Multi-Country Ricardian Model

Eaton and Kortum (2002)

International Trade (PhD), Spring 2024

Ahmad Lashkaripour

Indiana University

Roadmap

- Today, we will cover **Eaton and Kortum (2002)**.
- The Eaton-Kortum model extends DFS1977 by allowing for
  - arbitrarily many countries
  - arbitrary trade costs
- The Eaton-Kortum model provides a Ricardian foundation for the gravity equation

$$\text{Ricardian specialization} \quad \longrightarrow \quad \text{Trade Value} \propto \frac{\text{Exporter's GDP} \times \text{Importer's GDP}}{\text{Distance}^{\beta}}$$

Environment

- The global economy consist of $N \geq 2$ countries
- We use $i, j, n \in \{1, .., N\}$ to index countries
- There is a continuum of homogeneous goods $\omega \in [0, 1]$
- Each good ω which is sourced from the chepast supplier.
- Labor is the only factor of production:
  - country i is populated by L_i workers
  - w_i denotes the wage rate in country i
- Perfect competition + constant returns to scale.

Demand

CES Utility Function

- The representative consumer in country i has a CES utility function:

$$U_i(\mathbf{q}) = \left[\int_{\omega} q_i(\omega)^{\frac{\sigma-1}{\sigma}}\, d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

- σ is the elasticity of substitution across goods.
- The Cobb-Douglas utility function in DFS1977 is a special case of the CES utility function, where $\sigma \to 1$.

Demand

CES Utility Function

- Utility maximization $\longrightarrow$ expenditure on good ω equals

$$p_i(\omega)q_i(\omega) = \frac{p_i(\omega)^{1-\sigma}}{\int_{\omega'} p_i(\omega')^{1-\sigma} d\omega} Y_i$$

- $p_i(\omega)$ is the price of good ω in country i.
- $Y_i = w_i L_i$ is total income in country i.

Supply

- The price of good ω in country i if sourced from country n

$$p_{ni}(\omega) = \tau_{ni}\, a_n\,(\omega)\; w_n$$

  - τ_{ni} is the iceberg trade cost
  - $a_n(\omega)$ is the unit labor cost of producing ω in country n

- Country i buys good ω from the cheapest supplier:

$$p_i(\omega) = \min\{p_{1i}(\omega), ..., p_{Ni}(\omega)\}$$

Technology

- Let $z_n(\omega) \equiv 1/a_n(\omega)$ denote productivity.
- Let $F_n(.)$ denote the distribution of country n's productivity:

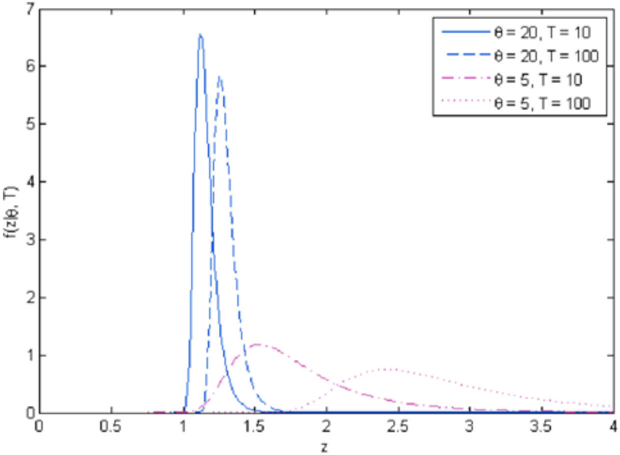
$$F_n(z) \equiv \Pr\left\{z_n(\omega) \leq z\right\}$$

- EK2002 assume $F_n(z)$ is Frechet:

$$F_n(z) = \exp\left(-T_n z^{-\theta}\right)$$

- Why Frechet? If ideas arrive with a Poisson distribution, and the technology of producing goods is determined by the best "idea," then the limiting distribution is Frechet, where T_n reflects the country n's stock of ideas.

The Frechet Distribution



Source: Fieler (2011, Econometrica)

Equilibrium Expenditure Shares

- The probability that $p_{ni}(\omega) \leq p$ is given by

$$\begin{aligned} G_{ni}(p) &\equiv \Pr\left(p_{ni}(\omega) \leq p\right) \\ &= \left\{\frac{w_n}{z_n(\omega)}\tau_{ni} \leq p\right\} = 1 - \exp\left\{-\Phi_{ni} p^{\theta}\right\} \end{aligned}$$

where $\Phi_{ni} \equiv T_n \left(w_n \tau_{ni}\right)^{-\theta}$.

Equilibrium Expenditure Shares

- The probability that $p_{ni}(\omega) \leq p$ is given by

$$
\begin{aligned}
G_{ni}(p) &\equiv \Pr\left(p_{ni}(\omega) \leq p\right) \\
&= \left\{\frac{w_n}{z_n(\omega)}\tau_{ni} \leq p\right\} = 1 - \exp\left\{-\Phi_{ni} p^{\theta}\right\}
\end{aligned}
$$

where $\Phi_{ni} \equiv T_n \left(w_n \tau_{ni}\right)^{-\theta}$.

- *Note*: The above probability is the same for all goods ω

Equilibrium Expenditure Shares

- The probability that good ω is supplied at a price lower than p in country i

$$G_i(p) = \Pr\left\{p_i(\omega) \leq p\right\}$$
$$= 1 - \prod_{n=1}^{N}(1 - G_{ni}(p)) = 1 - \exp\left\{-\Phi_i p^{\theta}\right\}$$

where Φ_i is defined as

$$\Phi_i = \sum_{n=1}^{N} \Phi_{ni}, \qquad \Phi_{ni} \equiv T_n \left(w_n \tau_{ni}\right)^{-\theta}$$

Equilibrium Expenditure Shares

- The probability that country n is the lowest cost supplier of good ω to country i is

$$\pi_{ni} \equiv \Pr\left\{p_{ni}(\omega) \leq \min_{j \neq n} p_{ji}(\omega)\right\} = \int_0^\infty \Pr\left\{\min_{j \neq n} p_{ji}(\omega) \geq p\right\} dG_{ni}(p)$$
$$= \int_0^\infty \prod_{j \neq n} \left(1 - G_{ji}(p)\right) dG_{ni}(p)$$

- Substituting $G_{ni}(p) = 1 - \exp\left(-\Phi_{ni} p^\theta\right)$ in the last line, yields:

$$\pi_{ni} = \frac{\Phi_{ni}}{\Phi_i} = \frac{T_n \left(\tau_{ni} w_n\right)^{-\theta}}{\sum_{j=1}^N T_j \left(\tau_{ji} w_j\right)^{-\theta}}$$

Equilibrium Expenditure Shares

- The probability that country n is the lowest cost supplier of good ω to country i is

$$\pi_{ni} \equiv \Pr\left\{ p_{ni}(\omega) \leq \min_{j \neq n} p_{ji}(\omega) \right\} = \int_0^\infty \Pr\left\{ \min_{j \neq n} p_{ji}(\omega) \geq p \right\} dG_{ni}(p)$$
$$= \int_0^\infty \prod_{j \neq n} \left(1 - G_{ji}(p)\right) dG_{ni}(p)$$

- Substituting $G_{ni}(p) = 1 - \exp\left(-\Phi_{ni} p^{\theta}\right)$ in the last line, yields:

$$\pi_{ni} = \frac{\Phi_{ni}}{\Phi_i} = \frac{T_n \left(\tau_{ni} w_n\right)^{-\theta}}{\sum_{j=1}^{N} T_j \left(\tau_{ji} w_j\right)^{-\theta}}$$

- Because (a) all goods receive *i.i.d.* draws and (b) there are a continuum of varieties, by the law of large numbers, this probability will be equal to the fraction of goods sourced from origin n.

Equilibrium Expenditure Shares

- *Claim*: The distribution of *realized* ($\mathcal{R}$) prices for goods purchased from origin n is independent of country n's characteristics!
- **Proof:** Define the distribution of realized prices from origin n as

$$G_{ni}^{\mathcal{R}}(p) \equiv \left\{ p_{ni}(\omega) \leq p \mid p_{ni}(\omega) \leq \min_{j \neq n} p_{ji}(\omega) \right\}$$

  - We can easily verify that $G_{ni}^{\mathcal{R}}(p)$ is independent of n:

$$G_{ni}^{\mathcal{R}}(p) = \frac{\int_0^p \prod_{j \neq n} \left(1 - G_{ji}(\tilde{p})\right) dG_{ni}(\tilde{p})}{\pi_{ni}} = G_i(p)$$

Equilibrium Expenditure Shares

- *Claim*: The distribution of *realized* ($\mathcal{R}$) prices for goods purchased from origin n is independent of country n's characteristics!

- **Proof:** Define the distribution of realized prices from origin n as

$$G^{\mathcal{R}}_{ni}(p) \equiv \left\{ p_{ni}(\omega) \leq p \mid p_{ni}(\omega) \leq \min_{j \neq n} p_{ji}(\omega) \right\}$$

  - We can easily verify that $G^{\mathcal{R}}_{ni}(p)$ is independent of n:

$$G^{\mathcal{R}}_{ni}(p) = \frac{\int_0^p \prod_{j \neq n} \left(1 - G_{ji}(\tilde{p})\right) dG_{ni}(\tilde{p})}{\pi_{ni}} = G_i(p)$$

- *Implication*: the fraction of goods sourced from origin n is equal to the fraction of income spent on goods from n:

$$\lambda_{ni} \sim \pi_{ni} \quad \longrightarrow \quad X_{ni} = \lambda_{ni} Y_i$$

Equilibrium Price Index

- The CES utility implies that the price index in country i is

$$P_i = \left(\int_\omega p_i(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} = \left(\int_0^\infty p^{1-\sigma} dG_i(p) \right)^{\frac{1}{1-\sigma}}$$

- Noting that (a) $G_i(p) = 1 - \exp\left(-\Phi_i p^\theta\right)$, and (b) $\Phi_i = \sum_{n=1}^N T_n \left(w_n \tau_{ni}\right)^{-\theta}$, the above expressions yields

$$P_i = C \left(\sum_{n=1}^N T_n \left(w_n \tau_{ni}\right)^{-\theta} \right)^{\frac{-1}{\theta}},$$

where $C \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$ (reminder: $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$).

General Equilibrium solution algorithm

For any given vector of exogenous parameters and variables $\{\tau_{in}, T_i, L_i, \theta\}_{i,n}$, equilibrium is a vector wage, $\{w_i\}_i$, such that labor markets clear in all countries. Namely,

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(w_1, ..., w_N) \times E_n\left(w_n\right)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = w_i L_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(w_1, ..., w_N) = \dfrac{T_i\left(\tau_{in} w_i\right)^{-\theta}}{\sum_{j=1}^{N} T_j\left(\tau_{jn} w_j\right)^{-\theta}} & (\forall i, j) \\ E_n(w_n) = w_n L_n & (\forall i, \text{ balance budegt}) \end{cases}$$

General Equilibrium—defined in terms of Y

For any given vector of exogenous parameters and variables $\{\tau_{in}, T_i, L_i, \theta\}_{i,n}$, equilibrium is a vector of GDP levels, $\{Y_i\}_i$, such that labor markets clear in all countries. Namely,

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(Y_1, ..., Y_N) \times E_n\left(Y_n\right)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = Y_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(Y_1, ..., Y_N) = \dfrac{T_i L_i^{\theta} (\tau_{in} Y_i)^{-\theta}}{\sum_{j=1}^{N} T_j L_j^{\theta} \left(\tau_{jn} Y_j\right)^{-\theta}} & (\forall i, j) \\ E_n(Y_n) = Y_n & (\forall i, \text{ balance budegt}) \end{cases}$$

An Overview of the Eaton-Kortum Model

- The Eaton-Kortum model belongs to the quantitative class of models reviewed earlier:

$$\tilde{\chi}_i \sim T_i L_i^{\theta}, \qquad \epsilon \sim \sigma - 1$$

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{Y_i}{P_i}, \qquad P_i = \left[\sum_{n=1}^{N} \int_{\omega \in \Omega_{ni}} p_{ni}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

An Overview of the Eaton-Kortum Model

- The Eaton-Kortum model belongs to the quantitative class of models reviewed earlier:

$$\tilde{\chi}_i \sim T_i L_i^{\theta}, \qquad \epsilon \sim \sigma - 1$$

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{Y_i}{P_i}, \qquad P_i = C \times \left[\sum_{n=1}^{N} T_n L_n^{\theta} \left(\tau_{ni} Y_n \right)^{-\theta} \right]^{-\frac{1}{\theta}}$$

An Overview of the Eaton-Kortum Model

- The Eaton-Kortum model belongs to the quantitative class of models reviewed earlier:

$$\tilde{\chi}_i \sim T_i L_i^{\theta}, \qquad \epsilon \sim \sigma - 1$$

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{Y_i}{P_i}, \qquad P_i = C \times \left[\sum_{n=1}^{N} T_n L_n^{\theta} \left(\tau_{ni} Y_n \right)^{-\theta} \right]^{-\frac{1}{\theta}}$$

C: encapsulates non-country-specific constants

Eaton-Kortum *vs.* Armington

- The Eaoton-Kortum model predicts similar ex post gains from trade (up-to a choice of trade elasticity) as the Armington model:

$$GT_i \equiv \frac{W_i - W_i^{(autarky)}}{W_i} \sim \text{gains from trade}$$

Eaton-Kortum *vs.* Armington

- The Eaoton-Kortum model predicts similar ex post gains from trade (up-to a choice of trade elasticity) as the Armington model:

$$GT_i = 1 - \lambda_{ii}^{\frac{1}{\theta}} \sim 1 - \lambda_{ii}^{\frac{1}{\epsilon}}$$

Eaton-Kortum *vs.* Armington

- The Eaoton-Kortum model predicts similar ex post gains from trade (up-to a choice of trade elasticity) as the Armington model:

$$GT_i = 1 - \lambda_{ii}^{\frac{1}{\theta}} \sim 1 - \lambda_{ii}^{\frac{1}{\epsilon}}$$

- It also predict the same *ex ante* welfare impacts in response to a trade cost shock $\{\hat{\tau}_{in}\}_{i,n}$:

$$\widehat{W}_i = \frac{\widehat{Y}_i}{\widehat{P}_i}, \qquad \widehat{P}_i = \left[\sum_n \lambda_{ni}\, \hat{\tau}_{ni}^{-\theta}\, \widehat{Y}_n^{-\theta}\right]^{-\frac{1}{\theta}}$$

where $\widehat{Y}_i$ can be calculated with data on baseline expenditure shares, $\{\lambda_{in}\}_{i,n}$, and GDP levels, $\{Y_i\}_i$, via the following system:

$$\widehat{Y}_i Y_i = \sum_{n=1}^{N} \left[\frac{\lambda_{in}\, \hat{\tau}_{in}^{-\theta}\, \widehat{Y}_i^{-\theta}}{\sum_{j=1}^{N} \lambda_{jn}\, \hat{\tau}_{jn}^{-\theta}\, \widehat{Y}_j^{-\theta}} \widehat{Y}_n Y_n\right]$$

Other Elements of Eaton & Kortum (2003)

- The Eaton-Kortum model satisfies the common macro-level representation covered earlier $\longrightarrow$ the same quantitative techniques apply

- Other elements of Eaton & Kortum (2002)
  - roundabout production (a special case of input-output extension covered later)
  - non-traded sector (a special case of the multi-sector extension covered later)
  - two approaches to estimating the trade elasticity, θ, which we will review later.

Additional Material

Algorithm for Calculating The Equilibrium Wages

Alvarez & Lucas (2007) rewrite the excess demand function as

$$f_i(\mathbf{w}) = \frac{1}{w_i}\left[\sum_{n=1}^{N} \frac{T_i\left(\tau_{in} w_i\right)^{-\theta}}{\sum_{j=1}^{N} T_j\left(\tau_{jn} w_j\right)} w_n L_n - w_i L_n\right],$$

and show that it satisfies the following properties for $\mathbf{w} \gg 0$

1. $f_i(.)$ is continuous.
2. $f_i(.)$ is homogeneous of degree zero: $f_i(\alpha\mathbf{w}) = f_i(\mathbf{w})$
3. $\sum_{i=1}^{N} Y_i f_i(\mathbf{w}) = 0$ (Walras' law)
4. There exists a $b > 0$ such that $f_i(\mathbf{w}) > -b, \ \ (\forall i)$.
5. Let $\bar{\mathbf{w}}$ be a vector of GDP where $\bar{w}_l = 0$ and $\bar{w}_n > 0$ for all $n \neq l$. Then, $\lim_{\mathbf{w}\to\bar{\mathbf{w}}} \max_i f_i(\mathbf{w}) = \infty$

Algorithm for Calculating The Equilibrium Wages

- Alvarez & Lucas (2007) also show that $f_i(.)$ satisfies the gross substitute property:

$$\frac{\partial f_i(\mathbf{w})}{\partial w_k} > 0 \quad \forall k \neq i$$

- The above property sates that if the wage in other countries rises, the demand for goods from country i increases.
- $f_i(.)$ satisfies the gross substitute property $\longrightarrow$ *unique* equilibrium.

Algorithm for Calculating The Equilibrium Wages

- To compute the equilibrium wages, define the following mapping:

$$M_i(\mathbf{w}) = w_i \left[1 + \lambda \frac{f_i(\mathbf{w})}{L_i}\right]$$

- If we start with a vector of wages that satisfy $\sum_i w_i L_i = 1$, then $\sum_i M_i(\mathbf{w}) L_i = 1$.
- Starting with an initial guess $\mathbf{w}^0$, and updating according to $\mathbf{w}^m = M_i(\mathbf{w}^{m-1})$, yields the unique equilibrium wage: $\mathbf{w}^* = M_i(\mathbf{w}^*)$.

return