The Krugman Model

International Trade (PhD), Fall 2024

Ahmad Lashkaripour

Indiana University

Overview

- The Krugman model is essentially a multi-country *Dixit-Stiglitz* model
- *Increasing returns to scale* is the driving force behind international trade.
- This is the simplest firm-level model that generates gravity.

Main references

- Krugman P., *"Scale Economies, Product Differentiation, and the Pattern of Trade."* American Economic Review. 1980.
- *The basics of "Dixit-Stiglitz lite"* by Jonathan Dingel

Environment

- Many countries indexed by $i,\ n = 1, ..., N$
- Each country hosts many symmetric firms
  - firms are indexed by ω
  - firms supply differentiated varieties and are monopolistically competitive
- Labor is the only factor of production
- Country i is endowed with L_i (inelastically-supplied) units of labor
- Trade is balanced: $D_i = 0 \longrightarrow E_i = Y_i \quad (\forall i)$

Demand

The representative consumer in country i has a CES utility function over differentiated firm-level varieties from various origin countries:

$$U_i\left(\mathbf{q}_{1i}, \dots, \mathbf{q}_{Ni}\right) = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_n} q_{ni}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

- $\sigma \geq 1$ is the elasticity of substitution between firm-level varieties.
- Ω_n is the set of all firms operating in origin n
- $q_{ni}\left(\omega\right)$ is the quantity of firm-level variety ω from origin country n.

Demand

- The representative consumer maximizes utility subject to their budget constraint:

$$\max_{\mathbf{q}_i} U_i(\mathbf{q}_{1i}, ..., \mathbf{q}_{Ni}) \qquad s.t. \qquad \sum_{n=1}^{N} \left[\int_{\omega \in \Omega_n} p_{ni}(\omega) \, q_{ni}(\omega) \right] \leq E_i$$

- The firm-level CES demand function implied by utility maximization:

$$\underbrace{\frac{p_{ni}(\omega) \, q_{ni}(\omega)}{E_i}}_{\text{expenditure share}} = \left(\frac{p_{ni}(\omega)}{P_i} \right)^{1-\sigma}, \qquad \underbrace{P_i = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_n} p_{ni}(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}}_{\text{CES price index}}$$

Supply – Cost Function

- Firm ω pays a sunk entry cost $w_i f_i$ to operate from country i

- Upon entry firm ω can sell to any country subject to a constant marginal cost

$$\text{MC}_{in}(\omega) = \underbrace{\frac{1}{\varphi_i(\omega)}}_{\text{productivity}} \times \underbrace{\tau_{in}}_{\text{trade cost}} \times \underbrace{w_i}_{\text{wage}}$$

- Assumption: Firms operating in country i are symmetric:

$$\varphi_i(\omega) = \varphi_i, \qquad q_{in}(\omega) = q_{in} \qquad (\forall \omega \in \Omega_i)$$

- The total cost faced by a typical firm from country i:

$$\text{TC}_i = w_i f_i + \sum_{n=1}^{N} \frac{1}{\varphi_i} \tau_{in} w_i q_{in}$$

Supply – Optimal Pricing

- The market structure is monopolistic competition

- Firm ω sets price to maximize variable profits

$$p_{in} = \arg\max_{p} \left[p - \frac{1}{\varphi_i} \tau_{in} w_i \right] q_{in}(p),$$

  where $q_{in}(p)$ is characterized by the CES demand function.

- The optimal price exhibits a constant markup over marginal cost

$$p_{in} = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \frac{1}{\varphi_i} \tau_{in} w_i$$

Supply – Number of Firms

- Let $M_i \equiv | \Omega_i |$ denote the mass of firms operating in country i.
- M_i is determined by free entry (*i.e.*, firms enter until profits are dissipated)

$$\text{net profits} = \sum_{n=1}^{N} \left[\underbrace{\left(p_{in} - \frac{1}{\varphi_i} \tau_{in} w_i \right) q_{in}}_{\text{variable profits from sales to } n} \right] - w_i f_i = 0$$

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$$p_{in} = \frac{\sigma}{\sigma - 1}\frac{1}{\varphi_i}\tau_{in}w_i \qquad \longrightarrow \qquad \frac{1}{\sigma}\sum_{n=1}^{N} p_{in}q_{in} = w_i f_i \qquad \text{(free entrty)}$$

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- Since total revenue from sales equates wage payments in country i, we can derive a closed-form expression for the mass of firms, M_i:

$$M_i \times \overbrace{\sum_n p_{in} q_{in}}^{\text{sales per firm}} = \overbrace{w_i L_i}^{\text{wage bill}} \quad \xrightarrow{\text{free entry}} \quad M_i = \frac{1}{\sigma f_i} L_i$$

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Aggregate Expenditure Shares

- Aggregate expenditure shares can be constructed from firm-level expenditure shares:

$$\lambda_{in} = \frac{M_i p_{in} q_{in}}{E_n} \quad \xrightarrow{\text{CES demand}} \quad \lambda_{in} = \frac{M_i p_{in}^{1-\sigma}}{\sum_{j=1}^{N} M_j p_{jn}^{1-\sigma}}$$

- Accounting for optimal pricing and free entry yields our familiar-looking gravity equation:

$$\begin{cases} p_{in} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_i}\tau_{in}w_i \\ M_i = L_i/\sigma f_i \end{cases} \longrightarrow \quad \lambda_{in} = \frac{\left(L_i/f_i\right)\varphi_i^{\sigma-1}\left(\tau_{in}w_i\right)^{1-\sigma}}{\sum_{j=1}^{N}\left(L_j/f_j\right)\varphi_j^{\sigma-1}\left(\tau_{jn}w_j\right)^{1-\sigma}}$$

General Equilibrium

For a given vector of parameters, $\{\tau_{in}, \varphi_i, f_i, L_i, \sigma\}_{i,n}$, equilibrium is a vector of wages, $\{w_i\}_i$, that clear labor markets in all countries:

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(w_1, ..., w_N) \times E_n(w_n)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = w_i L_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(w_1, ..., w_N) = \dfrac{(L_i/f_i)\varphi_i^{\sigma-1}(\tau_{in} w_i)^{1-\sigma}}{\sum_{j=1}^{N} \left(L_j/f_j\right) \varphi_j^{\sigma-1} \left(\tau_{jn} w_j\right)^{1-\sigma}} & (\forall i, j) \\ E_n(w_n) = w_n L_n & (\forall i, \text{ balance budegt}) \end{cases}$$

General Equilibrium (in terms of Y)

For a given vector of parameters, $\{\tau_{in}, \varphi_i, f_i, L_i, \sigma\}_{i,n}$, equilibrium is a vector of GDP levels, $\{Y_i\}_i$, that clear labor markets in all countries:

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(Y_1, ..., Y_N) \times E_n(Y_n)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = Y_i \quad ,\forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

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An Overview of the Krugman Model

- The Krugman model belongs to the quantitative class of models reviewed earlier:

$$\tilde{\chi}_i \sim f_i^{-1} \left(L_i^{\frac{\sigma}{\sigma-1}} \varphi_i \right)^{\sigma-1}, \qquad \epsilon \sim \sigma - 1$$

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{Y_i}{P_i}, \qquad P_i = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_n} p_{ni}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

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$$W_i = \frac{Y_i}{P_i}, \qquad P_i = C \times \left[\sum_{n=1}^{N} \varphi_n^{\sigma-1} L_n^{\sigma} \left(f_n \right)^{-1} \left(\tau_{ni} Y_n \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

C: encapsulates non-country-specific constants

Krugman *vs.* Neoclassical Trade Models

- The Krugman model predicts similar *ex post* gains from trade (up-to a choice of trade elasticity) as neoclassical models—*e.g.*, Armington, Eaton-Kortum

$$GT_i \equiv \frac{W_i - W_i^{(autarky)}}{W_i} \sim \text{gains from trade}$$

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$$GT_i = 1 - \lambda_{ii}^{\frac{1}{\sigma-1}} \sim 1 - \lambda_{ii}^{\frac{1}{\epsilon}}$$

- It also predict the same *ex ante* welfare impacts in response to a trade cost shock $\{\hat{\tau}_{in}\}_{i,n}$:

$$\widehat{W}_i = \frac{\widehat{Y}_i}{\widehat{P}_i}, \qquad \widehat{P}_i = \left[\sum_n \lambda_{ni}\, \hat{\tau}_{ni}^{1-\sigma}\, \widehat{Y}_n^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

where $\widehat{Y}_i$ can be calculated with data on basline expenditure shares, $\{\lambda_{in}\}_{i,n}$, and GDP levels, $\{Y_i\}_i$, via the following system:

$$\widehat{Y}_i Y_i = \sum_{n=1}^{N} \left[\frac{\lambda_{in}\, \hat{\tau}_{in}^{1-\sigma}\, \widehat{Y}_i^{1-\sigma}}{\sum_{j=1}^{N} \lambda_{jn}\, \hat{\tau}_{jn}^{1-\sigma}\, \widehat{Y}_j^{1-\sigma}} \widehat{Y}_n Y_n\right]$$

Real GDP and Size: Krugman *vs.* Armington

- Real GDP is characterized by technology parameters, size, and, trade openness:

$$\frac{Y_i}{P_i} = \underbrace{\frac{\varphi_i}{\tau_{ii}\left(\sigma f_i\right)^{\sigma-1}} \times L_i^{\frac{\sigma}{\sigma-1}} \times \lambda_{ii}^{\frac{1}{1-\sigma}}}_{\text{Krugman}}, \qquad \frac{Y_i}{P_i} = \underbrace{\frac{A_i \beta_i}{\tau_{ii}} \times L_i \times \lambda_{ii}^{\frac{1}{1-\sigma}}}_{\text{Armington}}$$

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- The Krugman model predicts that, all else equal, real GDP (Y/P) increases more than proportional with population size (L):

$$\underbrace{\frac{\partial \ln\left(Y_i/P_i\right)}{\partial \ln L_i} = \frac{\sigma}{\sigma-1} > 1}_{\text{Krugman}}; \qquad \underbrace{\frac{\partial \ln\left(Y_i/P_i\right)}{\partial \ln L_i} = 1}_{\text{Armington}}$$

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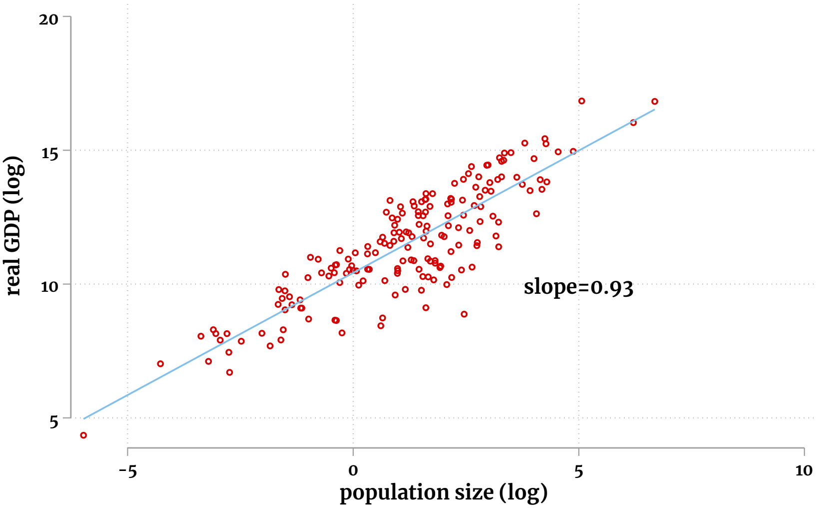
- **Why?** because of *increasing-returns to scale*, driven by love-for-variety.

Are the Scale Effects Predicted by Krugman Consistent with Data?



Source: Penn World Tables 10, year = 2019

Are the Scale Effects Predicted by Krugman Consistent with Data?

- Cross-sectional data suggests that $\frac{\partial \ln(Y_i/P_i)}{\partial \ln L_i} < 1$, but does it reject the presence of scale economies?

- Not necessarily, because larger countries have higher domestic expenditures shares ($\frac{\partial \ln \lambda_{ii}}{\partial \ln L_i} > 0$) and presumably larger domestic trade frictions ($\frac{\partial \ln \tau_{ii}}{\partial \ln L_i} > 0$):

$$\frac{d \ln (Y_i/P_i)}{d \ln L_i} = \frac{\partial \ln (Y_i/P_i)}{\partial \ln L_i} + \underbrace{\frac{\partial \ln (Y_i/P_i)}{\partial \ln \tau_{ii}}}_{=-1} \underbrace{\frac{d \ln \tau_{ii}}{d \ln L_i}}_{>0} + \underbrace{\frac{\partial \ln (Y_i/P_i)}{\partial \ln \lambda_{ii}}}_{=1/(1-\sigma)<0} \underbrace{\frac{d \ln \lambda_{ii}}{d \ln L_i}}_{>0}$$

- See Ramondo, Rodríguez-Clare, & Saborío-Rodríguez (2016, AER) for a formal exploration of the income-size elasticity puzzle.