The Melitz-Pareto Model

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Overview

- Melitz (2003) introduces firm heterogeneity & fixed export costs into Krugman (1980).
- Trade values respond to external shocks along two margins:
  - intensive margin: average sales per firm
  - extensive margin: number of firms that can profitably serve each market
- Despited the added richness, the Melitz model delivers the *gravity equation* if firm productivity levels exhibit a Pareto distribution.

- **Main references**:
  1. Melitz (2003), *"The impact of trade on intra-industry reallocations and aggregate industry productivity."* Econometrica.
  2. Chaney (2008), *"Distorted Gravity: The Intensive and Extensive Margins of International Trade."* American Economic Review.

Why was the Melitz Model Developed?

- From the lens of the Krugman model:
  - firms in a given country have similar productivity levels
  - all firms export to international markets
- Firm-level data suggests otherwise:
  - there is great across-firm heterogeneity in productivity levels.
  - most firms do not export: only 4% of U.S. firms exported in 2000.
  - exporters are more productive that non-exporters.
- The Melitz model extends Krugman to accommodate these data regularities.

The Central Insight of the Melitz Model

Neo-classical trade models

- trade enables countries to re-allocate resources from less-productive to more-productive (comparative advantage) industries.
- trade boosts aggregate productivity $\longrightarrow$ gains from trade

The Melitz model

- import competition crowds out less-productive firms, and reallocates resources from *less*- to *more-productive* firms.
- trade boosts aggregate productivity $\longrightarrow$ gains from trade

Environment

- Many countries indexed by $i,\ n = 1, ..., N$
- Many heterogeneous firms operate in each country
  - firms are indexed by ω
  - firms supply differentiated varieties and are monopolistically competitive
  - firms must incur a fixed overhead cost to serve each market
- Labor is the only factor of production
- Country i is endowed with L_i (inelastically-supplied) units of labor
- Trade is balanced: $D_i = 0 \longrightarrow E_i = Y_i \quad (\forall i)$

Demand

The representative consumer in country i has a CES utility function over differentiated firm-level varieties from various origin countries:

$$U_i\left(\mathbf{q}_{1i}, \ldots, \mathbf{q}_{Ni}\right) = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_{ni}} q_{ni}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

- $\sigma \geq 1$ is the elasticity of substitution between firm-level varieties.
- $\Omega_{ni} \subset \Omega_i$ is the sub-set of firms located in origin i that serve market i
- $q_{ni}\left(\omega\right)$ is the quantity of firm-level variety ω from origin country n.

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Demand

- The representative consumer maximizes utility subject to their budget constraint:

$$\max_{\mathbf{q}_i} U_i(\mathbf{q}_{1i}, ..., \mathbf{q}_{Ni}) \qquad s.t. \qquad \sum_{n=1}^{N} \left[\int_{\omega \in \Omega_{ni}} p_{ni}(\omega)\, q_{ni}(\omega) \right] \leq E_i$$

- The firm-level CES demand function implied by utility maximization:

$$\underbrace{\frac{p_{ni}(\omega)\, q_{ni}(\omega)}{E_i}}_{\text{expenditure share}} = \left(\frac{p_{ni}(\omega)}{P_i} \right)^{1-\sigma}, \qquad \underbrace{P_i = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_{ni}} p_{ni}(\omega)^{1-\sigma}\, d\omega \right]^{\frac{1}{1-\sigma}}}_{\text{CES price index}}$$

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Supply: Total Cost Function

- Firm ω located in country i, faces three types of cost
  - A sunk *entry cost* paid in terms of domestic labor: $w_i f_i^e$
  - *variable* + *fixed cost* of supplying $q_{in}(\omega)$ units to market n

$$\frac{1}{\varphi_i(\omega)} d_{in} w_i q_{in}(\omega) \; - \; \underbrace{w_n f_{in} \times \mathbb{1}_{q_{in}(\omega)>0}}_{\text{fixed export cost}}$$

- The total cost of operations for firm ω located in country i:

$$\text{TC}_i(\omega) = w_i f_i^e + \sum_{n=1}^{N} \left(\frac{d_{in} w_i}{\varphi_i(\omega)} q_{in}(\omega) \; + \; w_n f_{in} \times \mathbb{1}_{q_{in}(\omega)>0} \right).$$

Supply: Entry Scheme

- There is a pool of *ex-ante* identical firms in country n, each of which can pay an entry cost ($w_i f_i^e$) to independently draw a productivity φ from distribution $G_i(\varphi)$.
- Productivity, φ, uniquely determines the firm-level outcomes $\longrightarrow$ we can specify firm-level variables in terms of φ.

- Firms in country i enter until *expected* profits are dissipated to zero:

$$\mathbb{E}_\varphi \left[\sum_n \pi_{in}(\varphi) \right] - w_i f_i^e = 0$$

- After entry, firms serves market n if it's profitable given their realized productivity φ:

$$\pi_{in}(\varphi) \equiv \pi_{in}^{V}(\varphi) - w_n f_{in} \geq 0 \qquad \longrightarrow \qquad q_{in}(\varphi) > 0$$

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- After entry, firms serves market n if it's profitable given their realized productivity φ:

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Supply: *Optimal Pricing*

- The market structure is monopolistic competition

- A firm with productivity φ sets price to maximize variable profits

$$p_{in}(\varphi) = \arg\max_{p} \left[p - \frac{1}{\varphi} d_{in} w_i \right] q_{in}(p)$$

  where $q_{in}(p)$ is characterized by the CES demand function.

- The optimal price exhibits a constant markup over marginal cost

$$p_{in}(\varphi) = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \frac{1}{\varphi} d_{in} w_i$$

Entry and Selection into Markets

Zero Profit Cut-off Condition: firms with productivity $\varphi \geq \varphi_{in}^*$ export from i to market n:

$$\pi_{in}^V(\varphi_{in}^*) - w_n f_{in} = 0 \qquad \text{(Zero Profit Cut-off)}$$

Free Entry Condition

- Let M_i denote the mass of firms that pay the entry cost to operate from country i
- M_i is implicitly determined by the *free entry* condition:

$$\mathbb{E}_{\varphi}\left[\sum_{i=1}^{N}\left(\pi_{in}^V(\varphi) - w_n f_{in}\right) \times \mathbb{1}_{\varphi \geq \varphi_{in}^*}\right] - w_i f_i^e = 0 \qquad \text{(Free Entry)}$$

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sales per firm depends on $\{M_i\}_i$

The Effect of Trade on Aggregate Productivity

- Trade has two (interrelated) effects on aggregate productivity:

  1. import competition crowds out the least productive firms in each country

     $$\varphi_{ii}^{*}\left[\text{trade}\right] > \varphi_{ii}^{*}\left[\text{autarky}\right]$$

  2. the most productive (high-φ) firms can profitably export to foreign markets $\longrightarrow$ trade allows the most productive firms to grow in size

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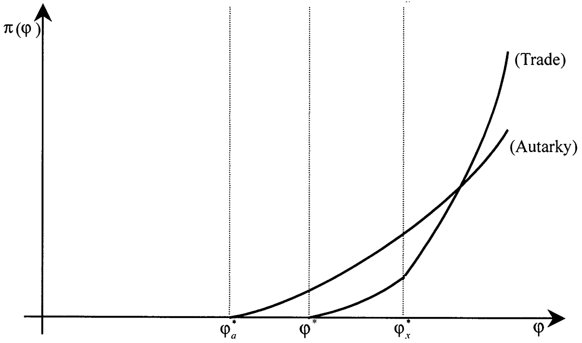
- Effects (1) & (2) $\longrightarrow$ trade reallocates resources from less- to more-productive firms $\longrightarrow$ trade increases aggregate productivity

Illustration of Inter-Firm Reallocation in Melitz (2003)



Note: this graph is based on the original Melitz (2003) paper featuring N symmetric countries, with $\pi(\varphi)$ denoting total profits net of fixed costs:

$$\varphi_a^* \sim \varphi_{ii}^* \text{ [autarky]} \qquad \varphi^* \sim \varphi_{ii}^* \text{ [trade]} \qquad \varphi_x^* \sim \varphi_{in}^* \quad (\forall n \neq i)$$

Key Assumption for Obtaining Gravity

- Following Chaney (2008, AER), assume that $G(.)$ is ***Pareto***:

$$G_i(\varphi) = 1 - (A_i/\varphi)^{\gamma}$$

- γ represents the degree of firm-level heterogeneity.
- A_i is a measure of country i's aggregate productivity.

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- γ represents the degree of firm-level heterogeneity.
- A_i is a measure of country i's aggregate productivity.
- **Note:** the Pareto assumption is necessary for obtaining a gravity equation.

Deriving the Gravity Equation

Step 1: Aggregating Firm-Level Sales

- Export sales from country i to n are the sum of all firm-level sales:

$$X_{in} = M_i \int_{\varphi_{in}^*}^{\infty} p_{in}(\varphi) q_{in}(\varphi) dG_i(\varphi)$$

- Appealing to CES demand, we can re-write the above equation as

$$\begin{aligned} X_{in} =& M_i \int_{\varphi_{in}^*}^{\infty} \left(\frac{p_{in}(\varphi)}{P_n} \right)^{1-\sigma} E_n dG_i(\varphi) = \gamma M_i A_i^{\gamma} \left(\frac{p_{in}(\varphi_{in}^*)}{P_i} \right)^{1-\sigma} E_n \int \left(\frac{p_{in}(\varphi)}{p_{in}(\varphi_{in}^*)} \right)^{1-\sigma} \varphi^{-\gamma-1} d\varphi \\ =& M_i A_i^{\gamma} \left(\frac{p_{in}(\varphi_{in}^*)}{P_i} \right)^{1-\sigma} E_n \int_{\varphi_{in}^*}^{\infty} \left(\frac{\varphi_{in}^*}{\varphi} \right)^{1-\sigma} \varphi^{-\gamma-1} d\varphi = \gamma\sigma M_i A_i^{\gamma} w_n f_{in} \int_{\varphi_{in}^*}^{\infty} \left(\frac{\varphi_{in}^*}{\varphi} \right)^{1-\sigma} \varphi^{-\gamma-1} d\varphi \end{aligned}$$

where the last line follows from the ZPC condition: $\left(\frac{p_{in}(\varphi_{in}^*)}{P_n} \right)^{1-\sigma} E_n = \sigma w_n f_{in}$.

Deriving the Gravity Equation

Step 1: Aggregating Firm-Level Sales

The last expression on the previous slide, can be simplified in 3 steps:

1. simplify the integral by a change in variables, $\nu \equiv \varphi / \varphi^*_{in}$

$$X_{in} = M_i A_i^{\gamma} w_n f_{in} \left(\varphi^*_{in}\right)^{-\gamma} \underbrace{\int_1^{\infty} \nu^{-\sigma-\gamma} d\nu}_{=1}$$

2. appeal to the ZPC condition to characterize φ^*_{in}:

$$\left(\frac{p_{in}(\varphi^*_{in})}{P_n}\right)^{1-\sigma} E_n = \sigma w_n f_{in} \implies \varphi^*_{in} = \frac{\sigma}{\sigma - 1} d_{in} w_i \left(\frac{E_n P_n^{\sigma-1}}{\sigma w_n f_{in}}\right)^{\frac{1}{1-\sigma}}$$

3. gather all *destination-specific* terms into one term, Θ_n:

$$X_{in} = \Theta_n \times M_i A_i^{\gamma} f_{in}^{1-\frac{\gamma}{\sigma-1}} \left(d_{in} w_i\right)^{-\gamma}$$

Deriving the Gravity Equation

Step 1: Aggregating Firm-Level Sales

- Combining our equation for aggregate sales, $X_{in} = \Theta_n A_i N_i f_{in}^{1-\frac{\gamma}{\sigma-1}} \left(\tau_{in} w_i\right)^{-\gamma}$, with the national-level budget constraint, $\sum_i X_{in} = E_n$, delivers

$$X_{in} = \frac{M_i A_i^{\gamma} \left(d_{in} w_i\right)^{-\gamma} f_{in}^{1-\frac{\gamma}{\sigma-1}}}{\sum_{j=1}^{N} M_j A_j^{\gamma} \left(d_{jn} w_j\right)^{-\gamma} f_{jn}^{1-\frac{\gamma}{\sigma-1}}} E_n$$

  where $E_n = Y_n = w_n L_n$ since there are no trade imbalances.

- The next step is to characterize $\{M_i\}_i$ as a function of *structural* parameters using the free-entry condition.

Deriving the Gravity Equation

Step 2: Characterizing the Mass of Entrants (M_i)

- The free entry condition yields a closed-form solution for the number of firms

$$\sum_{n=1}^{N}\left(\frac{1}{\sigma}X_{in} - \underbrace{M_{in}\, w_n\, f_{in}}_{\text{fixed overhead cost}}\right) - \underbrace{M_i\, w_i\, f^e}_{\text{entry cost}} = 0 \qquad \text{(Free Entry)}$$

where the mass of entrants serving market n is $M_{in} \equiv [1 - G_i(\varphi^*_{in})]\, M_i$.

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- After some tedious algebra, the above equation implies that

$$M_i = \frac{\sigma - 1}{\sigma\gamma}\frac{L_i}{f^e}$$

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- After some tedious algebra, the above equation implies that

$$M_i = \frac{\sigma - 1}{\sigma\gamma}\frac{L_i}{f^e} \qquad \longrightarrow \qquad X_{in} = \frac{L_i A_i^{\gamma}\left(d_{in} f_{in}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}} w_i\right)^{-\gamma}}{\sum_{j=1}^{N} L_j A_j^{\gamma}\left(d_{jn} f_{jn}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}} w_j\right)^{-\gamma}} E_n$$

General Equilibrium

For any given vector of parameters $\{d_{in}, f_{in}, f^e, L_i, \sigma, \gamma\}_{i,n}$, equilibrium is a vector of wages, $\{w_i\}_i$, such that labor markets clear in all countries:

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(w_1, ..., w_N) \times E_n(w_n)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = w_i L_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(w_1, ..., w_N) = \dfrac{A_i^{\gamma} L_i \left(d_{in} f_{in}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}} w_i \right)^{-\gamma}}{\sum_{j=1}^{N} A_j^{\gamma} L_j \left(d_{jn} f_{jn}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}} w_j \right)^{-\gamma}} & (\forall i, j) \\ E_n(w_n) = w_n L_n & (\forall i, \text{ balance budegt}) \end{cases}$$

General Equilibrium (in terms of Y)

For any given vector of exogenous parameters and variables $\{d_{in}, f_{in}, f^e, L_i, \sigma, \gamma\}_{i,n}$, equilibrium is a vector of GDP levels, $\{Y_i\}_i$, such that labor markets clear in all countries.

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(Y_1, ..., Y_N) \times E_n(Y_n)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = Y_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

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An Overview of the Melitz-Pareto Model

- The Melitz-Pareto model belongs to the class of quantitative models reviewed earlier:

$$\tilde{\chi}_i \sim L_i^{1+\gamma} A_i^{\gamma}, \qquad \tau_{in} \sim d_{in} f_{in}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}}, \qquad \epsilon \sim \gamma$$

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{Y_i}{P_i}, \qquad P_i = C \times \left[\sum_{n=1}^{N} A_n^{\gamma} L_n^{1+\gamma} \left(d_{ni} f_{ni}^{\frac{\gamma-\sigma+1}{(\sigma-1)\gamma}} w_n \right)^{-\gamma} \right]^{-\frac{1}{\gamma}}$$

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C: encapsulates non-country-specific constants

Melitz *vs.* Krugman and Neoclassical Trade Models

- The Melitz-Pareto model predicts similar gains from trade (up-to a choice of trade elasticity) as Krugman, Armington, or Eaton-Kortum:

$$GT_i = 1 - \lambda_{ii}^{\frac{1}{\gamma}} \sim 1 - \lambda_{ii}^{\frac{1}{\epsilon}}$$

- It also predicts similar welfare impacts *w.r.t.* to a trade cost shock $\{\hat{\tau}_{in}\}_{i,n} \sim \left\{\hat{d}_{in}\right\}_{i,n}$:

$$\widehat{W}_i = \frac{\widehat{Y}_i}{\widehat{P}_i}, \qquad \widehat{P}_i = \left[\sum_n \lambda_{ni}\, \hat{\tau}_{ni}^{-\gamma}\, \widehat{Y}_n^{-\gamma}\right]^{-\frac{1}{\gamma}}$$

where $\widehat{Y}_i$ can be calculated with data on the expenditure matrix,$\{\lambda_{in}\}_{i,n}$, and GDP levels, $\{Y_i\}_i$, using the following system:

$$\widehat{Y}_i Y_i = \sum_{n=1}^{N} \left[\frac{\lambda_{in}\, \hat{\tau}_{in}^{-\gamma}\, \widehat{Y}_i^{-\gamma}}{\sum_{j=1}^{N} \lambda_{jn}\, \hat{\tau}_{jn}^{-\gamma}\, \widehat{Y}_j^{-\gamma}} \widehat{Y}_n Y_n\right]$$

Auxiliary slides

Extensive vs. Intensive Margins in Melitz-Pareto

Why is the trade elasticity $\epsilon \equiv \frac{\partial \ln X_{in}}{\partial \ln d_{in}}$ in the Melitz-Pareto model independent of σ?

- Applying the Leibniz rule we can decompose the trade elasticity into extensive and intensive margin components:

$$\frac{\partial \ln X_{in}}{\partial \ln d_{in}} = \underbrace{\frac{\int_{\varphi_{in}^*}^{\infty} x_{in}\left(\varphi\right) \frac{\partial \ln x_{in}(\varphi)}{\partial \ln d_{in}} \tau_{in} dG_i\left(\varphi\right)}{X_{in}}}_{\text{intensive margin}=\sigma-1} + \underbrace{\frac{x_{in}\left(\varphi_{in}^*\right) \varphi_{in}^* \frac{\partial \ln \varphi_{in}^*}{\partial \ln d_{in}} dG_i\left(\varphi_{in}^*\right)}{X_{in}}}_{\text{extensive margin }=\gamma-\sigma+1} = \gamma,$$

where $x_{in}\left(\varphi\right) \equiv p_{in}\left(\varphi\right) q_{in}\left(\varphi\right)$ denotes sales by a firm with productivity φ.

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  where $x_{in}\left(\varphi\right) \equiv p_{in}\left(\varphi\right) q_{in}\left(\varphi\right)$ denotes sales by a firm with productivity φ.

- The contribution of σ to the extensive and intensive margin elasticities nullify each other—i.e., a greater σ implies that trade adjusts more aggressively on the intensive margin but less aggressively on the extensive margin.

Final Remarks

- Arkolakis et. al. (2018, ReStud) show that one can obtain gravity without CES if the firm-level productivity distribution is Pareto and demand exhibits the following functional-form:
$$q_\omega(\mathbf{p}, y) = \mathcal{D}\left(p/P\left(\mathbf{p}, y\right)\right) \mathcal{Q}\left(\mathbf{p}, y\right)$$

- The ACDR demand system admits variable & heterogeneous markups, but the distribution of markups is independent of $\{\tau_{in}\}_i$ and the origin country.

- The gains from trade under ACDR preferences are

$$GT_i = 1 - \lambda_{ii}^{\frac{1}{\epsilon}(1-\eta)}, \qquad \begin{cases} \eta = 0 & \text{if preferences are homothetic} \\ \eta \neq 0 & \text{if preferences are non-homothetic} \end{cases}$$

- under *non-homothetic* preferences, trade can influence relative demand for low- versus high-markup varieties $\longrightarrow$ trade modifies the extent of misallocation form markups