The Armington Model

International Trade (PhD), Fall 2024

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Overview

- *International product differentiation* is the driving force behind international trade.
- The simplest model that generates the gravity equation and labor-service exchange economy reviewed in the last lecture.

Main references

- Anderson and van Wincoop, *"Gravity with gravitas: A solution to the border puzzle."* American Economic Review. 2003.

Environment

- Many countries indexed by $i,\ n = 1, ..., N$
- Each country supplies one differentiated product
- Labor is the only factor of production
- Country i is endowed with L_i (inelastically-supplied) units of labor
- Trade is balanced $(D_i = 0$, for all $i)$

Prefferences and Demand

The representative consumer in country i has a CES utility function over goods sourced from different origin countries:

$$U_i\left(Q_{1i}, \ldots, Q_{Ni}\right) = \left[\sum_{n=1}^{N} \beta_n^{\frac{1}{\sigma}} Q_{ni}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- goods are differentiated by country of origin.
- Index ni corresponds to variables associated with *origin* $n \times$ *destination* i
- $\sigma \geq 1$ is the inter-national elasticity of substitution.
- β_n is a constant demand shifter that reflects the quality or appeal of goods originating from country n (exogenous)

Prefferences and Demand

- The representative consumer in country i maximizes her utility given prices (P) and total expendable income (E):

$$\max_{\mathbf{Q}_i} U_i(Q_{1i}, ..., Q_{Ni}) \qquad s.t. \qquad \sum_{n=1}^{N} P_{ni} Q_{ni} \leq E_i$$

- Utility maximization delivers the following CES demand function:

$$\underbrace{\lambda_{ni} \equiv \frac{P_{ni} Q_{ni}}{E_i}}_{\text{expenditure share}} = \beta_n \left(\frac{P_{ni}}{P_i} \right)^{1-\sigma} \qquad \text{where} \qquad \underbrace{P_i = \left[\sum_{n=1}^{N} \beta_n P_{ni}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}}_{\text{CES price index}}$$

Production and Supply

- The unit labor cost of producing goods in origin i and delivering them to destination n:

$$C_{in} = \frac{1}{A_i} \times \tau_{in}$$

- The perfectly competitive price of produce in:

$$P_{in} = \frac{1}{A_i} \tau_{in} w_i$$

  - w_i denotes the wage rate in country i (endogenous).
  - $\tau_{in} \geq 1$ is the iceberg trade cost b/w origin i and destination n.

General Equilibrium

For any given vector of exogenous parameters and variables $\{\tau_{in}, A_i, \beta_i, L_i, \sigma\}_{i,n}$, equilibrium is a vector of wages, $\{w_i\}_i$, such that labor markets clear in all countries. Namely,

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(w_1, ..., w_N) \times E_n(w_n)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = w_i L_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(w_1, ..., w_N) = \frac{\beta_i A_i^{\sigma-1} (\tau_{in} w_i)^{1-\sigma}}{\sum_{j=1}^{N} \beta_j A_j^{\sigma-1} \left(\tau_{jn} w_j\right)^{1-\sigma}} & (\forall i, j) \\ E_n(w_n) = w_n L_n & (\forall i, \text{ balance budegt}) \end{cases}$$

General Equilibrium (in terms of Y)

For any given vector of exogenous parameters and variables $\{\tau_{in}, A_i, \beta_i, L_i, \sigma\}_{i,n}$, equilibrium is a vector of GDP levels, $\{Y_i\}_i$, such that labor markets clear in all countries. Namely,

$$\sum_{n=1}^{N} \underbrace{\lambda_{in}(Y_1, ..., Y_N) \times E_n\left(Y_n\right)}_{\text{country } n\text{'s demand for } i\text{'s labor services}} = Y_i \quad , \forall i$$

where the expenditure shares (λ_{in}) and total national expenditure (E_n) are given by

$$\begin{cases} \lambda_{in}(Y_1, ..., Y_N) = \dfrac{\beta_i (A_i L_i)^{\sigma-1} (\tau_{in} Y_i)^{1-\sigma}}{\sum_{j=1}^{N} \beta_j \left(A_j L_j\right)^{\sigma-1} \left(\tau_{jn} Y_j\right)^{1-\sigma}} & (\forall i, j) \\ E_n(Y_n) = Y_n & (\forall i, \text{ balance budegt}) \end{cases}$$

An Overview of the Model

- The Armington model is a special case of the generic model reviewed in lectures 1 and 2:

$$\tilde{\chi}_i \sim \beta_i \left(A_i L_i\right)^{\sigma-1}, \qquad\qquad \epsilon \sim \sigma - 1$$

- So, the quantitative strategies for model estimation and counterfactual analysis (covered in lectures 1 and 2) apply to this model.

- However, since we have introduced a formal notion of prefferences, we can determine the indirect utility (or welfare) of representative consumer in country i:

$$W_i = \frac{Y_i}{P_i}, \qquad \text{where} \qquad P_i = \left[\sum_{n=1}^{N} \beta_n P_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Welfare Impact Evaluation

- Consider an external (and possibly large) shock to trade costs: $\{\hat{\tau}_{in}\}_{i,n}$
- Given $\{\hat{\tau}_{in}\}_{i,n}$ we can calculate the change in GDPs by solving the following system:

$$\hat{Y}_i Y_i = \sum_{n=1}^{N} \left[\frac{\lambda_{in} \left(\hat{\tau}_{in} \hat{Y}_i\right)^{1-\sigma}}{\sum_{j=1}^{N} \lambda_{jn} \left(\hat{\tau}_{jn} \hat{Y}_j\right)^{1-\sigma}} \times \hat{Y}_n Y_n \right]$$

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- Given $\left\{\hat{\tau}_{in}\right\}_{i,n}$ and $\left\{\hat{Y}_i\right\}_i$, we can calculate the change in welfare as

$$\hat{W}_i = \frac{\hat{Y}_i}{\hat{P}_i}, \qquad \text{where} \qquad \hat{P}_i = \left[\sum_{n=1}^{N} \lambda_{ni} \hat{P}_{ni}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

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Growth Accounting in the Armington Model

- The welfare impacts of a generic growth shock to productivity and trade cost parameters, $\{\mathrm{d}\ln A_i\}_i$, and $\{\mathrm{d}\ln \tau_{in}\}_{i,n}$, can be specified as

$$\mathrm{dln} W_i = \mathrm{d}\ln Y_i - \mathrm{d}\ln P_i$$

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- The above formula is the *Divisia* index for measuring welfare effects and holds non-parametrically if preferences are stable and homothetic.

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- We can simplify the Divisia index by appealing to the CES demand structure whereby

$$\mathrm{d}\ln\lambda_{ni} - \mathrm{dln}\lambda_{ii} = (1-\sigma)\left(\mathrm{d}\ln P_{ni} - \mathrm{d}\ln P_{ii}\right),$$

allowing us to specify the change in import prices as

$$\mathrm{d}\ln P_{ni} = \mathrm{d}\ln P_{ii} + \frac{1}{1-\sigma}\left(\mathrm{d}\ln\lambda_{ni} - \mathrm{dln}\lambda_{ii}\right)$$

Growth Accounting in the Armington Model

- Plugging the expression for $\mathrm{d}\ln P_{ni}$ into the Divisia index yields

$$
\begin{aligned}
\mathrm{dln} W_i &= \mathrm{d}\ln Y_i - \sum_{n=1}^{N} \lambda_{ni} \mathrm{d}\ln P_{ni} \\
&= \mathrm{d}\ln Y_i - \mathrm{d}\ln P_{ii} \;-\; \frac{1}{1-\sigma} \sum_{n} \left[\lambda_{ni} \left(\mathrm{dln}\, \lambda_{ni} - \mathrm{d}\ln \lambda_{ii}\right)\right] \\
&= \mathrm{d}\ln Y_i - \mathrm{d}\ln P_{ii} \;-\; \frac{1}{1-\sigma} \left(\sum_{n} \left[\lambda_{ni} \mathrm{dln}\, \lambda_{ni}\right] - \sum_{n} \left[\lambda_{ni}\right] \mathrm{d}\ln \lambda_{ii} \right)
\end{aligned}
$$

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\begin{aligned}
\mathrm{dln} W_i &= \mathrm{d}\ln Y_i - \sum_{n=1}^{N} \lambda_{ni} \mathrm{d}\ln P_{ni} \\
&= \mathrm{d}\ln Y_i - \mathrm{d}\ln P_{ii} - \frac{1}{1-\sigma} \sum_n \left[\lambda_{ni} \left(\mathrm{dln}\, \lambda_{ni} - \mathrm{d}\ln \lambda_{ii}\right)\right] \\
&= \mathrm{d}\ln Y_i - \mathrm{d}\ln P_{ii} - \frac{1}{1-\sigma} \left(\sum_n \left[\lambda_{ni} \mathrm{dln}\, \lambda_{ni}\right] - \sum_n \left[\lambda_{ni}\right] \mathrm{d}\ln \lambda_{ii} \right)
\end{aligned}
$$

- Noting that $\begin{cases} \sum_n \lambda_{ni} \mathrm{d}\ln \lambda_{ni} = 0 \\ \sum_n \lambda_{ni} = 1 \end{cases}$, the last line in the above equation simplifies to

$$
\mathrm{dln} W_i = \mathrm{d}\ln Y_i - \mathrm{d}\ln P_{ii} + \frac{1}{1-\sigma} \mathrm{d}\ln \lambda_{ii} \qquad (*)
$$

Growth Accounting in the Armington Model

- Considering that $P_{ii} = \frac{\tau_{ii}}{A_i} w_i$, where $w_i = \frac{Y_i}{L_i}$, we can specify $\mathrm{d}\ln P_{ii}$ as

$$\mathrm{d}\ln P_{ii} = \mathrm{d}\ln Y_i + \mathrm{dln}\tau_{ii} - \mathrm{d}\ln A_i,$$

- Plugging the above expression for $\mathrm{d}\ln P_{ii}$ into Equation $(*)$ from the previous slides, yields:

$$\mathrm{dln} W_i = \mathrm{d}\ln A_i - \mathrm{d}\ln \tau_{ii} + \frac{1}{1-\sigma}\mathrm{d}\ln \lambda_{ii}$$

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$$\mathrm{d}\ln W_i = \underbrace{\mathrm{d}\ln A_i - \mathrm{d}\ln \tau_{ii}}_{\text{domestic}} + \underbrace{\frac{1}{1-\sigma}\mathrm{d}\ln \lambda_{ii}}_{\text{trade}}$$

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$$\mathrm{d}\ln W_i = \underbrace{\mathrm{d}\ln A_i - \mathrm{d}\ln \tau_{ii}}_{\text{domestic}} + \underbrace{\frac{1}{1-\sigma}\mathrm{d}\ln \lambda_{ii}}_{\text{trade}}$$

Key insight:

- the change in the overall trade-to-GDP ratio, $\mathrm{d}\ln \lambda_{ii} \sim \mathrm{d}\ln\left(1 - \frac{\text{Trade}_i}{\text{GDP}_i}\right)$, is the sufficient statistics for measuring trade-related welfare effects.
- how the shock modifies trade relations with specific partners is inconsequential for welfare.

The Gains From Trade in the Armington Model

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i \equiv \frac{W_i - W_i^{(autarky)}}{W_i} = 1 - \exp\left(-\int_\tau^\infty \mathrm{d}\ln W_i\right)$$

- We can calculate the gains from trade using our previous accounting formula by noting that autarky corresponds to $\lambda_{ii}^{(autarky)} = 1$:

$$\mathrm{GT}_i = 1 - \exp\left(-\int_{\lambda_{ii}}^1 \frac{1}{1-\sigma} \mathrm{d}\ln\lambda_{ii}\right) = 1 - \exp\left(\frac{1}{\sigma - 1}\int_{\lambda_{ii}}^1 \mathrm{d}\ln\lambda_{ii}\right)$$

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$$\mathrm{GT}_i = 1 - \lambda_{ii}^{\frac{1}{\sigma-1}}$$

- The realized gains from trade depend on two *sufficient statics*: λ_{ii} and $1 - \sigma$