand quantities. Therefore,

$$\left(\frac{\partial \sum_n Z_n(\mathbb{P}_i;\bar{\mathbf{w}})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} = 0 \tag{A.15}$$

Merging the Price, Income, and Emission Effects. Putting together Equations (A.13), (A.14), and (A.15), the F.O.C. with respect to $\tilde{P}_{ji,k}$ conditional on holding domestic demand quantities and other policy variables fixed, equals zero.

$$\frac{\partial V_i(E_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} + \left(\frac{\partial V_i(E_i,\tilde{\mathbf{P}}_i)}{\partial E_i}\frac{\partial\left[T_i(\mathbb{P}_i;\bar{\mathbf{w}})+\Pi_i(\mathbb{P}_i;\bar{\mathbf{w}})\right]}{\partial \ln \tilde{P}_{ji,k}} - \delta_i\frac{\partial \sum_n Z_n(\mathbb{P}_i;\bar{\mathbf{w}})}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{C}_i} = 0 \tag{A.16}$$

An immediate implication of the above finding is that the F.O.C *w.r.t.* $\tilde{P}_{ji,k}$ collapses to the second line of Equation (A.12). This observation delivers our envelope result:

Lemma 2. *Suppose the necessary F.O.C.s for optimality are satisfied for* $\left\{\tilde{P}_{ji,k}\right\}_{j,k} \in \tilde{\mathbb{P}}_i$. *Then,*

$$\frac{\partial W_i\left(\mathbb{P}_i, \bar{\boldsymbol{w}}, \mathbf{C}_i\right)}{\partial \ln \mathbf{C}_i} \equiv \left[\frac{\partial V_i}{\partial E_i}\frac{\partial\left(T_i + \Pi_i\right)}{\partial \ln \mathbf{C}_i} - \delta_i \frac{\partial \sum_n Z_n}{\partial \ln \mathbf{C}_i}\right] = 0, \tag{A.17}$$

indicating that for all policy instruments $\mathscr{P} \in \mathbb{P}_i$, *the terms in F.O.C.s accounting GE demand effects in the domestic economy (second line of Equation (A.12)) reduce to zero.*

Proof. The proof follows trivially from Equation (A.12), which presents the generic F.O.C. with respect to any policy variable including $\tilde{P}_{ji,k}$, and Equation (A.16) that shows that the impact of $\tilde{P}_{ji,k}$ conditional on holding demand quantities fixed is zero. Hence, Equation (A.17) must be satisfied to ensure optimality w.r.t. $\left\{\tilde{P}_{ji,k}\right\}_j \subset \mathbb{P}_i$, eliminating the need to characterize $\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \mathscr{P}}$ in Equation Equation (A.12) irrespective of which policy variable's F.O.C. is being specified. □

Lemma 2 simplifies the task of characterizing optimal policy formulas in two crucial ways. First, it asserts that in solving the F.O.C.s *w.r.t.* any of the policy instrument, $\mathscr{P} \in \mathbb{P}_i$, the general equilibrium elasticity of domestic demand quantities, $\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \mathscr{P}}$, need not be characterized. Second, it implies that in the F.O.C.s *w.r.t.* $\mathscr{P} \in \mathbb{P}_i - \left\{\tilde{P}_{ji,k}\right\}_{j,k}$, quantities sold in the domestic economy can be treated as if they were constant. This is a remarkable result since general equilibrium effects that alter the domestic demand schedule can be extremely difficult to be dealt with.

A.6.2 Proof of Proposition 1

Proposition 1 in the main text immediately follows from Lemma 1 and Lemma 2. Specifically, Lemma 1 indicates that if the government of country i has access to all policy instruments, then the GE elasticities of wages *w.r.t.* policy are redundant; and Lemma 2 indicates that, if in addition, the policy choice is optimal, the GE elasticities of domestic demand quantities *w.r.t.* policy are redundant.

A.7 First-Order Condition *w.r.t.* $\tilde{P}_{ji,k}$

In this section, we complete our solution to the F.O.C.s *w.r.t.* the price instruments $\tilde{P}_{ji,k} \in \mathbb{P}_i$ which are associated with a good sold to home consumers or producers. Recall, Equation (A.16) already shows that the impact of $\tilde{P}_{ji,k}$, conditional on holding demand quantities fixed, is zero—where the consumer price effects cancel out other direct effects. Consequently, the F.O.C. *w.r.t.* $\tilde{P}_{ji,k}$ reduces to:

$$\left[\frac{\partial\left(T_i + \Pi_i\right)}{\partial \ln \mathbf{C}_i} - \tilde{\delta}_i \frac{\partial \sum_n Z_n}{\partial \ln \mathbf{C}_i}\right] \times \frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \tilde{P}_{ji,k}} = 0 \tag{A.18}$$

where $\tilde{\delta}_i = \delta_i \times \left(\frac{\partial V_i}{\partial E_i}\right)^{-1}$. Below, we unpack the F.O.C.s *w.r.t.* $\tilde{P}_{ji,k}$ separately for income effects and carbon emission effects that are present in Equation (A.18).

Income Effects. Net of the direct impact of the policy, which we included in Equation (A.16), the impact of policy on income $Y_i = \bar{w}_i \bar{L}_i + T_i + \Pi_i$, can be expressed as:

$$\frac{\partial\left(T_i+\Pi_i\right)}{\partial \ln \mathbf{C}_i} \frac{\mathrm{d} \ln \mathbf{C}_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} = \frac{\partial \mathit{\Pi}_i\left(P_{ii,0}, w_i\right)}{\partial \ln P_{ii,0}} \frac{\partial \ln P_{ii,0}}{\partial \ln Q_{i,0}} \frac{\partial \ln Q_{n,0}(.)}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}$$
$$+\sum_n \sum_g\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right) C_{ni,g} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]-\sum_n\left[P_{ni,0} C_{ni,0} \frac{\partial \ln \mathcal{P}_{nn,0}(.)}{\partial \ln Q_{n,0}} \frac{\partial \ln Q_{n,0}(.)}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]$$
$$+\sum_g\left[\alpha_{i,g} P_{ii,g} Q_{i,g} \frac{\partial \ln Q_{i,g}(.)}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]-\sum_{g \neq 0}\left[\tilde{P}_{i,0} Z_{i,g} \frac{\partial \ln \mathcal{Z}_{i,g}(.)}{\partial \ln Q_{i,g}} \frac{\partial \ln Q_{i,g}(.)}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]=0 \quad \text{(A.19)}$$

The first term on the right-hand side of the first line represents the change in tax revenues due to policy-led demand adjustments (given the baseline producer prices). The first term on second line of Equation (A.19) contains revenue effects due to scale-driven changes to energy producer prices. The remaining terms in Equation (A.19) account for the change in carbon tax revenues. The elasticities featuring on the second line of (A.19) can be expressed as

$$\frac{\partial \ln \mathcal{P}_{nn,0}\left(w_n, Q_{n,0}\right)}{\partial \ln Q_{n,0}}=\frac{\phi_n}{1-\phi_n} \qquad (\forall n), \qquad \frac{\partial \mathit{\Pi}_i\left(P_{ii,0}, w_i\right)}{\partial \ln P_{ii,0}}=P_{ii,0} C_{ii,0}, \qquad (\forall g \neq 0)$$

These relationships reveal that $\left[\frac{\partial \mathit{\Pi}_i(.)}{\partial \ln P_{ii,0}} \frac{\partial \ln P_{ii,0}}{\partial \ln Q_{i,0}}-P_{ni,0} C_{ni,0} \frac{\partial \ln \mathcal{P}_{nn,0}(.)}{\partial \ln Q_{n,0}}\right]=0$, eliminating the first term on the right-hand side of Equation (A.19) with the domestic element of the second sum in the second line. The last line in Equation (A.19) can be simplified by noting that $\frac{\partial \ln \mathcal{Z}_{i,g}(.)}{\partial \ln Q_{i,g}}=1$, per Equation (Z) in Table A.1. Plugging the above relationships back into Equation (A.19) and appealing to our previously-derived relationship, $\frac{\partial \ln Q_{n,0}(.)}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}=\sum_{\ell \neq i}\left[\psi_{n\ell}^{(i,0)} \rho_{\ell i,0} \frac{\mathrm{d} \ln C_{\ell i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]$, simplifies the term accounting for income effect as:

$$\frac{\partial\left(T_i+\Pi_i\right)}{\partial \ln \mathbf{C}_i} \frac{\mathrm{d} \ln \mathbf{C}_i}{\mathrm{d} \ln \tilde{P}_{ji,k}}=\sum_n \sum_{g \neq 0}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right) C_{ni,g} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]+\left(\tilde{P}_{ii,0}-P_{ii,0}\right) C_{ii,0} \frac{\mathrm{d} \ln C_{ii,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}$$
$$+\sum_{n \neq i}\left[\left(\tilde{P}_{ni,0}-\left(1+\omega_{ni,0}\right) P_{ni,0}\right) C_{ni,0} \frac{\mathrm{d} \ln C_{ni,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right]+\sum_{g \neq 0}\left[\left(\alpha_{i,g} P_{ii,g} Q_{i,g}-\tilde{P}_{i,0} Z_{i,g}\right) \frac{\partial \ln Q_{i,g}}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}\right] \quad \text{(A.20)}$$

where $\omega_{ni,0}$ collects all the direct and indirect linkages of good $ni,0$'s export supply to the *producer* price of energy goods associated with country i's economy. That is, $\omega_{ni,0}$ is the inverse export supply elasticity of energy (for flows from n to i), which is equals to:

$$\omega_{ni,0}=\frac{\phi_n}{1-\phi_n} \sum_{\ell \neq i}\left[\rho_{\ell i,0} \tilde{\psi}_{\ell n}^{(i,0)}\right],$$

where $\rho_{\ell i,0}=\frac{P_{\ell i,0} C_{\ell i,0}}{Y_{\ell,0}}$, $Y_{n,0} \equiv P_{nn,0} Q_{n,0}$ and $\tilde{\psi}_{\ell n}^{(i,0)}=\psi_{\ell n}^{(i,0)} Y_{\ell,0} / Y_{n,0}$, with $\psi_{\ell n}^{(i,0)}$ denoting the entry (ℓ, n) of matrix $\mathbf{\Psi}^{(i,0)}$as specified by Equation (A.5). Before moving on to other terms in the F.O.C, two remarks about Equation (A.20) are in order. First, this equation specifies the partial derivative of income *w.r.t.* policy holding wages, $\mathbf{w}$, fixed. As demonstrated in Section A.4, general equilibrium wage effects end up being welfare neutral, and need not to be unpacked. Second, $\mathrm{d} \ln C_{ni,g} / \mathrm{d} \ln \tilde{P}_{ji,k}$ encompasses demand changes that channel through price and income effects. As such, it implicitly depends on $\partial Y_i\left(\tilde{\mathbb{P}}_i ; \mathbf{w}\right) / \partial \ln \tilde{P}_{ji,k}$. We purposely avoid making this dependence explicit to keep the notation compact, as the parametrization of $\mathrm{d} \ln C_{ni,g} / \mathrm{d} \ln \tilde{P}_{ji,k}$ (or its value) has no bearings for the optimal policy choice, $\mathbb{P}_i^*$.

Carbon Emission Effects. The change in carbon emissions associated with $\tilde{P}_{ji,k} \in \tilde{\mathbb{P}}_i$ are two-fold. First, a change in $\tilde{P}_{ji,k}$ alters the entire non-energy demand schedule in country i, which in turn modifies the demand for energy inputs. Second, the resulting change in the scale of energy production, alters energy prices from all countries other than i (since energy prices at home are fully determined by policy, $\tilde{\mathbb{P}}_i$). We can formally characterize these two effects as follows:

$$\delta_i \frac{\partial Z^{(global)}}{\partial \ln \mathbf{C}_i} \frac{\mathrm{d} \ln \mathbf{C}_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} = \delta_i \sum_n \sum_{g \neq 0} Z_{n,g} \left[\frac{\partial \ln \mathcal{Z}_{n,g}\left(w_n, \tilde{P}_{n,0}, Q_{n,g}\right)}{\partial \ln Q_{n,g}} \frac{\partial \ln \mathcal{Q}_{n,g}\left(.\right)}{\partial \ln C_{ni,g}} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right]$$
$$+ \delta_i \sum_{n \neq i} \sum_{\ell \neq i} \left[Z_n \frac{\partial \ln \mathcal{Z}_n\left(w_n, \tilde{P}_{n,0}, \mathbf{Q}_n\right)}{\partial \ln \tilde{P}_{n,0}} \frac{\partial \ln \tilde{\mathcal{P}}_{n,0}\left(\tilde{\mathbf{P}}_{n,0}\right)}{\partial \ln \tilde{P}_{\ell n,0}} \frac{\partial \ln \mathcal{P}_{\ell n,0}\left(..., Q_{\ell,0}\right)}{\partial \ln Q_{\ell,0}} \frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right] \tag{A.21}$$

Considering that $\partial \ln \mathcal{Z}_{n,k}\left(.\right) / \partial \ln Q_{n,k} = 1$, the first term on the right-hand side of the above equation can be unpacked as

$$\sum_n \sum_{g \neq 0} Z_{n,g} \left[\frac{\partial \ln \mathcal{Z}_{n,g}\left(.\right)}{\partial \ln Q_{n,g}} \frac{\partial \ln \mathcal{Q}_{n,g}\left(.\right)}{\partial \ln C_{ni,g}} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right] = Z_i \frac{\partial \ln Q_{i,g}}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} + \sum_{n \neq i} \sum_{g \neq 0} \left[Z_{n,g} \rho_{ni,g} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right]$$
$$= Z_i \frac{\partial \ln Q_{i,g}}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} + \sum_{n \neq i} \sum_{g \neq 0} \left[v_{n,g} P_{ni,g} C_{ni,g} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right],$$

where the last line uses $v_{n,g} = \frac{Z_{n,g}}{Y_{n,g}}$ and $Y_{n,g} = P_{ni,g} C_{ni,g} / \rho_{ni,g}$. We purposely separate the term $Z_i \frac{\partial \ln Q_{i,g}}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}}$ in the above equation from the proceeding sum, since it cancels out with terms relating to carbon tax revenues. The last term on the right-hand side of Equation (A.21) can be unpacked by noting that $\frac{\partial \ln \mathcal{Z}_n(.)}{\partial \ln \tilde{P}_{n,0}} \sim \zeta_n$, $\frac{\partial \ln \tilde{\mathcal{P}}_{n,0}(.)}{\partial \ln \tilde{P}_{\ell n,0}} = \lambda_{\ell n,0}$ per Shephard's lemma, and $\frac{\partial \ln \mathcal{P}_{\ell n,0}(.)}{\partial \ln Q_{\ell,0}} = \frac{\phi_\ell}{1-\phi_\ell}$. In particular,

$$\sum_{n \neq i} \sum_{\ell \neq i} \left[\frac{\partial \ln \mathcal{Z}_n\left(.\right)}{\partial \ln \tilde{P}_{n,0}} \frac{\partial \ln \tilde{\mathcal{P}}_{n,0}\left(.\right)}{\partial \ln \tilde{P}_{\ell n,0}} \frac{\partial \ln \mathcal{P}_{\ell n,0}\left(.\right)}{\partial \ln Q_{\ell,0}} \frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right] = \frac{\phi}{1-\phi} \sum_{n \neq i} \sum_{\ell \neq i} \left[\zeta_n \lambda_{\ell n,0} \frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right]$$
$$= \frac{\phi}{1-\phi} \sum_{\ell \neq i} \left(\sum_{n \neq i} \left[Z_n \zeta_n \lambda_{\ell n,0} \right] \frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right) = \frac{\phi}{1-\phi} \sum_{n \neq i} \left[\left(\sum_{\ell \neq i} \sum_{m \neq i} \left[\frac{\rho_{\ell m,0} \zeta_m}{\tilde{P}_{m,0}} \right] \tilde{\psi}_{\ell n}^{(i,0)} \right) P_{ni,0} C_{ni,0} \frac{\mathrm{d} \ln C_{ni,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right],$$

where the last line uses the accounting identity, $Z_n \lambda_{\ell n,0} = P_{\ell n,0} Q_{\ell n 0} / \tilde{P}_{n,0}$, the definition $\tilde{\psi}_{\ell n}^{(i,0)} = \psi_{\ell n}^{(i,0)} Y_{\ell,0} / Y_{n,0}$, and the previously-derived expression, $\frac{\partial \ln Q_{\ell,0}}{\partial \ln \mathbf{C}_{i,0}} \frac{\mathrm{d} \ln \mathbf{C}_{i,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} = \sum_{n \neq i} \left[\psi_{\ell n}^{(i,0)} \rho_{ni,0} \frac{\mathrm{d} \ln C_{ni,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right]$ if $\ell \neq i$ (see Section A.2). Plugging the above expressions back into Equation (A.21) delivers the following characterization of emissions effects:

$$\delta_i \frac{\partial Z^{(global)}}{\partial \ln \mathbf{C}_i} \frac{\mathrm{d} \ln \mathbf{C}_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} = \delta_i \left(Z_i \frac{\partial \ln Q_{i,g}}{\partial \ln C_{ii,g}} \frac{\mathrm{d} \ln C_{ii,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} + \sum_{n \neq i} \sum_{g \neq 0} \left[v_{n,g} P_{ni,g} C_{ni,g} \frac{\mathrm{d} \ln C_{ni,g}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right] \right)$$
$$+ \delta_i \frac{\phi}{1-\phi} \sum_{n \neq i} \left[\left(\sum_{\ell \neq i} \sum_{m \neq i} \left[\rho_{\ell m,0} \zeta_m / \tilde{P}_{m,0} \right] \tilde{\psi}_{\ell n}^{(i,0)} \right) P_{ni,0} Q_{ni,0} \frac{\mathrm{d} \ln C_{ni,0}}{\mathrm{d} \ln \tilde{P}_{ji,k}} \right]. \tag{A.22}$$

Merging the Income and Emission Effects. Recall that $\tilde{\delta}_i = \left(\frac{\partial \ln V_i}{\partial \ln E_i} \right)^{-1} \times \delta_i$ is the CPI-adjusted climate damage cost, and define $\tilde{\omega}_{ni,0}$ as the augmented (inverse) export supply elasticity that measures the welfare gains from export supply contraction that channel through both terms-of-trade and carbon reduction. This elasticity is described by

$$\widetilde{\omega}_{ni,0} = \frac{\phi}{1-\phi}\sum_{\ell\neq i}\left[\tilde{\psi}^{(i,0)}_{\ell n}\left(\rho_{\ell i,0} + \tilde{\delta}_i \sum_{m\neq i}\frac{r_{\ell m,0}\zeta_m}{\tilde{P}_{m,0}}\right)\right] = \omega_{ni,0} + \tilde{\delta}_i \sum_{\ell\neq i}\sum_{m\neq i}\left[\tilde{\psi}^{(i,0)}_{\ell n}\rho_{\ell m,0}\frac{\zeta_m}{\tilde{P}_{m,0}}\right] \tag{A.23}$$

Without climate damage the above elasticity simply coincides with the standard notion of (inverse) export supply elasticity in general equilibrium (i.e., $\widetilde{\omega}_{ni,0} = \omega_{ni,0}$ if $\tilde{\delta}_i = 0$)

Merging Equations (A.20) and (A.22), under the definition of $\widetilde{\omega}_{ni,0}$ as in Equation (A.23), delivers the unpacked version of Equation (A.18) as:

$$\begin{aligned}\left(\frac{\partial\left\{T_i+\Pi_i\right\}}{\partial \ln \mathbf{C}_i} - \tilde{\delta}_i\frac{\partial Z^{(global)}}{\partial \ln \mathbf{C}_i}\right)\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln \tilde{P}_{ji,k}} = \; & \left(\tilde{P}_{ii,0} - P_{ii,0}\right)C_{ii,0}\frac{\mathrm{d}\ln C_{ii,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}} \\ & + \sum_{n\neq i}\left[\left(\tilde{P}_{ni,0} - \left(1+\widetilde{\omega}_{ni,0}\right)P_{ni,0}\right)C_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right] \\ & + \sum_{n}\sum_{g\neq 0}\left[\left(\tilde{P}_{ni,g} - \left(1+\tilde{\delta}_i v_{n,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right] \\ & + \sum_{g\neq 0}\left[\left(\alpha_{i,g}P_{ii,g}Q_{i,g} - \left(\tilde{P}_{i,0}+\tilde{\delta}_i\right)Z_{i,g}\right)\frac{\partial \ln Q_{i,g}\left(.\right)}{\partial \mathrm{ln} C_{ii,g}}\frac{\mathrm{dln} C_{ii,g}}{\mathrm{dln}\tilde{P}_{ji,k}}\right] = 0.\end{aligned} \tag{A.24}$$

A.8 First-Order Condition *w.r.t.* $\alpha_{i,k}$

This section derives the F.O.C.s that are associated with the choice of energy input shares, $\left\{\alpha_{i,k}\right\}_{k>0}$. Following Equation (A.11), or equivalently Equation (A.12), the F.O.C.s consist of separate terms accounting for *consumer price effect*, *income effect*, and *emission effect*, with the general equilibrium wage effects being redundant.

Consumer Prices Effects. The policy choice $\alpha_{i,k}$ has trivially no consumer prices effects.

Income Effects. We now characterize the income effects associated with the energy input share instrument. Considering Equation (A.1), perturbing $\alpha_{i,k}$, holding all other elements of $\tilde{\mathbb{P}}_i$ and $w_i \in \mathbf{w}$ constant, directly modifies two tax revenue-related terms: $\left(1-\alpha_{i,k}\right)P_{ii,k}Q_{i,k}$ and $\tilde{P}_{i,0}Z_{i,k}$ (note, however, that only $Z_{i,k}$ in the latter terms is affected, as $\tilde{P}_{i,0}$ is pinned down by the price elements in $\tilde{\mathbb{P}}_i$). The noted effects can in turn modify the entire (income-elastic) demand schedule in the home economy, thereby changing T_i and Π_i. Letting $\mathbf{C}_i \equiv \left\{C_{ni,k}\right\}_{n,i,k}$ denote the demand schedule at home, we can write the income effects associated with the noted perturbation as:

$$\begin{aligned}\frac{\mathrm{d}\ln\left[T_i\left(\mathbb{P}_i\right)+\Pi_i\left(\mathbb{P}_i\right)\right]}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)} = & \left(\frac{\partial\left\{\left(1-\alpha_{i,k}\right)P_{ii,k}Q_{i,k}\right\}}{\partial\ln\left(1-\alpha_{i,k}\right)} - \tilde{P}_{i,0}Z_{i,k}\frac{\partial\ln Z_{i,k}}{\partial\ln\left(1-\alpha_{i,k}\right)}\right)_{\mathbf{C}_i} \\ & + \underbrace{\frac{\partial\ln\left(T_i+\Pi_i\right)}{\partial\ln \mathbf{C}_i}\cdot\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)}}_{\text{income effects: final + energy demand}}\end{aligned}$$

The first terms on the right-hand side specify the direct effects of perturbing energy input shares given the demand schedule ($\mathbf{C}_i$). The last term represents general equilibrium income effects associated with final and input demand. First, the policy-led change in consumer expenditure, $E_i = Y_i$, alters the entire non-energy demand schedule in economy i and, consequently, tax revenues, T_i. Second, the policy-led change in total energy demand (Z_i) modifies the entire energy demand schedule, $\mathbf{C}_{i,0}$, and, consequently, T_i and Π_i. We do not unpack the term encompassing income effects because, according to Lemma 2, they are redundant in the neighborhood of the optimum policy. The direct terms in the above equation can be unpacked as follows:[56]

$$\begin{aligned}\left(\frac{\partial\left\{(1-\alpha_{i,k})\,P_{ii,k}Q_{i,k}\right\}}{\partial \ln(1-\alpha_{i,k})}\right)_{\mathbf{C}_i} &= (1-\alpha_{i,k})\,P_{ii,k}Q_{i,k}\left(1+\frac{\partial \ln P_{ii,k}(.)}{\partial \ln(1-\alpha_{i,k})}\right) \\ &= (1-\alpha_{i,k})\,P_{ii,k}Q_{i,k}\times\frac{\alpha_{i,k}}{1-\alpha_{i,k}}\frac{\partial \ln Z_{i,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})},\end{aligned}\tag{A.25}$$

where the last line follows from combining and contrasting two intermediate relationships as we demonstrate next. Let $\tilde{\tau}_{i,k}$ denote the ad valorem carbon tax rate, such that $Z_{i,k} = \alpha_{i,k}\frac{P_{ii,k}}{\tilde{\tau}_{i,k}}Q_{i,k}$. Note that $\tilde{\tau}_{i,k}$ is an implicit function of $\alpha_{i,k}$, per cost minimization; and $\partial \ln P_{ii,k}/\partial \ln \tilde{\tau}_{i,k} = \alpha_{i,k}$, per Shephard's lemma. Taking these intermediate identities into account, we can produce the following expression for $\frac{\partial \ln Z_{i,k}}{\partial \ln(1-\alpha_{i,k})}$ in Equation (A.25):

$$\frac{d \ln Z_{i,k}}{d \ln(1-\alpha_{i,k})} = \frac{\partial \ln \alpha_{i,k}}{\partial \ln(1-\alpha_{i,k})}\left[1+\left(\frac{\partial \ln P_{ii,k}}{\partial \ln \tilde{\tau}_{i,k}}-1\right)\frac{\partial \ln \tilde{\tau}_{i,k}}{\partial \ln \alpha_{i,k}}\right] = \frac{\partial \ln \alpha_{i,k}}{\partial \ln(1-\alpha_{i,k})}\left[1+(\alpha_{i,k}-1)\frac{\partial \ln \tilde{\tau}_{i,k}}{\partial \ln \alpha_{i,k}}\right].$$

Appealing to the same logic, we can specify $1+\frac{\partial \ln P_{ii,k}}{\partial \ln(1-\alpha_{i,k})}$ in Equation (A.25) as follows:

$$\begin{aligned}1+\frac{\partial \ln P_{ii,k}}{\partial \ln(1-\alpha_{i,k})} &= 1+\frac{\partial \ln P_{ii,k}}{\partial \ln \tilde{\tau}_{i,k}}\frac{\partial \ln \tilde{\tau}_{i,k}}{\partial \ln(1-\alpha_{i,k})} \\ &= \frac{\alpha_{i,k}}{1-\alpha_{i,k}}\frac{\partial \ln \alpha_{i,k}}{\partial \ln(1-\alpha_{i,k})}\left(1+\left[\frac{1-\alpha_{i,k}}{\alpha_{i,k}}\right]\alpha_{i,k}\frac{\partial \ln \tilde{\tau}_{i,k}}{\partial \ln \alpha_{i,k}}\right) = \frac{\alpha_{i,k}}{\alpha_{i,k}-1}\frac{\partial \ln Z_{i,k}}{\partial \ln(1-\alpha_{i,k})},\end{aligned}$$

which establishes the relationship presented under Equation A.25.[57] Now, we can plug the relationship specified by Equation A.25 back into our initial expression for $\frac{\partial Y_i(.)}{\partial \ln(1-\alpha_{i,k})}$ to obtain a simplified expression for income effects:

$$\frac{\partial Y_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})} = \left[\alpha_{i,k}P_{ii,k}Q_{i,k}-\tilde{P}_{i,0}Z_{i,k}\right]\frac{\partial \ln Z_{i,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})}+\frac{\partial \ln\{T_i+\Pi_i\}}{\partial \ln \mathbf{C}_i}\cdot\frac{d \ln \mathbf{C}_i(.)}{d \ln(1-\alpha_{i,k})}.\tag{A.26}$$

Emission Effects. Next, we specify the impact on carbon emissions. The choice of $\alpha_{i,k}$, notice, does not directly influence the demand for energy in foreign (holding other elements of $\mathbb{P}_i$ constant). Hence, it does not prompt carbon leakage beyond general equilibrium income effects that alter the demand schedule. The direct impact of $\alpha_{i,k}$ on carbon emissions is limited to domestic production. Stated formally,

$$\delta_i\frac{\partial Z^{(global)}(\mathbb{P}_i;\bar{\mathbf{w}})}{\partial \ln(1-\alpha_{i,k})} = \tilde{\delta}_i\frac{\partial V_i(.)}{\partial E_i}\left[\sum_k\left(\frac{\partial Z_{i,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})}\right)_{\mathbf{C}_i}+\frac{\partial Z^{(global)}}{\partial \ln \mathbf{C}_i}\cdot\frac{d \ln \mathbf{C}_i}{d \ln(1-\alpha_{i,k})}\right],\tag{A.27}$$

[56] Recall that $Q_{i,k} = \sum d_{in,k}C_{in,k}$, where $C_{ii,k} \in \mathbf{C}_i$ and $\frac{\partial C_{in,k}(\mathbb{P}_i,\mathbf{w})}{\partial \ln(1-\alpha_{i,k})} = 0$, since foreign prices and income levels all fully-determined by price elements of $\mathbb{P}_i$ and foreign wages, $\mathbf{w}_{-i}$.

[57] Note that if the production function has a CES parametrization with a substitution elasticity ς, then $\frac{\partial \ln Z_{i,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})} = \frac{\varsigma}{\varsigma-1}\frac{1-\alpha_{i,k}}{\alpha_{i,k}}$.

where $\tilde{\delta}_i \equiv \delta_i \left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}$ is the CPI adjusted climate damage parameter. Consolidating Equations (A.26) and (A.27) and invoking our earlier result about the neutrality of wages ($\frac{\partial W_i(\mathbb{P}_i;\mathbf{w})}{\partial \ln \mathbf{w}} \cdot \frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln \mathscr{P}} = 0$ for all $\mathscr{P} \in \mathbb{P}_i$), delivers the following F.O.C.s *w.r.t.* energy input shares $\alpha_{i,k}$, or equivalently $(1-\alpha_{i,k})$, as a policy instrument:

$$\left[\alpha_{i,k}P_{ii,k}Q_{i,k} - \left(\tilde{\delta}_i + \tilde{P}_{i,0}\right) Z_{i,k}\right] \frac{\partial \ln Z_{i,k}}{\partial \ln\left(1-\alpha_{i,k}\right)} + \left(\frac{\partial\left(T_i + \Pi_i\right)}{\partial \ln \mathbf{C}_i} - \tilde{\delta}_i \frac{\partial \sum_n Z_n}{\partial \ln \mathbf{C}_i}\right) \cdot \frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)} = 0, \tag{A.28}$$

where $\frac{\mathrm{d}\ln \mathbf{C}_i}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)} = \left\{\frac{\mathrm{d}\ln C_{ni,k}}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)}\right\}_{n,k}$ represents the change in home's domestic demand schedule through general equilibrium income effects. Note that, following Lemma 2, the last term in Equation A.28 reduces to zero under the optimal choice of $\{\tilde{P}_{ji,k}\}_{j,k}$.

A.9 First-Order Condition *w.r.t.* $\tilde{P}_{ij,k}$

Next, we derive the F.O.C. *w.r.t.* a generic export price instrument $\tilde{P}_{ij,k} \in \tilde{\mathbb{P}}_i$, where $j \neq i$. We must reemphasize that the F.O.C.s consist of partial derivatives that represent the marginal effect of perturbing $\tilde{P}_{ji,k}$, holding $\mathbb{P}_i - \left\{\tilde{P}_{ij,k}\right\}$ and wages, $\mathbf{w}$, constant. We again break down our derivation of the F.O.C. based on the decomposition presented under Equation (A.11), or equivalently Equation (A.12).

Income Effects. To specify the income effects associated with policy $\tilde{P}_{ij,k}$ if $j \neq i$, we take the partial derivative of Equation A.1 w.r.t. $\tilde{P}_{ij,k} \in \tilde{\mathbb{P}}_i$ treating demand quantities in the domestic economy ($\mathbf{C}_i$) and the global wage rates ($\mathbf{w}$) to be constant, per Lemmas 1 and 2. In the case of non-energy export prices ($k \neq 0$), the noted partial derivative becomes[58]

$$\begin{aligned}
\left(\frac{\partial Y_i\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i} = & \tilde{P}_{ij,k}C_{ij,k} + \sum_{g\neq 0}\left[\left(\tilde{P}_{ij,g} - P_{ij,g}\right) C_{ij,g}\frac{\partial \ln \mathcal{D}_{ij,g}\left(..., \tilde{P}_{ij,k}\right)}{\partial \ln \tilde{P}_{ij,k}}\right] \\
& + \underbrace{\left[\frac{\partial \Pi_i\left(.\right)}{\partial \ln P_{ii,0}}\frac{\partial \ln \mathcal{P}_{ii,0}\left(.\right)}{\partial \ln Q_{i,0}} - P_{ii,0}Q_{i,0}\frac{\partial \ln \mathcal{P}_{ii,0}\left(.\right)}{\partial \ln Q_{i,0}}\right]}_{=0}\left(\frac{\partial \ln Q_{i,0}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i} \\
& + \sum_{g\neq 0}\left[\left(\alpha_{i,g}P_{ii,g}Q_{i,g} - \tilde{P}_{i,0}Z_{i,g}\frac{\partial \ln \mathcal{Z}_{i,g}\left(\alpha_{i,g}, Q_{i,g}\right)}{\partial \ln Q_{i,g}}\right)\left(\frac{\partial \ln Q_{i,g}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i}\right].
\end{aligned} \tag{A.29}$$

The first line accounts for mechanical revenue generation and cross-demand effects. Note that by assumption *A1* and *A2* foreign country j's national expenditure, $E_j = \left(1+\bar{\pi}_{j,0}\right)\bar{w}_j L_j$, remains unchanged in response to home's policy. The second line represents the impact of country i's export policy on the global demand for energy and thus energy tax revenues. Perturbing $\tilde{P}_{ij,k}$, in particular, modifies the output sold by various countries to market j, which in turn modifies the amount of energy inputs demanded by these goods. The third line in the above equation collapses to zero per Hotelling and Shephard's Lemmas. And the last line accounts for the change in carbon tax revenues.

[58] Notice that $\left(\frac{\partial \ln C_{ij,0}\left(\tilde{\mathbb{P}}_i;\mathbf{w}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i} = 0$, because (a) there are is cross-substitutability between energy and non-energy variaties, and (b) the assumption regarding the invariance of $\Pi_n/w_n L_n$ to $\mathscr{P} \in \tilde{\mathbb{P}}_i$ ensures that total energy expenditure in foreign is constant holding $\mathbf{w}$ and $\mathbf{C}_i$ fixed.

Likewise, we can characterize the income effects associated with energy export prices ($\tilde{P}_{ij,0} \in \tilde{\mathbb{P}}_i$) as

$$\left(\frac{\partial Y_i\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,0}}\right)_{\mathbf{C}_i} = \tilde{P}_{ij,0}C_{ij,0} + \left[\left(\tilde{P}_{ij,0}-P_{ij,0}\right)C_{ij,0}\frac{\partial \ln \widetilde{\mathcal{D}}_{ij,0}\left(.\right)}{\partial \ln \tilde{P}_{ij,0}}\right]$$
$$+\sum_{n\neq i}\left[P_{nj,0}C_{nj,0}\widetilde{\omega}_{ni,0}\frac{\partial \ln \widetilde{\mathcal{D}}_{nj,0}\left(.\right)}{\partial \ln \tilde{P}_{ij,0}}\right] - \sum_{g\neq 0}\sum_{n\neq i}\left[\frac{\partial \ln \mathcal{P}_{ni,g}\left(.\right)}{\partial \ln \tilde{P}_{ij,0}}P_{ni,g}C_{ni,g}\right]$$
$$+\underbrace{\left[\frac{\partial \Pi_i\left(.\right)}{\partial \ln P_{ii,0}}\frac{\partial \ln P_{ii,0}\left(.\right)}{\partial \ln Q_{i,0}} - P_{ii,0}Q_{i,0}\frac{\partial \ln \mathcal{P}_{ii,0}\left(.\right)}{\partial \ln Q_{i,0}}\right]}_{=0}\left(\frac{\partial \ln Q_{i,0}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,0}}\right)_{\mathbf{C}_i}, \tag{A.30}$$

where the second line uses $P_{nj,0}C_{nj,0}\widetilde{\omega}_{ni,0} = P_{ni,0}C_{ni,0}\widetilde{\omega}_{nj,0}$.[59] The above equation differs from Equation A.29 in two important details. First, alternations to energy demand modify energy prices in the rest of the world (holding wages fixed). These price adjustments are captured by the first term on the second line. Second, energy export prices influence the cost of production and prices in the rest of the world. The first term in the last line of the above equation collects these input-output effects.[60]

CPI Effects. Export prices do not enter the indirect utility from consumption at home. Hence, the CPI effect associated with them is trivially zero. In particular,

$$\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i \qquad \Longrightarrow \qquad \frac{\partial V_i\left(E_i,\tilde{\mathbf{P}}_i\right)}{\partial \tilde{P}_{ij,k}} = 0.$$

Carbon Emissions. The effect of export taxes on carbon emissions are comprised of *scale effects* whereby shrinking the export demand for non-energy goods ($k \neq 0$) modifies the amount of energy inputs used for producing these goods, and *technique effects* whereby a change in the price of energy exports ($k = 0$) modifies the intensity by which foreign producers employ energy inputs. In the case of non-energy export prices ($k \neq 0$), only the former effect is relevant and can be characterized as

$$\begin{aligned}\delta_i\left(\frac{\partial Z^{(global)}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i} =& \delta_i\sum_n\sum_{g\neq 0}\left[Z_{n,g}\frac{\partial \ln \mathcal{Z}_{n,g}\left(.\right)}{\partial \ln Q_{n,g}}\left(\frac{\partial \ln Q_{n,g}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial \ln \tilde{P}_{ij,k}}\right)_{\mathbf{C}_i}\right]\\ =&\delta_i\sum_n\sum_{g\neq 0}\left[\frac{\partial \ln \mathcal{Q}_{n,g}\left(...,Q_{nj,g}\right)}{\partial \ln Q_{nj,g}}\frac{\mathrm{d}\ln C_{nj,g}}{\mathrm{d}\ln \tilde{P}_{ij,k}}\right]\\ =&\delta_i\sum_n\sum_{g\neq 0}\left[\rho_{nj,g}\frac{\partial \ln \mathcal{D}_{nj,g}\left(...,\tilde{P}_{ij,k}\right)}{\partial \ln \tilde{P}_{ij,k}}\right] = \delta_i\sum_n\sum_{g\neq 0}\left[v_{n,g}P_{nj,g}Q_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right]\end{aligned} \tag{A.31}$$

For energy goods ($k = 0$), the latter effect is relevant,. That is, the export price directly influences the price of energy in the country j and in the rest of the world through indirect general equilibrium

[59] The above equation precludes second-order terms corresponding to the product of cross-demand and cross-supply effects—since, $\frac{\partial \ln \widetilde{\mathcal{D}}_{ij,0}(.)}{\partial \ln \tilde{P}_{\ell j,0}}\left(\frac{\partial \ln P_{\ell j,0}(.)}{\partial \ln \tilde{P}_{ij,0}}\right)_{\mathbf{C}_i} \approx 0$.

[60] Notice that there are no cross-demand effects between energy and non-energy goods, which is why the non-energy export tax revnues are insensetive to $\tilde{P}_{ij,0}$ holding $\mathbf{w}$ fixed. The implicit asumption here is that aggregate energy prices in the rest of world are invaraint to third-country cross-demand effects.

linkages. In particular,

$$\delta_i\left(\frac{\partial Z^{(global)}\left(\tilde{\mathbb{P}}_i;\mathbf{w}\right)}{\partial\ln\tilde{P}_{ij,0}}\right)_{\mathbf{C}_i}=\delta_i\sum_{g\neq 0}\left[Z_{j,g}\frac{\partial\ln\mathcal{Z}_{j,g}\left(w_j,\tilde{P}_{j,0},\mathbf{Q}_j\right)}{\partial\ln\tilde{P}_{j,0}}\frac{\partial\ln\tilde{\mathcal{P}}_{j,0}\left(...,\tilde{P}_{ij,0}\right)}{\partial\ln\tilde{P}_{ij,0}}\right]$$
$$+\delta_i\sum_{n\neq i}\sum_{\ell\neq i}\left[Z_n\frac{\partial\ln\mathcal{Z}_n\left(w_n,\tilde{P}_{n,0},\mathbf{Q}_n\right)}{\partial\ln\tilde{P}_{n,0}}\frac{\partial\ln\tilde{\mathcal{P}}_{n,0}\left(.\right)}{\partial\ln\tilde{P}_{\ell n,0}}\frac{\partial\ln\mathcal{P}_{\ell n,0}\left(.\right)}{\partial\ln Q_{\ell,0}}\frac{\partial\ln Q_{\ell,0}\left(\tilde{\mathbb{P}}_i;\bar{\mathbf{w}}\right)}{\partial\ln\tilde{P}_{ij,0}}\right].\tag{A.32}$$

Considering that $\frac{\partial\ln\tilde{\mathcal{P}}_{j,0}(...)}{\partial\ln\tilde{P}_{ij,0}}=\lambda_{ij,0}$ and $\frac{\partial\ln\mathcal{Z}_{j,g}(.)}{\partial\ln\tilde{P}_{j,0}}\sim\zeta_{j,g}$, we can simplify the first term on the right-hand side of the above equation as

$$\delta_i\sum_{g\neq 0}\left[Z_{j,g}\frac{\partial\ln\mathcal{Z}_{j,g}\left(.\right)}{\partial\ln\tilde{P}_{j,0}}\frac{\partial\ln\tilde{\mathcal{P}}_{j,0}\left(...\right)}{\partial\ln\tilde{P}_{ij,0}}\right]=\delta_i\lambda_{ij,0}\underbrace{\sum_{g\neq 0}\left[\frac{Z_{j,g}}{Z_j}\zeta_{j,g}\right]}_{\sim\zeta_j}Z_j,$$

where $\zeta_j\equiv\sum_{g\neq 0}\left[\frac{Z_{j,g}}{Z_j}\zeta_{j,g}\right]$ is the aggregate energy input demand elasticity in country i. Considering that $\frac{\partial\ln\mathcal{P}_{\ell n,0}(.)}{\partial\ln Q_{\ell,0}}=\frac{\phi_\ell}{1-\phi_\ell}$ and appealing to our previously-derived expression for $\frac{\partial\ln Q_{\ell,0}\left(\tilde{\mathbb{P}}_i;\mathbf{w}\right)}{\partial\ln\tilde{P}_{ij,0}}$ (from Section A.2), we can unpack the second term on the right-hand side of Equation A.32 as follows:

$$\sum_{n\neq i}\sum_{\ell\neq i}\left[Z_n\frac{\partial\ln\mathcal{Z}_n\left(.\right)}{\partial\ln\tilde{P}_{n,0}}\frac{\partial\ln\tilde{\mathcal{P}}_{n,0}\left(.\right)}{\partial\ln\tilde{P}_{\ell n,0}}\frac{\partial\ln\mathcal{P}_{\ell n,0}\left(.\right)}{\partial\ln Q_{\ell,0}}\frac{\partial\ln Q_{\ell,0}\left(\tilde{\mathbb{P}}_i;\mathbf{w}\right)}{\partial\ln\tilde{P}_{ij,0}}\right]$$
$$=\sum_{n\neq i}\sum_{\ell\neq i}\left(Z_n\zeta_n\lambda_{n\ell,0}\frac{\phi}{1-\phi}\frac{\partial\ln Q_{\ell,0}\left(\tilde{\mathbb{P}}_i;\mathbf{w}\right)}{\partial\ln\tilde{P}_{ij,k}}\right)=\sum_{n\neq i}\sum_{\ell\neq i}\left(Z_n\zeta_n\lambda_{n\ell,0}\frac{\phi}{1-\phi}\sum_{m\neq i}\left[\psi_{\ell m}^{(i,0)}\rho_{mj,0}\frac{\mathrm{d}\ln C_{mj,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]\right)$$
$$=\sum_{n\neq i}\sum_{\ell\neq i}\left(\frac{\zeta_n}{\tilde{P}_{n,0}}P_{\ell n,0}Q_{\ell n,0}\frac{\phi}{1-\phi}\sum_{m\neq i}\left[\psi_{\ell m}^{(i,0)}\rho_{mj,0}\frac{\mathrm{d}\ln C_{mj,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]\right)=\sum_{m\neq i}\left[\sum_{\ell\neq i}\left(\tilde{\psi}_{\ell m}^{(i,0)}\sum_{n\neq i}\frac{\zeta_n}{\tilde{P}_{n,0}}\rho_{\ell n,0}\right)P_{mj,0}Q_{mj,0}\frac{\mathrm{d}\ln C_{mj,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]$$

where the last line implicitly imposes that $\psi_{\ell m}^{(i,0)}\rho_{mn,0}\frac{\mathrm{d}\ln C_{mn,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\approx 0$ if $n\neq j$ and $\tilde{\psi}_{\ell m}^{(i,0)}\equiv\psi_{\ell m}^{(i,0)}\frac{Y_{\ell,0}}{Y_{m,0}}$, as defined earlier. Also, note that energy demand effects in market j is

$$\frac{\mathrm{d}\ln C_{mj,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}=\frac{\partial\ln\widetilde{\mathcal{D}}_{mj,0}\left(.\right)}{\partial\ln\tilde{P}_{ij,0}}+\frac{\ln\widetilde{\mathcal{D}}_{mj,0}\left(.\right)}{\partial\ln E_{i,0}}\left(\frac{\mathrm{d}\ln E_{j,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right)_{\mathbf{w}}\approx\tilde{\varepsilon}_{mj,0}^{(ij,0)},$$

where last line follows from assumption *A2*, *i.e.*, the constancy of Π_n/w_nL_n in foreign countries ($n\neq i$), implying that $\left(\mathrm{d}\ln E_{j,0}/\mathrm{d}\ln\tilde{P}_{ij,0}\right)_{\mathbf{w}}\approx 0$. Consolidating Equation A.29 (income effects relating to energy prices) and Equations A.31 (carbon emissions relating to non-energy prices) and dividing all the terms by E_j delivers the following F.O.C. for *non-energy* export prices

$$e_{ij,k}+\sum_{g\neq 0}\left[\left(1-\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]-\sum_{n\neq i}\sum_{g\neq 0}\left[\tilde{\delta}_i v_{n,g}e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]$$
$$+\frac{1}{E_j}\left\{\sum_{g\neq 0}\left[\left(\alpha_{i,g}P_{ii,g}Q_{i,g}-\left(\tilde{P}_{i,0}+\tilde{\delta}_i\right)Z_{i,g}\right)\left(\frac{\partial\ln Q_{i,g}}{\partial\ln\tilde{P}_{ij,k}}\right)_{\mathbf{C}_i}\right]+\left(\frac{\partial\left(T_i+\Pi_i\right)}{\partial\ln\mathbf{C}_i}-\tilde{\delta}_i\frac{\partial Z}{\partial\ln\mathbf{C}_i}\right)\cdot\frac{\mathrm{d}\ln\mathbf{C}_i}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right\}=0\tag{A.33}$$

Similarly, we can consolidate Equations A.30 (income effects relating to energy prices) and Equation A.32 (carbon emissions relating to energy prices) and divide all the resulting terms by total energy expenditure, $\tilde{P}_{j,0}Z_j$, to characterize following F.O.C. for *energy* export prices to market j,

$$(1-\Lambda_{ij,0})\,\lambda_{ij,0}+\left(1-\frac{P_{ij,0}}{\tilde{P}_{ij,0}}\right)\lambda_{ij,0}\tilde{\varepsilon}_{ij,0}^{(ij,0)}-\sum_{n\neq i}\left[\widetilde{\omega}_{ni,0}\lambda_{nj,0}\tilde{\varepsilon}_{nj,0}^{(ij,0)}\right]$$
$$-\frac{\tilde{\delta}_i}{\tilde{P}_{j,0}}\lambda_{ij,0}\zeta_j+\frac{1}{\tilde{P}_{j,0}Z_j}\left(\frac{\partial\left(T_i+\Pi_i\right)}{\partial\ln\mathbf{C}_i}-\tilde{\delta}_i\frac{\partial Z}{\partial\ln\mathbf{C}_i}\right)\cdot\frac{\mathrm{d}\ln\mathbf{C}_i}{\mathrm{d}\ln\tilde{P}_{ij,0}}=0, \tag{A.34}$$

where $\widetilde{\omega}_{ji,k}$ is defined under Equation A.23. Following Lemma 2, the last term in Equations A.33 and A.34 must equal zero if the first-order optimality condition for $\left\{\tilde{P}_{ji,k}\right\}_{j,k}$ is satisfied. In the following step we combine the F.O.C.s to solve for optimal taxes taking note of this property.

A.10 Solution to the System of First-Order Conditions

We now put together the F.O.C.s we derived for $\mathscr{P}=\left\{\tilde{P}_{ji,k}\right\}_{j,k}$ (Equation A.24), for $\mathscr{P}=\left\{\alpha_{i,k}\right\}_k$ (Equation A.28) and $\mathscr{P}=\left\{\tilde{P}_{ij,k}\right\}_{j\neq i,k}$ (Equations A.33 and A.34). These F.O.C.s are specified in terms of consumer-to-producer price wedges. We can recover the explicit tax rates corresponding to these wedges as follows:

$$\tau_{i,k}=\frac{\alpha_{i,k}P_{i,k}Q_{i,k}}{Z_{i,k}}-\tilde{P}_{i,0},\qquad\left(1+t_{ji,k}\right)=\frac{\tilde{P}_{ji,k}}{P_{ji,k}}\quad(j\neq i),\qquad\frac{1}{1+s_{i,k}}=\frac{\tilde{P}_{ii,k}}{P_{ii,k}},\qquad\frac{1}{1+x_{ij,k}}=\frac{\tilde{P}_{ij,k}}{P_{ij,k}}\Big/\frac{\tilde{P}_{ii,k}}{P_{ii,k}}.$$

where, for completeness, we allow for $s_{i,k}$ to denote the production subsidy, which, unlike export subsidies, is applied irrespective of the final location of sale. We introduce $s_{i,k}$ as an explicit policy instrument in our derivation to demonstrate its redundancy—as claimed in the main text. Considering the above equations, we can write the system of F.O.C.s specified by Equation A.28, **??**, A.33, and A.34 in terms of tax rates as

$$\left[\tilde{P}_{ji,k}\right]\qquad\sum_{n\neq i}\sum_{g\neq 0}\left[\left(1-\frac{1+\tilde{\delta}_i v_{n,g}}{1+t^*_{ni,g}}\right)e_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]+\sum_{g\neq 0}\left[\left(1-\frac{1}{1+s^*_{i,g}}\right)e_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]$$
$$+\left(1-\frac{1}{1+s^*_{i,0}}\right)\lambda_{ii,0}\frac{\tilde{P}_{i,0}Z_i}{E_i}\frac{\mathrm{d}\ln C_{ii,0}}{\mathrm{d}\ln\tilde{P}_{ji,k}}+\sum_{n\neq i}\left[\left(1-\frac{1+\omega_{ni,0}}{1+t^*_{ni,0}}\right)\lambda_{ni,0}\frac{\tilde{P}_{i,0}Z_i}{E_i}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]=0$$

$$\left[\alpha_{i,k}\right]\qquad\left[\left(\tilde{P}_{i,0}+\tau^*_{i,k}\right)-\left(\tilde{\delta}_i+\tilde{P}_{i,0}\right)\right]Z_{i,k}\frac{\partial\ln Z_{i,k}}{\partial\ln\left(1-\alpha_{i,k}\right)}=0$$

$$\left[\tilde{P}_{ij,k}\right]\qquad e_{ij,k}-\sum_{g\neq 0}\left[x^*_{ij,g}e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right]-\sum_{n\neq i}\sum_{g\neq 0}\left[\tilde{\delta}_i v_{n,g}e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]=0$$

$$\left[\tilde{P}_{ij,0}\right]\qquad\left(1-\Lambda_{ij,0}\right)\lambda_{ij,0}-x^*_{ij,0}\lambda_{ij,0}\varepsilon_{ij,0}^{(ij,0)}-\sum_{n\neq i}\left[\widetilde{\omega}_{ni,0}\lambda_{nj,0}\varepsilon_{nj,0}^{(ij,0)}\right]-\frac{\tilde{\delta}_i}{\tilde{P}_{j,0}}\lambda_{ij,0}\zeta_j=0$$

where the F.O.C.s for $\alpha_{i,k}$, $\tilde{P}_{ij,k}$, and $\tilde{P}_{ij,0}$ invoke Lemma 2 to eliminate the redundant terms. Also, notice that we need not to unpack the terms $\frac{\partial\ln Z_{i,k}(.)}{\partial\ln(1-\alpha_{i,k})}$ and $\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}$, as they are irrelevant to the resulting optimal tax solution, which involves a carbon tax that is *not* supplemented with any production tax or subsidy (i.e., $s^*_{i,k}=s^*_{i,0}=0$),

$$\tau^*_{i,k}\sim\tau^*_i=\tilde{\delta}_i \tag{A.35}$$

and import tariffs that are plain carbon border adjustments for non-energy imports and a sum of terms-of-trade- and climate-related restrictions for energy imports,

$$t^*_{ni,k} = \tau^*_i v_{n,k} \qquad\qquad t^*_{ni,0} = \omega_{ni,0} + \tau^*_i \sum_{\ell \neq i} \sum_{j \neq i} \left[\tilde{\psi}^{(i,0)}_{\ell n} \rho_{\ell j,0} \frac{\zeta_j}{\tilde{P}_{j,0}} \right]. \tag{A.36}$$

The above formulas hold irrespective of the underlying consumption utility aggregator. The optimal export tax can be recovered non-parametrically by plugging the above optimal tax formulas into the corresponding F.O.C. and inverting the resulting system. As a practical step, we first derive the optimal export tax formula in the semi-parametric case where preferences are additively separable across industries and generalized separable within industries. In this case, $\varepsilon^{(ij,g)}_{ij,k} = 0$ if $g \neq k$. Moreover, per Cournot aggregation, $-\left(1 - \lambda_{ij,k}\right) \varepsilon^{(ij,k)}_{nj,k} = \lambda_{ij,k} \left(1 + \varepsilon_{ij,k}\right)$, where $\lambda_{ij,k}$ denotes the within industry expenditure share. Plugging these relationships into the F.O.C.s presented above, yields the following optimal export subsidy formulas for non-energy and energy goods:

$$\left(1 + x^*_{ij,k}\right) = \frac{1 + \varepsilon_{ij,k}}{\varepsilon_{ij,k}} \sum_{n \neq i} \left[\left(1 + \tilde{\delta}_i \nu_{n,k}\right) \grave{\lambda}_{nj,k} \right] \sim \frac{1 + \varepsilon_{ij,k}}{\varepsilon_{ij,k}} \sum_{\ell \neq i} \left[\left(1 + t^*_{ni,k}\right) \grave{\lambda}_{nj,k} \right] \tag{A.37}$$

$$\left(1 + x^*_{ij,0}\right) = \frac{1 + \varepsilon_{ij,0}}{\varepsilon_{ij,0}} \sum_{n \neq i} \left[\left(1 + t^*_{ni,0}\right) \grave{\lambda}_{nj,0} \right] - \left(\Lambda_{ij,0} + \tau^*_i \frac{\zeta_j}{\tilde{P}_{j,0}} \right) \frac{1}{\varepsilon_{ij,0}} \tag{A.38}$$

where $\varepsilon_{ij,k} \sim \tilde{\varepsilon}^{(ij,k)}_{ij,k}$ denotes the own-price elasticity of demand and $\grave{\lambda}_{nj,k} \equiv \lambda_{nj,k} / \left(1 - \lambda_{ij,k}\right)$, which satisfies the adding up property, $\sum_{\ell \neq i} \grave{\lambda}_{nj,k} = 1$. The above formulas describe the optimal policy non-parametrically in terms of generic final and input demand elasticity values. In the CES case, these elasticity values are given by $\varepsilon_{ij,k} = -\sigma_k + (\sigma_k - 1)\lambda_{ij,k}$, implying the following formulas for export subsidies:

$$\text{[CES preferences]} \qquad \left(1 + x^*_{ij,k}\right) = \frac{\sigma_k - 1}{1 + (\sigma_k - 1)\left(1 - \lambda_{ij,k}\right)} \sum_{\ell \neq i} \left[\left(1 + t^*_{ni,k}\right) \lambda_{nj,k} \right].$$

Non-Separable Preferences. Previously, we derived the optimal export tax formula for the case where preferences are additively separable across industries. Here, we provide the formula for the general case. Appealing to the aggregation property, $e_{ij,k} + \sum_{n,g} e_{nj,g} \varepsilon^{(ij,k)}_{nj,g} = 0$, of Marshallian demand functions, we can express the first-order condition w.r.t. the export price $\tilde{P}_{ij,k}$, as follows:

$$-\sum_{g \neq 0} \left[\left(1 + x^*_{ij,g}\right) e_{ij,g} \varepsilon^{(ij,k)}_{ij,g} \right] - \sum_{n \neq i} \sum_{g \neq 0} \left[\left(1 + \tilde{\delta}_i \nu_{n,g}\right) e_{nj,g} \varepsilon^{(ij,k)}_{nj,g} \right] = 0,$$

Noting that $t^*_{ni,g} = \tilde{\delta}_i \nu_{n,g}$, we can re-write the above equation in matrix notation as

$$\underbrace{\begin{bmatrix} e_{ij,1}\varepsilon^{(ij,1)}_{ij,1} & \ldots & e_{ij,K}\varepsilon^{(ij,1)}_{ij,K} \\ \vdots & \ddots & \vdots \\ e_{ij,1}\varepsilon^{(ij,K)}_{ij,1} & \ldots & e_{ij,K}\varepsilon^{(ij,K)}_{ij,K} \end{bmatrix}}_{\mathbf{E}_{ij}} \underbrace{\begin{bmatrix} 1 + x^*_{ij,1} \\ \vdots \\ 1 + x^*_{ij,K} \end{bmatrix}}_{\mathbf{1}+\mathbf{x}^*_{ij}} = -\underbrace{\begin{bmatrix} \mathbf{E}_{1j} & \cdots & \mathbf{E}_{Nj} \end{bmatrix}}_{\mathbf{E}_{-ij}} \underbrace{\begin{bmatrix} \mathbf{1} + \mathbf{t}^*_{1j} \\ \vdots \\ \mathbf{1} + \mathbf{t}^*_{Nj} \end{bmatrix}}_{\mathbf{1}+\mathbf{t}^*_i},$$

where $\mathbf{E}_{nj}$ is defined analogous to $\mathbf{E}_{ij}$ for all n and $\mathbf{t}^*_{nj} = \left[t^*_{nj,k}\right]_k$ is a $K \times 1$ vector consisting of optimal tariffs on origin n varieties. Since $\mid e_{ij,k}\varepsilon^{(ij,k)}_{ij,k} \mid - \sum_{k \neq j} e_{ij,g}\varepsilon^{(ij,k)}_{ij,g} = e_{ij,k} + \sum_{n \neq i} \sum_g e_{ij,g}\varepsilon^{(ij,k)}_{nj,g} > 0$, then $\mathbf{E}_{ij}$

is strict diagonally dominant. Hence, given the Lèvy-Desplanques Theorem, $\mathbf{E}_{ij}$ is invertible (Horn and Johnson (2012)) and the above system recovers $\mathbf{1}+\mathbf{x}_{ij}^{*}$ as

$$\mathbf{1}+\mathbf{x}_{ij}^{*}=-\mathbf{E}_{ij}^{-1}\mathbf{E}_{-ij}\left(\mathbf{1}+\mathbf{t}_{i}^{*}\right). \tag{A.39}$$

A.11 Small Open Economy + CES-Cobb-Douglas

The small open case of our formulas can be helpful for obtaining intuition. Consider a small open economy for which $\rho_{ni,k}\approx\lambda_{in,k}\approx0$ for all k. In addition, suppose preferences have a CES-Cobb-Douglas parametrization. In that case, the optimal policy schedule becomes:

$$\begin{cases}\tau_{i,k}^{*}=\tau_{i}^{*}=\tilde{\delta}_{i}/\tilde{P}_{i,0} & \text{[carbon tax]}\\ t_{ni,k}^{*}=\tilde{\delta}_{i}v_{n,k}\qquad t_{ni,0}^{*}=0 & \text{[import tax]}\\ 1+x_{in,k}^{*}=\frac{\sigma_{k}-1}{\sigma_{k}}\left(1+\tilde{\delta}_{i}\sum_{n\neq i}v_{n,k}\lambda_{nj,k}\right) & \text{[export subsidy (non-energy)]}\\ 1+x_{in,0}^{*}=\frac{\sigma_{0}-1}{\sigma_{0}}+\frac{\zeta_{n}}{\sigma_{0}}\left(\tilde{\delta}_{i}/\tilde{P}_{n,0}\right) & \text{[export subsidy (energy)]}\end{cases}$$

where, if the non-energy production function is also CES, then $\zeta_{n}=-\varsigma\left(1-\alpha_{n}\right)$, where $\alpha_{n}=\sum\alpha_{n,g}\frac{Z_{n,g}}{Z_{n}}$ is the average carbon intensity in country n.

B Optimal Cooperative Policies

This section characterizes optimal policy formulas for the globally first best. In this scenario, a global planner maximizes a weighted average of national welfare values, as the *global welfare*, subject to the availability of lump-sum transfers (the outcome of this scenario is equivalent to a Nash bargaining game with side payments).

B.1 First-Best: Globally Optimal Carbon Taxes

We consider a planning problem where the planner maximizes the global welfare, as the weighted average of of national welfare values, by setting prices and implementing income transfers. The planner's choice of transfers determine the share of national expenditure (π_i) from global income, i.e., $E_i=\pi_iY$, where $Y=\sum_i Y_i$. The optimal policy $\tilde{\mathbb{P}}\equiv\left\{\tilde{\mathbf{P}},\boldsymbol{\alpha},\boldsymbol{\pi}\right\}$ consisting of consumers prices, energy input shares, and inter-country transfers, can be obtained as the solution to following planning problem

$$\max_{\tilde{\mathbb{P}}}W\left(\tilde{\mathbb{P}};\mathbf{w}\right)=\sum_{n}\vartheta_{n}\log\left(V_{n}\left(E_{i},\tilde{\mathbf{P}}_{n}\right)-\delta_{n}Z\left(\tilde{\mathbb{P}};\mathbf{w}\right)\right)$$

subject to equilibrium constraints and the availability of lump-sum international transfers, whereby $E_i=\pi_iY\left(\tilde{\mathbb{P}};\mathbf{w}\right)$. The Pareto weights, ϑ_n, are exogenous policy parameters that add up to one, $\sum_n\vartheta_n=1$. As before, let $W_n=V_n\left(.\right)-\delta_nZ^{(global)}$ denote country n's climate-adjusted welfare.

First-order Condition *w.r.t.* $\tilde{P}_{ji,k}$. Appealing to Roy's identity, whereby $\frac{\partial V_i(.)}{\partial E_i} / \frac{\partial V_i(.)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} C_{ji,k}$, we can write the F.O.C. *w.r.t.* $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}$ as:

$$\frac{\partial W(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} = \sum_n \left(\frac{\vartheta_n}{W_n} \frac{\partial V_n(.)}{\partial E_n} \pi_n \right) \frac{\partial Y(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} - \frac{\vartheta_i}{W_i} \frac{\partial V_i(.)}{\partial E_i} \tilde{P}_{ji,k} C_{ji,k}$$
$$- \sum_n \left[\frac{\vartheta_n}{W_n} \delta_n \right] \frac{\partial Z(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} + \frac{\partial W(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \mathbf{w}} \cdot \frac{\mathrm{d} \ln \mathbf{w}}{\mathrm{d} \ln \tilde{P}_{ji,k}} = 0. \tag{B.1}$$

The neutrality of wages applies in this case, , $\frac{\partial W(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \mathbf{w}} = \mathbf{0}$, using virtually the same steps as we have taken in Section A.4. So, the last term in the above equation can be discarded. We can specify the first-term on the right-hand side by noting that global income is given by:

$$Y = \sum_i [w_i L_i + \Pi_i] + \sum_{n,i} \sum_k \left[\left(\tilde{P}_{ni,k} - [1 - \alpha_{i,k}] P_{ni,k} \right) C_{ni,k} \right] - \sum_i \sum_k \tilde{P}_{i,0} Z_{i,k}.$$

Extrapolating from the derivations presented in Section A and noting that $\tau_{i,k} Z_{i,k} \equiv \alpha_{i,k} P_{i,k} Q_{i,k} - \tilde{P}_{i,0} Z_i$, we can write the change in global income as,

$$\frac{\partial Y(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} = \sum_g \sum_{n,i} \left[\left(\tilde{P}_{ni,g} - P_{ni,g} \right) C_{ni,g} \frac{d \ln C_{ni,g}}{d \ln \tilde{P}_{ji,k}} \right] + \sum_n \left[\tau_{n,k} \frac{\partial Z_{n,k}(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right] = 0.$$

Noting that any policy-driven changes to energy producer prices are neutralized by a proportional change to energy reserves surplus, per Hotelling's lemma, $\left[\frac{\partial \Pi_i(P_{i,0}, w_i)}{\partial \ln P_{ii,0}} - P_{ii,0} Q_{i,0} \right] \frac{\partial \ln P_{ii,0}}{\partial \ln \mathscr{P}} = 0$ for all $\mathscr{P} \in \tilde{\mathbb{P}}$. Plugging the expression for $\frac{\partial Y(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}}$ back into Equation B.1, and noting that $\delta_i = \tilde{\delta}_i \frac{\partial V_i(.)}{\partial E_i}$, yields

$$\frac{1}{Y} \left[\sum_n \left(\vartheta_n \frac{V_n}{W_n} \frac{\partial \ln V_n(.)}{\partial \ln E_n} \right) - \frac{\vartheta_i}{\pi_i} \frac{V_i}{W_i} \frac{\partial \ln V_i(.)}{\partial \ln E_i} \right] \tilde{P}_{ji,k} C_{ji,k} + \sum_g \sum_{n,i} \left[\left(\tilde{P}_{ni,g} - P_{ni,g} \right) C_{ni,g} \frac{d \ln C_{ni,g}}{d \ln \tilde{P}_{ji,k}} \right]$$
$$\frac{1}{Y} \sum_{\ell,k} \left\{ \left[\sum_n \left(\vartheta_n \frac{V_n}{W_n} \frac{\partial \ln V_n(.)}{\partial \ln E_n} \right) \tau_{\ell,k} - \sum_n \frac{\vartheta_n}{\pi_n} \frac{V_n}{W_n} \tilde{\delta}_n \right] \frac{\partial Z_{\ell,k}(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \tilde{P}_{ji,k}} \right\} = 0. \tag{B.2}$$

Here, for expositional purposes, preferences are assumed to be homothetic, implying that $\frac{\partial \ln V_n(.)}{\partial \ln E_n} = 1$. Next, we characterize the F.O.C. *w.r.t.* the energy input share policy.

First-order Condition *w.r.t.* $\alpha_{i,k}$. Fixing consumer prices via the policy choice, $\tilde{\mathbf{P}}$, the choice vis-a-vis the energy input share $\alpha_{i,k}$ has no bearings on the consumer price index. Accordingly, the F.O.C. *w.r.t.* this policy choice can be specified as:

$$\frac{\partial W(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln(1-\alpha_{i,k})} = \sum_n \left(\frac{\vartheta_n}{W_n} \frac{\partial V_n(.)}{\partial E_n} \pi_n \right) \frac{\partial Y(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln(1-\alpha_{i,k})}$$
$$- \sum_n \left[\frac{\vartheta_n}{W_n} \delta_n \right] \frac{\partial Z^{(global)}(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln(1-\alpha_{i,k})} + \frac{\partial W(\tilde{\mathbb{P}};\mathbf{w})}{\partial \ln \mathbf{w}} \cdot \frac{\mathrm{d} \ln \mathbf{w}}{\mathrm{d} \ln(1-\alpha_{i,k})} = 0. \tag{B.3}$$

To determine the change in global income in Equation (B.3), we appeal to the relationship derived in Section A.8, $\left(\frac{\partial \ln\left([1-\alpha_{i,k}] P_{ii,k} Q_{i,k} \right)}{\partial \ln(1-\alpha_{i,k})} \right)_{\mathbf{C}} = \frac{\alpha_{i,k}}{1-\alpha_{i,k}} \frac{\partial \ln Z_{i,k}(\mathbb{P}_i;\mathbf{w})}{\partial \ln(1-\alpha_{i,k})}$. Invoking this relationship and following similar steps to those delineated in Section A.8 delivers:

$$\frac{\partial Y\left(\tilde{\mathbb{P}};\mathbf{w}\right)}{\partial\ln\left(1-\alpha_{i,k}\right)}=\sum_{g}\sum_{n,i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)}\right]+\sum_{n}\left[\tau_{n,k}\frac{\partial Z_{n,k}\left(\tilde{\mathbb{P}};\mathbf{w}\right)}{\partial\ln\left(1-\alpha_{i,k}\right)}\right]=0.$$

Plugging the above equation back into Equation (B.3) and noting that $\delta_i=\tilde{\delta}_i\frac{\partial V_i(.)}{\partial E_i}$, yields

$$\begin{aligned}\frac{\partial W\left(\tilde{\mathbb{P}};\mathbf{w}\right)}{\partial\ln\left(1-\alpha_{i,k}\right)}=&\sum_{g}\sum_{n,i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\left(1-\alpha_{i,k}\right)}\right]\\&+\frac{1}{Y}\sum_{n}\left\{\left[\sum_{n}\left(\vartheta_n\frac{V_n}{W_n}\frac{\partial\ln V_n\left(.\right)}{\partial\ln E_n}\right)\tau_{\ell,k}-\sum_{n}\frac{\vartheta_n}{\pi_n}\frac{V_n}{W_n}\tilde{\delta}_n\right]\frac{\partial Z_{n,k}\left(\tilde{\mathbb{P}};\mathbf{w}\right)}{\partial\ln\left(1-\alpha_{i,k}\right)}\right\}=0.\end{aligned}\tag{B.4}$$

Recovering the Optimal Policy. The optimal policy can be obtained by jointly solving the system of equations specified by (B.2) and (B.4). Without loss of generality, and for a clearer exposition, suppose preferences are homothetic, i.e., $\frac{\partial\ln V_n(.)}{\partial\ln E_n}=1$. Then, the trivial solution to Equation B.2 entails that $\frac{\vartheta_i}{\pi_i}\frac{V_i}{W_i}\frac{\partial\ln V_i(.)}{\partial\ln E_i}=\sum_n\left(\vartheta_n\frac{V_n}{W_n}\frac{\partial\ln V_n(.)}{\partial\ln E_n}\right)$, from which we could recover the optimal income shares, π_i^*. The trivial solution to (B.2) and (B.4) also requires zero good-specific taxes—i.e., $\tilde{P}_{ji,k}=P_{ji,k}$ for all ji,k. The optimal carbon tax meanwhile is a Pigouvian tax that internalizes the global externality of carbon emissions. Consolidating these points, the optimal border taxes are zero and the optimal income shares and carbon tax rates are given by:

$$\pi_i^{\star}=\frac{\vartheta_i\frac{V_i}{W_i}}{\sum_n\vartheta_n\frac{V_n}{W_n}},\qquad\qquad\tau_i^{\star}=\sum_{n}\tilde{\delta}_i.\tag{B.5}$$

Alternative Objective Function Specification. In the above, we chose the commonly-used weighted sum of logs as a social welfare function. We emphasize that a different welfare aggregation used by the planner may alter the globally optimal policy choice. Suppose the planner maximizes the following welfare function,

$$\max_{\tilde{\mathbb{P}}}\prod_{n}\left\{\left(V_n\left(E_i,\tilde{\mathbf{P}}_n\right)/\vartheta_n\right)^{\vartheta_n}\right\}-\delta Z^{(global)};$$

subject to equilibrium constraints, and the availability of lump-sum international transfers, whereby $E_i=\pi_iY$. In this specification, $\delta=\Delta\left(\delta_1,...,\delta_N\right)$ aggregates over the country-specific climate damage parameters, with a simple sum, $\delta=\sum_n\delta_n$, constituting a special case. Under this welfare function, the same derivation steps, as outlined before, deliver the following optimal policy formula:

$$\pi_i^{\star}\sim\frac{E_i}{Y}=\vartheta_i,\qquad\qquad\tau_n^{\star}=\left(\prod_{n}\tilde{P}_n^{\vartheta_n}\right)\delta\tag{B.6}$$

and, as before, the optimal border taxes are zero. Note the two subtle differences compared to Equation (B.5). First, the optimal carbon tax, as before, internalizes the global carbon externality, but it employs a different CPI deflator. Second, a country's income share becomes precisely equal to the weight assigned in the planner's objective function.

C Alternative Optimal Policy Designs

This section examines optimal policy under alternative objective functions and policy constraints. First, we derive the globally optimal border taxes that maximize global welfare under second-best conditions where carbon taxes are unavailable. Second, we characterize the *unilateral policy frontier* by maximizing a weighted combination of domestic and foreign welfare using unilateral policy tools. Third, we generalize our optimal policy formulas to explicitly include energy extraction taxes.

C.1 Second-Best: Globally Optimal Border Taxes

Consider a second-best cooperative scenario in which carbon taxation (or plain output taxation) is not politically feasible. The optimal policy, in this case, is obtained as the solution to a planning problem where the global planner selects border taxes and lump-sum transfers to maximize, $W \equiv \sum_n \vartheta_n \log W_n\left(\tilde{\mathbb{T}};\mathbf{w}\right)$. The policy set $\tilde{\mathbb{T}} = \{\mathbf{t}, \boldsymbol{\pi}\}$ includes the global vector of trade taxes $\mathbf{t} = \left\{t_{n\ell,k}\right\}_{n\neq\ell}$ and each country's share from global income $\boldsymbol{\pi} = \{\pi_i\}$ that determines lump-sum international transfers. As in previously-examined optimal policy scenarios, equilibrium outcomes can be expressed as an explicit function of policy and wages $\left(\tilde{\mathbb{T}};\mathbf{w}\right)$.

As an intermediate step, we appeal to the Lerner symmetry to show that wage effects are welfare neutral given the available policy instruments. Recall that a policy-wage combination is feasible iff the vector of wage rates, $\mathbf{w}$, satisfies the labor-market clearing conditions worldwide given $\tilde{\mathbb{T}}$. Moreover, let $\mathcal{E}$ denote the set of feasible policy-wage combinations. Let $\mathbf{t}_{ji}$ and $\mathbf{t}_{ij}$ denote the subset of tariffs collected and paid by country i. And let $\mathbf{t}_{-i} \equiv \mathbf{t} - \left\{\mathbf{t}_{ji}, \mathbf{t}_{ij}\right\}$ denote all other tariff variables. The Lerner symmetry asserts that for any $a \in \mathbb{R}_+$,

$$\begin{cases} \left(\mathbf{t}_{ji}, \mathbf{t}_{ij}, \mathbf{t}_{-i}, \boldsymbol{\pi}; w_i, \mathbf{w}_{-i}\right) \in \mathcal{E} \implies \left(a\mathbf{t}_{ji}, \frac{1}{a}\mathbf{t}_{ij}, \mathbf{t}_{-i}, \boldsymbol{\pi}; aw_i, \mathbf{w}_{-i}\right) \in \mathcal{E} \\ W_n\left(\mathbf{t}_{ji}, \mathbf{t}_{ij}, \mathbf{t}_{-i}, \boldsymbol{\pi}; w_i, \mathbf{w}_{-i}\right) = W_n\left(a\mathbf{t}_{ji}, \frac{1}{a}\mathbf{t}_{ij}, \mathbf{t}_{-i}, \boldsymbol{\pi}; aw_i, \mathbf{w}_{-i}\right) \qquad \forall n \end{cases}.$$

The above result immediately indicates that any possible welfare gains from perturbing $\mathbf{w}$, can be perfectly mimicked with an appropriate adjustment to the global tariff vector $\mathbf{t}$. That is, any possible welfare gains via wage adjustments will be already internalized by the optimal policy choice $\tilde{\mathbb{T}}^*$, amounting to $\frac{\partial W\left(\tilde{\mathbb{T}}^*;\mathbf{w}^*\right)}{\partial \mathbf{w}} = 0$.[61] In addition to using this result, we reformulate the optimal policy problem into a problem where the central planner chooses prices rather than tariffs. Since the central planner can set border taxes but not domestic taxes/subsidies, she has control over the consumer prices of all goods that cross international borders—namely, $\tilde{\mathbf{P}}^{(border)}$where $\tilde{\mathbf{P}}^{(border)} \equiv \left\{\tilde{P}_{ni,k}\right\}_{n\neq i}$. Using this correspondence, we can reformulate all equilibrium variables as function of$\left\{\tilde{\mathbf{P}}_{-nn}\right\}_n$, income shares, and wages, $\mathbf{w}$, and solve the optimal policy problem accordingly.

Lemma 3. *The global optimal border taxes can be obtained as the optimal choice of a central planer selecting the "consumer" price of traded varieties and lump-sum transfer given wages ($\bar{w}$). Namely,*

$$\max_{\tilde{\boldsymbol{P}}^{(border)},\boldsymbol{\alpha}} \sum_{i=1}^{N} \vartheta_n \log W_i\left(\tilde{\boldsymbol{P}}^{(border)}, \boldsymbol{\alpha}; \overline{\boldsymbol{w}}\right),$$

[61] Also, note that the rental rate of energy reserves, r_i, in the isomorphic equilibrium co-varies with the wage rate, i.e., $r_i' = ar_i$ and $\mathbf{r}_{-i}' = \mathbf{r}_{-i}$. Hence, the neutrality of wages extends to rental rates and the implied energy surplus, $\Pi_i = r_i\bar{R}_{i,0}$. Stated formally, $\partial W/\partial \mathbf{r} = 0$ in the neighborhood of the optimum policy, $\tilde{\mathbb{T}}^*$.

the solution to which determines the globally optimal border tax on good ji,k *as the optimal wedge between consumer and producer price:* $1+t^{\star}_{ji,k}=\tilde{P}^{\star}_{ji,k}/P_{ji,k}$.

Next, we derive the necessary F.O.C.s for optimality w.r.t. each price instrument borrowing from our previous derivations. Appealing to Roy's identity, $\frac{\partial V_i(.)}{\partial E_i}/\frac{\partial V_i(.)}{\partial \ln \tilde{P}_{ji,k}}=-\tilde{P}_{ji,k}C_{ji,k}$, and treating wages as given, the F.O.C. with respect to $\tilde{P}_{ji,k}$ can be specified as

$$\frac{\partial W\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\partial \ln \tilde{P}_{ji,k}}=\sum_n\left(\frac{\vartheta_n}{W_n}\frac{\partial V_n\left(.\right)}{\partial E_n}\pi_n\right)\frac{\partial Y\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\partial \ln \tilde{P}_{ji,k}}-\frac{\vartheta_i}{W_i}\frac{\partial V_i\left(.\right)}{\partial E_i}\tilde{P}_{ji,k}C_{ji,k}-\sum_n\left[\frac{\vartheta_n}{W_n}\delta_n\right]\frac{\partial Z\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\partial \ln \tilde{P}_{ji,k}}. \tag{C.1}$$

Global income Y consists of factor payments plus tax revenues. Namely,

$$Y=\sum_i\left(w_iL_i+\Pi_i\right)+\sum_{g,i}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\right]+\sum_n\bar{\tau}_nZ_n$$

The last term represents carbon tax revenues based on pre-determined national tax rate $\bar{\tau}_n$, not chosen by the central planner. The logic for including this term is to accommodate settings where carbon taxes can be exogenously in place, but governments cannot elevate it to the globally optimal rate, $\tau^{\star}$. Hence, the first term in Equation C.1 consisting of income effects can be unpacked as:

$$\frac{\partial Y\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\partial \ln \tilde{P}_{ji,k}}=\sum_{g,i}\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]+\sum_n\left[\bar{\tau}_n\frac{\mathrm{d}Z_n\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\mathrm{d}\tilde{P}_{ji,k}}\right]=0.$$

Since wages and input prices of energy are fixed given $\bar{\mathbf{w}}$ and $\bar{\boldsymbol{\tau}}$, the change in emissions due to policy is driven purely by scale effects and can be characterized as:

$$\begin{aligned}\frac{\partial Z\left(\tilde{\mathbb{T}};\mathbf{w}\right)}{\partial \ln \tilde{P}_{ji,k}}&=\sum_n\sum_g\left[Z_{n,g}\frac{\partial \ln Z_{n,g}}{\partial \ln Q_{n,g}}\frac{\partial \ln Q_{n,g}\left(.\right)}{\partial \ln C_{ni,g}}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]\\&=\sum_{n,g}\left[Z_{n,g}r_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]=\sum_{n,g}\left[Z_{n,g}\frac{P_{ni,g}C_{ni,g}}{Y_{i,g}}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]=\sum_{n,g}\left[v_{n,g}P_{ni,g}C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right],\end{aligned}$$

where $v_{n,g}=Z_{n,g}/Y_{n,g}$ where $Y_{n,g}=\sum_{n'}P_{nn',g}C_{nn'g}$. Plugging the above expressions back onto Equation (C.1) and considering that optimal transfers satisfy $\pi^{\star}_i=\frac{\vartheta_i\frac{V_i}{W_i}}{\sum_n\vartheta_n\frac{V_n}{W_n}}$ from our previous derivation, we obtain the following necessary first-order condition for optimality:

$$\sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-\left[1+\left(\tau^{\star}-\bar{\tau}_n\right)v_{n,g}\right]P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]-\left(\tau^{\star}-\bar{\tau}_i\right)\sum_g\left[\nu_{i,g}P_{ii,g}Q_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln \tilde{P}_{ji,k}}\right]=0,$$

where $\tau^{\star}=\sum_n\tilde{\delta}_n$ represents the globally optimal carbon tax under the first-best allocation. In our previous derivations we did not have to characterize the general equilibrium demand elasticities, $\mathrm{d}\ln C/\mathrm{d}\ln \tilde{P}$, because we did not have a missing policy problem. Here, however, we must characterize the noted elasticities, and to make progress we assume that demand for non-energy goods is income inelastic. Under this assumption, $\mathrm{d}\ln C_{ni,g}/\mathrm{d}\ln \tilde{P}_{ji,k}=\varepsilon^{(ji,k)}_{ni,g}$, which simplifies the first-order condition as follows after dividing all the terms by E_i and noting that $e_{ni,g}=\tilde{P}_{ni,g}C_{ni,g}/E_i$

$$\sum_{n\neq i}\sum_g\left[\left(1-\left[1+\left(\tau^{\star}-\bar{\tau}_n\right)v_{n,g}\right]\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)e_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right]-\left(\tau^{\star}-\bar{\tau}_i\right)\sum_g\left[v_{i,g}e_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right]=0.$$

To simplify the above equation further, we appeal to a corollary of the Slutsky equation, $e_{ni,g}\varepsilon^{(ji,k)}_{ni,g}=$

$e_{ji,k}\varepsilon_{ji,k}^{(ni,g)}$ and note that demand is homogeneous of degree zero, whereby $\sum_{n\neq i}\sum_g \varepsilon_{ji,k}^{(ni,g)} = -\sum_g \varepsilon_{ji,k}^{(ii,g)}$. Invoking these properties of demand simplifies the first-order condition as:

$$\sum_{n\neq i}\sum_{g}\left[\left(1+\left(\tau^{\star}-\bar{\tau}_n\right)v_{n,g}\right)\frac{1}{1+t_{ni,g}^{\star}}\varepsilon_{ji,k}^{(ni,g)}\right]-\sum_{g}\left[\left(1+\left(\tau^{\star}-\bar{\tau}_i\right)v_{i,g}\right)\varepsilon_{ji,k}^{(ii,g)}\right]=0. \tag{C.2}$$

The first-order condition described by Equation (C.2) represents a system of equations that can be condensed using matrix notation. In particular, invert the following matrix-equivalent system to obtain the $N(K-1)\times 1$ vector of optimal import tariffs $\mathbf{T}_{-ii}^{\star} = \left[\frac{1}{1+t_{ji,g}^{\star}}\right]_{j,k}$ per destination i,

$$\mathbf{T}_{-ii}^{\star} = \left(\tilde{\mathbf{E}}_{-ii}^{(-ii)}\right)^{-1}\tilde{\mathbf{E}}_{-ii}^{(ii)}\mathbf{1}_K,$$

where$\mathbf{1}$ is a $K\times 1$ column vector of ones; and $\tilde{\mathbf{E}}_{-ii}^{(-ii)}$ and $\tilde{\mathbf{E}}_{-ii}^{(ii)}$ are respectively $(N-1)K\times N(K-1)$ and $N(K-1)\times K$ matrixes of untaxed-carbon-adjusted demand elasticities:

$$\tilde{\mathbf{E}}_{-ii}^{(-ii)} \equiv \left[\left(1+\left(\tau^{\star}-\bar{\tau}_n\right)v_{n,g}\right)\varepsilon_{ji,k}^{(ni,g)}\right]_{jk,ng}; \qquad \tilde{\mathbf{E}}_{-ii}^{(ii)} \equiv \left[\left(1+\left(\tau^{\star}-\bar{\tau}_i\right)v_{i,g}\right)\varepsilon_{ji,k}^{(ii,g)}\right]_{jk,g}.$$

CES Preferences with Additive Separability Across Industries. We can derive simple formulas for the globally optimal carbon border taxes in the special case where preferences are additively separable across industries and CES within industries. In that case, Marshallian demand elasticities are given by:

$$\varepsilon_{ji,k}^{ni,g}=0 \text{ if } g\neq k; \qquad \varepsilon_{ji,k}^{(ni,k)}=(\sigma_k-1)\lambda_{ni,k} \text{ if } n\neq j; \qquad \varepsilon_{ji,k}^{(ji,k)}=-1-(\sigma_k-1)(1-\lambda_{ji,k}).$$

Plugging these elasticity values back into Equation (C.2) delivers the following first-order condition w.r.t. the price of good ji,k:

$$\sum_{n\neq i}\left[\frac{1}{1+t_{ni,k}^{\star}}\left(1+\left(\tau^{\star}-\bar{\tau}_n\right)\nu_{n,k}\right)\left[(\sigma_k-1)\lambda_{ni,k}-\mathbb{1}_{n=j}\sigma_k\right]\right]=\left[1+\left(\tau^{\star}-\bar{\tau}_i\right)\nu_{i,k}\right](\sigma_k-1)\lambda_{ii,k}$$

The symmetry in the above equation asserts that that $\mathscr{T}_{ni,k}^{\star}\equiv\left(1+\left(\tau^{\star}-\bar{\tau}_n\right)\nu_{n,k}\right)/\left(1+t_{ni,k}^{\star}\right)$ must be independent of origin subscripts n and uniform across all export partners—i.e., $\mathscr{T}_{ni,k}^{\star}=\mathscr{T}_{i,k}^{\star}$. Invoking this observation, it is straightforward to solve for $\mathscr{T}_{ni,k}^{\star}$, which yields:

$$1+t_{ni,k}^{\star}=\frac{1+(\sigma_k-1)\lambda_{ii,k}}{1+\left[1+\left(\tau^{\star}-\bar{\tau}_i\right)v_{i,k}\right](\sigma_k-1)\lambda_{ii,k}}\left[1+\left(\tau^{\star}-\bar{\tau}_n\right)v_{n,k}\right].$$

To provide intuition, the second term (on the right-hand side) is a border carbon tax based on the difference between the applied carbon tax and globally optimal rate in origin n. The first term is adjustment to mitigate substitutability between the taxable traded varieties and the non-traded variety ii,k. This term collapses to zero when when there is no substitutability (i.e., $\sigma_k=1$) or when the non-traded variety is already taxed at the optimal rate (i.e., $\bar{\tau}_i=\tau^{\star}$).

C.2 Country i's Unilateral Policy Frontier

This section characterizes an alternative unilateral policy design in which the home government maximizes its national welfare augmented by a weighted average of foreign welfare values. This characterization, by varying the weights assigned to foreign countries, traces out country i's *unilateral policy frontier*, representing the spectrum of welfare outcomes achievable through the unilateral policy instruments, $\mathbb{P}_i$. Each point on country i's unilateral policy frontier is the solution to the following planning problem:

$$\max_{\mathbb{P}_i} V_i\left(E_i, \tilde{\mathbf{P}}_i\right) - \delta_i Z^{(global)} + \sum_{n \neq i} \left[\vartheta_{ni}\left(V_n\left(.\right) - \delta_n Z^{(global)}\right)\right],$$

subject to equilibrium constraints. Here, ϑ_{ni} is the weight that country i assigns to country n's welfare relative to its own welfare. For a given set of weights $\boldsymbol{\vartheta}_i \equiv \{\vartheta_{ni}\}_{n \neq i}$, we denote the solution by $\mathbb{P}_i^{\ast}\left(\boldsymbol{\vartheta}_i\right)$. It is important to note that the unilateral policy frontier does *not* include the globally first-best outcome due to the fact that country i does not have access to policy instruments of other countries. However, it encompasses the following canonical policy scenarios:

1. If $\vartheta_{ni} = 0$ for all $n \neq i$. Then, the solution $\mathbb{P}_i^{\ast}$ corresponds with the *unilaterally optimal* policy, $\mathbb{P}_i^*$, which we derived earlier.

2. If $\vartheta_{ni} < 0$ for a subset of countries $n \in \tilde{\mathbb{C}} \subset \mathbb{C}/\{i\}$, the solution $\mathbb{P}_i^{\ast}$ imposes a *sanction* on countries in $\tilde{\mathbb{C}}$. In that case, country i manipulates its terms-of-trade vis-a-vis countries in $\tilde{\mathbb{C}}$ to impose extra penalty (relative to the above case) on them, as in Sturm Becko (Forthcoming).

3. The weigh assignment $\{\vartheta_{ni}\}_{n \neq i}$ is such that the foreign welfare, $W_{-i} = \sum_{n \neq i} \vartheta_{ni}\left(V_n\left(.\right) - \delta_n Z^{(global)}\right)$ is preserved. In that case, the solution $\mathbb{P}_i^{\ast}$ aligns with the *externality-free* unilaterally-optimal policy, as studied in Kortum and Weisbach (2020).

Methodologically, we take the same steps as in our earlier derivation of unilaterally optimal policy. Here, the only difference is that we should trace out the policy effects on foreign welfare through its inclusion in the objective function. For simplicity, we focus on the case of Cobb-Douglas-CES preferences for a small open economy, for which we obtain $\mathbb{P}_i^{\ast}$ for any set of welfare weights, ϑ_{ni}, as follows:

$$\begin{cases} \tau_i^{\ast} = \tilde{\delta}_i + \sum_{n \neq i} \tilde{\vartheta}_{ni}\tilde{\delta}_n & \text{[carbon tax]} \\ 1 + t_{ni,k}^{\ast} = (1 + \bar{t}_i) + \tau_i^{\ast} v_{i,k}, \qquad t_{ni,0}^{\ast} = \bar{t}_i & \text{[import tax]} \\ 1 + x_{in,k}^{\ast} = \frac{\sigma_k - 1}{\sigma_k}\left(1 + \bar{t}_i\right) + \frac{1}{\sigma_k}\tilde{\vartheta}_{ni} + \frac{\sigma_k - 1}{\sigma_k}\tau_i^{\ast}\sum_{j \neq i}\left[v_{j,k}\lambda_{nj,k}\right] & \text{[export subsidy (final good)]} \\ 1 + x_{in,0}^{\ast} = \frac{\sigma_0 - 1}{\sigma_0}\left(1 + \bar{t}_i\right) + \frac{1}{\sigma_0}\tilde{\vartheta}_{ni} + \frac{\zeta_n}{\sigma_0}\frac{\tau_i^{\ast}}{\tilde{P}_{n,0}} & \text{[export subsidy (energy)]} \end{cases}$$

where $\tilde{\vartheta}_{ni}$ is defined as: $\tilde{\vartheta}_{ni} = \vartheta_{ni}\frac{\tilde{P}_i}{\tilde{P}_n}$, reflecting the fact that a nominal income transfer between two countries translates to a welfare transfer up to their relative consumer price indexes. (Recall that the consumer price index is given by $\tilde{P}_n \equiv \partial V_n / \partial E_n$, and $\tilde{\delta}_n \equiv \tilde{P}_n \delta_n$). In a special case with quasi-linear demand and a large enough freely-traded sector, it is implied that $\tilde{P}_i = \tilde{P}_n$, and so, $\tilde{\vartheta}_{ni} = \vartheta_{ni}$.

Compared to the unilaterally optimal policy, which we have derived in details and extensively discussed earlier, the above policy outline is generically different in two ways: *(i)* The carbon tax, $\tau_i^{\ast}$, equals the domestic externality, $\tilde{\delta}_i$, plus a weighted sum of foreign externalities, $\sum_n \tilde{\vartheta}_{ni}\tilde{\delta}_n$. *(ii)*

The terms-of-trade components of border taxes are adjusted according to the welfare weights. For example, for final-good export subsidies, the optimal border policy formula is:

$$1 + x^{*}_{ni,k} = \underbrace{\frac{\sigma_k - 1}{\sigma_k}(1 + \bar{t}_i) + \frac{1}{\sigma_k}\tilde{\vartheta}_{ni}}_{\text{terms of trade}} + \underbrace{\frac{\sigma_k - 1}{\sigma_k}\tau^{*}_i \sum_{j \neq i}\left[v_{j,k}\lambda_{nj,k}\right]}_{\text{carbon border adjustment}}$$

where $\frac{1}{\sigma_k}\tilde{\vartheta}_{ni}$ is now part of the terms of trade manipulation, and the carbon border adjustment is itself regulated by the welfare weights implicit in $\tau^{*}_i = \tilde{\delta}_i + \sum_{n \neq i} \tilde{\vartheta}_{ni}\tilde{\delta}_n$.

Consider a point on the frontier that corresponds to the case where the weights assigned to foreign countries adjusted for the consumer price indices are one, wherein $\tilde{\vartheta}_{ni} = 1$ for all $n \neq i$. In that case, the home country taxes carbon at the globally optimal rate, $\tau^{*}_i = \sum_n \tilde{\delta}_n$, and exerts no terms of trade externality on foreign countries. However, as emphasized in the main text, governments acting in their own self interest often veer away from this ideal policy point. This tendency mirrors the ongoing issue of *free riding* in climate action, which has been our main motivation for brining in trade policy to the issue of international climate agreements.

Lastly, we use Figure A.5 to illustrate the policy frontier when country i is the EU. To construct the figure, we have used our calibrated model, running simulations by varying the weight that the EU assigns to non-EU countries. For simplicity, a common weight is assigned to all non-EU countries. The peak point, where the EU's welfare is maximized, corresponds to zero weights assigned to non-EU countries. When the weight that the EU assigns to non-EU countries becomes positive, the EU exerts a lower level of terms-of-trade transfers from non-EU countries. Therefore, relative to the peak point, the EU's welfare falls and the welfare of non-EU countries rises. In contrast, when the weight is negative, the EU's border policy becomes more punitive against non-EU countries, but that comes at a welfare cost to the EU (relative to its welfare under the optimal unilateral policy, and not necessarily relative to the status quo). The two points labelled as "Externality-Free" and "Maximal Sanction" on the figure highlight these two alternatives.

In addition, Figure A.6 shows the effects of EU's unilateral policy on welfare and carbon emissions along the weight that the EU assigns to non-EU countries. A higher weight raises the non-EU's welfare at the cost of the EU's welfare. Also note that, a higher weight leads to a larger emission reduction at the global level, which comes from the emission reduction in the EU, partially offset by the carbon leakage via the increase in emissions in non-EU regions.

C.3 Optimal Policy Formulas with Energy Extraction Taxes

Our optimal policy framework accommodates extraction subsidies or taxes as the wedge between the producer and consumer price of energy (as the good that the energy extraction industry produces), with the corresponding subsidy rate denoted by $(1 + s_{i,0}) = P_{ii,0}/\tilde{P}_{ii,0}$. Our derivations in Appendix A yielded $s^{*}_{i,0} = 0$, indicating that extraction tax-cum-subsidies are unnecessary for obtaining the unilaterally or globally optimal outcomes. Nonetheless, we are able to reformulate our optimal policy formulas to explicitly include extraction taxes. Below, we present these formulas and explain the logic for why extraction taxes are redundant.

Unilaterally Optimal Policy with Extraction Taxes. To present the unilaterally optimal policy formulas with extraction taxes, we introduce $\mathscr{T}_{i,0}$ to directly denote the *ad valorem* extraction tax rate.

More formally,

$$1+\mathscr{T}_{i,0} \equiv \frac{\tilde{P}_{ii,0}}{P_{ii,0}} = (1+x_{in,0})\,\frac{\tilde{P}_{in,0}}{P_{in,0}} = \frac{1}{1+s_{i,0}}.$$

Proposition 2 implicitly shows that the optimal extraction tax rate can be set to zero ($s^*_{i,0} = \mathscr{T}^*_{i,0} = 0$). That is, when demand-side carbon taxes and energy border taxes are available, extraction taxes become redundant. We first demonstrate this redundancy and then use it to obtain optimal policy formulas allowing for an arbitrary extraction tax. Suppose the government seeks to implement an extraction tax $\mathscr{T}_{i,0} > 0$, yielding the following domestic and foreign energy prices (without other taxes):

$$\tilde{P}_{i,0} = \tilde{\mathcal{P}}_{i,0}\left((1+\mathscr{T}_{i,0})\,P_{ii,0}, \mathbf{P}_{-ii,0}\right), \qquad \tilde{P}_{n,0} = \tilde{\mathcal{P}}_{n,0}\left((1+\mathscr{T}_{i,0})\,P_{in,0}, \mathbf{P}_{-in,0}\right) \quad (\text{for } n \neq i)$$

These prices can alternatively be reproduced without extraction taxes using the following mix of energy border taxes and demand-side carbon taxes:

$$1+t_{ni,0} = \frac{1}{1+\mathscr{T}_{i,0}}, \qquad 1+x_{ni,0} = \frac{1}{1+\mathscr{T}_{i,0}}, \qquad \tau_i = \mathscr{T}_{i,0}\tilde{P}_{i,0}.$$

To show this, we need to prove that the above tax combination yields the same after-tax energy prices as the extraction tax. For the domestic energy price, the equivalence can be shown as follows:

$$\begin{aligned}\tilde{P}_{i,0} = \tilde{\mathcal{P}}_{i,0}\left(P_{ii,0}, (1+\mathbf{t}_{i,0})\,\mathbf{P}_{-ii,0}\right) + \tau_i = \tilde{\mathcal{P}}_{i,0}\left(P_{ii,0}, \frac{1}{1+\mathscr{T}_{i,0}}\mathbf{P}_{-ii,0}\right) + \mathscr{T}_{i,0}\tilde{P}_{i,0} \\ = (1+\mathscr{T}_{i,0})\,\tilde{\mathcal{P}}_{i,0}\left(P_{ii,0}, \frac{1}{1+\mathscr{T}_{i,0}}\mathbf{P}_{-ii,0}\right) = \tilde{\mathcal{P}}_{i,0}\left((1+\mathscr{T}_{i,0})\,P_{ii,0}, \mathbf{P}_{-ii,0}\right),\end{aligned}$$

where $\mathbf{t}_{i,0}$ collects the energy import taxes and the second line uses the fact that the price aggregator, $\tilde{\mathcal{P}}_{i,0}(.)$, is a homogeneous of degree one function. The first term, above, is the price under the energy import and demand-side carbon tax mix. The last line shows that this price equals the price under the extraction tax. Next, consider the foreign energy price. Under country i's energy export subsidy, the foreign price is:

$$\tilde{P}_{n,0} = \tilde{\mathcal{P}}_{n,0}\left(\frac{1}{1+x_{in,0}}P_{in,0}, \mathbf{P}_{-in,0}\right) = \tilde{\mathcal{P}}_{n,0}\left((1+\mathscr{T}_{i,0})\,P_{in,0}, \mathbf{P}_{-in,0}\right),$$

which, by construction, equals the foreign energy price under the extraction tax. These equalities reveal that any energy price vector can be obtained without extraction taxes and by using demand-side carbon taxes and energy border taxes/subsidies alone. In other words, extraction taxes are redundant when these other instruments are available. Leveraging the noted redundancy, the unilaterally optimal policy schedule could be more generally represented to include an extraction tax. We demonstrate this for the small open economy case under CES preferences. For an arbitrary choice of extraction tax, $\mathscr{T}^*_{i,0}$, the unilaterally optimal policy can be alternatively represented as

$$\begin{cases} \tau^*_i = \tilde{\delta}_i - \mathscr{T}^*_{i,0}\tilde{P}_{i,0}, & & \text{[carbon tax]} \\ t^*_{ni,k} = \bar{t}_i + \tau^*_i v_{n,k} & 1+t^*_{ni,0} = (1+\bar{t}_i)\left(1+\mathscr{T}^*_{i,0}\right) & \text{[import tax]} \\ 1+x^*_{in,k} = (1+\bar{t}_i)\,\frac{\sigma_k-1}{\sigma_k} + \tau^*_i \sum_{j\neq i}\left[\lambda_{jn,k}v_{j,k}\right]\frac{\sigma_k-1}{\sigma_k} & & \text{[export subsidy (non-energy)]} \\ 1+x^*_{in,0} = \left[(1+\bar{t}_i)\,\frac{\sigma_0-1}{\sigma_0} + \tau^*_i \frac{1}{\sigma_0}\frac{\zeta_n}{\tilde{P}_{n,0}}\right]\left(1+\mathscr{T}^*_{i,0}\right) & & \text{[export subsidy (energy)]} \end{cases}$$

Our baseline representation set $\mathscr{T}_{i,0}^{*} = 0$, but we could have alternatively set $\tau_i^{*} = 0$ and load the carbon tax entirely on extraction $\mathscr{T}_{i,0}^{*} = \tilde{\delta}_i / \tilde{P}_{i,0}$.

Globally Optimal Policy. Our baseline model shows that the globally optimal outcome requires setting a Pigouvian wedge, represented by $\sum_n \tilde{\delta}_n$, between the producer and consumer price of energy worldwide. This wedge and the optimal allocation can be achieved with demand-side carbon taxes ($\tau_i^{\star}$) plus lump-sum transfers or through extraction taxes ($\mathscr{T}_{i,0}^{\star}$) plus lump-sum transfers. The optimal tax rate in each case is given by

$$\tau_i^{\star} = \sum_n \tilde{\delta}_n, \qquad \text{or} \qquad \mathscr{T}_{i,0}^{\star} = \sum_n \left[\tilde{\delta}_n\right] / \tilde{P}_{i,0}.$$

Both of the the above policies deliver the optimal Pigouvian wedge as demonstrated below:

$$\overbrace{\tilde{\mathcal{P}}_{i,0}\left(\left(1+s_{1,0}^{\star}\right)P_{1i,0},\ldots,\left(1+s_{N,0}^{\star}\right)P_{Ni,0}\right)}^{\text{price under extraction tax}} = \left(1+\frac{\sum_n\left[\tilde{\delta}_n\right]}{\tilde{P}_{i,0}}\right)\tilde{\mathcal{P}}_{i,0}\left(P_{1i,0},\ldots,P_{Ni,0}\right)$$
$$= \tilde{\mathcal{P}}_{i,0}\left(P_{1i,0},\ldots,P_{Ni,0}\right) + \sum_n\left[\tilde{\delta}_n\right] = \underbrace{\tilde{\mathcal{P}}_{i,0}\left(P_{1i,0},\ldots,P_{Ni,0}\right)+\tau_i^{\star}}_{\text{price under input-side carbon tax}}.$$

More generally, any mix of demand-side carbon taxes and extraction taxes that satisfy $\tau_i^{\star} + \mathscr{T}_{i,0}^{\star}\tilde{P}_{i,0} = \sum_n \tilde{\delta}_n$, along with lump-sum transfers, could implement the globally optimal outcome. Importantly, without transfers, $\tau_i^{\star}$ and $\mathscr{T}_{i,0}^{\star}$, do not deliver identical welfare outcomes at the national level. Hence, the lump-sum transfers that supplement each tax choice are different. The reason is that carbon tax revenues accrue primarily to major energy consumers under demand-side energy taxes and to major producers under extraction taxes. So, the optimal transfers should be adjusted based on revenue streams. However, the choice of transfers does not affect the overall effectiveness of global carbon taxes, which is our main focus.

D Equilibrium in Changes

This section outlines the equations describing the change in non-policy variables as a function of policy change $\mathscr{R}^T = \left\{x'_{ij,k}, t'_{ji,k}, \tau'_{i,k}\right\}$ and the sufficient statistics $\mathscr{B}$ as specified in Section 4 of the paper. Let z' denote the value of a generic variable z in the counterfactual equilibrium, with $\widehat{z} \equiv z'/z$ denoting the corresponding change using the exact hat-algebra notation. The change to variety-specific producer prices and CES and Cobb-Douglas consumer price indices, for energy ($k = 0$) and final goods ($k = 1, ..., K$), are given by

$$\begin{cases} \widehat{P}_{ji,k} = \widehat{P}_{jj,k} = \left[\left(1-\alpha_{j,k}\right)\widehat{w}_j^{1-\varsigma} + \alpha_{j,k}\widehat{\tilde{P}}_{j,0k}^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}} & \text{a) producer price } (ij,\ k \geq 1) \\ \widehat{P}_{ji,0} = \widehat{P}_{jj,0} = \widehat{w}_j^{1-\phi_j}\widehat{r}_j^{\phi_j} & \text{b) producer price } (ij,\ k = 0) \\ \widehat{\tilde{P}}_{ji,k} = \left(\widehat{1+t_{ji,k}}\right)\left(\widehat{1+x_{ji,k}}\right)^{-1}\widehat{P}_{ji,k} & \text{c) consumer price } (ij,\ k \geq 0) \\ \widehat{\tilde{P}}_{i,k} = \left[\sum_{j=1}^N \lambda_{ji,k}\widehat{\tilde{P}}_{ji,k}^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}} & \text{d) consumer price index } (i,\ k > 0) \\ \widehat{\tilde{P}}_{i,0} = \left[\sum_{j=1}^N \lambda_{ji,0}\widehat{\tilde{P}}_{ji,0}^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}} & \text{e) distribution-level energy price } (i,\ k = 0) \\ \widehat{\tilde{P}}_{i,0k} = \widehat{\tilde{P}}_{i,0}\tilde{P}_{i,0} + \tau'_{i,k} & \text{e) after-carbon-tax energy price } (i,\ k = 0) \\ \widehat{\tilde{P}}_i = \prod_k \left(\widehat{\tilde{P}}_{i,k}^{\beta_{i,k}}\right) & \text{f) final consumer price } (i) \end{cases} \tag{D.1}$$

Note that the change to the producer price of each final good, $\widehat{P}_{ji,k}$, is governed by the change to the wage rate, $\widehat{w}_j$, and final price of energy inputs, $\widehat{\tilde{P}}_{j,0k}$, which itself depends on the change to international producer prices, $\{\widehat{P}_{jj,0}\}_j$, baseline energy expenditures shares, $\{\lambda_{ji,0}\}_j$, and optimal policy choices. The change in industry-level labor and energy input cost shares, carbon emissions, carbon intensities, and output quantities are given by

$$\begin{cases} \left(\widehat{1-\alpha_{i,k}}\right) = \left(\widehat{w}_i/\widehat{P}_{ii,k}\right)^{1-\varsigma},\ \widehat{\alpha}_{i,k} = \left(\widehat{\tilde{P}}_{i,0k}/\widehat{P}_{ii,k}\right)^{1-\varsigma} & \text{a) labor and energy cost share } (i, k \geq 1) \\ \widehat{Q}_{i,k} = \widehat{\ell}_{i,k} \times \left(\widehat{1-\alpha_{i,k}}\right)^{\frac{\varsigma}{1-\varsigma}} & \text{b) final good output quantity } (i, k \geq 1) \\ \widehat{Q}_{i,0} = \widehat{r}_i/\widehat{P}_{ii,0} & \text{c) energy output quantity } (i, k = 0) \\ \widehat{Z}_{i,k} = \widehat{\alpha}_{i,k}^{\frac{\varsigma}{\varsigma-1}} \times \widehat{Q}_{i,k} & \text{d) industry-level carbon emission } (i, k \geq 1) \\ \widehat{v}_{i,k} = \widehat{\alpha}_{i,k}^{\frac{\varsigma}{\varsigma-1}}/\widehat{P}_{ii,k} & \text{e) industry-level carbon intensity } (i, k \geq 1) \\ \widehat{Z}_i = \sum_{k=1}^K \left[(Z_{i,k}/Z_i) \times \widehat{Z}_{i,k}\right] & \text{f) national carbon emission from country } (i) \\ \widehat{Z}^{(global)} = \sum_{i=1}^N \left[\left(Z_i/Z^{(global)}\right) \times \widehat{Z}_i\right] & \text{g) global carbon emission} \end{cases} \tag{D.2}$$

The noted changes in prices and quantities determine the change in international trade shares and flows ($\tilde{X}_{ij,k} \equiv \tilde{P}_{ij,k}C_{ij,k}$ and $X_{ij,k} \equiv P_{ij,k}C_{ij,k}$). In particular,

$$\begin{cases} \widehat{\lambda}_{ji,k} = \left(\widehat{\tilde{P}}_{ji,k}/\widehat{\tilde{P}}_{i,k}\right)^{1-\sigma_k} & \text{a) international expenditure shares } (ji,\ k \geq 0) \\ \tilde{X}'_{ij,k} \equiv \tilde{P}'_{ij,k}C'_{ij,k} = \widehat{\lambda}_{ij,k}\lambda_{ij,k}\beta_{j,k}\widehat{Y}_jY_j & \text{b) after-tax trade flows of final goods } (ij, k \geq 1) \\ X'_{ij,k} \equiv P'_{ij,k}C'_{ij,k} = \left(1+t'_{ij,k}\right)^{-1}\left(1+x'_{ij,k}\right)\tilde{X}'_{ij,k} & \text{c) before-tax trade flows of final goods } (ij, k \geq 1) \\ \tilde{X}'_{ij,0} = \hat{\lambda}_{ij,0}\lambda_{ij,0}\sum_{k=1}^K \left[\frac{\hat{\alpha}_{j,k}\alpha_{j,k}}{1+\tau'_{j,k}}\sum_{n=1}^N \left(X'_{ij,k}\right)\right] & \text{d) after-tax trade flows of energy } (ij, 0) \\ X'_{ij,0} = \left(1+t'_{ij,0}\right)^{-1}\left(1+x'_{ij,0}\right)\tilde{X}'_{ij,0} & \text{d) before-tax trade flows of energy } (ij, 0) \end{cases} \tag{D.3}$$

The change in wages and industry-level labor shares are governed by the labor market clearing (LMC) conditions in the counterfactual equilibrium:

$$\begin{cases} \widehat{\ell}_{i,0} = \widehat{r}_i / \widehat{w}_i & \text{a) LMC } (i, k = 0) \\ \widehat{\ell}_{i,k} \ell_{i,k} \widehat{w}_i w_i \bar{L}_i = \sum_{j=1}^{N} \widehat{(1 - \alpha_{i,k})} X'_{ij,k} & \text{b) LMC } (i, k \geq 1) \\ \hat{\ell}_{i,0} \ell_{i,0} + \sum_{k=1}^{K} \hat{\ell}_{i,k} \ell_{i,k} = 1 & \text{c) National LMC } (i) \end{cases} \tag{D.4}$$

The first two conditions ensure that the industry-level wage bill equals payments to workers in the energy and final good industries. The third line ensures that labor markets clear at the national level. The change in the rental rate of carbon reserves is, accordingly, governed by the energy market clearing condition that connects the global energy demand to energy extraction in each country:

$$\hat{r}_i r_i \bar{R}_i = \phi_i \sum_j \left(X'_{ij,0} \right) \tag{D.5}$$

The change to tax revenues equals the net change to revenues from import tariffs (the first term on the right-hand side of the following equation), export subsidies (second term), and carbon taxes (third term),

$$\begin{aligned} \widehat{T}_i T_i = & \sum_{k=0}^{K} \sum_{n \neq i} \left[\frac{t'_{ni,k}}{1 + t'_{ni,k}} \tilde{X}'_{ni,k} \right] \\ & + \sum_{k=0}^{K} \sum_{n=1}^{N} \left[\frac{\left[1 - \left(1 + x'_{in,k} \right) \right]}{\left(1 + t'_{in,k} \right)} \tilde{X}'_{in,k} \right] \\ & + \sum_{k=1}^{K} \sum_{n=1}^{N} \left[\left(\frac{\tau'_{i,k}}{1 + \tau'_{i,k}} \right) \widehat{\alpha}_{i,k} \alpha_{i,k} \frac{\left(1 + x'_{in,k} \right)}{\left(1 + t'_{in,k} \right)} \tilde{X}'_{in,k} \right]. \end{aligned} \tag{D.6}$$

Finally, the change in national income, $\widehat{Y}_i$, is governed by the representative consumer's budget constraint:

$$\widehat{Y}_i Y_i = \widehat{w}_i w_i \bar{L}_i + \widehat{r}_i r_i \bar{R}_i + \widehat{T}_i T_i. \tag{D.7}$$

Solving Equations (D.1)-(D.7) in conjunction with the optimal policy equations in each policy scenario determines counterfactual equilibrium outcomes, $\mathscr{R} \equiv \{\mathscr{R}^T, \mathscr{R}^V\}$, which in turn determine the change to real consumption and welfare as

$$\widehat{V}_i = \frac{\widehat{Y}_i}{\widehat{\tilde{P}}_i}, \qquad \widehat{W}_i = \underbrace{\left(\frac{Y_i}{Y_i - \tilde{\delta}_i Z^{(global)}} \right) \widehat{V}_i}_{\text{Indirect Utility}} - \underbrace{\left(\frac{\tilde{\delta}_i Z^{(global)}}{Y_i - \tilde{\delta}_i Z^{(global)}} \right) \widehat{Z}^{(global)}}_{\text{Climate Damage Disutility}}. \tag{D.8}$$

We solve the general equilibrium system using a nested fixed point approach with two tiers. In the inner tier, given a preliminary guess of taxes, all non-tax variables are solved to satisfy the equilibrium conditions stipulated by Equations (D.1-(D.7). The outer tier solves for optimal taxes conditional on the fixed point achieved in the inner tier.

E Extensions

E.1 Increasing Returns to Scale Industries à la Krugman

In this section, we derive the unilaterally optimal policy in an extension where production technologies of final goods feature increasing-returns-to-scale. For this purpose, we incorporate firm-level product differentiation and love-for-variety à la Krugman into our baseline model. Scale economies, and the inefficiency they introduce to the market outcome, occur in this setting because firms fail to fully internalize the social gains from new varieties when making entry decisions. Despite this micro-foundation, this setting is also isomorphic to one in which there are external economies of scale, as we explain below. In any case, the resulting scale economies present an additional rationale for policy intervention, influencing the optimal design of carbon border taxes. We employ the optimal policy formulas derived here to assess the sensitivity of our baseline quantitative findings (pertaining to the effectiveness of Proposals 1 and 2) to the inclusion of increasing returns to scale industries.

E.1.1 The Economic Setting

The representative consumer maximizes a non-parametric utility aggregator $U_i\left(\{C_{ni,k}\}_{n,k}\right)$, where each composite consumption bundle $C_{ni,k}$ (corresponding to origin n–destination i–industry k) aggregates over firm-level quantities, $q_{ni,k}(\omega)$. Specifically,

$$C_{ni,k} = \left(\int_{\omega\in\Omega_{n,k}} q_{ni,k}(\omega)^{\frac{\gamma_k-1}{\gamma_k}} d\omega\right)^{\frac{\gamma_k}{\gamma_k-1}},$$

where $\gamma_k > 1$ denotes the elasticity of substitution between firm varieties from the same origin country and industry. We assume that $\gamma_0 \to \infty$, which, as will become evident shortly, retains no love of variety for energy products that originate from an exporter. Firm ω operating in country n–industry k is characterized by productivity $\varphi_{n,k}(\omega)$. As in our baseline model, a prototypical firm combines labor and energy inputs using a CES aggregator with elasticity ς, which yields the following marginal cost based on cost minimization:

$$c_{ni,k}(\omega) = \frac{\bar{d}_{ni,k}}{\varphi_{n,k}(\omega)} \times c_{n,k}; \qquad c_{n,k} = \left[(1-\bar{\kappa}_{n,k})^{\varsigma} w_n^{1-\varsigma} + \bar{\kappa}_{n,k}^{\varsigma}\tilde{P}_{n,0k}^{1-\varsigma}\right]^{1/(1-\varsigma)}$$

The cost share of carbon input is, accordingly, given by $\alpha_{n,k} = \bar{\kappa}_{n,k}^{\varsigma}\left(\tilde{P}_{n,0k}/c_{n,k}\right)^{1-\varsigma}$. Firms compete under monopolistic competition and charge a constant markup over marginal cost, i.e., $p_{ni,k}(\omega) = \frac{\gamma_k}{\gamma_k-1}c_{ni,k}(\omega)$. The CES producer price index associated with output bundle $C_{ni,k}$ can be, accordingly, expressed as

$$P_{ni,k} = M_{n,k}^{\frac{1}{\gamma_k-1}}\frac{\gamma_k}{\gamma_k-1}\frac{\bar{d}_{ni,k}}{\overline{\varphi}_{n,k}}c_{n,k}, \qquad \text{where} \qquad \overline{\varphi}_{n,k} \equiv \left[\int_{\omega\in\Omega_{n,k}} \varphi_{n,k}(\omega)^{1-\gamma_k} d\omega\right]^{\frac{1}{1-\gamma_k}}$$

The mass of entrants is governed by the free entry condition. Each firm incurs a sunk entry cost $c_{n,k}f_{n,k}^{(e)}$, upon which its productivity is realized. The mass of entrants, $M_{n,k}$, ensures that the ex-ante profit per firm $\frac{1}{\gamma_k}P_{nn,k}Q_{n,k}/M_{n,k}$ equals the entry cost, $c_{n,k}f_{n,k}^{(e)}$, in each location and industry. Namely,

$$M_{n,k} = \frac{P_{nn,k}Q_{n,k}}{\gamma_k c_{n,k}f_{n,k}^{(e)}}, \qquad \text{where} \qquad Q_{n,k} = \sum_n \bar{d}_{ni,k}C_{ni,k}$$

Plugging $M_{n,k}$ from the above equation back into our earlier expression for $P_{ni,k}$ determines the mass of entrants in terms of total output, $Q_{n,k}$. Plugging the implied expression for $M_{n,k}$ back into the CES producer price index and noting that $c_{n,k} \propto w_n (1-\alpha_{n,k})^{\frac{1}{\varsigma-1}}$, delivers

$$P_{ni,k} = \overline{d}_{ni,k}\overline{p}_{nn,k}w_n (1-\alpha_{n,k})^{\frac{1}{\varsigma-1}} \; Q_{n,k}^{-\frac{1}{\gamma_k}}$$

where $\overline{p}_{nn,k}$ collects all the constant price shifters apart from the iceberg trade cost, with the term $Q_{n,k}^{-\frac{1}{\gamma_k}}$ accounting for economies scale driven by love for variety. As before, the producer price of energy in country i is given by:

$$P_{in,0} = P_{ii,0} = \bar{p}_{ii,0} w_i \, Q_{i,0}^{\frac{\phi_i}{1-\phi_i}},$$

consistent with the implicit assumption that $\gamma_0 \to \infty$. The carbon emissions associated with final production can be measured as $Z_{n,k} = \alpha_{n,k} P_{nn,k} Q_{n,k} / \tilde{P}_{n,0k}$, per cost minimization. Noting that $P_{nn,k} \propto c_{n,k} Q_{n,k}^{-\frac{1}{\gamma_k}}$ and $\tilde{P}_{n,0k} \propto c_{n,k} \alpha_n^{\frac{1}{\varsigma-1}}$, we then obtain

$$Z_{i,k} = \bar{z}_{i,k} \times \alpha_{i,k}^{\frac{\varsigma}{\varsigma-1}} \times Q_{i,k}^{1-\frac{1}{\gamma_k}},$$

where $\overline{z}_{i,k}$ encompasses constant emissions shifters.

Isomorphism with *External* Economies of Scale. Consider an alternative formulation in which production technologies of final goods ($k = 1, ..., K$) feature external economies of scale that are operative at the industry level. Specifically, there is a measure one of symmetric firms in each country-industry (n, k), each with total factor productivity that equals $\varphi_{n,k} = \bar{\varphi}_{n,k} \, Q_{n,k}^{\mu_k}$. An individual firm does not internalize the impact of its production on the aggregate total factor productivity, and so, the producer price is given by $P_{nn,k} = \frac{c_{n,k}}{\bar{\varphi}_{n,k}} Q_{n,k}^{-\mu_k}$. Per cost minimization as before,

$$c_{n,k} = \left[(1-\bar{\kappa}_{n,k})^{\varsigma} \, w_n^{1-\varsigma} + \bar{\kappa}_{n,k}^{\varsigma} \tilde{P}_{n,0k}^{1-\varsigma} \right]^{1/(1-\varsigma)} \propto w_n (1-\alpha_{n,k})^{\frac{1}{\varsigma-1}},$$

indicating that the cost share of energy input equals $\alpha_{n,k} = \bar{\kappa}_{n,k}^{\varsigma} \left(\tilde{P}_{n,0k} / c_{n,k} \right)^{1-\varsigma}$, and the carbon emission equals $Z_{n,k} = \alpha_{n,k} P_{nn,k} Q_{n,k} / \tilde{P}_{n,0k}$. Taking note of these points, $P_{ni,k} = \overline{d}_{ni,k} \overline{p}_{nn,k}^{(ext)} w_n (1-\alpha_{n,k})^{\frac{1}{\varsigma-1}} \; Q_{n,k}^{-\mu_k}$, and $Z_{n,k} = \bar{z}_{n,k}^{(ext)} \times \alpha_{n,k}^{\frac{\varsigma}{\varsigma-1}} \times Q_{n,k}^{1-\mu_k}$. This setting, therefore, is isomorphic to the above Krugman-type extension provided that $\overline{p}_{nn,k}^{(ext)} = \overline{p}_{nn,k}$, $\bar{z}_{n,k}^{(ext)} = \bar{z}_{n,k}$, and $\mu_k = 1/\gamma_k$.

E.1.2 Unilaterally Optimal Policy Problem

As in our baseline model, we determine country i's unilaterally optimal policy as follow: The government in country i selects $\mathbb{P}_i = \left\{ \tilde{P}_{ij,k}, \tilde{P}_{ji,k}, \alpha_{i,k} \right\}_{j,k}$ to maximize the climate-adjusted national welfare, $V_i \left(E_i, \tilde{\mathbf{P}}_i \right) - \delta_i Z^{(global)}$,subject to equilibrium constraints.

We retrieve the unilaterally optimal taxes from the optimal policy solution, $\mathbb{P}_i^*$, similar to our baseline but with an added policy instrument, $s_{n,k}$, denoting production subsidies. With the added instrument, the wedges between consumer and producer prices are given by:

$$\tilde{P}_{ni,k} = \frac{(1+t_{ni,k})}{(1+s_{n,k})(1+x_{ni,k})} \times P_{ni,k}, \qquad \tilde{P}_{i,0k} = \tilde{P}_{i,0} + \tau_{i,k},$$

where $t_{ii,k} = x_{ii,k} = 0$, by definition. The optimal tax rates can be, therefore, determined as

$$1 + t^*_{ji,k} = \frac{\tilde{P}^*_{ji,k}}{P_{ji,k}}, \qquad \left(1 + x^*_{ij,k}\right)^{-1} = \frac{\tilde{P}^*_{ij,k}/P_{ij,k}}{P^*_{ii,k}/P_{ii,k}}, \qquad \left(1 + s^*_{i,k}\right)^{-1} = \frac{\tilde{P}^*_{ii,k}}{P_{ii,k}} \qquad \tau^*_{i,k} = \frac{\alpha^*_{i,k} Y_{i,k}}{Z_{i,k}} - \tilde{P}_{i,0}.$$

Our derivation here deals with the small open economy case, whereby producer prices in the rest of world (and aggregate variables, $\mathbf{w}_{-i}$, and $\mathbf{E}_{-i}$,) are invariant to $\mathbb{P}_i$. It is straightforward to verify that our domestic wage-neutrality result continues to hold in this setting. Hence, we can derive the first-order conditions while disregarding general equilibrium wage effects, as they are welfare neutral in the reformulated problem. With this background in mind, we derive the F.O.C.s *w.r.t.* each element of $\mathbb{P}_i$.

E.1.3 First-Order Conditions

To guide the derivations, we produce the balance of budget, which requires total expenditure (E_i) to be equal to total income (Y_i), as the sum of factor income and tax revenues:

$$\begin{aligned} Y_i = w_i L_i + \Pi_i + \sum_{n \neq i} \sum_k \left[\left(\tilde{P}_{ni,k} - P_{ni,k} \right) C_{ni,k} \right] \\ + \sum_n \sum_k \left[\left(\tilde{P}_{in,k} - [1 - \alpha_{i,k}] P_{in,k} \right) C_{in,k} \right] - \tilde{P}_{i,0} \sum_k Z_{i,k}. \end{aligned}$$

We derive the F.O.C.s with respect to various elements of $\mathbb{P}_i$ borrowing heavily from our baseline derivation in Appendix A. We begin with the F.O.C.s w.r.t. the energy input share, $\alpha_{i,k}$.

(i) **Carbon input shares, $\alpha_{i,k}$.** Note that the choice of $\alpha_{i,k}$ modifies welfare only through its influence on emission levels and income. As in our baseline derivation, the impacts on income can be specified as:

$$\frac{\partial \ln Y_i \left(\mathbb{P}_i; \mathbf{w} \right)}{\partial \ln \left(1 - \alpha_{i,k} \right)} = \left(\frac{\partial \left[\left(1 - \alpha_{i,k} \right) P_{ii,k} Q_{i,k} \right]}{\partial \ln \left(1 - \alpha_{i,k} \right)} - \tilde{P}_{i,0} Z_{i,k} \frac{\partial \ln Z_{i,k}}{\partial \ln \left(1 - \alpha_{i,k} \right)} \right)_{\mathbf{C}_i} + \frac{\partial \ln \left(\Pi_i + T_i \right)}{\partial \ln \mathbf{C}_i} \cdot \frac{\mathrm{d} \ln \mathbf{C}_i\,(.)}{\mathrm{d} \ln \left(1 - \alpha_{i,k} \right)},$$

where $T_i + \Pi_i$ represents tax revenues plus surplus payments to the energy reserves. The first term on the right-hand side represents welfare effects holding demand quantities constant. The second term accounts for the change in non-wage income through changes in demand quantities. Foreign demand quantities are invariant to $\alpha_{i,k}$ holding $\mathbb{P}_i - \{\alpha_{i,k}\}$ fixed, since they are fully determined by *(i)* foreign national expenditure and producer prices, which are invariant to $\alpha_{i,k}$ by the small open economy assumption, and *(ii)* after tax prices of varieties purchased from country i, which are set by policy, $\mathbb{P}_i$. Following the same steps as in Appendix A.8, the first term on the right-hand side can be specified as:

$$\left(\frac{\partial \left(\left(1 - \alpha_{i,k} \right) P_{ii,k} Q_{i,k} \right)}{\partial \ln \left(1 - \alpha_{i,k} \right)} \right)_{\mathbf{C}_i} = \alpha_{i,k} P_{ii,k} Q_{i,k} \left(\frac{\partial \ln Z_{i,k}}{\partial \ln \left(1 - \alpha_{i,k} \right)} \right)_{\mathbf{C}_i}$$

With the same logic, the emission effects associated with the policy choice of $\alpha_{i,k}$, can be decomposed as

$$\frac{\partial \ln Z_i \left(\mathbb{P}_i; \mathbf{w} \right)}{\partial \ln \left(1 - \alpha_{i,k} \right)} = \left(\frac{\partial \ln Z_{i,k}}{\partial \ln \left(1 - \alpha_{i,k} \right)} \right)_{\mathbf{C}_i} + \sum_{n \neq i} \sum_{g \neq 0} \left[Z_{n,g} \frac{\partial \ln Z_{n,g}}{\partial \ln Q_{n,g}} \frac{\mathrm{d} \ln Q_{n,g}}{\mathrm{d} \ln \left(1 - \alpha_{i,k} \right)} \right],$$

where the first terms represents the change in emissions holding demand quantities fixed (*technique effects*), while the last sum accounts for the change in emissions though adjustments to demand quantities (*scale and composition effects*). Consolidating the above equations delivers the following F.O.C. *w.r.t.* $\alpha_{i,k}$:

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln(1-\alpha_{i,k})} = \frac{\partial V_i(.)}{\partial E_i}\Bigg\{\left(\alpha_{i,g}P_{ii,g}Q_{i,g} - \left[\tilde{P}_{i,0}+\tilde{\delta}_i\right]Z_{i,g}\right)\left(\frac{\partial \ln Z_{i,k}}{\partial \ln(1-\alpha_{i,k})}\right)_{\mathbf{C}_i}$$
$$+\sum_g\left[\left(\alpha_{i,g}\left(1+\frac{\partial\ln P_{ii,g}}{\partial\ln Q_{i,g}}\right)P_{ii,g}Q_{i,g} - \left[\tilde{P}_{i,0}Z_{i,g}+\tilde{\delta}_iZ_{i,g}\right]\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}}\right)\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]$$
$$-\tilde{\delta}_i\sum_{n\neq i}\sum_{g\neq 0}\left[Z_{n,g}\frac{\partial\ln Z_{n,g}}{\partial\ln Q_{n,g}}\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]+\sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]$$
$$+\left(\frac{\partial \Pi_{i,0}(.)}{\partial\ln P_{ii,0}}-P_{ii,0}Q_{i,0}\right)\frac{\partial\ln P_{ii,0}}{\partial\ln Q_{i,0}}\frac{\mathrm{d}\ln Q_{i,0}}{\mathrm{d}\ln(1-\alpha_{i,k})}-\sum_g\sum_n\left[P_{in,g}C_{in,g}\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]\Bigg\}=0,$$

where $\tilde{\delta}_i \equiv \delta_i\left(\frac{\partial V_i}{\partial E_i}\right)^{-1}$ represents the CPI adjusted climate disutility parameter, while the elasticities of producer prices and emissions w.r.t. scale are $\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}} = \frac{\gamma_g-1}{\gamma_g}$ and $\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}} = -\frac{1}{\gamma_g}$. We can further simplify the above expression by noting that $\frac{\partial\Pi_{i,0}(.)}{\partial\ln P_{ii,0}} - P_{ii,0}Q_{i,0} = 0$, per Hotelling's lemma, and, $\alpha_{i,g}P_{ii,g}Q_{i,g} = \tilde{P}_{i,0g}Z_{i,g}$, per cost minimization. Additionally, $\alpha_{i,k}$ only affects output through its impact on the demand schedule in country i, so by the chain rule we get

$$\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln(1-\alpha_{i,k})} = \frac{\partial\ln Q_{n,g}}{\partial\ln C_{ni,g}}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})} = \rho_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})},$$

where $\rho_{ni,g} \equiv \frac{P_{ni,g}C_{ni,g}}{Y_{n,g}}$. Taking into account these intermediate relationships, the F.O.C. *w.r.t.* $\alpha_{i,k}$ can be simplified as follows,

$$\sum_g\left[\left(\tilde{P}_{i,0g}Z_{i,g}-\left[\tilde{P}_{i,0}Z_{i,g}+\tilde{\delta}_iZ_{i,g}\right]\right)\left\{\left(\frac{\partial\ln Z_{i,k}}{\partial\ln(1-\alpha_{i,k})}\right)_{\mathbf{C}_i}+\frac{\gamma_g-1}{\gamma_g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right\}\right]$$
$$+\sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]-\tilde{\delta}_i\sum_{n\neq i}\sum_{g\neq 0}\left[\frac{Z_{n,g}}{Y_{n,g}}\frac{\gamma_g-1}{\gamma_g}P_{ni,g}C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g}-P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]-\sum_g\sum_n\left[\frac{P_{in,g}C_{in,g}}{Y_{i,g}}\left(-\frac{1}{\gamma_g}P_{ii,g}C_{ii,g}\right)\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]=0.$$

After rearranging the terms and considering that $\sum_n\frac{P_{in,g}C_{in,g}}{Y_{i,g}} = \sum_n\rho_{in,g} = 1$, we obtain our final representation of the F.O.C.,

$$\sum_g\left[\left(\tilde{P}_{i,0g}Z_{i,g}-\left[\tilde{P}_{i,0}Z_{i,g}+\tilde{\delta}_iZ_{i,g}\right]\right)\left\{\left(\frac{\partial\ln Z_{i,k}}{\partial\ln(1-\alpha_{i,k})}\right)_{\mathbf{C}_i}+\frac{\gamma_g-1}{\gamma_g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right\}\right]$$
$$+\sum_{n\neq i}\sum_{g\neq 0}\left[\left(\tilde{P}_{ni,g}-\left(1+\frac{\gamma_g-1}{\gamma_g}\tilde{\delta}_iv_{n,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]$$
$$+\sum_{n\neq i}\left[\left(\tilde{P}_{ni,0}-P_{ii,0}\right)C_{ii,0}\frac{\mathrm{d}\ln C_{ii,0}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]+\sum_g\left[\left(\tilde{P}_{ii,g}-\frac{\gamma_g-1}{\gamma_g}P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln(1-\alpha_{i,k})}\right]=0 \qquad \text{(E.1)}$$

(ii) **Domestic and import consumer prices, $\tilde{P}_{ji,k}$.** The domestic and import consumer price instrument, $\tilde{P}_{ji,k}$, explicitly features in the the indirect utility. So, in addition to income and emissions effects, perturbing $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$ exerts a direct welfare effect. This point aside, we can apply the same logic as above to obtain the income and emissions effects associated with policy $\tilde{P}_{ji,k}$, noting that $\tilde{P}_{ji,k}$ (unlike

$\alpha_{i,k}$) influences emissions only *indirectly* through its effect on demand quantities. These considerations deliver the following F.O.C.,-

$$\begin{aligned}\frac{\mathrm{d}W_i}{\mathrm{d}\ln\tilde{P}_{ji,k}} =& \frac{\partial V_i\,(.)}{\partial\ln\tilde{P}_{ji,k}} + \frac{\partial V_i\,(.)}{\partial E_i}\Bigg\{\tilde{P}_{ji,k}C_{ji,k} + \sum_{n\neq i}\sum_{g}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right] \\ &+\sum_{g}\left[\left(\tilde{P}_{ii,g}-P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]-\sum_{g}\sum_{n}\left[P_{in,g}C_{in,g}\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]+\left(\frac{\partial\Pi_{i,0}\,(.)}{\partial\ln P_{ii,0}}-P_{ii,0}Q_{i,0}\right)\frac{\partial\ln P_{ii,0}}{\partial\ln Q_{i,0}}\frac{\mathrm{d}\ln Q_{i,0}}{\mathrm{d}\ln\tilde{P}_{ji,k}} \\ &-\tilde{\delta}_i\sum_{n\neq i}\sum_{g}\left[Z_{n,g}\frac{\partial\ln Z_{n,g}}{\partial\ln Q_{n,g}}\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]+\sum_{g\neq 0}\left[\left(\alpha_{i,g}\left(1+\frac{\partial\ln P_{ii,g}}{\partial\ln Q_{i,g}}\right)P_{ii,g}Q_{i,g}-\left[\tilde{P}_{i,0}Z_{i,g}+\tilde{\delta}_iZ_{i,g}\right]\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}}\right)\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]\Bigg\} =\end{aligned}$$

where the elasticities of producer prices and emissions *w.r.t.* scale are given by $\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}} = -\frac{1}{\gamma_g}$ and $\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}} = \frac{\gamma_g-1}{\gamma_g}$. We can simplify the above expression by invoking Roy's identity and Hotelling's lemma, whereby

$$\frac{\partial V_i\left(E_i,\tilde{\mathbf{P}}_i\right)}{\partial\ln\tilde{P}_{ji,k}} = -\frac{\partial V_i\left(E_i,\tilde{\mathbf{P}}_i\right)}{\partial E_i}\tilde{P}_{ji,k}C_{ji,k}, \qquad \frac{\partial\Pi_{i,0}\left(P_{ii,0},w_i\right)}{\partial\ln P_{ii,0}} = P_{ii,0}Q_{i,0}.$$

The first equation (Roy's identity) indicates that the direct consumption loss from perturbing $\tilde{P}_{ji,k}$ is exactly offset by the mechanical effect of $\tilde{P}_{ji,k}$ on tax revenues. Moreover, since $\tilde{P}_{ji,k}$ only impact output quantities through its effect on country i's demand schedule, we can apply the chain rule to obtain

$$\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}} = \frac{\partial\ln Q_{n,g}}{\partial\ln C_{ni,g}}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}} = \rho_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}.$$

Lastly, we note that per cost minimization, $\alpha_{i,g}P_{ii,g}Q_{i,g} = \tilde{P}_{i,0g}Z_{i,g}$. Collecting these relationships and plugging them into our initial formulation of the first-order condition, delivers

$$\begin{aligned}&\sum_{n\neq i}\sum_{g}\left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]-\tilde{\delta}_i\sum_{n\neq i}\sum_{g\neq 0}\left[\frac{\gamma_g-1}{\gamma_g}\frac{Z_{n,g}}{Y_{n,g}}P_{ni,g}C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right] \\ &+\sum_{g}\left[\left(\tilde{P}_{ii,g}-P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]-\sum_{g}\sum_{n}\left[\frac{P_{in,g}C_{in,g}}{Y_{i,g}}\left(-\frac{1}{\gamma_g}P_{ii,g}C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right)\right] \\ &+\sum_{g}\left[\left(\frac{\gamma_g-1}{\gamma_g}\tilde{P}_{i,0g}Z_{i,g}-\left[\tilde{P}_{i,0}Z_{i,g}+\tilde{\delta}_iZ_{i,g}\right]\frac{\gamma_g-1}{\gamma_g}\right)\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ji,g}}\right]=0.\end{aligned}$$

Rearranging the above equation and noting that $\frac{Z_{n,g}}{Y_{n,g}} \sim v_{n,g}$ and $\frac{P_{in,g}C_{in,g}}{Y_{i,g}} \sim \rho_{in,g}$, we obtain our final representation for the F.O.C.,

$$\begin{aligned}&\sum_{n\neq i}\sum_{g}\left[\left(\tilde{P}_{ni,g}-\left(1+\frac{\gamma_g-1}{\gamma_g}\tilde{\delta}_iv_{i,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]+\sum_{n\neq i}\left[\left(\tilde{P}_{ni,0}-P_{ni,0}\right)C_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right] \\ &+\sum_{g}\left[\left(\tilde{P}_{ii,g}-\frac{\gamma_g-1}{\gamma_g}P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]+\sum_{g}\left[\frac{\gamma_g-1}{\gamma_g}\left(\tilde{P}_{i,0g}-\left[\tilde{P}_{i,0}+\tilde{\delta}_i\right]\right)Z_{i,g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}}\right]=0,\end{aligned} \tag{E.2}$$

where the second sum on the right-hand side second is obtained by invoking the adding up constraint, $\sum_n\rho_{in,g}=1$.

(iii) **Non-Energy export prices, $\tilde{P}_{ij,k}$.** The F.O.C. *w.r.t.* export price $\tilde{P}_{ij,k}$ can be derived following the same logic, but keeping in mind two important distinctions. First, the export price $\tilde{P}_{ij,k}$ does not directly enter the indirect utility function of the home country, $V_i(.)$. Second, a change in $\tilde{P}_{ij,k}$ affects output quantities not only through its general equilibrium effect on the domestic demand schedule, $\mathbf{C}_i$, but also through adjustments to country j's demand schedule via cross-substitution effects. Considering this, the F.O.C. w.r.t. $\tilde{P}_{ij,k}$ can be expressed as:

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln\tilde{P}_{ij,k}} = \frac{\partial V_i(.)}{\partial E_i}\Bigg\{\tilde{P}_{ij,k}C_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - P_{ij,g}\right)C_{ij,g}\frac{\mathrm{d}\ln C_{ij,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g} - P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g} - P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] - \sum_g\sum_n\left[P_{in,g}C_{in,g}\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \left(\frac{\partial\Pi_{i,0}(.)}{\partial\ln P_{ii,0}} - P_{ii,0}Q_{i,0}\right)\frac{\partial\ln P_{ii,0}}{\partial\ln Q_{i,0}}\frac{\mathrm{d}\ln Q_{i,0}}{\mathrm{d}\ln\tilde{P}_{ij,k}}$$
$$-\tilde{\delta}_i\sum_{n\neq i}\sum_g\left[Z_{n,g}\frac{\partial\ln Z_{n,g}}{\partial\ln Q_{n,g}}\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_{g\neq 0}\left[\left(\alpha_{i,g}\left(1+\frac{\partial\ln P_{ii,g}}{\partial\ln Q_{i,g}}\right)P_{ii,g}Q_{i,g} - \left[\tilde{P}_{i,0}Z_{i,g} + \tilde{\delta}_i Z_{i,g}\right]\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}}\right)\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right]\Bigg\} =$$

where $\mathrm{d}\ln C_{ij,g}/\mathrm{d}\ln\tilde{P}_{ij,k} = \varepsilon^{(ij,k)}_{ij,g}$, since aggregate nominal expenditure in the rest of the world is invariant to country i's export policy by assumption. Relatedly, the impact of country i's export policy on the scale of output quantity can be decomposed as:

$$\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}} = \rho_{nj,g}\frac{\mathrm{d}\ln C_{nj,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}} + \rho_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}} \sim \rho_{nj,g}\varepsilon^{(ij,k)}_{nj,g} + \rho_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ji,k}},$$

where $\mathrm{d}\ln C_{ni,g}/\mathrm{d}\ln\tilde{P}_{ij,k}$ encompassed the effect in domestic consumption via general equilibrium income effect. With the above condition in mind, we simplify the first-order condition using the same intermediate relationships utilized earlier, obtaining:

$$\tilde{P}_{ij,k}C_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - \frac{\gamma_g-1}{\gamma_g}P_{ij,g}\right)C_{ij,g}\varepsilon^{(ij,k)}_{ij,g}\right] - \tilde{\delta}_i\sum_{n\neq i}\sum_g\left[\frac{\gamma_g-1}{\gamma_g}v_{n,g}P_{nj,g}C_{nj,g}\varepsilon^{(ij,k)}_{nj,g}\right]$$
$$+\sum_n\left[\left(\tilde{P}_{ni,0} - P_{ni,0}\right)C_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g} - \left(1+\frac{\gamma_g-1}{\gamma_g}\tilde{\delta}_i v_{i,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right]$$
$$+\sum_{g\neq 0}\left[\left(\tilde{P}_{ii,g} - \frac{\gamma_g-1}{\gamma_g}P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_g\left[\left(\tilde{P}_{i,0g} - \left[\tilde{P}_{i,0} + \tilde{\delta}_i\right]\right)Z_{i,g}\frac{\gamma_g-1}{\gamma_g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] = 0.$$

With additively separable preferences, we can simplify the above condition further by noting that $\varepsilon^{(ij,k)}_{nj,g} = -\frac{\lambda_{ij,k}}{1-\lambda_{ij,k}}\left(1+\varepsilon^{(ij,k)}_{ij,k}\right)$, delivering:

$$\left[\left(1+\frac{1}{\varepsilon_{ij,k}}\right)\tilde{P}_{ij,k} - \frac{\gamma_k-1}{\gamma_k}P_{ij,k}\right]C_{ij,k}\varepsilon_{ij,k} - \tilde{\delta}_i\frac{\gamma_k-1}{\gamma_k}\sum_{n\neq i}\left[v_{n,k}\lambda_{nj,k}\right]\tilde{P}_{ij,k}C_{ij,k}\left(1+\varepsilon_{ij,k}\right)$$
$$+\sum_n\left[\left(\tilde{P}_{ni,0} - P_{ni,0}\right)C_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_{n\neq i}\sum_{g\neq 0}\left[\left(\tilde{P}_{ni,g} - \left(1+\frac{\gamma_g-1}{\gamma_g}\tilde{\delta}_i v_{i,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g} - \frac{\gamma_g-1}{\gamma_g}P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] + \sum_g\left[\left(\tilde{P}_{i,0g} - \left[\tilde{P}_{i,0} + \tilde{\delta}_i\right]\right)Z_{i,g}\frac{\gamma_g-1}{\gamma_g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,k}}\right] = 0. \tag{E.3}$$

As before, $\varepsilon_{ij,k} \sim \varepsilon^{(ij,k)}_{ij,k}$ represents the own-price elasticity of demand to condense the notation.

(iv) **Energy export prices , $\tilde{P}_{ij,0}$.** The F.O.C. w.r.t. energy export price differs from the non-energy variant in one important detail. The price of energy exports from country i to j, influences the price of the composite energy bundle, $\tilde{P}_{j,0}$, in the destination country. As such, the F.O.C. features an additional term (the first term in the last line):

$$\frac{\mathrm{d}W_i}{\mathrm{d}\ln\tilde{P}_{ij,0}} = \frac{\partial V_i(.)}{\partial E_i}\Bigg\{\tilde{P}_{ij,0}C_{ij,0} + \sum_g\left[\left(\tilde{P}_{ij,g} - P_{ij,g}\right)C_{ij,g}\frac{\mathrm{d}\ln C_{in,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right] + \sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g} - P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g} - P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right] - \sum_g\sum_n\left[P_{in,g}C_{in,g}\frac{\partial\ln P_{in,g}}{\partial\ln Q_{i,g}}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]$$
$$+\left(\frac{\partial\Pi_{i,0}(.)}{\partial\ln P_{ii,0}} - P_{ii,0}Q_{i,0}\right)\frac{\partial\ln P_{ii,0}}{\partial\ln Q_{i,0}}\frac{\mathrm{d}\ln Q_{i,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}} - \tilde{\delta}_i\sum_{n\neq i}\sum_g\left[Z_{n,g}\frac{\partial\ln Z_{n,g}}{\partial\ln Q_{n,g}}\frac{\mathrm{d}\ln Q_{n,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]$$
$$-\tilde{\delta}_i\left[Z_j\frac{\partial\ln Z_j}{\partial\ln\tilde{P}_{j,0}}\frac{\partial\ln\tilde{P}_{j,0}}{\partial\ln\tilde{P}_{ij,0}}\right] + \sum_{g\neq 0}\left[\left(\alpha_{i,g}\left(1+\frac{\partial\ln P_{ii,g}}{\partial\ln Q_{i,g}}\right)P_{ii,g}Q_{i,g} - \left[\tilde{P}_{i,0}Z_{i,g} + \tilde{\delta}_i Z_{i,g}\right]\frac{\partial\ln Z_{i,g}}{\partial\ln Q_{i,g}}\right)\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]\Bigg\} = 0.$$

We rewrite the new term by noting that $\zeta_n \equiv \frac{\partial\ln Z_j}{\partial\ln\tilde{P}_{j,0}}$ and that $\frac{\partial\ln\tilde{P}_{j,0}}{\partial\ln\tilde{P}_{ij,0}} = \lambda_{ij,0}$ per Shephard's lemma. Consolidating these relationships and noting that $\lambda_{ij,0} = \tilde{P}_{ij,0}C_{ij,0}/\tilde{P}_{j,0}Z_j$, yields

$$Z_j\frac{\partial\ln Z_j}{\partial\ln\tilde{P}_{j,0}}\frac{\partial\ln\tilde{P}_{j,0}}{\partial\ln\tilde{P}_{ij,0}} = Z_j\zeta_j\lambda_{ij,0} = \frac{\zeta_j}{\tilde{P}_{j,0}}\tilde{P}_{ij,0}C_{ij,0}.$$

Plugging the above expression back into our original representation of the F.O.C., noting zero-cross substitutability between energy and non-energy goods (i.e., $\mathrm{d}\ln C_{ij,g}/\mathrm{d}\ln\tilde{P}_{ij,0} = 0$ if $g \neq 0$), and following the same steps as earlier to simplify, delivers:

$$\tilde{P}_{ij,0}C_{ij,0}\left(1 - \tilde{\delta}_i\frac{\zeta_j}{\tilde{P}_{j,0}}\right) + \left[\left(\tilde{P}_{ij,0} - \frac{\gamma_0 - 1}{\gamma_0}P_{ij,0}\right)C_{ij,0}\varepsilon_{ij,0}^{(ij,0)}\right]$$
$$+\sum_n\left[\left(\tilde{P}_{ni,0} - P_{ni,0}\right)C_{ni,0}\frac{\mathrm{d}\ln C_{ni,0}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right] + \sum_{n\neq i}\sum_g\left[\left(\tilde{P}_{ni,g} - \left(1 + \frac{\gamma_g - 1}{\gamma_g}\tilde{\delta}_i v_{i,g}\right)P_{ni,g}\right)C_{ni,g}\frac{\mathrm{d}\ln C_{ni,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right]$$
$$+\sum_g\left[\left(\tilde{P}_{ii,g} - \frac{\gamma_g - 1}{\gamma_g}P_{ii,g}\right)C_{ii,g}\frac{\mathrm{d}\ln C_{ii,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right] + \sum_g\left[\left(\tilde{P}_{i,0g} - \left[\tilde{P}_{i,0} + \tilde{\delta}_i\right]\right)Z_{i,g}\frac{\gamma_g - 1}{\gamma_g}\frac{\mathrm{d}\ln Q_{i,g}}{\mathrm{d}\ln\tilde{P}_{ij,0}}\right] = 0. \tag{E.4}$$

Since we have assumed that there is no love-of-variety for energy, i.e., $\gamma_0 \to \infty$, we can set $\frac{\gamma_0 - 1}{\gamma_0} = 1$ in the first line.

E.1.4 Jointly Solving the system of First-Order Conditions

Solving the system of F.O.C.s with respect to all elements of $\mathbb{P}_i$ (Equations E.1, E.2, E.3, and E.4) and following the same steps as in our baseline derivation (Appendix A) yields the following characterization of the unilaterally optimal policy with increasing-returns-to-scale final-good industries:

$$\begin{cases} \tau_i^* = \tilde{\delta}_i \sim \delta_i\tilde{P}_i, \qquad s_{i,k}^* = \frac{1}{\gamma_k - 1} & \text{[carbon tax \& domestic subsidy]} \\ t_{ni,k}^* = \bar{t}_i + \frac{\gamma_k - 1}{\gamma_k}\tau_i^* v_{n,k} \qquad t_{ni,0}^* = \bar{t}_i & \text{[import tax (energy and non-energy)]} \\ 1 + x_{in,k}^* = (1 + \bar{t}_i)\frac{\sigma_k - 1}{\sigma_k} + \frac{\gamma_k - 1}{\gamma_k}\tau_i^*\sum_{j\neq i}\left[\lambda_{jn,k}v_{j,k}\right]\frac{\sigma_k - 1}{\sigma_k} & \text{[export subsidy (non-energy)]} \\ 1 + x_{in,0}^* = (1 + \bar{t}_i)\frac{\sigma_0 - 1}{\sigma_0} + \tau_i^*\frac{1}{\sigma_0}\frac{\zeta_n}{\tilde{P}_{n,0}} & \text{[export subsidy (energy)]} \end{cases}$$

In the above representation, $\bar{t}_i$ is an arbitrary tax shifter, which accounts for the multiplicity of optimal policy schedules, according to Lerner symmetry. This tax shifter scales up all nominal variables associated with country i by a factor of $(1+\bar{t}_i)$. Less visible in the expressions, the shifter also scales the carbon tax and associated carbon border adjustments through its effect on the consumer price index $\tilde{P}_i$.

The above formulas differ from the baseline constant-returns-to-scale version of our model in two aspects. First, they incorporate domestic subsidies addressing the distortions that arise from different degrees of scale economies across industries. However, these subsidies are carbon-blind, since carbon is already optimally priced through τ_i^*. Second, the carbon border adjustment includes a scale adjustment, $\frac{\gamma_k-1}{\gamma_k} \in (0,1)$. The rationale is that carbon border taxes curb emissions by reducing the scale of output. Under increasing-returns-to-scale industries, the carbon intensity (Z/Q) increases with a reduction in output (Q), according to: $\frac{Z_{i,k}}{Q_{i,k}} = \bar{z}_{i,k} \times \alpha_{i,k}^{\frac{\varsigma}{\varsigma-1}} \times Q_{i,k}^{-\frac{1}{\gamma_k}}$. Thus, the optimal carbon border tax must strike a balance between a lower output and a higher unit carbon content. This tradeoff leads to the carbon border tax adjustment, $\frac{\gamma_k-1}{\gamma_k}$.

E.2 Melitz Model with Firm Selection

This appendix extends the Krugman-type model presented in Appendix E.2 to accommodate firm selection into export markets à la Melitz (2003). We show that the Melitz-type model is isomorphic to the Krugman-type model under a set of standard assumptions. The setting is akin to that described in Appendix E.1, but with two alterations: *(i)* A pool of potential firms can pay an entry cost $c_{i,k}f_{i,k}^{(e)}$ to draw their total factor productivity from a Pareto distribution, $G_{i,k}(\varphi) = 1 - \varphi^{-\theta_k}$. *(ii)* After realizing their total factor productivity, firms must pay a fixed export cost $y_n \bar{f}_{in,k}^{(x)}$ to serve market n, which is paid in terms of the income per worker, y_n, in the destination market.

As before, $M_{i,k}$ denotes the mass of firms that pay the fixed entry cost to operate from (i,k). Given the fixed export cost, only firms with a productivity above $\varphi_{in,k}^*$ serve market n. The CES price index of the national-level composite (in,k) is, thus, $P_{in,k} = \left[M_{i,k}\int_{\varphi_{in,k}^*}^{\infty}\left(\frac{1}{\varphi}\bar{\gamma}_k\bar{d}_{in,k}c_{i,k}\right)^{1-\gamma_k} \mathrm{d}G_{i,k}(\varphi)\right]^{\frac{1}{1-\gamma_k}}$, which can be written in terms of the productivity cutoff, $\varphi_{in,k}^*$, if $\theta_k > \gamma_k - 1$. Specifically,

$$P_{in,k}^{1-\gamma_k} = \frac{\theta_k}{\theta_k - \gamma_k + 1} M_{i,k}\left(\bar{\gamma}_k\bar{d}_{in,k}c_{i,k}\right)^{1-\gamma_k}\left(\varphi_{in,k}^*\right)^{\gamma_k-\theta_k-1} \tag{E.5}$$

To determine the mass of operating firms, note that the profits of a firm with productivity φ collected from sales in market n are given by $\pi_{in,k}(\varphi) = \frac{1}{\gamma_k}p_{in,k}(\varphi)\,q_{in,k}(\varphi) - y_n\bar{f}_{in,k}^{(x)}$. The CES demand function facing the firm is $q_{in,k}(\varphi) = p_{in,k}(\varphi)^{-\gamma_k} P_{in,k}^{\gamma_k-1} X_{in,k}$, where we use $X_{in,k} \equiv P_{in,k}C_{in,k}$ to compactly denote aggregate sales.[62] The productivity cutoff, $\varphi_{in,k}^*$, is determined by the zero cut-off profit condition, $\pi_{in,k}\left(\varphi_{ij,k}^*\right) = 0$, and can be written as a function of the price index, $P_{in,k}$,

$$\varphi_{in,k}^* = \frac{\frac{\gamma_k}{\gamma_k-1}\bar{d}_{in,k}c_{i,k}}{P_{in,k}}\left(\frac{X_{in,k}}{\gamma_k y_n \bar{f}_{in,k}^{(x)}}\right)^{\frac{1}{1-\gamma_k}}. \tag{E.6}$$

[62] More specifically, demand is determined by after-tax consumer prices as $q_{in,k}(\varphi) = \tilde{p}_{in,k}(\varphi)^{-\gamma_k}\tilde{P}_{in,k}^{\gamma_k-1}\tilde{X}_{in,k}$, but since all varieties associated with triplet (ni,k) are subjected to the same tax, we can write demand alternatively in terms of pre-tax prices.

It follows from combining Equations (E.5) and (E.6) that $(\text{Aggregate Marketing Costs})_{in,k} = \frac{\theta_k - \gamma_k + 1}{\theta_k \gamma_k} \times X_{in,k}$. Therefore, the gross ex-ante profits of a firm operating from (i,k) are $\sum_n \left[\left(\frac{1}{\gamma_k} - \frac{\theta_k - \gamma_k + 1}{\theta_k \gamma_k} \right) X_{in,k} \right] = \frac{\gamma_k - 1}{\theta_k \gamma_k} P_{ii,k} Q_{i,k}$. The free entry condition, which equates the gross ex-ante profits to the entry costs, therefore, yields $M_{i,k} = \frac{\gamma_k - 1}{\theta_k \gamma_k} \frac{P_{ii,k} Q_{i,k}}{c_{i,k} \bar{f}^{(e)}_{i,k}}$. Consolidating these points with Equations (E.5) and (E.6) and noting that $y_i \equiv Y_i / \bar{L}_i$, the aggregate price index of national-level varieties can be obtained as:

$$P_{in,k}^{1-\sigma_k} = \bar{\Gamma}_{in,k} \times \left(\bar{d}_{in,k} c_{i,k} \right)^{-(1+\theta_k)\rho_k} \times Q_{i,k}^{\rho_k} \times P_{n,k}^{-\frac{1-\sigma_k}{1-\gamma_k}(\gamma_k - \theta_k - 1)\rho_k} \tag{E.7}$$

where $\bar{\Gamma}_{in,k}$ is a constant and $\rho_k \equiv \left[1 + \frac{\theta_k + 1}{\sigma_k - 1} - \frac{\theta_k}{\gamma_k - 1} \right]^{-1}$.[63] To establish that the present model is isomorphic to the Krugman-type model examined in Section E.1, we define the following composite elasticities:

$$\sigma_k^{(Melitz)} \equiv 1 + (1 + \theta_k)\rho_k, \qquad \gamma_k^{(Melitz)} \equiv (1 + \theta_k) \tag{E.8}$$

Noting that $(1 - \sigma_k) + \frac{1-\sigma_k}{1-\gamma_k}(\gamma_k - \theta_k - 1)\rho_k = -(1 + \theta_k)\rho_k$, we define the following variety-level auxiliary price index,

$$\mathscr{P}_{in,k} = \bar{\Gamma}_{in,k}^{1/\left(1-\sigma_k^{(Melitz)}\right)} \times \bar{d}_{in,k} \times c_{i,k} \times Q_{i,k}^{-1/\gamma_k^{(Melitz)}} \tag{E.9}$$

The auxiliary price index, described by Equation (E.9), is closely related to the true price index, $P_{ij,k}$, according to:

$$\left(\frac{\mathscr{P}_{in,k}}{P_{n,k}} \right)^{1-\sigma_k^{(Melitz)}} = \left(\frac{P_{in,k}}{P_{n,k}} \right)^{1-\sigma_k} = \lambda_{in,k}$$

Using the above expressions and noting that $P_{n,k}^{1-\sigma_k} = \sum_i P_{in,k}^{1-\sigma_k}$, we obtain $P_{n,k} = \sum_i \left[\mathscr{P}_{in,k}^{1-\sigma_k^{(Melitz)}} \right]^{\frac{1}{1-\sigma_k^{(Melitz)}}}$. Next, we introduce taxes under the standard assumption that they are applied prior to the markup, acting as a cost-shifter. Taking similar steps as in the above derivations, we get the following formulation for consumer prices indexes and trade shares:

$$\text{(Aggregate Price Index- Melitz)} \quad \tilde{P}_{n,k} = \sum_i \left[\tilde{\mathscr{P}}_{in,k}^{1-\sigma_k^{(Melitz)}} \right]^{\frac{1}{1-\sigma_k^{(Melitz)}}}$$

$$\text{(Aggregate Demand Function - Melitz)} \quad \mathcal{D}_{in,k}\left(E_i, \tilde{\mathscr{P}}_i \right) = \frac{\tilde{\mathscr{P}}_{in,k}^{1-\sigma_k^{(Melitz)}}}{\sum_j \tilde{\mathscr{P}}_{ij,k}^{1-\sigma_k^{(Melitz)}}} \beta_{i,k} E_i,$$

where $\tilde{\mathscr{P}}_{in,k}$ is the consumer price index which is determined by the producer price index and taxes as below.

$$\tilde{\mathscr{P}}_{in,k} = \frac{(1 + t_{in,k})}{(1 + x_{in,k})(1 + s_{i,k})} \mathscr{P}_{in,k}, \qquad \mathscr{P}_{in,k} = \bar{\Gamma}_{in,k}^{1/\left(1-\sigma_k^{(Melitz)}\right)} \bar{d}_{in,k} c_{i,k} Q_{i,k}^{-1/\gamma_k^{(Melitz)}}$$

[63] Specifically, $\bar{\Gamma}_{in,k}$ has the following representation:

$$\bar{\Gamma}_{in,k} = \left[\frac{1}{\theta_k - \gamma_k + 1} \frac{\left[\gamma_k / (\gamma_k - 1) \right]^{-(1+\theta_k)}}{\bar{f}^{(e)}_{i,k}} \gamma_k^{-\frac{\gamma_k - \theta_k - 1}{1-\gamma_k}} \right]^{\rho_k} \times \left(\frac{\beta_{n,k} \bar{L}_n}{\gamma_k \bar{f}^{(x)}_{in,k}} \right)^{\rho_k \frac{\gamma_k - \theta_k - 1}{1-\gamma_k}}$$

Lastly, the balance-of-budget condition entails that $E_i = Y_i$, where total income is given by following

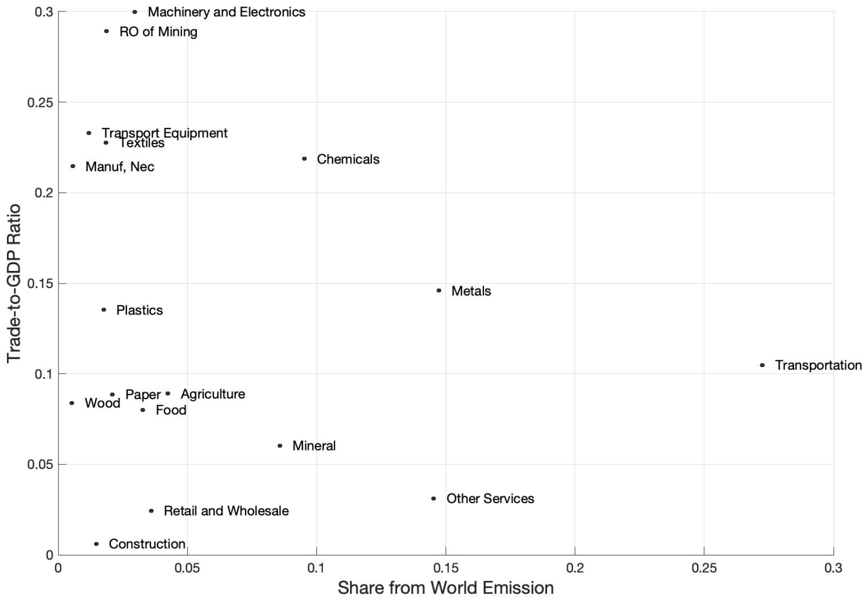
$$\text{(Balance of Budget - Melitz)} \quad Y_i = \varrho_i \left[w_i \bar{L}_i + r_i \bar{R}_i + T_i \right]$$

with $\varrho_n \equiv \left(1 - \sum_k \left[\frac{\theta_k - \gamma_k + 1}{\theta_k \gamma_k} \beta_{n,k}\right]\right)^{-1}$ denoting a correction that accounts for income from fixed cost payments. The wage and rental rates are determined by factor market clearing conditions as in earlier models, and so are the tax revenues. Letting $\tilde{P}_i = \prod_k \tilde{P}_{i,k}^{\beta_{i,k}}$ denote the consumer price index, the unilaterally optimal policy maximizes welfare, $W_i = \varrho_i \left(w_i \bar{L}_i + r_i \bar{R}_i + T_i \right) / \tilde{P}_i - \delta_i Z^{(global)}$ subject to the equilibrium conditions specified above. Considering the exact correspondence between the equilibrium conditions in the Melitz-Pareto model and the Krugman model studied in Appendix E.2, we can deduce that the unilaterally optimal policy for a small open economy are described by:

$$\begin{cases} \tau_i^* = \tilde{\delta}_i^{(Melitz)} \sim \frac{\delta_i}{\varrho_i} \tilde{P}_i, \qquad s_{i,k}^* = \frac{1}{\gamma_k^{(Melitz)} - 1} & \text{[carbon tax \& domestic subsidy]} \\ t_{ni,k}^* = \bar{t}_i + \frac{\gamma_k^{(Melitz)} - 1}{\gamma_k^{(Melitz)}} \tau_i^* v_{n,k} \qquad t_{ni,0}^* = \bar{t}_i & \text{[import tax (energy \& non-energy)]} \\ 1 + x_{in,k}^* = \left(1 + \bar{t}_i\right) \frac{\sigma_k^{(Melitz)} - 1}{\sigma_k^{(Melitz)}} + \frac{\gamma_k^{(Melitz)} - 1}{\gamma_k^{(Melitz)}} \tau_i^* \sum_{j \neq i} \left[\lambda_{jn,k} v_{j,k}\right] \frac{\sigma_k^{(Melitz)} - 1}{\sigma_k^{(Melitz)}} & \text{[export subsidy (non-energy)]} \\ 1 + x_{in,0}^* = \left(1 + \bar{t}_i\right) \frac{\sigma_0 - 1}{\sigma_0} + \tau_i^* \frac{1}{\sigma_0} \frac{\zeta_n}{\tilde{P}_{n,0}} & \text{[export subsidy (energy)]} \end{cases}$$

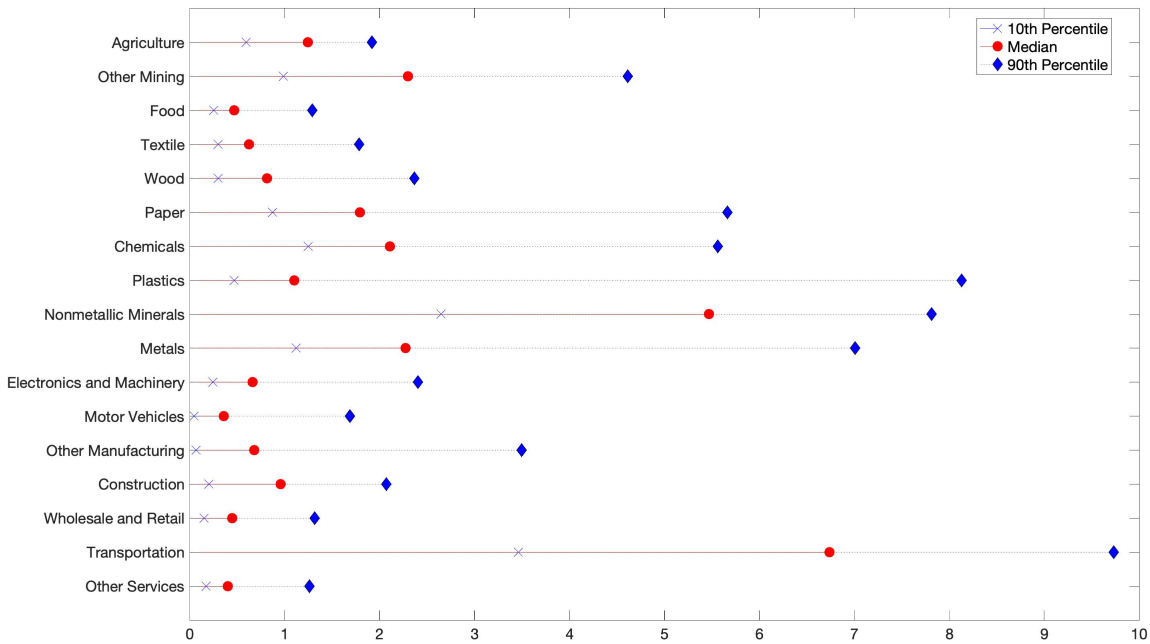
F Additional Figures and Tables

Figure A.1: Industry-level Share of Global Emissions vs Trade-to-GDP Ratio



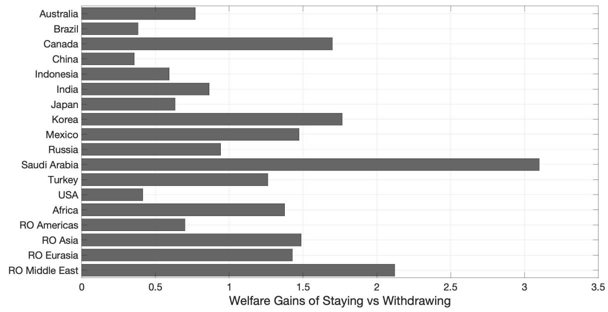
Notes: This figure shows the scatter plot of CO_2 emission for each industry as a share of total emissions against industry-level trade-to-GDP ratios.

Figure A.2: Unilaterally Optimal Carbon Import Taxes of the EU

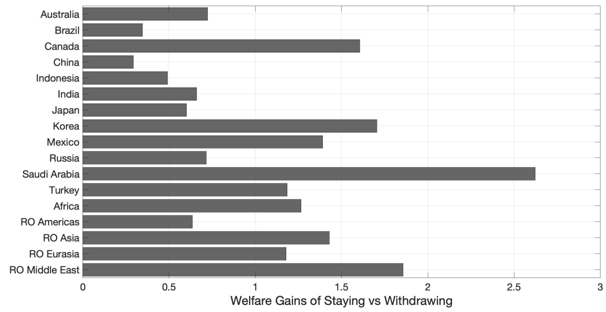


Notes: This figure shows for every industry the carbon import taxes adopted optimally and unilaterally by the EU. Holding an industry fixed, the unilaterally optimal import taxes differ across exporting countries since the carbon intensity of imported goods (carbon content per dollar of sales) varies across exporting countries. For each industry, the figure shows the 10th percentile, median, and 90th percentile of these carbon border tax rates across countries. These numbers are produced using the carbon disutility cost, equivalent to 53.2 $/tCO_2$, for the EU. In the absence of general equilibrium effects, adopting a higher carbon disutility cost for the EU proportionately scales up each point in the figure.

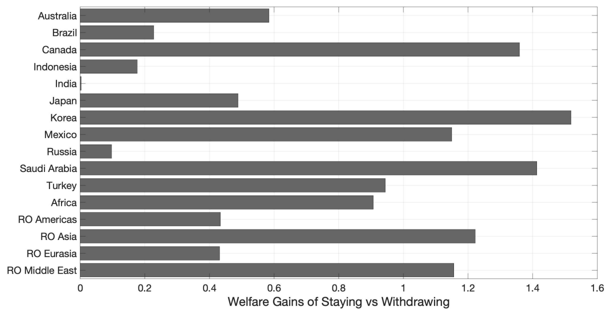
Figure A.3: Welfare Gains of Staying vs Leaving the Club-of-all-nations



(a) Core: EU



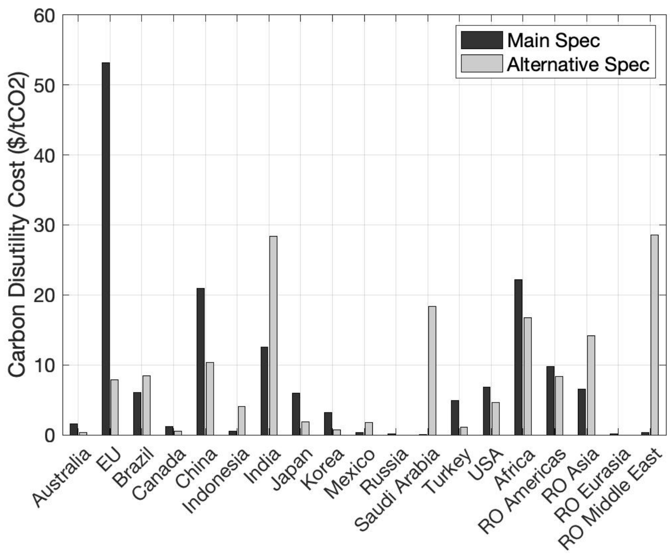
(b) Core: EU+US



(c) Core: EU+US+China

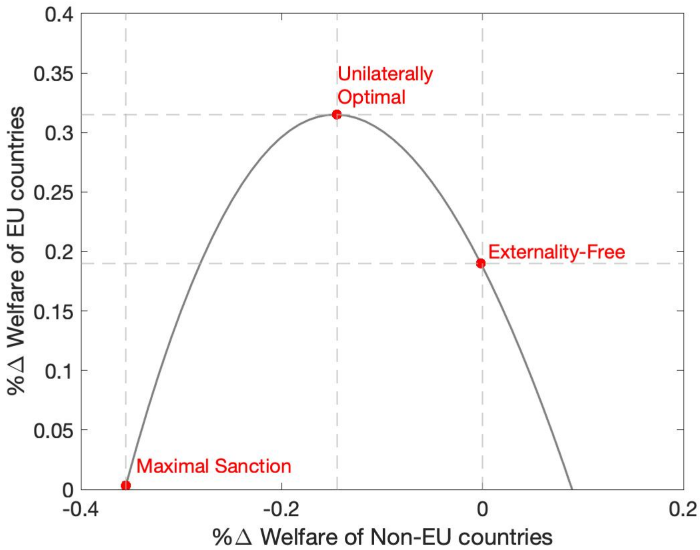
Notes: This figure shows the percentage change to welfare of staying relative to withdrawing unilaterally for each non-core country in different scenarios of the climate club: In Panel (a), the EU is the sole core member and the carbon tax target is 37 ($/tCO2). In Panel (b), the EU and US are the core members and the carbon tax target is 53 ($/tCO2). In Panel (c), the EU, US, and China are the core members and the carbon tax target is 90 ($/tCO2). In all three cases, the carbon tax target is the maximal target under which the club-of-all-nations emerges as the unique Nash equilibrium. In Panel (a) and (b), if we raised the tax target, the cub-of-all-nations would be still an equilibrium but not the unique equilibrium. But in Panel (c), if we raised the tax target, the club-of-all-nations would not be an equilibrium anymore. As shown in Panel (c), India is a marginal country that would withdraw if we raised the tax target. In addition, we evaluate the gains for core countries by comparing their welfare in the final outcome of the club relative to the status quo (*not* relative to the case where they unilaterally withdraw). Relative to the status quo: in case (a), the EU's welfare increases by 0.69%; in case (b), the welfare of the EU and US, respectively, increases by 0.85% and 0.09%; and in case (c), the welfare of the EU, US, and China increases, respectively, by 1.06%, 0.01%, and 0.39%.

Figure A.4: Carbon Disutility Costs: Baseline vs Alternative Specification



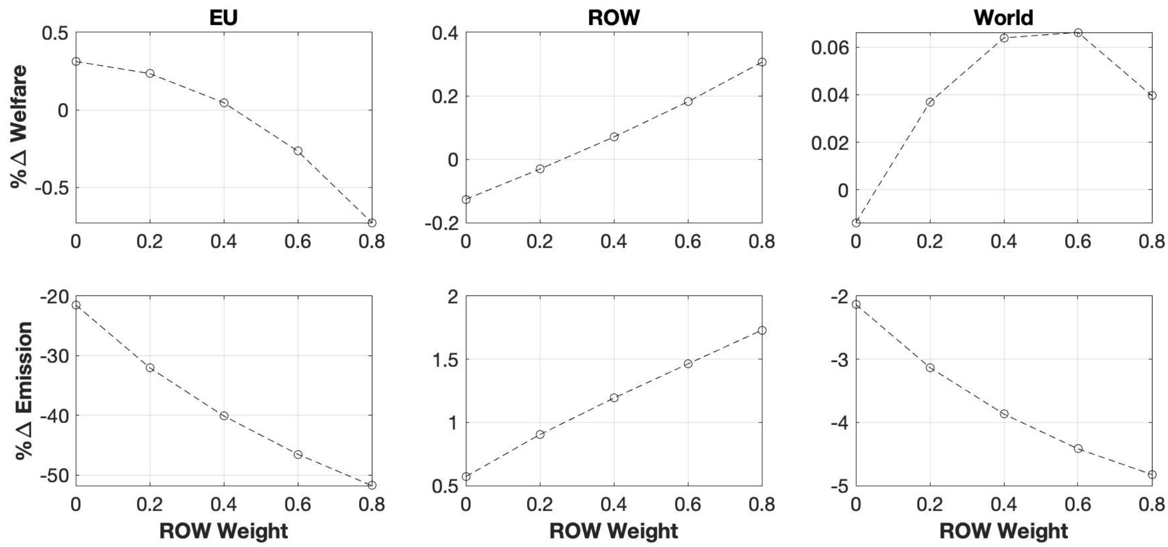
Notes: This figure shows the carbon disutility cost, $\tilde{\delta}_i$, for each country in our main specification versus an alternative specification, as discussed in Section 6.1. In both specifications, the sum, $\sum_i \tilde{\delta}_i$, equals the social cost of carbon at 156 ($/tCO2). In our main specification, the relative value of $\tilde{\delta}_i$ is larger for more populated countries, and controlling for population size, it is proportional to countries' environmentally-related taxes per unit of GDP. In the alternative specification, the relative value of $\tilde{\delta}_i$ is set based on the estimates of country-level social cost of carbon taken from Ricke et al. (2018).

Figure A.5: The Unilateral Policy Frontier of the EU



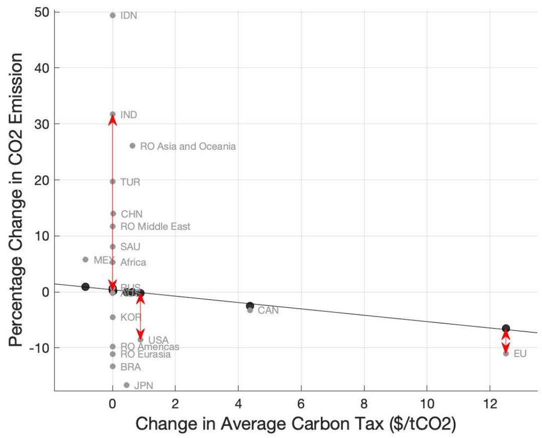
Notes: This figure shows the frontier of the EU's unilateral policy (EU as "Home") obtained from varying the weights that the EU assigns to the welfare of non-EU countries. The frontier illustrates the percentage change in the EU's welfare on y-axis against the percentage change in the ROW's welfare on x-axis (as an aggregation over the welfare of all non-EU countries). Each point on the frontier corresponds to a common weight that the EU assigns to non-EU countries. The maximum possible change in the EU's welfare corresponds to the point labelled as "Unilaterally Optimal" which is obtained when the EU assigns a zero weight to the ROW. By increasing ROW's weight from zero to positive values, we move along the frontier toward the right-hand side of the Unilaterally Optimal point. The point labelled as "Externality-Free" corresponds to the case where the EU's unilateral policy preserves the ROW's welfare relative to the status quo. By decreasing ROW's weight from zero to negative values, we move along the frontier to the left-hand side of the Unilaterally Optimal point. The point labelled as "Maximal Sanction" corresponds the case where the EU's policy maximally hurts the ROW without reducing its own welfare. See Appendix C.2 for details including the policy formulas.

Figure A.6: The EU Unilateral Policy's Impact on Welfare and Emissions Along the Frontier

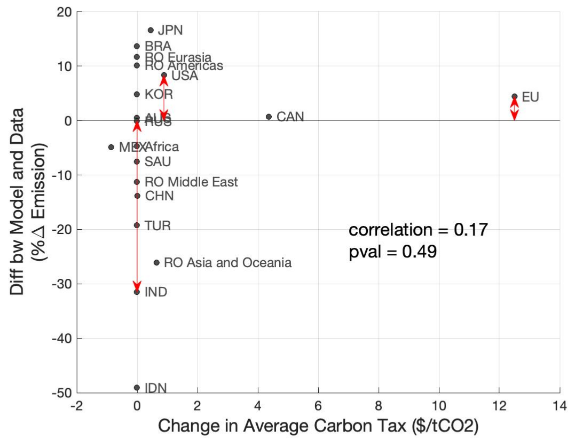


Notes: These figures show welfare and carbon emission changes in response to EU's unilateral policy (EU as "Home") obtained from varying the weight that the EU assigns to the welfare of non-EU countries. The x-axis shows the common weight that the EU assigns to non-EU countries. The y-axis shows the percentage change in welfare (three top figures) and carbon emissions (bottom three figures) in the EU, ROW as aggregate of non-EU countries, and the world. See Appendix C.2 for details including the policy formulas.

Figure A.7: Model Predictions vs Actual Emissions Response to Observed Carbon Tax Changes



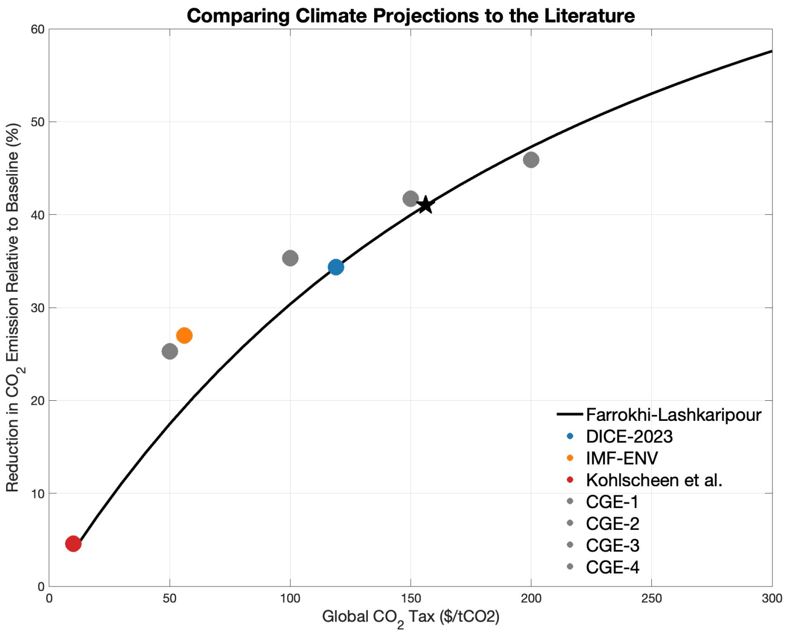
(a)



(b)

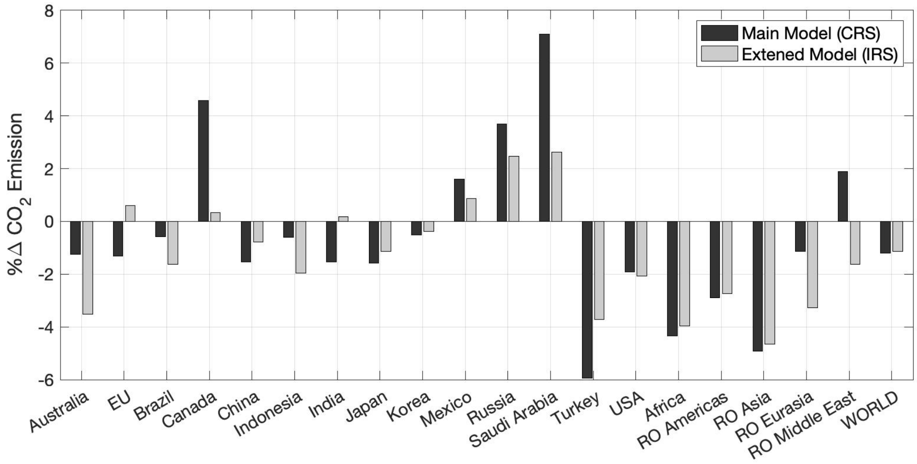
Notes: Panel (a) displays the model-predicted changes in CO2 emissions ($\Delta\mathbf{Z}^{(model)}$) for each country in response to the observed changes in average carbon taxes from 2014 to 2022 ($\Delta\boldsymbol{\tau}^{(data)}$). The predictions are based on our model calibrated to 2014 data as the baseline year; and, the year 2022 is the most recent year with available carbon tax and emissions data. The average carbon tax is calculated as the total carbon tax revenue in a country divided by the country's total CO2 emissions, using data from the World Bank Carbon Pricing Dashboard. This differs from implied carbon prices of specific policies—for example, the EU Emissions Trading System (EU-ETS) permit price, since EU countries have other climate policies and the EU-ETS covers only part of EU emissions. Panel (b) shows the difference between the predicted and actual CO2 emission changes over this period ($\Delta\mathbf{Z}^{(model)} - \Delta\mathbf{Z}^{(data)}$). This difference is visually illustrated using the red double arrows for the USA, EU, and India. The difference between observed and predicted changes can help validate our model in the spirit of the IV-based test proposed by Adao et al. (2023). Our exercise remains suggestive since data limitations prevent us to run the exact test proposed by Adao et al. (2023). With that caveat, if the correlation between $\left(\Delta\mathbf{Z}^{(model)} - \Delta\mathbf{Z}^{(data)}\right)$ and $\Delta\boldsymbol{\tau}^{(data)}$ is indistinguishable from zero, we cannot reject the null that our model is misspecified. The correlation between the noted variables is 0.17 with a p-value of 0.49, which is statistically indistinguishable from zero.

Figure A.8: CO2 Emission Reduction in Response to Globally Optimal Carbon Tax at Different Values of the SC-CO2



Notes: This figure compares our model's predicted global emission reductions to projections from other leading studies. The solid black line shows the percent reduction in global CO_2 emissions under the globally optimal carbon tax in our model, evaluated at different social costs of CO_2. The star indicates our model's predicted reduction under a social cost of $156/tCO2, which is our preferred value. The other dots represent reductions predicted by other studies: (1) *DICE-2023* represents the predicted emissions reduction in 2050 relative to a baseline without counterfactually elevated carbon prices (Barrage and Nordhaus, 2023). (2) *CGE-1* to *CGE-4* are average projected reductions across 10 computable general equilibrium (CGE) models, each projecting global emission reduction in 2030 relative to baseline—See Table 1 in Böhringer et al. (2021) for the list of models and Makarov et al. (2021) and Chepeliev et al. (2021) for further illustrations. (3) *IMF-ENV* represents projected emissions reduction under the International Monetary Fund's recursive dynamic neoclassical model for 2030 relative to baseline (Chateau et al., 2022). (4) *Kohlscheen et al* represents empirical estimate of emission reductions from various climate policies' implied carbon pricing across 121 countries (Kohlscheen et al., 2021).

Figure A.9: Emission Changes in Proposal 1: CRS vs IRS



Notes: This figure shows CO2 emission changes from carbon border taxes under Proposal 1 for each country and at the level of the world, for the main model with constant-returns-to-scale (CRS) final-goods technologies and in the extended model with increasing-returns-to-scale (IRS) final-goods technologies.

Table A.2: Accounting of CO_2 Emissions

		Direct Emission		Total Emission	
		MtCO2	Share	MtCO2	Share
Energy Types					
	Coal	155	0.5%	0	0.0%
	Crude Oil	282	0.9%	0	0.0%
	Natural Gas	188	0.6%	0	0.0%
	Refined Oil	1033	3.4%	0	0.0%
	Electricity and Gas Manuf.	12920	42.6%	0	0.0%
	Aggregate	14578	48.1%	0	0.0%
Downstream Industries					
1	Agriculture	512	1.7%	945	3.1%
2	RO of Mining	158	0.5%	411	1.4%
3	Food	333	1.1%	731	2.4%
4	Textiles	101	0.3%	415	1.4%
5	Wood	33	0.1%	116	0.4%
6	Paper	174	0.6%	469	1.5%
7	Chemicals	890	2.9%	2106	6.9%
8	Plastics	130	0.4%	390	1.3%
9	Mineral	1369	4.5%	1895	6.2%
10	Metals	1392	4.6%	3260	10.7%
11	Machinery and Electronics	149	0.5%	660	2.2%
12	Transport Equipment	60	0.2%	266	0.9%
13	Manuf, Nec	57	0.2%	125	0.4%
14	Construction	175	0.6%	329	1.1%
15	Retail and Wholesale	163	0.5%	807	2.7%
16	Transportation	5089	16.8%	6082	20.1%
17	Other Services	908	3.0%	3256	10.7%
	Aggregate	11693	38.6%	22261	73.4%
Households		4056	13.4%	8066	26.6%
Global		30327	100.0%	30327	100.0%

Notes: This table shows CO2 emissions by industries and households, based on direct and total emissions, with "total emissions" including direct and indirect CO2 emissions associated with *purchases of energy*. The carbon accounting shown in this table requires us to exclude non-CO2 greenhouse gas emissions and CO2 emissions associated with manufacturing process that do not arise from using fossil fuel energy. See Section 4.2 for a detailed description, and note that our procedure ensures the accounting of carbon flows.

Table A.3: Estimates of Trade Elasticity Parameters

Industry	Industry Code	(σ_k)	Obs.
Agriculture	1	4.80 (0.41)	28,228
Other Mining	2	11.16 (1.17)	49,255
Food	3	4.80 (0.41)	28,228
Textiles	4	5.25 (0.79)	13,418
Wood	5	7.50 (1.94)	7,424
Paper	6	7.55 (2.00)	8,728
Chemicals	7	9.60 (0.93)	25,464
Plastic	8	9.60 (0.93)	25,464
Minerals	9	6.27 (1.60)	9,482
Metals	10	6.99 (2.17)	12,548
Electronics & Machinery	11	4.98 (1.69)	13,148
Motor Vehicles	12	5.88 (1.36)	12,742
Other Mfg	13	5.80 (1.05)	12,498
Energy	101-105	11.16 (1.17)	49,255

Notes: This table reports our estimated trade elasticity parameters based on the specification described in Section 4.2.

Table A.4: Climate Club Game with the EU as Core and Maximum Carbon Tax Target of 37 ($/tCO2)

Round 1	Brazil, Korea, RO Eurasia
Round 2	China, Turkey
Round 3	Australia, Indonesia, India, Japan, Russia, United States, Africa, RO Asia, RO Middle East
Round 4	Canada, Mexico, Saudi Arabia, RO of Americas

Notes: This table shows the convergence of our solution method via successive rounds to a club of all nations, for the case in which the EU is the sole core member and the carbon tax target is at its maximal value of 37 $/t$CO_2$. A country unilaterally evaluates to join or leave at each round given the club configuration at its previous round.

Table A.5: Climate Club Game with the EU, US & China as Core and Maximum Carbon Tax Target of 90 ($/tCO2)

Round 1	Australia, Brazil, Canada, Japan, Korea, Mexico, Turkey, Africa, RO Americas, RO Eurasia
Round 2	Indonesia, Russia, Saudi Arabia, RO Asia, RO Middle East
Round 3	India

Notes: This table shows the convergence of our solution method via successive rounds to a club of all nations, for the case in which the EU, US, and China are core members and the carbon tax target is at its maximal value of 90 $/tCO_2$. A country unilaterally evaluates to join or leave at each round given the club configuration at its previous round.

Table A.6: Noncooperative Outcomes under Alternative Specifications

	Non-Cooperative			Globally
	Carbon+Border Tax	Carbon Tax	Border Tax	First Best
Main Specification	-6.5%	-5.4%	-1.2%	-41.0%
Alt SCC	-4.4%	-3.3%	-1.2%	-28.6%
Alt Carbon Disutility	-4.5%	-3.5%	-1.0%	-41.4%
Alt Trade Elast	-6.5%	-5.4%	-1.2%	-40.9%
Alt Energy Demand Elast	-8.0%	-6.6%	-1.5%	-48.9%
Alt Energy Supply Elast	-4.1%	-3.3%	-0.9%	-33.8%

Notes: This table shows outcomes under Proposal 1 (carbon border taxes) for five alternative specifications regarding the social cost of carbon, disutility parameters from carbon emissions, trade elasticity parameters, energy input demand elasticity and energy supply elasticity. See Section 6.1 for details of of these alternative parameterizations. The table shows the percentage change in global carbon emission under noncooperative carbon & border taxes (first column) and under only carbon taxes (second column). The implied difference between these two outcomes corresponds to the column under "Border Tax." As a benchmark for comparison, the emission reduction under the globally first best is reported in the last column.

Table A.7: Climate Club Outcomes under Alternative Specifications

Specification	Core Members	Max Tax ($/tCO2) (1)	Δ CO2 Climate Club (2)	Δ CO2 First Best (3)	Col 2 divided by Col 3 (4)
Alt SCC	EU	25	-9.7%	-28.6%	0.34
	EU+US	54	-18.9%	-28.6%	0.66
	EU+US+CHN	89	-28.0%	-28.6%	0.98
Alt Carbon Disutility	EU	31	-11.7%	-41.4%	0.28
	EU+US	72	-23.8%	-41.4%	0.58
	EU+US+CHN	95	-29.4%	-41.4%	0.71
Alt Trade Elasticty	EU	36	-13.3%	-40.9%	0.33
	EU+US	66	-22.2%	-40.9%	0.54
	EU+US+CHN	89	-27.9%	-40.9%	0.68
Alt Energy Demand Elasticity	EU	35	-15.7%	-48.9%	0.32
	EU+US	49	-21.0%	-48.9%	0.43
	EU+US+CHN	83	-32.0%	-48.9%	0.65
Alt Energy Supply Elasticity	EU	25	-6.1%	-33.8%	0.18
	EU+US	52	-12.4%	-33.8%	0.37
	EU+US+CHN	82	-19.1%	-33.8%	0.57

Notes: This table shows outcomes under Proposal 2 (Climate Club), for three scenarios of core countries (EU, EU+USA, EU+USA+China), and for five alternative specifications regarding the social cost of carbon, disutility parameters from carbon emissions, trade elasticity parameters, energy input demand elasticity and energy supply elasticity. See Section 6.1 for details of these alternative parameterizations. The table shows the maximal carbon tax target that supports the club-of-all-nation, percentage change in global carbon emission, the percentage change emission reduction achieved under globally first best, and the fraction of the first-best emission reduction that the club replicates.

Table A.8: Non-Cooperative and Cooperative Policy Outcomes under Increasing Returns to Scale

	Non-Cooperative						Globally Cooperative		
	Carbon + Border Tax			Carbon Tax					
Country	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW
Australia	-1.8%	-0.7%	-0.6%	1.7%	-0.0%	0.1%	-36.1%	-1.8%	-1.1%
EU	-19.0%	-0.3%	-0.1%	-19.4%	-0.0%	0.2%	-34.7%	-0.5%	1.4%
Brazil	-2.4%	-0.1%	0.3%	-0.8%	0.0%	0.3%	-35.8%	0.7%	2.8%
Canada	4.1%	-2.0%	-2.0%	3.8%	-0.0%	0.0%	-40.0%	-1.6%	-1.1%
China	-8.2%	-0.2%	-0.0%	-7.5%	0.0%	0.1%	-36.0%	-1.4%	-0.4%
Indonesia	0.1%	-0.3%	-0.2%	2.1%	-0.0%	0.1%	-40.6%	-2.9%	-2.5%
India	-4.8%	-0.5%	0.3%	-5.0%	0.0%	0.7%	-43.4%	6.6%	12.6%
Japan	-1.7%	-0.3%	-0.2%	-0.5%	0.0%	0.1%	-35.7%	-1.9%	-1.1%
Korea	0.6%	0.1%	0.3%	0.9%	0.0%	0.2%	-36.5%	1.5%	2.9%
Mexico	3.8%	-1.7%	-1.7%	2.9%	-0.0%	0.0%	-38.9%	-1.1%	-0.9%
Russia	6.1%	-1.5%	-1.5%	3.6%	-0.2%	-0.2%	-41.8%	0.2%	0.2%
Saudi Arabia	8.7%	-4.0%	-4.0%	5.9%	-0.6%	-0.6%	-39.8%	-6.7%	-6.6%
Turkey	-3.7%	-0.7%	-0.0%	-0.0%	0.1%	0.7%	-36.4%	3.0%	8.3%
USA	-3.7%	-0.4%	-0.3%	-1.6%	0.0%	0.0%	-39.6%	-2.0%	-1.7%
Africa	-12.8%	-1.5%	-0.3%	-9.2%	-0.1%	1.1%	-39.8%	10.7%	22.1%
RO Americas	-5.5%	-0.7%	-0.3%	-2.8%	-0.0%	0.4%	-38.6%	2.9%	6.3%
RO Asia	-5.0%	-1.3%	-1.2%	-0.4%	0.0%	0.2%	-37.9%	-0.0%	1.2%
RO Eurasia	0.4%	-1.3%	-1.3%	3.8%	-0.1%	-0.1%	-43.4%	-0.9%	-0.7%
RO Middle East	2.7%	-2.8%	-2.8%	4.3%	-0.3%	-0.3%	-40.9%	-1.2%	-1.0%
Global	-5.9%	-0.6%	-0.3%	-4.8%	-0.0%	0.2%	-38.1%	-0.5%	1.0%

Notes: For the extended version of our model a la Krugman that features increasing returns to scale in final good industries, this table shows for every country the change to CO_2 emission, real consumption, and welfare under noncooperative and cooperative policy equilibrium. Here, the baseline corresponds to an equilibrium in which each country's import tariffs and domestic carbon taxes are set at their applied rates in 2014, export subsidies are zero, and production subsidies correct for markup misallocation. The Krugman-type extension of our model and the optimal policy formulas in that setting are presented briefly in Section 6.3, with all the details provided in Section E.1 of the appendix.

Table A.9: Climate Club Outcomes under Increasing Returns to Scale

Core	Max Carbon Price Target ($/tCO2)	Reduction in World CO_2
EU	34	14.8%
EU+USA	46	18.6%
EU+USA+CHN	86	29.3%

Notes: For the extended version of our model a la Krugman that features increasing returns to scale in final good industries, this table shows the climate club outcomes of the maximal carbon price target and the corresponding reduction in global CO_2 emissions for three scenario of the core countries (EU, EU+USA, EU+USA+China). The Krugman-type extension of our model and the optimal policy formulas in that setting are presented briefly in Section 6.3, with all the details provided in Section E.1 of the appendix.