

Isomorphism & Welfare Analysis

International Trade (PhD), Fall 2024

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General Setup

- The representative consumer in country i has a CES utility aggregator over composite goods sourced from various origin countries $n = 1, \dots, N$. Namely,

$$U_i(Q_{1i}, \dots, Q_{Ni}) = \left(Q_{1i}^{\frac{\sigma-1}{\sigma}} + \dots + Q_{Ni}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $Q_{ni} = \left(\int_{\omega \in \Omega_{ni}} q_{ni}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$ over individual goods indexed by ω .

- Utility maximization s.t. budget constraint ($\sum_n P_{ni}Q_{ni} \leq E_i$) implies

$$\lambda_{ni} \equiv \frac{P_{ni}Q_{ni}}{E_i} = \left(\frac{P_{ni}}{P_i} \right)^{1-\sigma}, \quad P_i = \left[\sum_{n'=1}^N P_{n'i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

- Trade is balanced + labor is the sole factor of production $\longrightarrow E_i = Y_i = w_i L_i$

A General Representation of Aggregate Price Indexes

Following [Costinot and Rodriguez-Clare \(2014\)](#) we can specify the price indexes implied by quantitative trade models including Krugman, Eaton-Kortum, and Melitz-Pareto as

$$P_{ni} = \tau_{ni} \omega_n \times \left(\left(\frac{L_i}{f_{ni}} \right)^{\frac{\delta}{1-\sigma}} \frac{\tau_{ni} \omega_n}{P_i} \right)^{\eta} \times \left(\frac{L_n}{f_n^e} \right)^{\frac{\delta}{1-\sigma}} \times \tilde{\zeta}_{ni}$$

- τ_{ni} : iceberg trade cost
- f_{ni} : fixed operating cost
- f_n^e : sunk entry cost
- $\tilde{\zeta}_{ni}$ is composed of structural parameters unrelated to τ_{ni}

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The Welfare Impacts of an Arbitrary Change to Trade Costs

- Let W_i denote welfare in country i

$$W_i = \frac{E_i}{P_i} \xrightarrow{\text{balanced trade}} W_i = \frac{Y_i}{P_i}$$

- The welfare impacts of a generic shock to trade costs, $\{d \ln \tau_{in}\}_{i,n}$:

$$d \ln W_i = d \ln Y_i - d \ln P_i$$

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- We can update the expression for $d \ln W_i$ by appealing to the CES demand structure:

$$d \ln \lambda_{ni} - d \ln \lambda_{ii} = (1 - \sigma) (d \ln P_{ni} - d \ln P_{ii})$$

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$$d \ln P_{ni} = d \ln P_{ii} + \frac{1}{1 - \sigma} (d \ln \lambda_{ni} - d \ln \lambda_{ii})$$

Growth Accounting in the Armington Model

- Plugging the expression for $d \ln P_{ni}$ into the welfare equation yields

$$\begin{aligned} d \ln W_i &= d \ln Y_i - \sum_{n=1}^N \lambda_{ni} d \ln P_{ni} \\ &= d \ln Y_i - d \ln P_{ii} - \frac{1}{1 - \sigma} \sum_n [\lambda_{ni} (d \ln \lambda_{ni} - d \ln \lambda_{ii})] \end{aligned}$$

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$$d \ln W_i = \frac{1}{1-\sigma} d \ln \lambda_{ii} + \left(\underbrace{d \ln Y_i}_{= d \ln w_i} - d \ln P_{ii} \right)$$

Dissecting the Welfare Gains from Trade Liberalization

The welfare gains from trade liberalization, $\{d \ln \tau_{ni}\}_{n,i} < 0$, can be decomposed as

$$d \ln W_i = \underbrace{\frac{1}{1-\sigma} d \ln \lambda_{ii}}_{\text{gains from variety}} + \underbrace{(d \ln w_i - d \ln P_{ii})}_{\text{productivity gains}}$$

- With CES preferences, a country always gains from importing differentiated varieties from the rest of the world.
- In some settings (e.g., Eaton-Kortum, Melitz) trade liberalization also increases aggregate labor productivity (TFP):

$$P_{ii} Q_i = w_i L_i \quad \longrightarrow \quad \frac{w_i}{P_{ii}} = \frac{Q_i}{L_i} \sim \text{TFP}_i \quad \longrightarrow \quad d \ln w_i - d \ln P_{ii} = d \ln \text{TFP}_i$$

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effective output adjusted for iceberg & fixed cost payments

A Special Case Reviewed Earlier: *THE ARMINGTON MODEL*

- Aggregate TFP in the Armington model is invariant to trade by assumption:

$$P_{ni} = \underbrace{\frac{1}{A_n} \tau_{ni}}_{\text{constant}} w_n \quad \longrightarrow \quad (d \ln w_i - d \ln P_{ii}) = 0$$

- The welfare gains from incremental trade liberalization are, therefore,

$$d \ln W_i = \frac{1}{1 - \sigma} d \ln \lambda_{ii}$$

- Considering that $\tau^{\text{autarky}} = \infty$ and $\lambda_{ii}^{\text{autarky}} = 1$, the overall gains from trade are

$$GT_i \equiv - \int_{\tau}^{\infty} d \ln W_i = - \int_{\lambda_{ii}}^1 \frac{1}{1 - \sigma} d \ln \lambda_{ii} = \frac{1}{1 - \sigma} \ln \lambda_{ii}$$

Welfare Impacts Beyond Armington

- To characterize $(d \ln w_i - d \ln P_{ii})$, appeal to our earlier expression for price indexes:

$$P_{ni} = \tau_{ni} \omega_n \times \underbrace{\left(\left(\frac{L_i}{f_{ni}} \right)^{\frac{\delta}{1-\sigma}} \frac{\tau_{ni} \omega_n}{P_i} \right)^\eta}_{\text{firm-selection effects}} \times \underbrace{\left(\frac{L_n}{f_n^e} \right)^{\frac{\delta}{1-\sigma}}}_{\text{entry effects}} \times \zeta_{ni}$$

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$$\frac{P_{ii}}{w_i} = \left(\frac{Y_i}{P_i}\right)^\eta \times \underbrace{\left(\left(\frac{L_i}{f_{ii}}\right)^{\frac{\delta}{1-\sigma}} \frac{1}{L_i}\right)^\eta \left(\frac{L_i}{f_i^e}\right)^{\frac{\delta}{1-\sigma}} \zeta_{ii}}_{\text{invariant to } d \ln \tau}$$

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- To characterize $(d \ln w_i - d \ln P_{ii})$, appeal to our earlier expression for price indexes:

$$(d \ln w_i - d \ln P_{ii}) = -\eta d \ln \left(\frac{Y_i}{P_i} \right) = -\eta d \ln W_i$$

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Beyond Armington: *The Gains from Trade*

- Plugging $(d \ln w_i - d \ln P_{ii}) = -\eta d \ln W_i$ back into our earlier formula for $d \ln W_i$, yields

$$d \ln W_i = -\frac{1}{\epsilon} d \ln \lambda_{ii} \sim \frac{1}{(1 - \sigma)(1 + \eta)} d \ln \lambda_{ii}$$

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where ϵ is the **trade elasticity** that is defined as

$$\epsilon \equiv -\frac{\partial \ln \left(\frac{\lambda_{ni}}{\lambda_{ii}} \right)}{\partial \ln \tau_{ni}} = -\frac{\partial \ln \left(\frac{\lambda_{ni}}{\lambda_{ii}} \right)}{\partial \ln \left(\frac{P_{ni}}{P_{ii}} \right)} \times \frac{\partial \ln \left(\frac{P_{ni}}{P_{ii}} \right)}{\partial \ln \tau_{ni}} = (\sigma - 1) \times (1 + \eta)$$

Procedure for Computing the Gains from Trade

- Use data on trade shares, $\{\lambda_{ji}\}$, and trade costs, $\{\tau_{ji}\}$, to estimate ϵ as

$$\log \left(\frac{\lambda_{ni}}{\lambda_{ii}} \right) = -\epsilon \log \tau_{ni} + \varepsilon_{ni}$$

- Use the estimated $\hat{\epsilon}$ and data on λ_{ii} , to compute the gains from trade as

$$GT_i = \lambda_{ii}^{-\frac{1}{\hat{\epsilon}}}$$

- **Note:** the above procedure is model-blind, but the interpretation of ϵ depends on the underlying model (*e.g.*, Krugman *vs.* Eaton-Kortum *vs.* Melitz)

Taking Stock

- Arkolakis, Costinot, Rodriguez-Clare (2012, ACR) where to first to popularize the sufficient statistics approach to the gains from trade.

Caveat 1:

- The ACR result is occasionally interpreted as gains from trade being blind to firm heterogeneity
- A different interpretation is that the ACR result speaks to strong distributional assumptions (like Pareto) rather than firm-heterogeneity per se.

Caveat 2:

- τ_{ni} is often unobservable; so ϵ is often estimated using tariff data
 - $\tilde{\epsilon} \equiv$ the elasticity of trade w.r.t. tariffs
 - without firm-election, $\epsilon = \tilde{\epsilon}$
 - with firm-election, $\epsilon \neq \tilde{\epsilon}$

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the choice of model determines
how the estimated $\tilde{\epsilon}$ maps into ϵ

Some Number Using Data from 2008 and $\epsilon = 5$

	λ_{ii}	% GT
Ireland	0.68	8%
Belgium	0.70	7.5%
Germany	0.80	4.5%
China	0.88	2.6%
U.S.	0.92	1.8%

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- Based on the above numbers ACR (2012) conclude that gains from trade are *small*.

The Gains from Trade: *Reduced-Form Evidence*

- Reduced-form evidence from **Frankel & Romer (1999)** indicate that

$$\ln(\text{Real GDP}_i) = 3.94 \underbrace{(1 - \lambda_{ii})}_{\frac{1}{2}\text{OPENNESS}} + \varepsilon_i$$

- Considering that $\ln \lambda_{ii} \approx -(1 - \lambda_{ii})$ for small λ_{ii} , quantitative trade model predict

$$\ln(\text{Real GDP}_i) \approx \frac{1}{\epsilon} (1 - \lambda_{ii}) + \tilde{\varepsilon}_i$$

- If we believe that $\epsilon \approx 5 \implies$ reduced-form evidence imply gains that are *20-times* larger than those predicted by quantitative trade models!

The Gains from Trade: *Reduced-Form Evidence*

- The gap between the gains predicted by quantitative trade models and the gains predicted by **Frankel & Romer (1999)** can be *partially* eliminated if we account for
 - multiple industries with different trade elasticities
 - intermediate input trade (input-output linkages)
 - trade-led technology adoption
- However, even after adding all the above elements, the gap still persists!