The Ricardo-Roy Model with Multiple Factors

International Trade (PhD), Fall 2024

Ahmad Lashkaripour

Indiana University

Overview of Lecture

- This lecture reviews a multi-industry **Ricardo-Roy** model with multiples types of workers.
  - international specialization across industries à la Ricardo
  - allocation of workers across industries à la Roy
- Main implications
  - The aggregate gains from trade are larger...
  - ... but trade creates winners and losers (complements a rich empirical literature).
- Main References:
  - parametric Ricardo-Roy model: *Galle, Rodrıguez-Clare, Yi (2023, ReStud)*
  - non-parametric Ricardo-Roy model: *Costinot and Vogel (2015, ARE)*

Background: The China Syndrome

- Autor, Dorn and Hanson’s paper in the AER 2013
  - over 5000 Google Scholar citations
  - Frequent mention in major newspapers and magazines
  - **Major finding:** decline in wages and employment for regions most exposed to import competition from China.

Background: The China Syndrome

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  - **Major finding:** decline in wages and employment for regions most exposed to import competition from China.
- However, the ADH methodology can only identify relative effects...
  - higher imports also imply gains via lower prices for all
  - what are the absolute effects? are groups better or worse off?
  - need a structural general equilibrium model to answer these questions

Environment

- $n,\ i = 1, ..., N$ countries
- $k = 1, ..., K$ industries
- Labor is the only factor of production
- $g = 1, ..., G$ groups of workers
- $\bar{L}_{i,g}$ denotes the total number of group g workers in Country i.

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Birdseye View of Model

Demand and Supply of Final Goods

- Governed by a multi-industry gravity model, à la Eaton-Kortum or equivalently Armington.

Key Departures from the Standard Multi-Industry Model

- Workers have heterogeneous abilities.
- Different industries within a country offer varying wages.
- Workers sort into industries to maximize their *productivity* × *wage*, following the Roy model.

Demand for Final Goods

- Cobb-Douglas utility aggregator across industries:

$$U_i(\mathbf{Q}_i) = \prod_k \left(\frac{Q_{i,k}}{\beta_{i,k}}\right)^{\beta_{i,k}}$$

  implying a constant share $\beta_{i,k}$ of expenditure on industry k goods.

- CES utility aggregator across a continuum of goods $\omega \in \Omega_k$ within industry k:

$$Q_k(\boldsymbol{q}) = \left(\int_{\omega\in\Omega_k} q(\omega)^{\frac{\sigma_k-1}{\sigma_k}} d\omega\right)^{\frac{\sigma_k}{\sigma_k-1}}$$

  - goods $\omega \in \Omega_k$ are internationally homogeneous $\longrightarrow$ each good is purchased from the country supplying it at the lowest price.

Supply of Final Goods (*Eaton & Kortum, 2002*)

- The price at which country n can supply good $\omega \in \Omega_k$ to market i

$$p_{ni,k}(\omega) = \tau_{ni,k} w_{n,k} / z_{n,k}(\omega)$$

where productivities, $z(\omega)$, are distributed *Fréchet*: $\Pr\{z_{n,k}(\omega) \leq z\} = \exp\left(-T_{n,k} z^{-\theta}\right)$.

- Country i buys good ω from the cheapest supplier $\longrightarrow$ the share of country i's spending on country n goods is

$$\lambda_{ni,k}\left(\mathbf{w}_k\right) = \frac{T_{n,k}\left(\tau_{ni,k} w_{n,k}\right)^{-\theta_k}}{\sum_\ell T_{\ell,k}\left(\tau_{\ell i,k} w_{\ell,k}\right)^{-\theta_k}}$$

where $\mathbf{w}_k \equiv \{w_{n,k}\}_n$ is a vector describing the **wage per efficiency units** across different countries in industry k.

Demand for Labor Services: *Ricardian*

- Demand for labor efficiency units in industry k of country i[1]

$$E_{i,k}^{D}(\mathbf{w}_k) \equiv \frac{1}{w_{i,k}} \sum_{n=1}^{N} \lambda_{in,k}\left(\boldsymbol{w}_k\right) \beta_{n,k} E_n$$

[1] E_n denotes total expenditure in country n, which, as we will see shortly, equates total income from wages, Y_i, since their is no deficit.

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- The above equation reflected that fact that labor markets are efficient, so

$$\text{Wage Payments} \sim w \times \text{efficiency units} = \text{Total Sales}$$

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Supply of Labor: *Roy Model*

Heterogeneous workers within countries:

- There is a constant measure $\bar{L}_{i,g}$ of group g workers in country i.
- Each individual ι from group g independently draws an efficiency vector $\boldsymbol{z}(\iota) = \{z_1(\iota), ..., z_K(\iota)\}$ from the following GEV distribution:

$$F_{i,g}(\boldsymbol{z}) = \exp\left(-\sum_{k=1}^{K} a_{i,gk} z_k^{-\kappa_g}\right)$$

Discrete choice problem facing individual ι:

- choose the industry where their wage income is maximized given their productivity, $\mathbf{z}(\iota)$:

$$\text{indidviual } \iota\text{'s income} = \max\{w_{i,1} z_1(\iota), ..., w_{i,K} z_K(\iota)\}$$

Supply of Labor: *Roy Model*

- Theorem of Extreme Value $\longrightarrow$ share of group g workers in country i that choose to work in industry k is

$$\pi_{i,gk}(\boldsymbol{w}_i) = \frac{a_{i,gk}\, w_{i,k}^{\kappa_g}}{\sum_s a_{i,gs}\, w_{i,s}^{\kappa_g}}$$

- **Intuition**: worker type g is more likely to choose industry k if
    - they are inherently more capable in that industry (reflected in a high-$a_{i,gk}$)
    - industry k pays higher wages (reflected in a high-$w_{i,k}$).

Supply of Labor: *Roy Model*

- The total supply of efficiency units by group g workers to industry k:

$$E^S_{i,kg} = \pi_{i,gk}(\mathbf{w}_i)\, e_{i,gk}(\mathbf{w}_i)\, \bar{L}_{i,g}$$

Supply of Labor: *Roy Model*

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  where the average productivity of group g workers that select industry k is

$$e_{i,gk}(\mathbf{w}_i) = \Gamma\left(\frac{\kappa_g - 1}{\kappa_g}\right) a_{i,gk}^{\frac{1}{\kappa_g}} \pi_{i,gk}(\mathbf{w}_i)^{-\frac{1}{\kappa_g}}$$

- **Intuition:** a higher $\pi_{i,gk}$ means that more group g workers are choosing industry k, which implies that *less productive* individuals are choosing industry k (presumably dues to higher wages) $\longrightarrow$ lower avg. productivity.

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$$e_{i,gk}(\mathbf{w}_i) = \Gamma\left(\frac{\kappa_g - 1}{\kappa_g}\right) a_{i,gk}^{\frac{1}{\kappa_g}} \pi_{i,gk}(\mathbf{w}_i)^{-\frac{1}{\kappa_g}}$$

- **A limitation of the model:** the avg. income of group g workers is equalized across industries:

$$y_{i,g} = w_{i,k} e_{i,gk} = \left(\sum_k a_{i,gk} w_{i,k}^{\kappa_g}\right)^{\frac{1}{\kappa_g}}$$

Labor Market Clearing Condition

- Equilibrium is a $N \times K$ vector of wage, $\mathbf{w} \equiv \{w_{i,k}\}$ that satisfy $N \times K$ labor market clearing conditions:

$$\underbrace{\sum_g e_{i,gk}(\mathbf{w}_i)\pi_{i,gk}(\mathbf{w}_i)\bar{L}_{i,g}}_{\text{Supply } (\sum_g E^S_{i,gk})} = \underbrace{\frac{1}{w_{i,k}}\sum_n \lambda_{in,k}(\mathbf{w}_k)\beta_{n,k}Y_n(\boldsymbol{w}_n)}_{\text{Demand } \left(E^D_{i,k}\right)}$$

- Total expenditure in country i equals wage income, $E_i = Y_i(\mathbf{w}_i)$, where

$$Y_i(\mathbf{w}_i) = \sum_k \sum_g w_{i,k} E^S_{i,gk}(\mathbf{w}_i)$$

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- Total expenditure in country i equals wage income, $E_i = Y_i(\mathbf{w}_i)$, where

$$Y_i(\boldsymbol{w}_i) = \sum_g y_{i,g}(\boldsymbol{w}_i)\bar{L}_{i,g} \qquad \textit{where} \qquad y_{i,g} = \left(\sum_k a_{i,gk} w_{i,k}^{\kappa_g}\right)^{\frac{1}{\kappa_g}}$$

Welfare Analysis

- The average real income per worker in group g is

$$W_{i,g} = \frac{y_{i,g}}{P_i}, \qquad where \qquad P_{i,k} = \prod_{k=1}^{K} P_{i,k}^{\beta_{i,k}}$$

  - $P_{i,k} = \xi_k \left[\sum_{n=1}^{K} T_{n,k} \left(\tau_{ni,k} w_{n,k}\right)^{-\theta_k}\right]^{-\frac{1}{\theta_k}}$ is the price index as in Eaton-Kortum
  - $\xi_k \equiv \Gamma\left(\frac{\theta_k - \sigma_k + 1}{\sigma_k - 1}\right)^{\frac{1}{\sigma_k - 1}}$ is a constant shifter

Note: the expenditure shares ($\beta_{i,k}$) and price index P_i are assumed to be common across groups:

- however, there is evidence that the share of expenditure (β) on tradable goods is higher among low-income groups (Fajgelbaum and Khandelwal, 2016 QJE)

Welfare Effects of Trade Shocks (group-specific)

- Change in domestic expenditure and employment shares:

$$\hat{\lambda}_{ii,k} = \frac{\hat{w}_{i,k}^{-\theta_k}}{\sum_n \lambda_{ni,k} \left(\hat{\tau}_{ni,k}\hat{w}_{n,k}\right)^{-\theta_k}} = \left(\frac{\hat{w}_{i,k}}{\hat{P}_{i,k}}\right)^{-\theta_k} \longrightarrow \frac{\hat{w}_{i,k}}{\hat{P}_{i,k}} = \hat{\lambda}_{ii,k}^{-\frac{1}{\theta_k}} \qquad [*]$$

$$\hat{\pi}_{i,gk} = \frac{\hat{w}_{i,k}^{\kappa_g}}{\sum_n \pi_{i,gk}\hat{w}_{i,g}^{\kappa_g}} = \left(\frac{\hat{w}_{i,k}}{\hat{y}_{i,g}}\right)^{\kappa_g} \longrightarrow \frac{\hat{y}_{i,g}}{\hat{w}_{i,k}} = \hat{\pi}_{i,gk}^{-\frac{1}{\kappa_g}} \qquad [**]$$

- Following equations [*] and [**], the welfare effects of a trade shock are determined by two sufficient ststaitics (changes in domestic expenditure and employment shares):

$$\hat{W}_{i,g} = \prod_k \left(\frac{\hat{w}_{i,k}}{\hat{P}_{i,k}} \frac{\hat{y}_{i,g}}{\hat{w}_{i,k}}\right)^{\beta_{i,k}} = \prod_k \hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \hat{\pi}_{i,kg}^{-\frac{\beta_{i,k}}{\kappa_g}}$$

Aggregate Welfare Effects of Trade Shocks

- The change in aggregate welfare is the income-weighted change in group-level welfare levels:

$$\hat{W}_i \equiv \frac{\hat{Y}_i}{\hat{P}_i} = \sum_{g \in G_i} \frac{Y_{i,g}}{Y_i} \hat{W}_{i,g}$$

- Plugging the previously-derived expression for $\hat{W}_{i,g}$ into the above equation yields

$$\hat{W}_i = \prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \right) \cdot \left(\sum_g \frac{Y_{i,g}}{Y_i} \prod_k \hat{\pi}_{i,kg}^{-\frac{\beta_{i,k}}{\kappa_g}} \right)$$

Special Case: $\kappa_g \to 1$

- If $\kappa_g \to 1$, the model has the same welfare and counterfactual implications as the model in which labour is sector specific.

- The effect of trade shocks on group g workers relative to the rest of the economy:

$$\lim_{\kappa \to 1} \frac{\hat{W}_{i,g}}{\hat{W}_i} \approx \left(\sum_k \pi_{i,kg} \hat{r}_{i,k} \right)^{\frac{1}{\kappa}} \qquad\qquad r_{i,k} \equiv \frac{Y_{i,k}}{Y_i}$$

- Exposure to the shock is determined by the employment share in various industries (π) interacted with how these industries expand or shrink ($\hat{r}$) in response to the shock

- The exposure measure, $\sum_k \pi_{i,kg} \hat{r}_{i,k}$, has a *shift-share* structure

Proposition *(Galle, Rodrıguez-Clare, Yi)*

Assume that $\kappa_g = \kappa$ for all g, then the aggregate gains from trade are strictly higher than those that arise in the single factor model (that arise in the limit as $\kappa \to \infty$)

Proposition (*Galle, Rodrıguez-Clare, Yi*)

Assume that $\kappa_g = \kappa$ for all g, then the aggregate gains from trade are strictly higher than those that arise in the single factor model (that arise in the limit as $\kappa \to \infty$)

$$\hat{W}_i = \prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \right) \cdot \left(\sum_g \frac{Y_{i,g}}{Y_i} \prod_k \hat{\pi}_{i,kg}^{\frac{\beta_{i,k}}{\kappa}} \right) > \underbrace{\prod_k \left(\hat{\lambda}_{ii,k}^{-\frac{\beta_{i,k}}{\theta_k}} \right)}_{\text{GT in single factor model}}$$

Performing Counterfactuals using Exact Hat-Algebra

- If the economy is exposed to a change in trade costs, $\{\hat{\tau}_{in,k}\}$ or technology levels $\{\hat{T}_{i,k}\}$, then counterfactual outcomes can be solved using the following system of NK equations and unknowns ($\hat{w}_{i,k}$):

$$\sum_g \hat{\pi}_{i,gk}\left(\hat{\mathbf{w}}_i\right)^{1-\frac{1}{\kappa_g}} Y_{i,gk} = \frac{1}{\hat{w}_{i,k}} \sum_n \hat{\lambda}_{in,k}\left(\hat{\mathbf{w}}_k\right) \lambda_{in,k} \beta_{n,k} \hat{Y}_n\left(\hat{\mathbf{w}}_i\right) Y_n$$

where $Y_{i,gk} \equiv w_{i,k}\pi_{i,gk}L_{i,g}$ and the hat-functions are given by:

$$\begin{cases} \hat{Y}_i(\hat{\mathbf{w}}_i)Y_i = \sum_g \left(\sum_k \pi_{i,kg}\hat{w}_{i,k}{}^{\kappa_g}\right) Y_{i,g} \\ \hat{\pi}_{i,gk}\left(\hat{\mathbf{w}}_k\right) = \dfrac{\hat{w}_{i,k}{}^{\kappa_g}}{\sum_s \pi_{i,gs}\hat{w}_{i,s}{}^{\kappa_g}} \\ \hat{\lambda}_{in,k}\left(\hat{\mathbf{w}}_k\right) = \dfrac{\hat{T}_{i,k}\left(\hat{\tau}_{in,k}\hat{w}_{n,k}\right)^{-\theta_k}}{\sum_j \lambda_{jn,k}\hat{T}_{j,k}\left(\hat{\tau}_{ji,k}\hat{w}_{j,k}\right)^{-\theta_k}} \end{cases}$$

First Application: The Rise of Chinese Exports ($\hat{T}_{\text{China}}$)[2]

Table 2: The Welfare Effects of the China Shock on the US

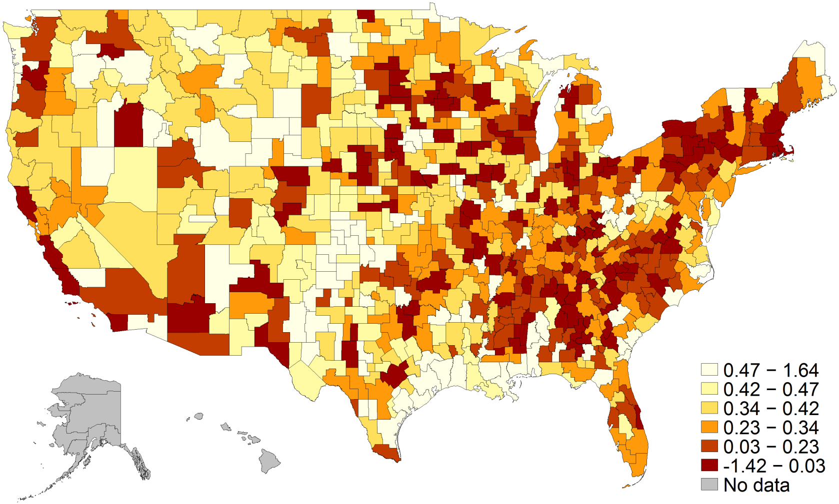
κ	Aggregate	Mean	CV	Min.	Max.	ACR
$\to 1$	0.24	0.30	1.40	-1.73	2.32	0.14
1.5	0.22	0.27	1.16	-1.42	1.64	0.15
3	0.20	0.24	0.80	-0.90	0.97	0.16
$\to \infty$	0.20	0.20	0	0.20	0.20	0.20

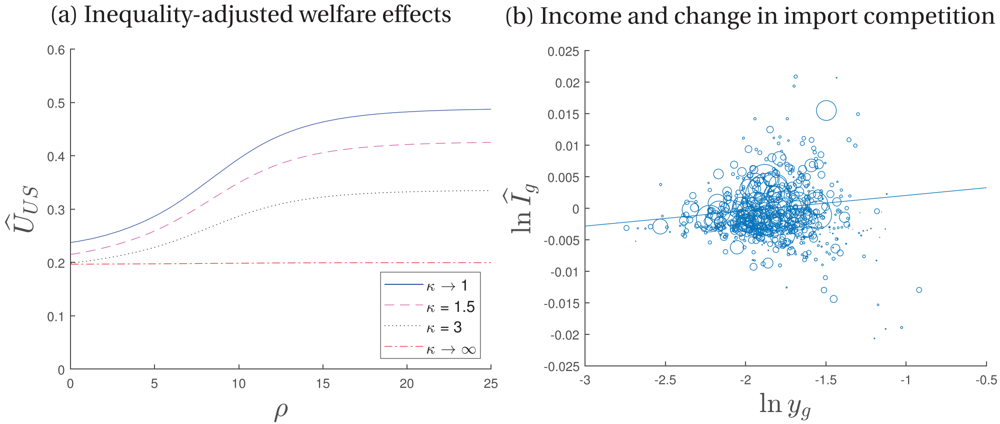
- $\hat{T}_{\text{China},k}$ is inferred from China's export growth to global markets.
- A worker group g is defined as a group of workers residing in one of the 722 commuting zones in the US.

[2] Source: *Galle, Rodriguez-Clare, Yi, 2018.*

First Application: The Rise of Chinese Exports ($\hat{T}_{China}$)

Figure 1: Geographical distribution of the welfare gains from the rise of China





- $\hat{U}_{US} = \left[\sum_g \omega_g \hat{W}_g^{1-\rho}\right]^{\frac{1}{1-\rho}}$ where ω_g is the theory-implied welfare weight
- ρ is the coefficient of relative risk aversion for the agent behind the veil of ignorance
- $\hat{I}_g = \sum_k \pi_{igk} \hat{r}_{i,k}$ is the model-implied measure of exposure to the China shock

Second Application: The Gains from Trade ($\hat{\tau} \to \infty$)

Table 3: Aggregate and Group-level Gains from Trade

κ	Aggregate	Mean	CV	Min.	Max.	ACR
$\to 1$	1.61	1.65	0.82	-6.98	3.72	1.45
1.5	1.56	1.59	0.58	-4.19	2.97	1.45
3	1.51	1.52	0.31	-1.38	2.22	1.45
$\to \infty$	1.45	1.45	0	1.45	1.45	1.45

- The gains for the US are calculated by setting $\hat{\tau}_{\text{USA},k} \to \infty$.
- A worker group g is defined as a group of workers residing in one of the 722 commuting zones in the US.

Second Application: The Gains from Trade ($\hat{\tau} \to \infty$)

Figure 5: Geographical Distribution of the Gains from Trade